Longitudinal/Cross-sectional Study of the Impact of *Mathematics in Context* on Student Performance

Instruction for 1998-1999

(Technical Report #31)

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INTRODUCTION

The purposes of the longitudinal/cross-sectional study of the impact of *Mathematics in Context* (MiC; National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998) on student performance are (a) to determine the mathematical knowledge, understanding, attitudes, and levels of student performance as a consequence of studying MiC for over three years; and (b) to compare student knowledge, understanding, attitudes, and levels of performance of students using MiC with those using conventional mathematics curricula. The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). For this study, information is being gathered on 14 variables over a 3-year period for three groups of students (those in Grades 5, 6, and 7 in 1997). The variables have been organized in five categories (prior, independent, intervening, outcome, and consequent). (See Figure 1 for variables and hypothesized relationships.)

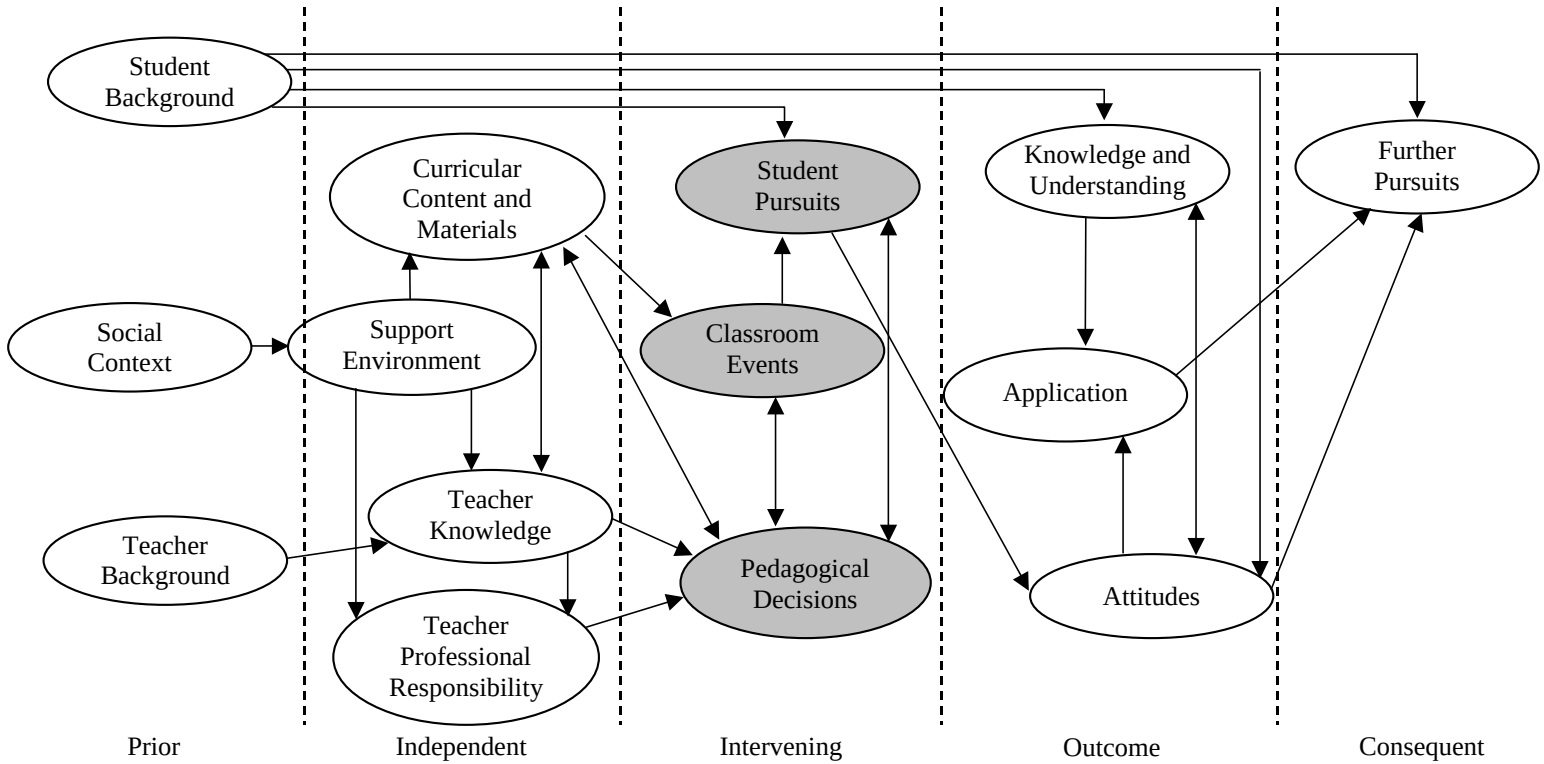

Figure 1. Revised model for the monitoring of school mathematics.

For analytical purposes, although the variation in each set of variables could be examined using structural equations, the number of classes at a given grade level is small, and collinearity across variables poses a serious interpretation problem. For this reason, a simplified research function will be used to make both the cross-sectional and longitudinal comparisons. Variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, can be attributed to variations in prior achievement (PA), method of instruction (I), opportunity to learn with understanding (OTL*u*), and school capacity (SC). This relationship can be expressed as—

$$CA = PA + I + OTL_u + SC.$$

Each of these composite indices is being specified from the variables in the original model. This paper details the analysis of the *Instruction* variable.

Overview

The purpose of this working paper is to summarize information for the composite variable *Instruction* collected during the first year of the longitudinal/cross-section study, the 1998–1999 school year, for sixth-, seventh-, and eighth grade teachers and students. The purpose of gathering this information was to document the variation in instruction that study students experienced as they studied either MiC or conventional mathematics curricula. The composite index *Instruction* is specified from data gathered on the intervening variables (*pedagogical decisions*, *classroom events*, and *student pursuits*) in the structural research model. In the simplified research function, *Instruction* includes five major categories: *unit planning*, *lesson planning*, *mathematical interaction during instruction*, *classroom assessment practice*,[1] and *student pursuits during instruction* (see Figure 2). Information on *unit planning*, *lesson planning*, and *classroom assessment practice* was gathered through the Teacher Interview: Instructional Planning and Classroom Interaction (see Appendix A; Shafer, Davis, & Wagner, 1998). Information on *lesson planning*, *mathematical interaction during instruction*, and *classroom assessment* was gathered through the Teacher Log (see Appendix B; Shafer, Wagner, & Davis, 1997). Information on *mathematical interaction during instruction*, *classroom assessment*, and *student pursuits* was gathered through the Classroom Observation Scale (see Appendix C; Davis, Wagner, & Shafer, 1998).

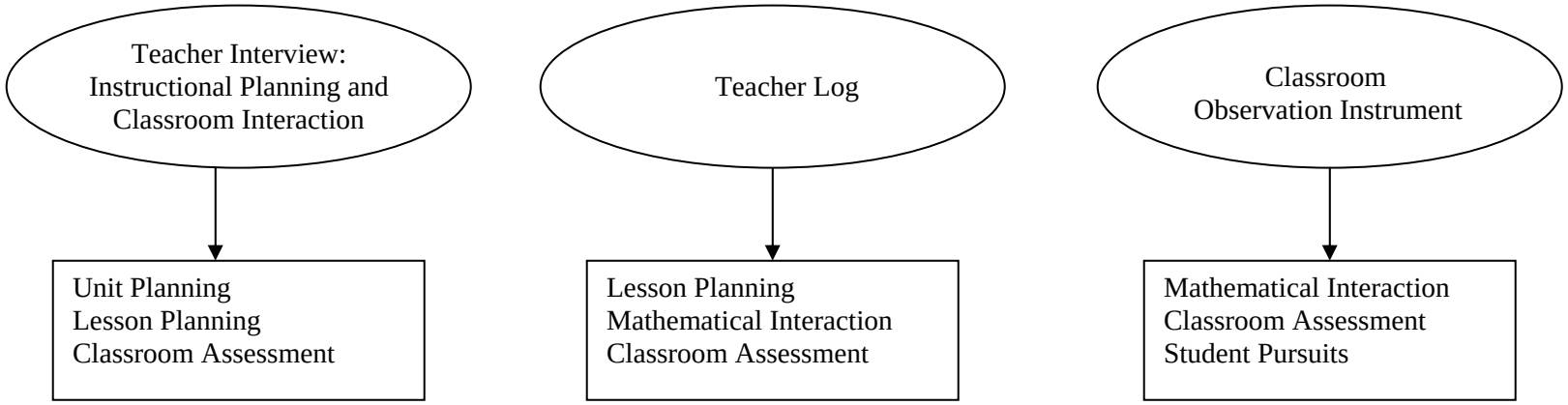

Figure 2. Categories of *Instruction* in the longitudinal/cross-sectional study of the impact of *Mathematics in Context* on student performance and their sources.

Fifty-three teachers and students in 94 classes from four school districts participated in the study. Districts are identified by number, and the classes by school and teacher (both pseudonyms). Also noted are the type of materials used (MiC materials or a conventional text).

[1] The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

The Sample

Districts 1 and 2

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. All MiC teachers used commercial versions of the units. Teachers using conventional curricula used whatever curriculum was already in place in the schools. The sample in Districts 1 and 2 consisted of 33 sixth-, seventh-, and eighth-grade teachers and their classes from diverse schools in Districts 1 and 2. These teachers taught a combined total of 71 classes involving 1487 students (see Tables 1 and 2).

Table 1
Summary of Study Participants, Districts 1 and 2, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
District 1			
MiC	11	26	521
Conventional	8[2]	10	245
District 2			
MiC	10	26	552
Conventional	4	9[3]	169
Total	33	71	1487

[2] In one class a Wacker Middle, Marin was replaced by Kewndall, who was replaced by DiMatteo.
[3] Cunningham at Newberry Middle taught classes at two grade levels.

Table 2
Characteristics of the Student Participants, Districts 1 and 2, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
District 1											
MiC	45	55	19	0	2	6	55	11	7	88	12
Conventional	44	56	20	0	0	7	55	11	7	88	12
District 2											
MiC	47	53	44	1	1	22	12	11	8	73	27
Conventional	56	44	20	0	0	7	55	11	7	85	16

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.

[2]Includes nonresponses.

District 1 was located in an urban region in the eastern part of the United States. In 13 elementary, 6 middle, and 4 high schools, 1325 teachers were responsible for teaching the district's 15,532 students. Four of the middle schools participated in the second year of the study. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. The teacher population was approximately 40% minority with 20% African American teachers and 16% Hispanic teachers. Approximately 30–40% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. The district provided teachers with nine paid in-service days for general professional development and two paid in-service days for professional development related to the teaching of mathematics. For preliminary teacher certification, 24 semester hours were recommended for fifth- and sixth-grade teachers; 24 semester hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary for continuing certification. District requirements were the same as the state requirements.

District 2, located in a large urban area in southeastern United States, had 19,536 teachers and 345,861 students housed in 201 elementary schools, 51 middle schools and numerous high schools. Four schools, one elementary and three middle schools, participated in the second year of the study. The district student population was predominantly minority with 32% African American students and 52% Hispanic students. The teacher population was more than half minority with 26% African American teachers and 32% Hispanic teachers. More than 50% of the students were eligible for government-funded lunch programs. Approximately 55–65% of the students had learned English as a second language. The district provided each teacher with 10 substitute days for professional development. For elementary teachers, preliminary teacher requirements mandated one methods course in elementary mathematics and one college level mathematics course. For middle-grade mathematics certification (Grades 5–9), 18 semester hours in mathematics, including geometry, pre-calculus, and calculus and nine semester hours of methods including History of Mathematics were required. District requirements for preparation of mathematics teachers were the same as state requirements. Continuing certification required the completion of six semester hours in mathematics or 120 district in-service credits in mathematics every five years.

Districts 3 and 4

Districts 3 and 4 agreed to participate in a modified research design in which all MiC teachers used MiC. All teachers used commercial versions of MiC units. The sample in Districts 3 and 4 consisted of 16 sixth-, seventh-, and eighth-grade teachers who taught a combined total of 44 classes involving 863 students (see Tables 3 and 4).

Table 3
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 3	
MiC	7	18	339
		District 4	
MiC	9	26	524
Total	16	44	863

Table 4
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
						District 3					
MiC	48	52	0	1	0	3	87	9	1	98	5
District 4											
MiC	49	51	27	2	3	21	2	13	33	85	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.

[2]Includes nonresponses.

District 3 was located in a suburban area of a large western state. In the district's two elementary and one middle school, 68 teachers taught 1451 students. Study participants are from one middle school in this district. Three of the study classes were self-contained classrooms. The rest of the study students had several subject-matter teachers. The school's student and teacher populations were predominately White. Approximately 10–20% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. School administrators provided three paid in-service training days for general professional development and one paid in-service day for

mathematics professional development. For preliminary teacher certification, the state mandated single-subject credentials for Grades 7–8. Teachers for Grades K–6 were required to complete a multiple-subjects credential including several mathematics courses. Although the district provided mathematics courses and staff development opportunities, it did not require additional certification or continuing education for experienced teachers.

District 4 was one of many districts located in a large urban area in the eastern part of the United States. The district's 1930 teachers were responsible for teaching the 28,000 students in 23 elementary schools, seven middle schools, and several high schools. In District 4, Grades 6–8 are contained in middle schools in which students have several subject-matter teachers. Study participants are from one middle school in this district. The student population was predominately minority with 27% African American students, 21% Hispanic students, and 46% multiracial/other students. Teacher ethnicity data were unavailable. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. For new mathematics teachers, 18 credits in mathematics were required by the state, but no specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements. Five paid in-service training days for professional development were provided to all teachers. The district's middle schools were organized by subject matter providing students with several subject-matter teachers.

The *Instruction* Composite Variable

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. The five categories of *Instruction* were further subdivided into 19 subcategories, and an index was created for each (see Figure 3). In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence*; and *pace of instruction*. Four subcategories characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies; and lesson reflection, summary, or closure*. Three subcategories characterized *classroom assessment practice: evidence sought during classroom assessment; purpose and coherence of feedback given in response to classroom assessment*; and *content of feedback* provided in response to classroom assessment. Finally, three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students*; and *level of student engagement*.

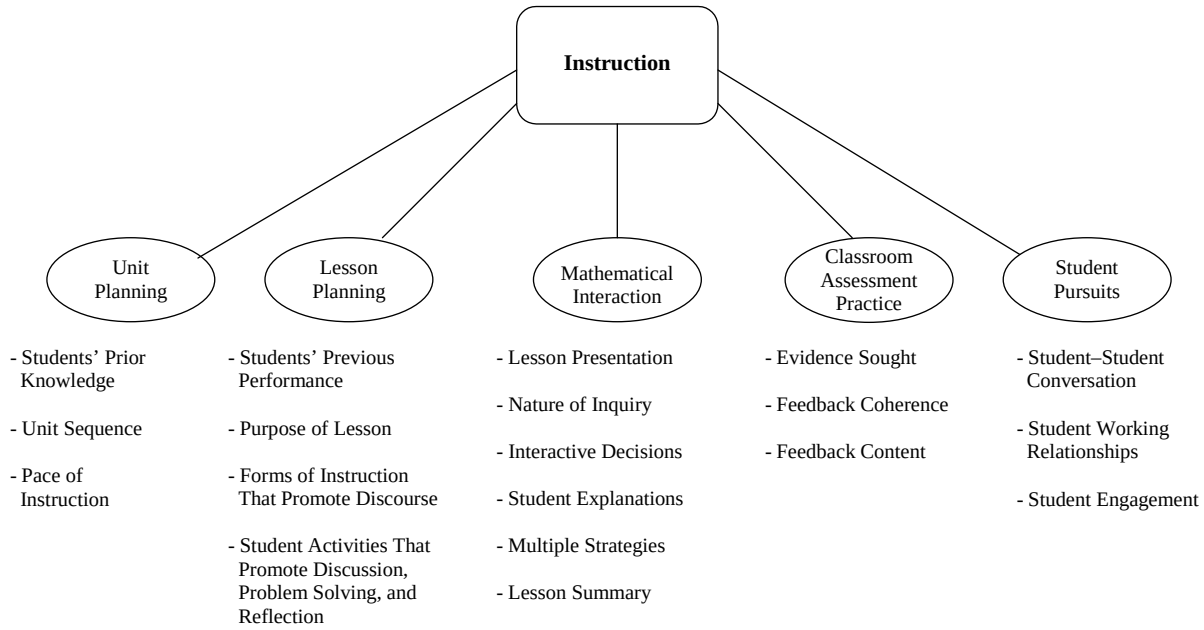

Figure 3. Major categories and subcategories of *Instruction*.

A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the simplified research function. Description of the theoretical framework that guided the analysis of instruction is contained in Appendix D, and description of the methodology used in analyzing the data is contained in Appendix E. For complete sets of data, see Appendices F, G, and H for fifth-, sixth-, and seventh-grade teachers, respectively.

Categories of Instruction

Unit Planning

In this study, three subcategories of instruction characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*. Forty-six percent of the teachers in Districts 1-4 based unit planning on their perceptions of students' prior knowledge with respect to mathematics skills and/or reading ability and vocabulary. Twenty-two percent planned units on the basis of information gathered through informal or formal assessment and included remedial skill-based activities to address weaknesses or extension activities for students who might be ready for such challenges. Thirty-three percent of the teachers planned units with little or no consideration of the prior knowledge of students in the current class. None of the teachers planned conceptually-based activities designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit. With respect to the sequence of units, 65% of the teachers did not vary from the unit/chapter sequence recommended in teacher support materials. Twenty-six percent of the teachers based decisions about unit sequence on external factors such as the content and dates of district or state standardized testing or on various seasonal events. Eleven percent of the teachers sequenced units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests. None of the teachers sequenced units to support the development of mathematics concepts. Finally, 80% of the teachers considered the recommendations for pacing in teacher support materials but planned to adjust the pace as the unit developed or as a result of collaboration with other teachers.

Lesson Planning

Four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Sixty-three percent of the teachers in Districts 1-4 made changes in the next day's plans because of students' performance in the previous lesson. The teachers extended the previous lesson to complete a task, disregarded time constraints, or added a review. Fifteen percent of the teachers made changes that focused on students' understanding of the mathematical content of the lesson. These teachers allowed for a more in-depth exploration of the mathematical content or introduced another approach to encourage students' understanding. Twenty-two percent of the teachers planned lessons with little or no regard for students' performance on the previous lesson. The teachers might have noted students' performance but lessons were planned in the same way as previous lessons. None of the teachers indicated that they made changes focused on encouraging thinking at higher levels by varying problem structure or setting or by emphasizing connections with related concepts. With respect to the purpose of the lesson, 52% of the teachers went beyond becoming familiar with the content, presentation, and context to make decisions for student learning, for example, potential student questions, possible misunderstandings, anticipation

of various solution strategies, accommodation of various ability levels, or conceptual development within a unit. Thirty-five percent of the teachers planned lessons from unit/chapter materials to become familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched. Thirteen percent of the teachers selected lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials included problem solving, applications of mathematics, or practice. None of the teachers planned lessons that emphasized higher order thinking, depth of knowledge, and/or understanding. For example, the teacher planned questions that engaged students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Planning *forms of instruction that promote classroom discourse for the purpose of the lesson* varied among all teachers in Districts 1-4. Forty-three percent of the teachers anticipated using whole-class discussion and small-group or pair work. Although these teachers planned for such lesson formats, the focus was primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. Forty-one percent planned for students' participation and collaboration for during instruction, but these still were not the primary focus of the lesson plan. Nine percent of the teachers rarely planned students' discourse in the classroom as part of the lesson. Instead, instruction focused on factual information or presentation of algorithms and procedures. Three MiC teachers planned forms of instruction that promoted substantive conversation. The teacher planned classroom activities that encouraged students to participate in discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. Similarly, planned *student activities that promoted discussion, problem solving, and reflection on the content of the lesson* varied among the teachers. Fifty-two percent of the teachers included investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) in lesson plans. Thirty-seven percent of the teachers included investigation of problems and discussion of answers and solution strategies as important elements in lesson plans, although questions or activities that encouraged students to reflect on or summarize lessons were rarely included. Seven percent of the teachers rarely planned investigation of problems and discussion of mathematical ideas for the lessons. Emphasis, instead, was placed on practicing routine calculations, and little discussion among students was anticipated. For one MiC teacher, investigation of problems and discussion of answers and solution strategies were dominant in lesson plans.

Mathematical Interaction during Instruction

Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*. With respect to *lesson presentation and development*, the results for the 33 teachers in Districts 1 and 2 revealed differences by grade level and type of curriculum taught. By grade level, more seventh-grade teachers taught mathematics for conceptual understanding than sixth- and eighth-grade teachers (5 of 12 seventh-grade teachers compared to 4 of 12 sixth-grade and 3 of 10 seventh-grade teachers). Six teachers (two at each grade level) emphasized conceptual understanding with active participation by students and teacher. Lesson presentations featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher. None of the teachers emphasized conceptual understanding with active participation by students with teacher support. Lesson presentations set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher. The greatest number of teachers, however, demonstrated procedures and strategies for students to

use. Five of the 12 sixth-grade teachers, 5 of 12 seventh-grade teachers, and 6 of 10 eighth-grade teachers presented and emphasized particular procedures or strategies. Furthermore, 6 teachers emphasized review. Three sixth-grade, 2 seventh-grade, and 1 eighth-grade teachers presented lessons that were underdeveloped because a major portion of the class time was devoted to review.

An additional pattern of variation was found when the levels of *lesson presentation* were reviewed by curriculum taught. Twelve of the 21 teachers using MiC either attempted to teach for or clearly emphasized conceptual understanding in contrast to none of the 12 teachers using conventional curricula. On the other hand, 10 of the 12 teachers using conventional curricula presented particular procedures or strategies in class in comparison to 6 of the 24 MiC teachers.

Teachers also varied in the *nature of inquiry during instruction*. Differences were noted by grade level and type of curriculum taught. Seven of 12 sixth-grade and 7 of 12 seventh-grade teachers, in comparison to 4 of 10 eighth-grade teachers, emphasized conceptual understanding, relationships among mathematical ideas, or linking procedural and conceptual knowledge. Also, more seventh-grade teachers (3 of 12 in comparison to 2 of 12 sixth-grade and 2 of 10 seventh-grade teachers) provided limited attention to conceptual understanding. The nature of inquiry for 4 of 10 eighth-grade teachers, in comparison to 2 of 12 seventh-grade and 3 of 12 sixth-grade teachers, was limited to lower order thinking. Lessons did not promote conceptual understanding, connections among mathematical ideas, and connections between mathematics and students' lives.

Teachers' interactive decisions also varied among the 35 teachers in Districts 1 and 2. Differences were revealed by grade level, by curriculum taught, and by district. By grade level, few teachers made interactive decisions that were most aligned with teaching mathematics for understanding (2 of 12 sixth-grade, 1 each seventh- and eighth-grade teachers). In contrast, more sixth-grade teachers made interactive decisions that were least aligned with teaching for understanding (7 of 12 compared to 5 of 12 seventh-grade teachers and 4 of 10 eighth-grade teachers). An additional pattern of variation was found when reviewed by curriculum taught. Eight of the 21 MiC teachers made interactive decisions that supported teaching mathematics for understanding in contrast to 2 of the 12 teachers using conventional curricula. On the other hand, 9 of the 12 teachers using conventional curricula made interactive decisions least aligned with teaching for understanding compared to 7 of the 21 MiC teachers. Differences by district were also apparent for teachers using MiC. In District 2, 7 of the 10 MiC teachers made interactive decisions that supported teaching mathematics for understanding in comparison to 4 of the 11 MiC teachers in District 1. Also, 1 of the 10 MiC teachers in District 2 made interactive decisions least aligned with teaching for understanding compared to 6 of the 11 MiC teachers in District 1.

Student explanations in most classes were focused on procedures or students' responses were limited to answers only. Although the means were higher for MiC teachers, no significant differences were found between teachers using MiC or conventional curricula. Multiple strategies were generally not elicited from students during instruction. By curriculum, multiple strategies were elicited more frequently during MiC lessons than in lessons using conventional curricula, but the differences were not statistically significant. Few opportunities, if any, were provided for reflection on or summary of lesson content. Four teachers provided some opportunities each week for students to reflect on the mathematics or summarize what they had learned in a lesson or in a series of lessons. None of the teachers frequently provided such opportunities for students.

Classroom Assessment Practice

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought during classroom assessment; purpose and coherence of feedback given in response to classroom assessment*; and *content of feedback* provided in response to classroom assessment. Fifty-eight percent of the 33 study teachers (9 of the 21 MiC teachers and 9 of the 12 teachers using conventional curricula) sought evidence of procedural competence based on student homework and classwork during their assessment practice. Six teachers (all MiC) sought student explanations in addition to procedural competence and answers. However, these explanations were often void of mathematical substance and were used to generate some form of communication rather than assess student understanding. Two MiC teachers were somewhat effective at eliciting student responses and orchestrating substantive whole-class discussions. However, the overriding focus of their classroom practice was to use correct answers and procedures as evidence of student learning. None of the teachers viewed student explanations as evidence of student learning or sought both process and product as evidence through verbal or written communication.

Eighty-five percent of the teachers (16 MiC and 11 using conventional curricula) provided feedback that was indirectly responsive to student needs. That is, teacher-directed feedback involved reteaching using more of the same type of instruction and practice sets that were used in the presentation of the lesson. Feedback may have also occurred in response to specific student questions or procedural errors when the teacher moved around the classroom during seatwork. Feedback provided by other students was minimal or non-existent. Four MiC teachers encouraged students to provide feedback to each other during small-group work, although the feedback consisted of sharing answers or procedures, not discussion to promote sense-making. One MiC teacher encouraged shared responsibility for feedback that promoted making sense of tasks, student responses, and mathematical conventions. Feedback was ongoing using verbal and written modes, sharing works in progress, and examples of revised responses. None of the teachers had open criteria for evaluating mathematical work, and students had the opportunity to create or modify such criteria.

The *content of feedback* varied widely among the teachers. Five MiC teachers provided clear and mathematically sound feedback that addressed skills, procedures, and concepts. Twelve teachers (8 MiC) directed feedback toward skills and procedures and the format of the answer (such as simplified form) rather than clarifying explanations or developing student understanding. Sixteen teachers (8 MiC) provided limited feedback (praise or criticism) that involved checking the correctness of answers but seldom addressed student misconceptions. One teacher who used a conventional curriculum provided feedback that was incoherent or illogical, consistently misleading, or lacked mathematical substance, and teachers were inattentive to student misconceptions. None of the teachers consistently provided feedback that emphasized conceptual understanding.

Students' Pursuits

Three subcategories of instruction characterized *students' pursuits during instruction*: *student–student conversation, collaborative working relationships among students*, and *level of student engagement*. Overall means for student–student conversation were statistically significant for MiC classes over classes using conventional curricula. During most class periods, however, student–student conversation was limited or was not encouraged. On only a few occasions did student–student conversation reach a substantive level characterized by reciprocal interaction that

involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level. Overall means were generally higher in District 2 than in District 1.

Study students rarely collaborated with one another during lessons. Few students shared ideas or discussed how problems could be solved. Even though they physically sat together, students worked on different problems at different paces. In only a few classes did some students exchange ideas or provide assistance to their classmates. When such interaction occurred, students did not make equal contributions to problem-solving efforts. Substantive collaboration among students was rarely noted.

Student engagement during instruction also varied among the teachers. On many occasions, student engagement was widespread. Most students were on task pursuing the substance of the lesson most of the time. They seemed to take the work seriously and put forth much effort. During other lessons, most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

Composite Index *Instruction*

Although teachers in all four research sites completed interviews, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index *Instruction*, therefore, was created only for teachers in Districts 1 and 2 for whom there was a complete set of ratings on all 19 indices. Thirty-two teachers were involved in the analysis. One teacher was not included because she did not teach a full semester during the study.

The composite index *Instruction* was created in a multiple-step process. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the Instruction Total.[4] Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subcategories and the Instruction Total (see Table 5).

[4] The sum of the weighted results was taken as a measure of the quality of instruction. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of instruction.

Table 5.

Correlation between the Instruction Total and the Subcategories of Instruction

Subcategory	Unit Planning			Lesson Planning				Mathematical Interaction						Classroom Assessment			Student Pursuits		
	SPK	US	PI	SPPL	PL	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE
SPK																			
US	0.092																		
PI	0.321**	0.017																	
SPPL	0.164	0.295**	0.051																
PL	0.136	-0.026	0.147	0.409***															
FIFD	0.038	-0.077	0.229*	0.177	0.326**														
SAPD	0.016	-0.181	0.134	0.118	0.474***	0.640***													
LPD	-0.023	0.104	0.176	0.157	0.312**	0.537***	0.546***												
NI	-0.026	-0.116	0.208	0.217*	0.506***	0.665***	0.549***	0.647***											
ID	-0.033	-0.015	0.285**	0.099	0.309**	0.568***	0.527***	0.676***	0.702***										
SE	-0.008	-0.072	0.141	0.175	0.260*	0.497***	0.468***	0.559***	0.692***	0.660***									
MS	-0.088	-0.060	0.048	0.025	0.227*	0.523***	0.426***	0.552***	0.635***	0.619***	0.788***								
LCS	-0.032	0.000	0.084	0.172	0.291**	0.476***	0.534***	0.556***	0.476***	0.495***	0.445***	0.471***							
ES	0.005	0.000	0.015	0.152	0.257*	0.408***	0.457***	0.650***	0.573***	0.602***	0.635***	0.599***	0.537***						
FCP	-0.036	0.055	0.108	0.079	0.266*	0.442***	0.423***	0.570***	0.520***	0.532***	0.561***	0.523***	0.480***	0.791***					
FC	-0.136	0.030	0.142	0.159	0.228*	0.398***	0.401***	0.624***	0.547***	0.613***	0.551***	0.469***	0.464***	0.695***	0.753***				
SC	0.015	-0.002	0.101	0.189	0.375**	0.493***	0.425***	0.436***	0.624***	0.532***	0.506***	0.475***	0.409***	0.483***	0.503***	0.578***			
SWR	0.073	0.008	0.063	0.109	0.336**	0.437***	0.422***	0.369***	0.532***	0.487***	0.521***	0.480***	0.385***	0.450***	0.486***	0.492***	0.845***		
OSE	-0.049	0.015	0.082	0.220	0.272*	0.322**	0.336**	0.525***	0.562***	0.512***	0.623***	0.572***	0.364***	0.548***	0.489***	0.518***	0.592***	0.593***	
Instr. Total	0.089	0.081	0.272*	0.309**	0.501**	0.697***	0.665***	0.801***	0.833***	0.803***	0.781***	0.721***	0.654***	0.787***	0.748***	0.751***	0.723***	0.667***	0.688***

*p<.05
**p<.01
***p<.001

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction
Instr. Total--Instruction Total

Five subcategories were not well correlated with the Instruction Total and other subcategories: *consideration of students' prior knowledge; unit sequence; pace of instruction; students' performance in the previous lesson*; and *the purpose of the lesson*. To verify these results, a principle component factor analysis was completed using SAS. Factors 1 and 2 accounted for a significant amount of the variance among the subcategories. Fourteen subcategories were included in Factors 1 and 2 (see Table 6). The five subcategories that had weak correlations to the Instruction Total were not influential in Factors 1 and 2 and were important only in the composition of other factors. Consequently, these subcategories were excluded from the analysis. The Instruction Total for each teacher was then recalculated.

Table 6.
Contribution of Subcategories to Principle Component Factors

Subcategory	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
SPK	-18	11	16	72*	9
US	4	-5	7	18	83*
PI	21	-5	-4	85*	7
SPPL	5	9	75*	0	46*
PL	19	21	78*	11	-12
FIFD	60*	15	33	23	-32
SAPD	59*	9	44*	9	-41*
LPD	83*	10	17	7	6
NI	65*	39	33	12	-23
ID	76*	28	8	19	-11
SE	68*	45*	5	5	-10
MS	68*	39	-2	-3	-17
LCS	68*	6	29	-4	-5
ES	79*	30	4	-10	12
FCP	75*	332	-1	-4	15
FC	74*	35	0	-8	17
SC	36	80*	22	6	-6
SWR	29	85*	15	8	-8
OSE	46*	65*	8	-5	9

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

Key

SPK--Consideration of Students' Prior Knowledge
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Using the revised Instruction Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into six groups. For each group of teachers, common characteristics from the subcategories of instruction were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each subcategory of instruction. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. Similar to the index for each subcategory, the underlying single dimension of the composite index was teaching mathematics for understanding. The levels of the composite index were on a continuum from least to most reflective of teaching mathematics for understanding. The six levels are summarized in Table 7.

Level 6: Most reflective of teaching for understanding
Level 5: Reflective of teaching for understanding
Level 4: Attempted to teach mathematics for understanding
Level 3: Limited attention to conceptual understanding
Level 2: Focus on procedures
Level 1: Underdeveloped lessons.

Table 7.
Summary of the Levels of the Composite Index for Instruction

Level 6: Most Reflective of Teaching for Understanding	**Level 5: Reflective of Teaching for Understanding**	**Level 4: Attempt to Teach for Conceptual Understanding**
Mathematical Interaction Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students with teacher support • Discussion of solutions, generalizations, connections Interactive decisions • Predominantly aligned with understanding • Frequent questions on articulation of thinking, understanding mathematics, or reasonable solutions *Classroom Assessment Practice* • Attention to mathematical processes • Ongoing, purposeful feedback from teacher, students • Feedback: making sense of mathematics, solutions • Student assessment of own work and others' work *Student Pursuits* • Occasional substantive conversation • Student-student conversation about procedures *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students and teacher • Discussion of solutions Interactive decisions • Attentive to teaching for understanding • Teacher explanations promote connections *Classroom Assessment Practice* • Student explanations as evidence of mathematical processes *or* procedural understanding • Feedback consistent with Level 6 *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Attempt for conceptual understanding, but focus on procedural understanding • General acceptance of teacher's procedures Interactive decisions • More attentive to good standard pedagogy • Additional exercises, mini-lessons, contexts, review *Classroom Assessment Practice* • Evidence from student explanations • Focus on procedural understanding • Teacher feedback related to concepts, contexts • Student-student feedback: answers, procedures *Student Pursuits* • Engagement mildly enthusiastic, teacher encouraged *Lesson Planning* • Student discussion, problem solving, planned

Table 7 (continued).
Summary of the Levels of the Composite Index for Instruction

Level 3: Limited Attention to Conceptual Understanding	**Level 2: Focus on Procedures**	**Level 1: Underdeveloped Lessons**
Mathematical Interaction Inquiry and lesson presentation • Students use invented or demonstrated strategies • Student explanations focused on procedures Interactive decisions • More reflective of good standard pedagogy • Some attention to articulation of thinking, reasonable solutions • Occasional addition of different context or review *Classroom Assessment Practice* • Evidence from homework, classwork, occasionally student explanations • Teacher feedback: concepts, contexts, *or* procedures, answer format • Student-student feedback: answers *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion of vocabulary, steps in procedures planned, not elaboration of thinking	*Mathematical Interaction* Inquiry and lesson presentation • Predominantly lower order thinking • Students expected to use demonstrated procedures Interactive decisions • Predominantly least aligned with understanding • Limited changes in response to student difficulties, misunderstanding *Classroom Assessment Practice* • Evidence from homework, classwork • Emphasis on procedures, format of answers • Teacher feedback indirectly responsive to students, inattentive to student misconceptions • Student-student feedback: minimal *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion anticipated but not planned	*Mathematical Interaction* Inquiry and lesson presentation • No formal lesson presentation • Procedures demonstrated to individual students • Student dependence on teacher for mathematical work • Frequent confusion or misunderstanding Interactive decisions • Least likely to support teaching for understanding • Teacher explanations preferred, no changes to address student needs *Classroom Assessment Practice* • Teacher feedback inattentive to student misconceptions, misleading, lacked mathematical substance • Student-student feedback: nonexistent *Student Pursuits*: Conversation not encouraged *Lesson Planning* • Student discussion, problem solving not considered

Results

In the second year of data collection, the results for all 32 teachers for the *Instruction* composite revealed differences by grade level, curriculum, and district. One-third of the teachers were at Levels 4 and 5, indicating that they attempted to or taught for understanding, but one-half of the teachers were at Levels 1 and 2, indicating that lessons were underdeveloped or focused on procedures, and no one was at Level 6 (see Figure 4a). The results for each grade level reflected the overall results. When reviewed by curriculum taught, half of the teachers using MiC and none of the teachers using conventional curricula were at Levels 4 and 5 (see Figure 4b). In contrast, 9 of the 11 teachers using conventional curricula were at Levels 1 or 2 compared with 8 of the 21 MiC teachers. When these results were reviewed by district, differences became apparent for MiC teachers. As in Year 1, MiC teachers in District 2 were more likely to teach mathematics for understanding than teachers in District 1 (see Figures 4c and 4d).

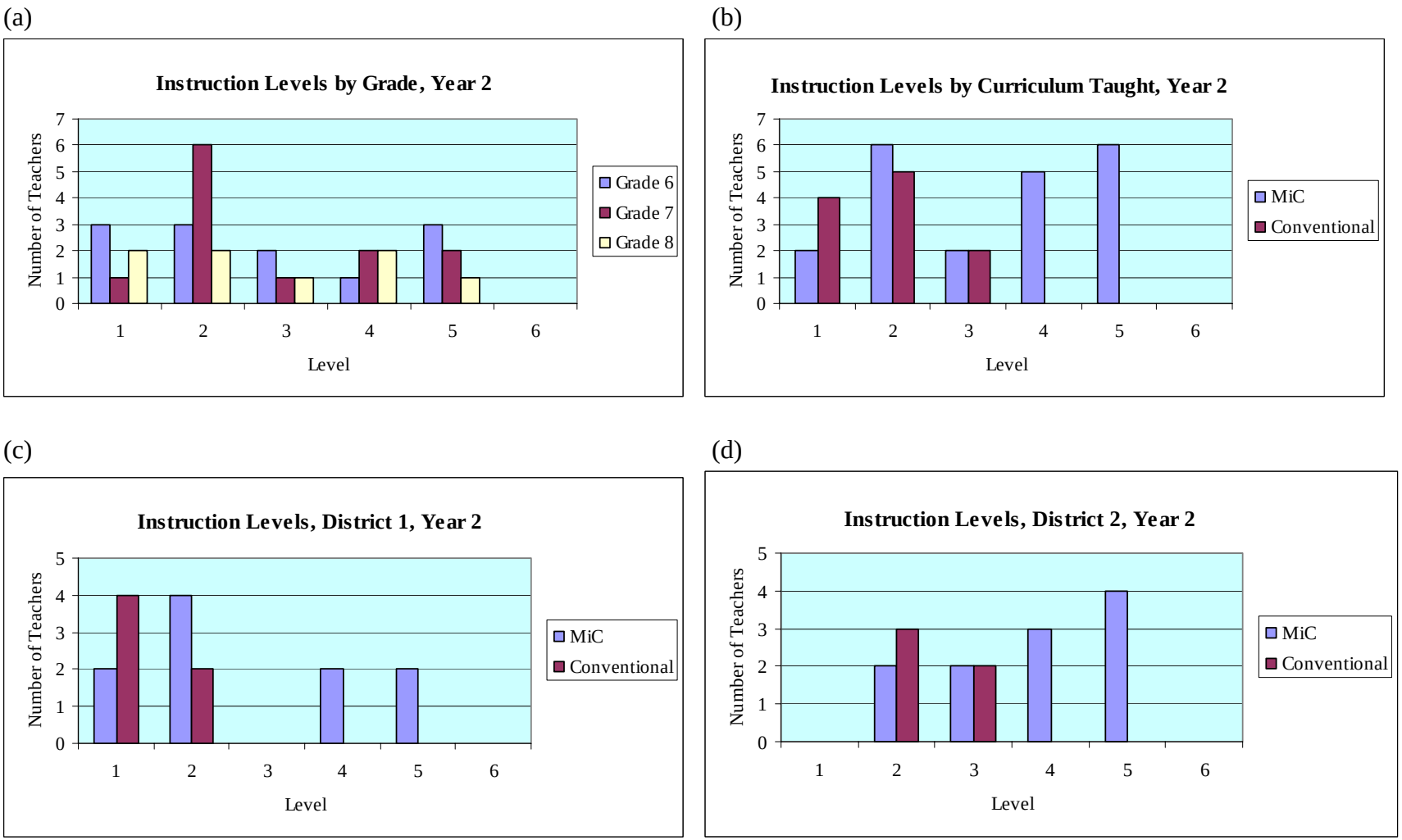

Figure 4. Teacher level on the *Instruction* composite index, Year 2: (a) by grade; (b) by curriculum taught; (c) by curriculum taught, District 1; (d) by curriculum taught, District 2

Conclusion

The development of the composite index *Instruction* was described. Based on 19 subcategories of *Instruction*, the composite index served as a useful tool in developing profiles of the instruction for each teacher. An extensive, diverse set of data from classroom observations, interviews, teaching logs, and journal entries was used to identify and scale variation in the instruction study students experienced.

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Appendix A

Teacher Interview: Instructional Planning and Classroom Interaction

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teacher Interview Protocol: Instructional Planning and Classroom Interaction

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Shafer, M. C., Davis, J., & Wagner, L. R. (1998) *Teacher interview: Instructional planning and classroom interaction*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 3). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

The development of this instrument was supported in part by the National Science Foundation #REC-9553889.

Description of Teacher Interview on
Instructional Planning and Classroom Interaction

The teacher interview on instructional planning and classroom interaction was the primary instrument used to gather information about the considerations teachers made when planning for teaching instructional units and individual lessons prior to instruction. The interview also gathered data on particular aspects of classroom interaction and differences between teaching MiC and teaching conventional mathematics curricula.

The interview was composed of five questions, one of which was reserved for teachers who used MiC. A list of probing questions accompanied each interview question. The probes were designed to enhance both the breadth and depth of teacher responses and to ensure consistency of the data gathered from all study teachers. Two interview questions included additional probes about the comparison of instructional planning and the nature of student conversations with MiC and conventional curricula. A brief introductory statement to the teacher provided a description of the interview, assurance that there were no right or wrong answers to interview questions, and notification that the interview would be audiotaped.

The first interview question asked teachers about their planning for each mathematics unit/chapter. The probes provided specific attention to (a) whether the teacher planned with others; (b) the considerations given in planning at the unit level, specifically, students' prior knowledge, textbook scope and sequence, district curriculum guidelines, state standards, district or state standardized tests, and other resources; and (c) setting the pace for instruction. An additional probe was reserved for teachers who used MiC. The focus of this probe was comparison of planning to teach MiC with planning to teach other mathematics textbooks used in the past.

The second interview question was dedicated to planning individual lessons. Probing questions provided attention to (a) considerations of students' performance in the previous lesson, and (b) whether the teacher solved unit/chapter problems or exercises before teaching, and, if so, the effect this preparation had on teaching the lesson.

The third interview question was designed to collect information on classroom interaction. In particular, the question addressed the issue of what counts as an acceptable answer. Related probes were (a) how students determine if their answers are acceptable; and (b) how the teacher determines if a student's answer is acceptable. The third probe elicited information about the ways in which students contribute to classroom discussions. A fourth probe was reserved for teachers who used MiC. The focus of this probe was differences in student participation when using MiC in comparison to student participation when using conventional mathematics curricula in the past. Additional probes for this question provided attention to differences in the types of conversation generated with each type of curriculum such as student conjectures, answers, and explanations.

The fourth interview question focused on the value of students working in small groups. Probing questions centered on (a) times when working in small groups was useful; (b) planning for small-group instruction; and (c) the ways grouping varied for different instructional purposes.

An additional interview question was reserved for teachers using MiC. This question focused on the ways that teaching MiC was different from teaching conventional mathematics curricula in the past for the students and the teacher. Probes provided attention to advantages and disadvantages of teaching MiC and difficulties in implementing MiC.

The interview protocol for instructional planning and classroom interaction was used during the spring semester of each study year. The interviews with teachers in Districts 1 and 2 were conducted by the on-site observer in each district. Audiotaped interviews were promptly sent to the research center. Interviews with teachers in Districts 3 and 4 were conducted via telephone by the project director or a project assistant at the research center. The interviews were conducted at times that were convenient for the

teacher and that did not interfere with classroom instruction such as during the teachers' planning time or before or after school. The interviews varied from 20-30 minutes, depending on teacher responses and the need to use probing questions. Interviewers were instructed to follow particular procedures, including asking probing questions and interjecting nonjudgmental comments when appropriate (see Instructions for Interviews in this appendix). Teachers received an honorarium of $25 per interview. Interviews were transcribed for analysis. All teachers in Districts 1 and 2 completed the interview on instructional planning and classroom interaction. One teacher in each of Districts 3 and 4 did not complete the interview. The combined completion rate for teachers in all four districts in 1997-1998 was 98%.

Teacher Interview Protocol: Instructional Planning and Classroom Interaction

Instructions for Interviewer

1. Be sure that you and the principal/teacher are alone and in a quiet room where responses can be recorded. Be sure that the tape is labeled with the principal/teacher's full name and the date of the interview.

2. Remember to follow the written protocol faithfully. You should always probe once if you think that the principal/teacher has not answered the question asked. In most cases, probes are given. In other cases, you may use one of the following.
   Anything else?
   Can you tell me more about ( )?
   Rephrase the question.

3. In responding to some questions, the principal/teacher may describe one aspect of the question in depth while not addressing the breadth of the question. Probes are provided to assist you in eliciting a broad response to each question. As you listen to the person's responses, glance at the list of probes and use the probe(s) that will give a more complete answer to the question. You might say, for example, "What about (probe)?" or "How important is (probe)?"

4. 3. If the principal/teacher has already answered a question you are about to ask, you should say: "The next question is ( ). I think you have already answered it. Do you think you have answered it? Is there anything else you want to add?

5. Your responses to the principal/teacher's statements should be non-committal and non-judgmental. Use responses such as "Thanks," "That's fine," "Alright," and "Okay."

6. If you forget to ask a question, make sure that you go back and ask it even if it is out of order.

7. Thank the principal/teacher for his/her time.

Instructions adapted from:
Fennema, E., Carpenter, T., & Loef, M. (1990). Belief Interview: CGI-2. Madison, WI: Wisconsin Center for Education Research, University of Wisconsin–Madison.

Teacher Interview Protocol: Instructional Planning and Classroom Interaction

Turn on the tape recorder and record the following:

This is (your first and last names). I am interviewing (teacher's first and last names) who teaches ( ) grade at (school name). Today is (month, date, year).

Say to teacher:

I will be asking you questions about how you decide what mathematics students should know and how you plan for your teaching. I am also interested in how you monitor student learning and how students contribute to classroom discussions. Please answer the following questions as truthfully as possible. There are no right or wrong answers to these questions. I am only interested in your opinions and ideas. Your responses will be audiotaped.

1. In general, how do you plan for each mathematics unit/chapter that you teach?
   - A. With whom do you plan?
   - B. Which of these do you take into consideration in your planning? Explain how.
     1. Students' prior knowledge
     2. Textbook scope and sequence
     3. District curriculum scope and sequence
     4. State standards
     5. District tests or other large-scale testing
     6. Other resources
   - C. How do you set the pace for instruction?
   - **D. For MiC teachers:**
     How does your *planning* for teaching MiC compare with your planning for other mathematics textbooks you have used?

2. How do you plan for *individual* lessons?
   - A. What considerations do you give in your planning to how students performed in previous lessons?
   - B. Do you work through the problems in the unit/chapter before teaching? How does this affect how you teach the lessons?

3. What counts as an acceptable answer?
   - A. How do students determine if their answers are acceptable?
   - B. How do you determine if a student's answer is acceptable?
   - C. In what ways do students contribute to whole class discussions?
   - **D. For MiC teachers:**
     In comparison to other mathematics curricula you have used, how does student participation in discussions differ when using MiC?
     1. **Type of answers and explanations**
     2. **Type of conversation (e.g., conjectures, support for their reasoning)**

4. Do you think it is valuable for students to work in small groups? Why?
   A. When is working in small groups useful?
   B. How do you plan for small group instruction?
   C. What type of grouping have you found to be the best for you and your students in terms of instruction (e.g., individual, occasional small groups, small groups that change over time, large group, etc.)?

5. For MiC teachers:
How is *teaching* MiC different than other mathematics curricula you have used?
   A. For the students?
   B. For you as the teacher?
   C. What advantages do you see in teaching the MiC curriculum?
   D. Do you see any disadvantages in using this curriculum?
   E. Have you encountered any difficulties implementing this curriculum?

Appendix B

Teaching Log

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teaching Log

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Shafer, M. C., Davis, J., & Wagner, L. R. (1997) *Teaching log*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 5). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

The development of this instrument was supported in part by the National Science Foundation #REC-9553889.

Description of the Teaching Log

The teaching log compiled by study teachers daily was one of the instruments designed to record information about daily instruction in study classes for the longitudinal/cross-sectional study. Information from the teaching logs was used in the analysis of the content of the actual curriculum, the use and modification of curricular materials, lesson planning, mathematical interaction during instruction, and classroom assessment. The teaching log was pilot-tested with nonstudy teachers during the spring semester prior to the study. Based on feedback from pilot teachers and input from district personnel involved in the study, the log was refined to clarify items and make the format easier for teachers to complete in five to ten minutes daily.

The teaching log consisted of Introductory Information, Daily Logs, and Journal Questions. The purpose of the Introductory Information was to document the unit/chapter taught, changes in class rosters, information about grouping for instruction, and the physical arrangement of the classroom. After indicating their name, the school, city, and date, teachers identified the text and the unit/chapter currently taught. Because the study was longitudinal, teachers noted names of students who were added (Item 1) or dropped from the class (Item 2). Over time, tables were used for teachers to record specific information. For students who were added to the class, teachers noted the approximate date students joined the class and the class periods to which the students were assigned. For students who dropped the class, teachers noted the approximate date students dropped and reasons they left the class. For Item 3, teachers indicated whether they intended for students to work in small groups or pairs during the teaching of the particular unit/chapter. They also described the reason for grouping students in that way and criteria for grouping the students. Finally, teachers sketched the physical arrangement of the classroom. The Introductory Information was completed once a month.

The daily log was printed on both sides of a single sheet of paper. The first side of the log was designed to document content taught, forms of instruction, and student activities. After noting the date, unit/chapter and pages taught on a particular day, teachers indicated if the lesson was a continuation of the previous lesson. If the lesson was continued, teachers were asked to indicate activities that were new to the current lesson. For Item 2, teachers checked whether all students in the class covered the same content. If they did not, they described the ways the content differed and the reasons for these differences. Item 3 was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). In Item 4, teachers noted whether the instructional activities precipitated changes in the physical arrangement of the classroom, and they described the reason for such changes. The last item on the first page was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. Items 2, 3, and 5 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

The second page of the log was designed to document supplemental materials, classroom assessment, homework, and formal assessment. For Item 6, teachers checked the additional materials used during the lesson: teacher-designed materials, work from text resource materials, work from other resources, quiz, calculators, or another resource specified by the teacher. Teachers were asked to date and attach teacher-designed materials, worksheets from other resources, and quizzes to the daily log page. Item 7 was designed to gather information about teachers' use of classroom assessment. If they assessed students

informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered. After the first semester of the study, parts (a) and (b) were changed to a checklist format for easier use by the teachers. In Item 8, teachers checked the type of homework assignment, if given: exercises form the text, completion of work begun in class, teacher-designed work, work from text resource materials, exercises from another text, supplementary practice, investigation or project, or other assignment specified by the teacher. After the first semester of the study, Item 8 was revised to be less time-intensive for teachers. In the original log, teachers were asked to list the pages and exercise numbers for text assignments and to attach exercises from supplemental resources and investigations or projects. For the revised item, a more inclusive checklist was used (adding teacher-designed materials, work from text resources, materials, and supplemental practice). Teachers were asked to briefly describe the content of teacher-designed and supplemental practice in lieu of attaching copies of such materials, and listing exercise numbers was eliminated. In the final item on the second page, for lessons during which a formal assessment was given, teachers checked the type of formal assessment used: end-of-unit or chapter test, district or state test, student presentation, or student projects. Teachers were asked to date and attach copies of end-of-unit or chapter tests that were representative of below average, average, and above average performance and copies instructions given for student presentations or projects. Items 6 and 8 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

Journal questions were printed on one side of a paper, and one sheet was inserted after each daily log sheet. The journal questions were designed to document lesson content that was emphasized or modified and notable classroom events. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them. In preparation for analysis, journal entries for each teacher were typed and collated by research staff.

Teachers were instructed to complete to complete a daily log sheet for each day of instruction as soon as possible after the lesson and complete at least one set of journal entries per week for the entire school year. If teachers taught multiple mathematics classes, they were asked to complete the log for the class that was observed monthly by the on-site observer. In this way, the information gathered through the log would add the teacher's perspective on the particular lessons for which observation reports were completed, thereby adding a means of triangulating data from observations and teaching logs. Each month the teaching log was a different color for ease in documenting the receipt of teacher logs. A binder was given to each teacher at the beginning of the school year. This binder contained the one-page Introductory Information, daily log sheets and journal questions for each instructional day for one month, a pocket folder for holding supplementary resources, quizzes, and formal assessments used by the teacher during instruction, and a postage-paid envelope for sending the log to the research team. Instructions for completing the teaching log and models of completed logs were reviewed with the teachers each August during the Summer Institute sponsored by the project for study teachers. District contact persons reviewed

the instructions with teachers who were unable to attend the Summer Institutes. Subsequent teaching logs with postage-paid envelopes were sent to each teacher monthly. Logs were sent to a contact teacher at each school for distribution. The contact teacher was given an honorarium of $50 per semester for distributing all study instruments to teachers on a timely basis. Teachers received an honorarium of $50 per teaching log upon receipt of the log at the research center. (As a result of negotiation with the teachers, the honorarium was increased to $125 per log during the second and third years of the study.)

Graduate project assistants were liaisons between the research staff and study teachers. Each project assistant read and commented on teacher logs received from one of two research sites (Districts 1 and 2). The numbers of teachers in Districts 1 and 2 who sent logs and journal entries to the research center monthly varied greatly (see Table B1), despite our extensive efforts to collect a full set of teaching logs from each teacher. During the first and second years of data collection, reminders were sent to teachers from the research staff, and graduate research assistants encouraged teachers to continue completing this important source of data through personalized letters of interest in the teachers' work.

Table B1
Number of Teaching Logs Received, by Grade and Year

Grade *(No. of Teachers*)*	Number of Teaching Logs Per Teacher	Percent of Teachers Submitting Teaching Logs			
		0-2 Logs	3-6 Logs	7-8 Logs	9 Logs
1997-1998					
5 *(13)*	0-9	23	15	23	38
6 *(12)*	0-9	33	42	8	17
7 *(10)*	0-9	10	20	30	40
1998-1999					
6 *(12)*	0-9	33	25	0	42
7 *(12)*	0-9	50	17	8	25
8 *(10)*	0-9	30	20	0	50
1999-2000					
7 *(9)*	0-9	44	22	0	33
8 *(9)*	0-9	22	11	11	56

*Includes teachers who taught portions of the school year

Reference:

Porter, A. C., Kirst, M. W., Osthoff, E. J., Smithson, J. L., & Schneider, S. A. (1993). *Reform up close: A classroom analysis*. Madison, WI: University of Wisconsin–Madison.

DAILY LOG PROCEDURES

The daily logs you complete are crucial components of the longitudinal study. These logs are designed to record daily practices in your mathematics classroom. No single instrument can characterize the complexities of classroom life, but the logs are intended to facilitate the general description of your teaching practices and your students' activities in the classroom. Your thoroughness in completing the daily logs is a most vital and appreciated aspect of this study.

INSTRUCTIONS:

At the beginning of month, please complete the introductory information.

The daily log is intended to reflect the character of your mathematics classroom. As its name implies, this log should be completed on a daily basis, as soon after the math class meets as possible. Most questions on the daily log can be completed with a check mark or brief descriptions.

The most crucial components of the daily log, the journal questions, require as thorough description as you can give to accurately reflect the lesson flow and classroom events. The importance of your thoroughness in answering these two questions cannot be underestimated. We would like you to respond to these questions as often as you notice events reflective of the suggested topics in your classroom, but we expect them to be completed at least once a week as these events present themselves.

If you do not have enough room to complete these, or any other questions, please complete them on an additional sheet of paper and attach the paper behind its corresponding entry.

In addition to filling in the daily log, where requested, we would like you to place hand-outs and/or student work in the folder provided and send these items along each month when you send your daily log entries to us. As you make copies of materials for students, please and date an additional copy to include with its corresponding daily log entry.

We estimate that it should take no more than 10 minutes each day to complete the daily log. The journal questions will require additional time to answer each week.

At the end of month, please staple or clip the log together with the introductory information in front, and return the completed log, the requested materials, and student work in the provided envelope.

If at any time you have questions about these procedures or the completion of the logs, please contact Lesley Wagner at 1-800-862-1055 or via e-mail at lrwagner@students.wisc.edu.

Thank you for your invaluable time in completing the daily log.

Please complete this information at the beginning of each month.

INTRODUCTORY INFORMATION

Name _____ Date ____/____/_____
School _____ Text _____
City _____ Unit/Chapter _____

1. Please list the names of students added to the class, the date they were added, and the period to which they were added:

Name of Student	Date Added to Class	Class Period

2. Please list the names of students dropped from the class, the date they were dropped, and the reason they were dropped (e.g., transferred to new class, transferred to new school):

Name of Student	Date Dropped from Class	Reason Dropped

3. If students will work in groups or pairs during the teaching of this unit or chapter, please describe how these groups or pairs are chosen and the reason for grouping the students this way.

4. Please sketch the physical arrangement of the classroom.

DAILY LOG

Date ____/___/_____ Pages _____ Unit/Chapter _____

1. __ Check here if today's lesson is a continuation of yesterday's. If so, only check activities which are new to today's lesson.

2. __ All students covered the same content.
   __ Content covered varied by student/group. Please describe how the instruction differed and the reasons for these differences.

3. Please check the instructional formats utilized during the lesson and circle the emphasis each received.

Instructional Formats	Emphasis Received			
__ Warm-up	1	2	3	4
__ Review of previous material	1	2	3	4
__ Teacher presentation of material	1	2	3	4
__ Whole class discussion during lesson	1	2	3	4
__ Small group or pair work	1	2	3	4
__ Independent practice	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary instructional format, used for more than 75% of the lesson.
3-Significant instructional format, used for 50-75% of the class period.
2-One of several instructional formats used with approximately equal emphasis.
1-An instructional activity used for a short period of time, approximately 15% or less of the class period.

4. If the activities in the lesson necessitated any physical change in the classroom or significant student movement, please describe.

5. Please check the student activities completed during the lesson and circle the emphasis each received.

Student Activities	Emphasis Received			
__ Listened to teacher and/or took notes	1	2	3	4
__ Investigated problem	1	2	3	4
__ Discussed answers and solution strategies	1	2	3	4
__ Participated in whole class discussion	1	2	3	4
__ Practiced computation	1	2	3	4
__ Took a test or quiz	1	2	3	4
__ Reflected on/summarized lesson concepts	1	2	3	4
__ Began homework	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary student activity, engaged in for more than 75% of the class period.
3-Significant student activity, engaged in for 50-75% of the class period.
2-One of several activity types, engaged in with approximately equal emphasis.
1-An activity engaged in for a short period of time, approximately 15% or less of the class period.

6. Please check the additional materials used during the lesson:

__ Teacher designed materials (Please date and attach)
__ Work from publisher resource materials
__ Worksheets or activities from sources other than the text or unit (Please date and attach)
__ Quiz (Please date and attach)
__ Calculators
__ Other (please specify) _____

7. If you informally assessed students during the class period, please answer the following questions:

a) Please check what you were assessing
__ Students' understanding of _____
__ Students' efforts in working as a group
__ Students' attitudes toward math (e.g., confidence, perseverance)
__ Other, please describe_____

b) Please check way(s) in which you informally assessed students
__ Observation
__ Listening during group work
__ Questioning
__ Checklists
__ Checking their work

c) Did the information you gained affect your instruction? __Yes __No
If yes, please describe.

8. Please check all student homework assignments that apply.
__ Exercises from text/unit
__ Completion of work begun in class
__ Teacher designed, please indicate content_____
__ Work from publisher resource materials
__ Exercises from source other than text
__ Supplementary practice, please indicate content _____
__ Investigation/Project
__ related to the unit, please describe_____
__ supplementary to the unit, please describe_____
__ Other, _____

9. If a formal assessment was part of the lesson, please indicate the type of assessment. Please attach copies of student assessments that are representative of below average, average, and above average performance as well as copies of student papers that show any interesting or unusual work.
__ Test
__ District or state developed test, please specify_____
__ Student presentations (Please date and attach the instructions or options given to students)
__ Student projects (Please date and attach the instructions or options given to students)

****Please remember to reflect on the following aspects of classroom instruction at least once a week.****

Date___/___/___

Journal Questions

1. Please describe the parts of the lesson you emphasized and any modifications you made in the lesson as compared to its presentation in the unit or chapter of the text. Please check and reflect on one or more of the following occurrences:

   ___ particular problems or aspects of the lesson that were emphasized and explain why they were emphasized
   ___ particular problems or aspects of the lesson that were deleted and explain why they were deleted
   ___ additional activities, problems, or procedures that were included in the lesson and explain why they were added
   ___ the order of presentation of lesson activities and/or content as compared to its presentation in the unit or chapter; if you changed the order of presentation, please describe how it was changed and explain why
   ___ other changes, please describe
   _____

2. Please describe any notable classroom event(s) related to the lesson.
   Please check and reflect on one or more of the following events:

   ___ the lesson or part of the lesson went exceptionally well
   ___ something surprising occurred
   ___ an idea was particularly difficult for the students
   ___ students seemed to comprehend an idea that had previously been troublesome
   ___ student misconceptions emerged
   ___ a student offered an unusual or unexpectedly sophisticated strategy
   ___ a student's question caused a modification in the lesson
   ___ other(s), please describe_____

Appendix C

Classroom Observation Instrument

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Observation Scale

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Description of the Observation Scale

The observation instrument for the longitudinal/cross-sectional study was designed to measure one independent variable (curricular content and materials—the actual curriculum) and the three intervening variables: pedagogical decisions, classroom events, and student pursuits. The observation instrument is composed of seven sections. In the first section, the observer recorded pertinent information related to the teacher and students: the name of the teacher, the school, and the grade level of the students in the class. The observer also recorded information pertinent to the particular lesson: the date of the observation, times the lesson began and ended, text used, unit/chapter taught, and the page numbers taught during the lesson. In the second section of the observation instrument, the observer conducted and recorded notes from a brief preobservation interview of the teacher during which the teacher was asked to identify the mathematical content to be explored or conveyed in the lesson and the location of the lesson with respect to the development of concepts in the instructional unit/chapter. In the third section, the observer recorded the flow of the lesson, which was a list of lesson activities along with the time allotted to each.

The next two sections of the observation instrument were collectively composed of 12 indices for various dimensions of instruction, which addressed the three intervening variables in the research model for the study. Nine of these indices focused on classroom events; the remaining three indices focused on student pursuits. Pedagogical decisions, although not presented in a separate section of the observation instrument, were central to both classroom events and student pursuits.

The indices used to characterize each dimension were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995), Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993), and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through a review of literature that was specific to each dimension. The indices were refined as a result of classroom observations of nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Three to four levels were identified for each dimension in order to identify differences in these variables between conventional and reform-based approaches to teaching and learning mathematics. The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the dimension being scaled. For example, levels of lessons that fostered conceptual understanding range from no attention to conceptual understanding during instruction to lessons in which the continual focus was on building connections among mathematical ideas.

The observation instrument was pilot-tested by project staff numerous times in both MiC and conventional classrooms in order to define and clarify descriptors for each item and to determine ways to achieve interrater reliability. Before the observation instrument was used in study classrooms, administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each dimension. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

In the sixth section of the observation instruction, the observer conducted and recorded notes from a brief postobservation interview of the teacher during which the teacher was asked to rate and comment on the degree to which the teacher felt the lesson achieved the purpose noted in the preobservation interview. The teacher was also asked whether any incidents occurred during the lesson which revealed student misunderstanding or provided opportunities to facilitated student understanding in any way. In this way, teachers had an opportunity to describe and explain modifications made during the lesson. In the final section of the observation instrument, the observer recorded any additional comments about the lesson.

Indices

Classroom Events

The lesson provided opportunities for students to make conjectures about mathematical ideas. In the conceptualization of conjectures in the longitudinal study, three types of student conjectures are described and sought in classroom interaction. First, students can make conjectures that are realizations of the connections between existing knowledge and the application of these concepts in new contexts. That is, students might see a connection between a new problems and problems they have already solved. Second, students may investigate the truthfulness of particular statements. Third, conjecturing may permeate a lesson. Given a pattern, for example, students are asked to devise a formula that captures the essence of the pattern in a concise form, which in turn leads to generalizations. Each type of conjecture is given a specific rating in the index, with an observation of the third type given the highest rating.

The lesson fostered the development of conceptual understanding. Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. Lower ratings in this category describe classrooms in which teaching for conceptual understanding occurs, but is often overshadowed by an emphasis on procedural knowledge. The highest rating describes a lesson in which links between conceptual and procedural understanding are the main emphasis of the instruction.

Connections within mathematics were explored. In this index, mathematical topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios which connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating is meant to reflect both the problems and instruction.

Connections between mathematics and students' daily lives were apparent in the lesson. This index measures whether connections between mathematics and students' daily lives were apparent in text problems or problems presented in class or were discussed by the teacher or students.

Students explained their responses or solution strategies. This index is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

Multiple strategies were encouraged and valued. This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. Higher ratings on this index refer to lessons in which discussion of alternative strategies is a frequent, important element of classroom instruction.

The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class. This index is intended to measure the ways in which the

teacher uses student responses during instruction. The highest rating is reserved for lessons in which the teacher not only probed individual students' thinking but also encouraged other students to comment on the solution strategies or used students' thinking processes to open discussions that encourage deeper understanding of mathematics.

The teacher used student inquiries as a guide for instructional mathematics investigation or as a guide to shape the mathematical content of the lesson. Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. This index measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

The teacher encouraged students to reflect on the reasonableness of their responses. This index is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

Student Pursuits

Student exchanges with peers reflected substantive conversation of mathematical ideas. Substantive conversation by students is characterized by interaction that is reciprocal, which involves listening carefully to others' ideas in order to understand them, building conversation on others' ideas, or extending an idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions. (Newmann, Secada, and Wehlage, 1995). While other items in this observation scale refer to the role of the teacher in mediating discourse, this item measures student discourse between peers in either large-group or small-group settings.

Interactions among students reflected collaborative working relationships. A low rating is given when students are physically sitting in groups but rarely working together. In contrast, the highest rating denotes a lesson in which students are actively involved in solving problems with their classmates and in which students made sure that all students in the group understood one problem before moving on to the next. N/A is reserved for lessons in which the goal is for students to work on problems independently.

The overall level of student engagement throughout the lesson was serious. This index measures the extent to which students remained on task during the lesson. Engagement is exemplified by behaviors in which students are attentive, complete assigned work, participate by raising questions, contribute to both large-group and small-group discussions, and help their peers (Secada and Byrd, 1993).

Observations

The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. Throughout the class period, the observer continually judged the levels of each dimension of classroom events and student pursuits. During each observation the observer took field notes that pertained to the 12 indices. Immediately after observing a lesson, the observer rated each item and recorded evidence from the lesson (consisting of dialogue or an artifact) to support the given rating. In general, a rating of 1 on a particular item indicated that the dimension was rarely or never seen in the lesson; the highest rating indicated that the dimension received major emphasis in the classroom. In practice, high ratings were rarely attained on every item during one observation. Ratings also varied in different observations of the same teacher.

The number of observations per teacher varied in each district (see Table C1). Most teachers in District 1 were observed once a month for a total of nine observations per teacher. During the first year of data collection, one teacher in District 1 accepted an administrative position in December; consequently, she was observed three times, and the newly assigned teacher was observed five times. During the second and third years of data collection, one eighth-grade control class had three teachers, and two seventh-grade experimental classes had two teachers over the course of the school year. As a result, each teacher was observed only a few times. Teachers in District 2 were observed a total of two to nine times each. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. In addition, four teachers from one school in District 2 withdrew from participation in the study during the spring semester of the first year of data collection; consequently, they were observed only three times. During the third year of data collection, two seventh-grade experimental classes were observed twice because the teacher had been on parental leave.

Table C1
Number of Observations Conducted, by Grade and Year

Grade *(No. of Teachers*)*	Number of Observations Per Teacher	Percent of Teachers Observed			
		1-3 Times	4-6 Times	7-8 Times	9 Times
1997-1998					
5 *(13)*	5-9	0	38	0	62
6 *(12)*	3-9	25	33	8	33
7 *(10)*	3-9	20	40	0	40
1998-1999					
6 *(12)*	5-9	0	33	25	42
7 *(12)*	5-9	0	42	25	33
8 *(10)*	2-9	20	40	30	10
1999-2000					
7 *(9)*	2-9	22	11	11	56
8 *(9)*	8-9	0	0	44	56

*Includes teachers who taught portions of the school year

Interrater Reliability

In the August prior to the study, each observer viewed two videotaped lessons with a graduate project assistant who developed the observation instrument and rated the lessons using the instrument. During these meetings discussions centered on consistency of ratings and descriptions of the types of conjectures observed, the nature of student–student conversation, and instances in which teachers used student inquiries to shape the lesson.

In the fall of 1997, each observer and a project assistant visited five classes in District 1 and nine classes in District 2. During the first few observations at each site, the project assistant's and observer's ratings of several items differed by one point. By the last observation, however, this disagreement had subsided considerably. The first dimension, student conjectures, initially caused difficulty for both observers. For example, observers initially categorized the repetitious practice of problems using a single prescribed algorithm as a first level conjecture, when the first level of conjecture is meant to describe the preponderance of students making conjectures that link concepts they have studied in the past with the same concept set within a new context. Another dimension, students' level of collaboration in the classroom, one observer tended to give the highest rating if students were physically sitting in groups. The project assistant emphasized the importance of circulating around the classroom to determine if

students actually worked in groups to support each other's learning. After this training, the observers began observing each study teacher once a month and completed a report for each observation. Completed reports were sent electronically to the research center for analysis. Each observer was compensated an amount per observation as part of a subcontract between the observer and the University of Wisconsin. The amount of payment varied according to the length of the class period observed. In September 1998, both observers worked on interrater reliability with a project assistant during on-site classroom observations in District 1. Because of the lack of funds, on-site work for interrater reliability between observers and a project assistant were not conducted in the fall of 1999.

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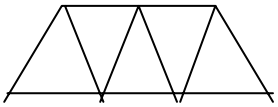
Observation Scale Descriptors

C.1. The lesson provided opportunities for students to make conjectures about mathematical ideas.

This scale measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas. There are three types of conjectures that students might make. One type of conjecture involves the student in making a guess about how to solve a particular problem based on experience solving problems with similar solution strategies. For example, students were solving problems in which they used properties of similar triangles. When asked to determine the height of a tree, students conjectured that an appropriate solution strategy would involve similar triangles. The students made a connection between the new problem and problems that they had previously solved. A second type of conjecture occurs when a student makes a guess about the truthfulness of a particular statement and subsequently plans and conducts an investigation to determine whether the statement is true or false. For example, a 12-year-old student disagreed with a statement that she was half as tall as she is now when she was 6-years old, and proceeded to support her argument by comparing her present height with heights of 6-year-old children. A third type of conjecture is a generalization. A generalization is created by reasoning from specific cases of a particular event, is tested in specific cases, and is logically reasoned to be acceptable for all cases of the event. For example, given that a beam is constructed of rods in the following configuration,



students are asked to describe the relation between the number of rods and the length of the beam1 (Wijers, Roodhardt, van Reeuwijk, Burrill, Cole, & Pligge, 1998). Using a table to organize their reasoning, students described the pattern that emerged, explained how the pattern fit the given diagram, and generated formulas for the relationship. In this situation, students reasoned from specific cases, tested and supported their ideas with evidence from drawings and the table, and described the relation in a formula.

1. No conjectures of any type were observed in the lesson. Students were not encouraged to make connections between a new problem and problems previously seen, investigate the validity of their own guesses, look for patterns, or make generalizations.

2. Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

3. Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

4. Students made generalizations about mathematical ideas.

---

1 The length of the beam is the number of rods on the bottom of the beam.

C.2. The lesson fostered the development of conceptual understanding.

Conceptual knowledge is described as the "facts and properties of mathematics that are recognized as being related in some way" (Hiebert & Wearne, 1986, p. 200), or as a network of relationships that link pieces of knowledge (Hiebert & Lefevre, 1986). In the primary grades, for example, students learn the labels for whole-number place-value positions. If this information is stored as isolated pieces of information, the knowledge is not conceptual. If this knowledge, however, is linked with other information about numbers, such as grouping objects into sets of ten or counting by tens or hundreds, then the information becomes conceptual knowledge. The network of relationships about place value grows as other pieces of knowledge related to place value, such as regrouping in subtraction, are recognized. Procedural knowledge, in contrast, is described as having two parts. One category comprises the written mathematical symbols, which are devoid of meaning and are acted upon through knowledge of the syntax of the system. A second category is composed of rules and algorithms for solving mathematics problems, step-by-step procedures that progress from problem statement to solution in a predetermined order. Procedural knowledge is rich in rules and strategies for solving problems, but it is not rich in relationships (Hiebert & Wearne, 1986).

Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. In addition and subtraction of decimals, for example, lining up the decimal points should be linked with combining like quantities. Instruction might explicitly bring out the relationships between lining up the decimal point in addition and subtraction and lining up whole numbers on the right side for the same operations (Hiebert & Wearne, 1986).

1. The lesson as presented did not promote conceptual understanding.

2. The lesson asked few questions that fostered students' conceptual development of mathematical ideas, or conceptual understanding was a small part of lesson design.

3. Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

4. The continual focus of the lesson was on building connections between disparate pieces of information or linking procedural knowledge with conceptual knowledge.

C.3. Connections within mathematics were explored in the lesson.

This scale measures the extent to which instruction addressed mathematical topics thoroughly enough to explore relationships and connections among them.2 A low rating is given when the mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning, and instruction treated this topic in isolation of other mathematical topics. A high rating is given when the mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. Rather than examining fragmented pieces of information, students looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical topics, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

Topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios, as in the following problem. Students are asked to determine the probability of a frog jumping from a cage and landing on white or black floor tiles and to express this probability as a fraction or percent (Jonker, van Galen, Boswinkel, Wijers, Simon, Burrill, & Middleton, 1998). In solving this problem, students use area, number, and probability concepts. Second, connections can be made among more narrowly defined areas such as a lesson involving the solution of quadratic equations. In this lesson, connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating should reflect both the problems and instruction.

1. The mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning. The mathematical topic was presented in isolation of other topics, and the teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

2. Connections among mathematical topics were present in the lesson. The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail by the teacher or the students.

3. Connections among mathematical topics were discussed by teacher and students during the lesson, or connections were clearly explained by the teacher.

4. The mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. During instruction, many students did at least one of the following: looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical relationships, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

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2 Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Deep Knowledge (pp. 31-35).

C.4. Connections between mathematics and students' daily lives were apparent in the lesson.

This scale measures whether connections between mathematics and students' daily lives were apparent in text problems or discussed by the teacher or students. Examples of problems that foster such connections are estimating the sale price of an item or determining the amount of ingredients required to serve four people when a recipe serves seven. In contrast, word problems such as "Bart is two years older than Lisa. In five years Bart will be twice as old as Lisa. How old are they now?" are devoid of connections between mathematics and students' lives.

1. Connections between mathematics and students' daily lives were not apparent in the lesson.

2. Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

3. Connections between mathematics and students' daily lives were clearly apparent in the lesson.

C.5. Students explained their responses or solution strategies.

This scale is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. Students simply stated answers to problems. They did not explain their responses or solution strategies orally or in written form.

2. Students explained how they arrived at an answer, but these explanations focused on the execution of procedures for solving problems rather than an elaboration on their thinking and solution path.

3. Students explained their responses or solution strategies. They elaborated on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results.

C.6. Multiple strategies were encouraged and valued.

This scale measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. In a classroom where multiple strategies are encouraged and valued, students spend much of their time discussing different strategies in a substantive manner, and this discourse is an important element within the classroom. Multiple strategies might be elicited by the teacher during whole-class or small-group discussion in which students explicitly share their strategies. The task itself might clearly involve students in solving the problem in different ways (e.g., find the discount in another way), or the task may require students to consider alternative approaches for successful completion (e.g., list as many ways as you can to calculate 15 x $1.98).

1. Multiple strategies were not elicited from students.

2. Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

3. Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

4. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

C.7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

This scale is intended to measure the ways in which the teacher uses student responses during instruction. Teachers can give credence to students' responses by inviting students to listen carefully to other students, to ask each other questions that clarify meaning, and to compare other students' strategies with their own. Teachers can also use student responses to pose questions that stimulate further discussion, to illustrate a point, or to relate them to other aspects of the lesson.

1. The teacher was interested only in correct answers. The majority of the teacher's remarks about student responses were neutral short comments such as "Okay," "All right," or "Fine." No attempt was made to use students' responses to further discussion.

2. The teacher established a dialogue with the student by asking probing questions in an attempt to elicit a student's thinking processes or solution strategies.

3. The teacher valued students' statements about mathematics by using them to foment discussion or to relate them to the lesson in some way. The teacher opened up discussion about the student response by asking other students questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

C.8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. A student's question about whether the sum of the angles of every triangle is always 180°, for example, might lead to a discussion of non-Euclidean geometry. This scale measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

Circle Yes, if the teacher used students' inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Circle No, if a student's comment or question potentially could have led to such a discussion, but the teacher did not pursue it.

Circle N/A, if no such opportunities came about during the lesson.

C.9. The teacher encouraged students to reflect on the reasonableness of their responses.

An unreasonable response refers to a response that is mathematically distant from the correct answer and might even be distant from an answer that students recognize as reasonable in contexts outside the classroom. One explanation for unreasonable responses is that students do not check the reasonableness of their answers. Although this may be true in some cases, unreasonable responses may also be the result of the lack of connections between symbols and their meaning. Evaluating the reasonableness of a solution involves connections between conceptual and procedural knowledge. These connections are especially significant at the end of the problem-solving process. Lining up decimal points when adding or subtracting decimals, for example, without connecting the process to place value concepts, may lead to unreasonable responses. Students might rely on rules or procedures to obtain correct answers and not have the conceptual knowledge to help them evaluate reasonableness of the answer (Hiebert & Wearne, 1986). This scale is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

1. The teacher rarely asked students whether their answers were reasonable. If a student gave an incorrect response, another student provided or was asked to provide a correct answer.

2. The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding.

3. The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

D.1. Student exchanges with peers reflected substantive conversation of mathematical ideas.

With this scale we are attempting to capture the quality of student communication. Substantive conversation by students is characterized by interaction that is reciprocal, involving listening carefully to others' ideas in order to understand them, building conversation on them, or extending the idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions.3 In contrast, student exchanges with little or no substantive conversation involve reporting facts or procedures in ways that do not encourage further discussion of ideas.

1. There were no exchanges between peers in small groups or as a formal part of the general discourse within a large-group setting.

2. Student exchanges with peers reflected little or no substantive conversation of mathematical ideas.

3. Most students only asked one another for a clarification of directions given by the teacher or simply accepted someone's answer without an explanation of how it was found. Few students asked how a solution was found or asked for a clarification of another student's answer.

4. Most of the students asked their classmates for a description of how they solved a particular problem, discussed alternative strategies, and/or questioned how classmates arrived at a solution.

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3 Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Substantive Conversation (pp. 35-40).

D.2. Interactions among students reflected collaborative working relationships.

The collaborative nature of the classroom can be thought of as students working together, exchanging ideas, and finding solutions to the same problem. This includes providing assistance to one another, making sure that everyone understands and is working on the same problem, exchanging ideas, and seeking help from each other when it is needed. Student collaboration can occur in a small-group or large-group setting. If the major focus of the lesson is on providing students with individual work, then N/A should be selected.

N/A. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Few students were sharing ideas or discussing how a problem should be solved in small groups or in a large-group setting. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and at different paces.

3. Some students were exchanging ideas, or providing assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.

4. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

D.3. The overall level of student engagement throughout the lesson was serious.[4]

This scale measures the extent to which students remained on task during the lesson.

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions by many. This was the central characteristic during much of the class.

2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task but not trying very hard.

3. Sporadic or episodic engagement. Most students, some of the time, were engaged in class activities, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

4. Widespread engagement. Most students, most of the time, were on task pursuing the substance of the lesson. Most students seemed to take the work seriously and were trying hard.

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4 Ideas were drawn from Secada & Byrd (pp. 14-15).

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Observation Scale

Observer: _____ Time Lesson Begins: _____
Teacher: _____ Time Lesson Ends: _____
School: _____ Textbook: _____
Grade: _____ Chapter/Unit: _____
Date of Observation: _____ Lesson (pages): _____

A. Pre-observation Interview With Teacher

1. What mathematical concept(s) or important ideas are being conveyed in this lesson?
   _____
   _____

2. Where is this activity generally situated in the development of a unit? (For example, day 1 (introduction) of 5 days needed to complete the unit)
   _____

B. Lesson Flow

Describe the main activities that occurred during the class period and the amount of time devoted to each activity. For example: warm-up—5 minutes, introduction of concept through context—7 minutes, large group discussion—10 minutes, group activity—25 minutes, summary by teacher—5 minutes.

For sections C and D please refer to the observation scale descriptors on the attached sheets. Please provide evidence supporting your rating.

C. Classroom Events

		Evidence
1. The lesson provided opportunities for students to make conjectures about mathematical ideas.	1 2 3 4	
2. The lesson fostered the development of conceptual understanding.	1 2 3 4	
3. Connections within mathematics were explored in the lesson.	1 2 3 4	
4. Connections between mathematics and students' daily lives were apparent in the lesson.	1 2 3	
5. Students explained their responses or solution strategies.	1 2 3	
6. Multiple strategies were encouraged and valued.	1 2 3 4	
7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.	1 2 3	
8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.	Yes No N/A	
9. The teacher encouraged students to reflect on the reasonableness of their responses.	1 2 3	

D. Pupil Pursuits

						Evidence
1. Student exchanges with peers reflected substantive conversation of mathematical ideas.		1	2	3	4	
2. Interactions among students reflected collaborative working relationships.	1	2	3	4	N/A	
3. The overall level of student engagement throughout the lesson was serious.	1	2	3	4		

E. Postobservation Interview With Teacher

1. On a scale of 1 to 4, how did the teacher rate the lesson as achieving the intended purpose cited in the pre-observation interview?

   Goals Not Met All Goals Met
   1 2 3 4

   Comments: _____
   _____
   _____

2. Did the teacher feel that any incidents occurred which revealed student misunderstanding or provided opportunities to facilitate student understanding in any way? Yes No If yes, please describe and explain if the lesson was modified in any way.
   _____
   _____

F. Additional Information

1. Please feel free to add any comments or information (on the back) that you think would be of interest in describing the classroom that you observed.

Appendix D

Theoretical Framework for *Instruction*

Theoretical Framework

The research function used to make both the cross-sectional and longitudinal comparisons attributes variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, to variations in prior achievement (PA), method of instruction (I), opportunity to learn with understanding (OTL*u*), and school capacity (SC). This relationship can be expressed as—

$$CA = PA + I + OTLu + SC.$$

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*,5 and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each (see Figure 1): (a) *consideration of students' prior knowledge during unit planning*; (b) *unit sequence*; (c) *pace of instruction*; (d) *consideration of students' performance in the previous lesson during lesson planning*; (e) *the purpose of the lesson*; (f) *forms of instruction that promote discourse for the purpose of the lesson*; (g) *student activities that promote discussion, problem solving, and reflection on the content of the lesson*; (h) *lesson presentation and development*; (i) *nature of mathematical inquiry during instruction*; (j) *interactive decisions during instruction*; (k) *nature of students' explanations*; (l) *elicitation of multiple strategies*; (m) *lesson reflection, summary, or closure*; (n) evidence sought during classroom *assessment*; (o) *purpose and coherence of feedback given in response to classroom assessment*; (p) *content of feedback provided in response to classroom assessment*; (q) *nature of student–student conversation*; (r) *collaborative working relationships among students*; and (s) *level of student engagement*. A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the research function. The focus of the remaining sections of this paper is the theoretical framework for the composite index *Instruction*.

The instruction envisioned in standards-based classrooms differs in character from conventional pedagogy in its emphasis on lessons that involve mathematical reasoning, conjectures, problem solving, verification of mathematical arguments, and connections among mathematical ideas (NCTM, 1991). When learning mathematics with understanding, students need the time and opportunity to develop relationships among mathematical ideas, extend and apply these ideas in new situations, reflect on and articulate their thinking, and make mathematical knowledge their own (Carpenter & Lehrer, 1999). Understanding mathematics requires reflection, the "conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed" (Battista, 1999, p. 429), and communication. Through reflection, students can look for connections between new mathematical ideas and ones they already know. Teachers can specifically ask students to identify and articulate these relationships or to discuss how a procedure is linked to a particular notation or underlying mathematical concept (Carpenter & Lehrer, 1999). Research in a growing number of studies underscores the importance of teaching mathematics for understanding, which is based on the principles that knowledge is constructed by the learner and is situated in the context of the learner's existing knowledge, skills, and beliefs; that the teacher's role is a guide for facilitating conceptual understanding; that mathematical tasks are nonroutine, accessible to all students, and engage students' thinking about important mathematics; that classrooms are communities of learners; and that mathematical tools are supports for learning (Cohen, McLaughlin, & Talbert, 1993; Fennema & Romberg, 1999; Hiebert et al., 1997). In contrast, when the aim of the lesson is primarily coverage of content, the emphasis is often on unconnected pieces of information and on the practice of repetitive procedures or heuristics determined by others (Battista, 1999). Such situations reduce cognitive demands on students.

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5 The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

Mathematical Interaction

Classroom discourse, including expressing thinking and representing mathematical ideas, is central to learning mathematics as a domain of inquiry (NCTM, 1991). Substantive discourse promotes shared understanding of mathematical ideas and emphasizes higher order thinking, which requires students to “combine facts and ideas in order to synthesize, generalize, explain, hypothesize or arrive at some conclusion or interpretation” (Newmann, Secada, & Wehlage, 1995, pp. 86–87). When students rethink their ideas in order to clarify their reasoning for others, they engage in “reflective abstract thinking” as they consider their solutions from the perspective of another student (Wood, 1996, p. 102). In addition, students learn through collaborative problem solving when they are confronted with “a surprise outcome (particularly when two alternative procedures lead to the same result), verbalize their mathematical thinking, and resolve conflicting points of view” (Yackel, Cobb, & Wood, 1991, p. 395). With appropriate guidance in large- and small-group settings, students discuss interpretations of problem situations, express their thinking, and react to the different levels and qualities of solution strategies shared in the group (Gravemeijer, 1994). During class discussions, for example, "solutions can be paralleled; ideas exchanged; approaches on different levels of schematizing considered and compared; arguments criticized, refuted, amended, supported . . ." (Streefland, 1992, p. 237). Such conversation is characterized by reciprocity. Students listen carefully to each other’s ideas, build conversation on those ideas, and mutually construct their understanding to a more abstract level (Newmann, Secada, & Wehlage, 1995). In order for students to participate in substantive mathematical discussions, classroom norms are important (Yackel & Cobb, 1996). Social norms form the basis of classroom cultures in which students explain their thinking, justify their approaches to problem solving, and support their mathematical arguments. Sociomathematical norms provide attention to facets of mathematical conversation that are related to students’ own mathematical cognitive activity, for example, “normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom . . . [as well as] what counts as an acceptable mathematical explanation and justification” (Yackel & Cobb, 1996, p. 461). In contrast, student conversation with little or no substantive content involves reporting facts and procedures in ways that do not encourage further discussion of ideas, or focusing primarily on correct answers. These situations can ultimately result in students' appropriating the answers or the solution strategies of the teacher or a dominant peer.

The types of exercises, problems, projects, and investigations used during instruction convey messages about the nature of mathematical activity and shape students' opportunities to learn mathematics with understanding. Tasks can provide ways for students to explore concepts and procedures, develop skills in meaningful contexts, seek connections among mathematical ideas, and apply mathematics in real-world situations. Tasks that promote depth of knowledge require that students "organize, interpret, evaluate, or synthesize complex information," allow students to consider alternative solutions or perspectives, and open opportunities for elaboration of thinking and support of conclusions (Newmann, Secada, & Wehlage, 1995, pp. 81, 83). Tasks are appropriate for students when they are genuinely problematic (i.e., they provide interesting situations in which students can extend and apply their knowledge of mathematics), promote connections to students’ existing knowledge, and allow for thinking about significant mathematics (Hiebert et al., 1997). The role of the teacher is the selection or creation and sequencing of these tasks, which provide opportunities for learning, reflection, and discussion. Tasks, however, can be adjusted to either reduce or encourage cognitive activity. The difference resides in enabling teachers to "proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task" (Henningsen & Stein, 1997).

Pedagogical Decisions

The pedagogical decisions teachers make prior to instruction have a direct impact on student learning. These decisions include deliberate advanced planning such as student grouping for instruction, time allotted to specific aspects of lessons, and emphases given during instruction. In order to orchestrate classroom discourse (both oral and written), the planning of various forms of instruction and student activities that promote discussion, problem solving, and reflection is imperative. The planning involved in teaching standards-based curricula requires more time than generally perceived necessary for teaching using conventional methods (Battista, 1999; Bay, Reys, & Reys, 1999). Standards-based curricula present mathematics in ways that may be unfamiliar to teachers and require advanced planning for successful instruction. Romberg (1997) outlined several features of the mathematical content of MiC that were problematic for teachers who used prepublication units. In MiC, topics traditionally reserved for high-school students are introduced to middle-school students in real-world contexts and with emphasis on student reasoning rather than on procedures. The sequence of topics throughout the grade levels in MiC units is also different from that in conventional middle-school mathematics curricula (e.g., concepts related to percent are introduced in fifth and sixth grades in MiC rather than, more conventionally, in eighth grade). In the prepublication study, this change in sequence sometimes meant that teachers had not taught the content previously. Furthermore, although MiC units primarily present mathematical ideas from one content strand, most involve topics from several strands (in order to emphasize the interrelated nature of mathematical ideas) and include some problems that can be approached in multiple ways (opening possibilities for students' access to problems). These changes provided challenges to these teachers, who were accustomed to teaching mathematics as isolated pieces of knowledge. In contrast, lesson planning with conventional curricula is often minimal because lesson content tends to be the same over many years, and lesson material is conveniently parceled into 2-page spreads.

Teachers also draw on pedagogical content knowledge, which includes information about difficulties students might encounter as they learn new topics, typical sequences students go through as they learn the topics, and potential ways of helping students overcome difficulties (Bransford, Brown, & Cocking, 1999). The thought required by teachers is demanding: “The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (Shulman, 1987, p. 15, original emphasis). This transformation of teacher knowledge into goals and expectations for student learning during the lesson is a distinguishing factor in planning quality instruction.

When instruction begins, however, teachers' interactive decisions become more important. Teachers' knowledge of mathematics and their own understanding of their students' thinking are also critical influences on teachers' interactive decision-making. As Heaton and Lampert (1993) pointed out:

> Interactive competence and a knowledge of mathematics are required if mathematics is going to be taught in a way that is responsive *both* to the way children think about number and shape *and* to the structure of the mathematical ideas under discussion. This knowledge is different from what one needs to know in order to competently teach a textbook lesson oriented toward having students remember how and when to do procedures. . . . The teacher’s knowledge of the subject must be used to make connections between what he or she understands and what students bring to the lesson. The teacher’s mathematical knowledge must be held in a form that enables it to be reshaped depending on the kinds of ideas students bring up in a discussion or in the activity of trying to solve a problem (p. 54, original emphasis).

The interactive decisions of teachers using prepublication versions of MiC became more complex as students assumed more active roles in their own learning. Clarke (1995) investigated the nature of critical incidents that developed as two teachers used MiC for the first time. In their efforts to understand students' solution strategies at various levels of sophistication, these teachers found it difficult both to structure lessons that allowed for some guidance

without limiting opportunities for student thinking and to determine the length of time to let students struggle with a problem before intervening. They also faced challenges of "valuing all genuine attempts at problems, while seeking to move students toward increasingly mathematically elegant methods" (p. 156), and they found that it was "very difficult to be interacting, to be listening in an attentive way, trying to understand students' solutions, and thinking of an appropriate response, many times during a lesson" (p. 163). Thus, the nature of teachers' interactive decision-making changes dramatically as teachers move away from conventional pedagogy toward teaching mathematics with curricula designed for students to learn mathematics with understanding.

Classroom Assessment

Classroom assessment is the process in which teachers gather, synthesize, and interpret information to make instructional decisions (Airasian, 1994). The content of classroom assessment should correspond to the mathematics content standards and emulate what it means to know and do mathematics. Multidimensional assessment practice attends to important instructional goals and the processes through which a student's understanding is developing and is more directly connected to ongoing instruction (Graue, 1993; Shepard, 2000; Wolf, Bixby, Glenn, & Gardner, 1991). Evidence gathered by teachers can be based on a taxonomy of problems and solution strategies such as that for whole number operations in Cognitively Guided Instruction (Carpenter, Fennema, Levi, & Empson, 1999) or particular statements that arise during classroom interaction such as "an interpretation of language or symbols, an assertion about a pattern, a proof that a pattern would continue beyond the observed data, or an interpretation of another student's assertion" (Lampert, 1988, p. 460). Evidence might also include students' plans to test conjectures, flexibility in working with mathematical tools, increased use of mathematical concepts and procedures, alternative approaches to problems, and sophistication of solution strategies (Shafer, 1996). Because each piece of evidence represents a sample of a student's performance at a particular point in time, it is important that the evidence a teacher collects for each student is representative of the student's achievements. Instructionally embedded assessments such as observing students as they work on tasks, listening to students' strategies, responses to teacher or student queries, and written assignments and quizzes provide a range of opportunities for students to demonstrate what they know about mathematics. The quality of such evidence also depends on the tasks in which students engage, the interactions among students and between the students and the teacher, and a learning environment in which social and sociomathematical norms have been developed. Over time, information from multiple assessments provides the foundation for a teacher to develop a broadly based understanding of each student's growth in mathematical domains (Gipps, 1994). Thus, the methods of assessment used and the types of evidence teachers actively seek during instruction affect information available for making instructional decisions.

Another aspect of classroom assessment practice is to providing multiple opportunities for feedback to encourage students' participation in mathematical discussion and to help them judge their own work against established performance standards and criteria. Ideally, verbal or written feedback is responsive to student statements in order to promote mathematical understanding and greater perspective of ideas. Providing feedback is not solely a teacher action. Students should be given the opportunity to share and critique the explanations, arguments, strategies and responses of other students. Substantive feedback provides students with information to improve their mathematical work and communication of their thinking, and it can be an important catalyst for students to extend their thinking to higher levels. Quality feedback includes references to mathematics principles and concepts (Bransford, Brown, & Cocking, 1999; NCTM, 1995). If feedback lacks mathematical substance, it is limited only to correct answers or superficial features of responses such as organization and neatness.

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Appendix E

Methodology for the Composite Index *Instruction*

The composite index *Instruction* is specified from data gathered on the intervening variables (*pedagogical decisions*, *classroom events*, and *student pursuits*) in the structural research model for the longitudinal/cross-sectional study. In the simplified research function, *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice,*6 and *student pursuits during instruction*. The five categories of *Instruction* were further subdivided into 19 subcategories, and an index was created for each (see Figure E1). In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence; and pace of instruction. Four subcategories characterized lesson planning: consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson;* and *student activities that promote discussion, problem solving, and reflection on the content of the lesson.* Six subcategories characterized the *mathematical interaction during instruction*: lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of *students' explanations; elicitation of multiple strategies;* and *lesson reflection, summary, or closure*. Three subcategories characterized

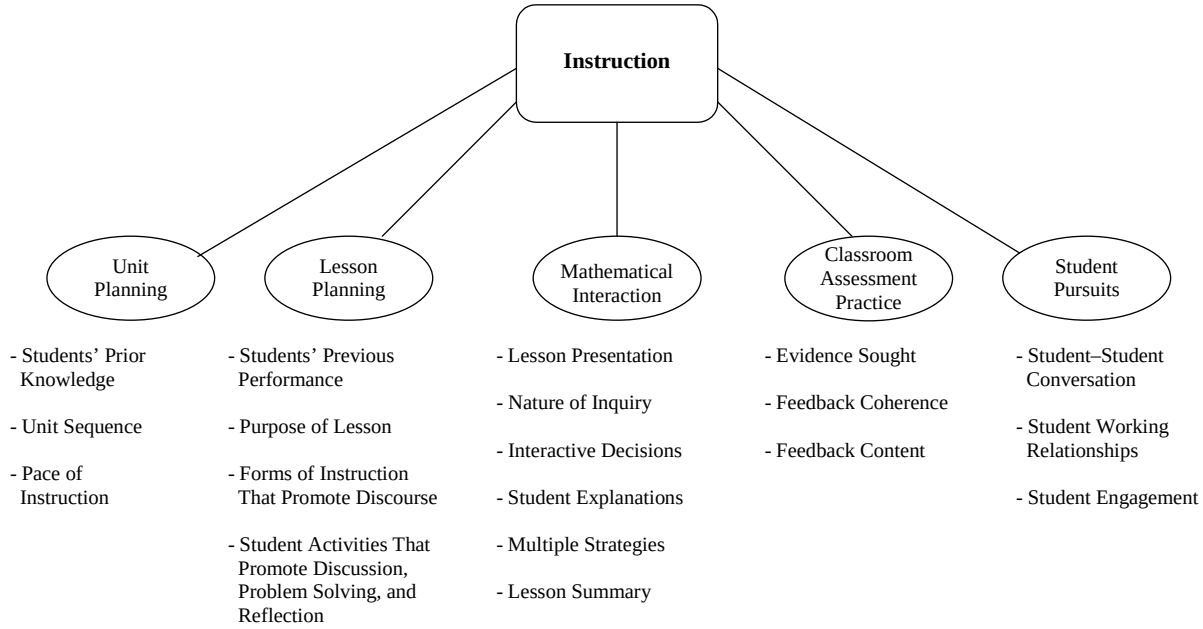

Figure E1. Major categories and subcategories of *Instruction*.

6 The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

classroom assessment practice: *evidence sought; purpose and coherence of feedback*; and *content of feedback*. Finally, three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students*; and *level of student engagement*.

A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the simplified research function. Description of the theoretical framework that guided the analysis of instruction is contained in Appendix D. For complete descriptions of teacher data, see Appendices F, G, and H for sixth-, seventh-, and eighth-grade teachers, respectively.

Sample

Districts 1 and 2

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. Data collection included teacher interviews, classroom observations, and teaching logs. Analysis focused on all five categories of instruction. All MiC teachers used commercial versions of the units, which became available during the summer before the study began. Teachers using conventional curricula used whatever curriculum was already in place in the schools. The sample in Districts 1 and 2 consisted of 35 sixth-, seventh-, and eighth-grade teachers and their classes from diverse schools in Districts 1 and 2. These teachers taught a combined total of 63 classes involving 1561 students (see Tables E1 and E2).

Table E1
Summary of Study Participants, Districts 1 and 2, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 1	
MiC	12	22	532
Conventional	6	11	240
		District 2	
MiC	12	23	584
Conventional	5	7	205
Total	35	63	1561

Table E2
Characteristics of the Student Participants, Districts 1 and 2, by Curriculum

Curriculum	Gender *(%)*		Ethnicity *(%)* *(self-idenified)*							Primary Language *(%)* *(self-idenified)*	
	Male	Female	African American	Native American	Asian	Hispanic	White	Multiracial	Other[1]	English	Other[2]
District 1											
MiC	49	51	21	1	1	7	54	10	5	92	7
Conventional	53	47	25	0	–	3	54	13	3	91	9
District 2											
MiC	47	53	9	1	–	49	17	17	7	85	15
Conventional	56	44	18	1	–	34	14	18	15	84	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.
[2]Includes nonresponses.

District 1 was located in an urban region in the eastern part of the United States. In 13 elementary, 6 middle, and 4 high schools, 1325 teachers were responsible for teaching the district's 15,532 students. Three elementary schools and four middle schools participated in the first year of the study. Six fifth-grade study classes were in self-contained elementary classrooms. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. Approximately 30–40% of the students in the district were eligible for government-funded lunch programs. Professional development to acquaint teachers of mathematics with reform-based curricula was offered in District 1, and monthly meetings were provided for teachers who were implementing such programs. For preliminary teacher certification, 24 credit hours were recommended for fifth- and sixth-grade teachers; 24 credit hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements.

District 2, located in a large urban area in southeastern United States, had 19,352 teachers and 342,996 students housed in 201 elementary schools, 51 middle schools and numerous high schools. Three elementary and three middle schools participated in the first year of the study. In District 2, two of the nine fifth-grade study classes were in self-contained settings in elementary schools. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district student population was predominantly minority, with 33% African American students and 52% Hispanic students. Over 50% of the students in the district were eligible for government-funded lunch programs. District 2 provided numerous possibilities for professional development. Each school was given six early-release days for general professional development. In addition, each school received 10 substitute days for professional development in mathematics and/or science, 12–18 days of in-service days in mathematics provided by (USI or Eisenhower) government funding (each involving 2–6 teachers), and 3–5 days of district-wide mathematics in-service. Teachers also had opportunities to participate in five days of paid in-service for mathematics during the summer. District requirements for preparation of mathematics teachers were the same as state requirements. For elementary teachers, preliminary teacher requirements mandated the study of arithmetic for the elementary school. For middle-grade mathematics certification (Grades 5–9), 18 semester hours in mathematics were required; for certification in mathematics for

Grades 6–12, 30 semester hours in mathematics were required. Continuing certification required the completion of six semester hours in mathematics or 120 district in-service credits in mathematics every 5 years.

Districts 3 and 4

Districts 3 and 4 agreed to participate in a modified research design in which all MiC teachers used MiC. Data collection included teacher interviews, but excluded classroom observations and teaching logs. Therefore, data analysis in Districts 3 and 4 was limited to teachers' unit and lesson planning. All teachers used commercial versions of MiC units. The sample in Districts 3 and 4 consisted of 18 fifth-, sixth-, and seventh-grade teachers who taught a combined total of 31 classes involving 665 students (see Tables E3 and E4).

Table E3
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 3	
MiC	13	21	438
		District 4	
MiC	5	10	227
Total	18	31	665

Table E4
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
					District 3						
MiC	48	52	--	--	--	2	87	9	--	98	5
					District 4						
MiC	49	51	31	--	1	20	3	15	29	85	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.
[2]Includes nonresponses.

District 3 was located in a suburban area in a large western state. In the district's two elementary and one middle school, 73 teachers taught 1480 students. In District 3, the six self-contained 5th grade study classes are in a school for Grades 3–5; Grades 6-8 were housed in a middle school. Three of the middle school study classes were in self-contained classrooms. The remaining middle-school study classes had several subject-matter teachers. Study participants included all fifth-, sixth-, and seventh-grade mathematics classes in the district. The schools' student and teacher populations were predominately White.

Approximately 10–20% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. School administrators provided paid monthly meetings for mathematics teachers who were implementing standards-based curricula for the first time. Teachers often met weekly without pay to prepare mathematics lessons. For preliminary teacher certification, the state mandated single-subject credentials for Grades 7–8. Teachers for Grades K–6 were required to complete a multiple-subjects credential including several mathematics courses. Although the district provided mathematics courses and staff development opportunities, it did not require additional certification and courses for experienced teachers.

District 4 was one of many districts located in a large urban area in the eastern part of the United States. The district's 1075 teachers were responsible for teaching the 20,000 students in 23 elementary schools, seven middle schools, and several high schools. In District 4, Grades 6–8 are contained in middle schools in which students have several subject-matter teachers. Study participants are from one middle school in this district. Because fifth-grade students in District 4 are dispersed among several middle schools, fifth-grade classes were not included in the data collection for District 4. Four sixth-grade and six seventh-grade classes from one middle school participated in the study. The student population was predominately minority with 64% African American students and 28% Hispanic students. Approximately 37% of the teacher population was minority with 31% African American teachers and 6% Hispanic teachers. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. For new mathematics teachers, 36 credits in mathematics or a mathematics major were required by the state, but no specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements. Professional development opportunities were provided to all mathematics teachers at both district and school levels, including personalized discussions with the assistant principal for mathematics and science. These discussions focused on reform recommendations in curriculum, instruction, and assessment; research in mathematics education; and applications of research in classroom practice.

Data Collection

Data were collected through the use of instruments designed to examine instruction in the longitudinal/cross-sectional study. Information on *unit planning, lesson planning*, and *classroom assessment practice* was gathered through the Teacher Interview: Instructional Planning and Classroom Interaction (see Appendix A; Shafer, Davis, & Wagner, 1998). Teachers in Districts 1-4 were interviewed twice during the school year. The interviews conducted in the spring semester focused on teachers' instructional planning and classroom interaction. The first interview question asked teachers about their planning for each mathematics unit/chapter. The probes provided specific attention to (a) whether the teacher planned with others; (b) the considerations given in planning at the unit level, specifically, students' prior knowledge, textbook scope and sequence, district curriculum guidelines, state standards, district or state standardized tests, and other resources; and (c) setting the pace for instruction. An additional probe was reserved for teachers who used MiC. The focus of this probe was comparison of planning to teach MiC with planning to teach other mathematics textbooks used in the past. The second interview question was dedicated to planning individual lessons. Probing questions provided attention to (a) considerations of students' performance in the previous lesson, and (b) whether the teacher solved unit/chapter problems or exercises before teaching, and, if so, the effect this preparation had on teaching the lesson. The third interview question was designed to collect information on classroom interaction and provided insight into teachers' classroom assessment practice. In particular, the question addressed the issue of what counts as an acceptable answer. Related probes were (a) how students determine if their answers are acceptable; and (b) how the teacher determines if a student's answer is acceptable. The third probe elicited information about the ways in which students contribute to classroom discussions. A fourth probe was reserved for teachers who used MiC. The focus of this probe was differences in student

participation when using MiC in comparison to student participation when using conventional mathematics curricula in the past. Additional probes for this question provided attention to differences in the types of conversation generated with each type of curriculum such as student conjectures, answers, and explanations.

Information on *lesson planning, mathematical interaction during instruction,* and *classroom assessment* was gathered through the Teacher Log Appendix B; Shafer, Wagner, & Davis, 1997). Each teacher in Districts 1 and 2 was asked to complete a daily teaching log. Three sections of the log are pertinent to data collection on instruction: lesson activities, student activities, and classroom assessment. Data from teacher log sheets, which were completed daily by each study teacher, were used to more fully describe the forms of instruction and the student activities teachers used. One item in the log was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). Another item was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. These data were summarized over all the days a particular study teacher completed a daily log sheet. Information from classroom observations generally supported the information teachers provided in these two items.

Data from teacher logs were also used to more fully describe teachers' use of *classroom assessment*. If teachers assessed students informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered.

Each teacher in Districts 1 and 2 was also asked to reflect on his/her teaching and respond to two journal questions at least once a week. Journal entries were submitted monthly as part of a daily teaching log (see Appendix B). The journal questions focused on emphases given during instruction, modification of curricular materials, and notable classroom events. The first journal question provided information about the interactive decisions teachers made during instruction. The second journal question provided additional insight into classroom interaction, especially when the journal entry was related to a lesson that was observed and teachers' classroom assessment practices. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them.

Information on *mathematical interaction during instruction, classroom assessment*, and *student pursuits* was gathered through the Classroom Observation Instrument (see Appendix C; Davis, Wagner, & Shafer, 1998). Classroom observations were conducted on each study teacher in Districts 1 and 2. Most teachers in District 1 were observed once a month for a total of nine observations per teacher. One teacher in District 1 accepted an administrative position in December; consequently, she was observed three times, and the newly assigned teacher was observed five times. Teachers in District 2 were observed a total of three to seven times each. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. Four teachers from one school in District 2 withdrew from participation in the study during the spring semester; consequently, they were observed only three times. The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. The research staff worked with both observers to establish interrater reliability. Completed observation reports were sent to the research center monthly.

The observation instrument contained six sections, four of which provided data about instruction: lesson activities, ratings on 12 indices about classroom events and student pursuits, postobservation interview, and observer comments. Two sections of the observation instrument were particularly useful in describing the mathematical interaction and student pursuits that transpired during class time. In one section, the observer recorded the flow of the lesson, which was a list of lesson activities along with the time allotted to each. This information illuminated the relative emphases teachers gave to particular forms of instruction and student activities. Another section of the observation instrument was collectively composed of 12 indices for various subcategories of instruction. For each index, the observer circled a rating and provided a statement of support for the given rating. Nine of these indices focused on classroom events, three of which provided particular attention to classroom assessment and teachers' interactive decisions: the use of student inquiries as a guide for instructional mathematics investigation or as a guide to shape the mathematical content of the lesson; encouragement for students to reflect on the reasonableness of their responses; and the use of student statements to build discussion or work toward shared understanding for the class. The remaining three indices focused on student pursuits. Ratings given by observers were checked by research staff for accuracy and were changed when appropriate. For example, on a few occasions, when the evidence provided by the observer strongly indicated another rating than the one circled, the rating was changed to reflect the evidence.

A brief postobservation interview was conducted during which the teacher was asked whether incidents occurred that revealed student misunderstanding or provided opportunities to facilitate student understanding in any way. If such an incident occurred, the teacher was asked to explain whether and in what way the lesson was modified. These data provided insights into teachers' interactive decisions and classroom assessment practices. In the final section of the observation instrument, the observer recorded any additional comments about the lesson. These comments included field notes that the observer deemed important for the research team or descriptions that more fully portrayed the lesson.

Indices

An index was created for each of the 19 subcategories of *Instruction*. The indices used to characterize each subcategory of *Instruction* were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through (a) review of literature that was specific to each subcategory (see Appendix D for a summary of the relevant literature) and (b) review of the teacher and observer data based on Strauss' (1987) system of open, axial, and selective coding, which involved repeated coding of the data for interpretive codes. These codes included both external codes identified prior to reviewing the data (e.g., demonstration of procedures and strategies) and internal codes that emerged from the data (e.g., students' inability to solve problems using a presented procedure). Categories were further refined, and additional categories were integrated for each index as data for fifth-grade study teachers in one district were reviewed. Indices were further revised during review of data from sixth- and seventh-grade teachers and teachers in other districts. As a result, three to five levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts. Indices included in the observation instrument were refined as a consequence of observing nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Before these indices were used in study classrooms, district and school administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms currently implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each subcategory. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the subcategory being scaled. For example, levels of *lesson presentation and development* range from no lesson presentation during instruction to lessons in which the teacher facilitated students' active participation in learning mathematics with understanding. One level in the some indices were subdivided further in order to more adequately describe the variation among teachers who demonstrated procedures or strategies to their students. For example, sublevels in one level of the index for lesson presentation and development categorized situations in which students were unable to complete exercises using the presented procedure and lessons in which students practiced the presented procedure in a rote fashion. In the remainder of this section, the indices for each subcategory of instruction, organized by major category, are described.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence*; and *pace of instruction*. Four levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts.

Students' Prior Knowledge. The following index measured the extent to which the teacher thought about and identified students' prior knowledge while planning to teach a unit:

1. *Little or no consideration of students' prior knowledge*. The teacher planned the unit with little or no consideration of the prior knowledge of students in the current class.

2. *Consideration of student abilities*. The teacher planned the unit based on perceptions of students' needs.
   (a) The teacher bases plans on perceptions of students' mathematics skills and/or reading ability and vocabulary.
   (b) The teacher bases plans on perceptions of students' needs related to the development of concepts.

3. *Informal or formal assessment of students' understanding*. The teacher planned the unit on the basis of information gathered through informal or formal assessment. The teacher might have, for example, planned remedial skill-based activities to address weaknesses or planned extension activities for students ready for such challenges.

4. *Conceptually-based activities planned*. The teacher planned unit activities that were designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence. The following index measured the extent to which the teacher might consider the sequence of instructional units:

1. *Little or no variance from the text sequence*. The teacher followed the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher based decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequenced units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequenced units to support the development of mathematics concepts.

Pace of Instruction. The following index measured the extent to which the teacher might consider the pace for instruction when planning to teach a unit:

1. *Little or no consideration of pacing*.
   (a) The teacher followed the recommendations for pacing in teacher support materials.
   (b) The teacher did not plan unit pacing because the curriculum was unfamiliar.
2. *Adjustment anticipated*. The teacher considered the recommendations for pacing in teacher support materials, but planned to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considered the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher planned substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Lesson Planning

Four subcategories characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Four levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts.

Students' Performance in the Previous Lesson. The following index measured the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might have noted students' performance, but the lesson was planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might have, for example, extended the previous lesson to complete a task, disregarded time constraints, or added a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might have used the information gathered to allow a more in-depth exploration of the mathematical content or to introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might have, for example, varied problem structure/setting to encourage thinking at higher levels or to emphasize connections with related concepts.

Purpose of the Lesson. This index measured the extent to which particular teachers thought about and identified the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose was to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction was to cover lessons in the textbook or curriculum; thus, no additional planning was deemed necessary.
2. *Overall curriculum plan in mind.*
   (a) The teacher selected lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might have included an additional focus on problem solving, applications of mathematics, or practice.
   (b) The teacher planned lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher made decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might have, for example, planned questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson. The following index measured the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention was focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might have, for example, planned for such work/discussion, but continued to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse was not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction.* The teacher encouraged such participation, but it was not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned.* The teacher might have, for example, planned classroom activities that encouraged students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson. The following index measured the extent to which a particular teacher might have included various student activities that promoted discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis was placed on practicing routine calculations, and little discussion among students was anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might have planned problem investigation or class discussion, but the significance of these activities was not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encouraged students to reflect on or summarize lessons, however, were not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher viewed reflection on or summarization of the lesson as an important element in instruction.

Mathematical Inquiry during Instruction

Because the data collection in Districts 1 and 2 included classroom observations and teacher logs, the remaining 12 indices were completed for only the study teachers in these districts. Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Lesson presentation and development. For *lesson presentation and development*, six levels were defined. Data from classroom observations and teacher journal entries provided multiple sources for which lesson presentation and development could be examined for each teacher. Two members of the research staff independently rated each piece of evidence according to the levels in the index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. An overall rating was then assigned for each teacher. Although variability did occur in the multiple pieces of evidence for each teacher

over the course of the school year, each teacher maintained basic elements of pedagogy that permitted the characterization of *lesson presentation and development* at one particular level. (See Shafer (2000b) for a detailed description of this index.)

This index measured the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding:

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.
2. *Emphasis on review*. The lesson was not well developed; consequently, students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.
   (a) A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.
   (b) Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.
3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.
   (a) Students were unable to solve problems using the presented procedure or strategy.
   (b) Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.
4. *Attempt made to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Although students were allowed to find their own solution strategies, they generally used a procedure or strategy presented by the teacher.
5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.
6. *Emphasis on conceptual understanding with active participation by students with teacher support*. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher.

The Nature of Inquiry. The index for the *nature of inquiry* is a composite of four indices that were included in the observation instrument: the development of conceptual understanding, the nature of students' mathematical conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. Three levels were identified for connections between mathematics and students' life experiences; four levels were identified for the other indices. The sum of the ratings given by the observers on these four indices for all observations of each study teacher was calculated. Cluster analysis permitted the classification of teachers. As a result, four levels were identified to capture the variation among teachers at different grade levels and in different districts.

This index characterizes the *nature of mathematical inquiry during instruction*:

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking; lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.
2. *Limited attention to conceptual understanding.* Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Emphasis on conceptual understanding.* Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.
4. *Emphasis on relationships among mathematical ideas or linking procedural and conceptual knowledge.* The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to make generalizations; and connections between mathematics and students' lives were discussed.

Teachers' interactive decisions. The index for *teachers' interactive decisions* is a composite of three indices, which included *explanation-oriented decisions, task-oriented decisions*, and *shifts in pedagogical approach*. The categories in the three indices are similar to the other indices in that they are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the subcategory being scaled. For example, categories of *shifts in pedagogical approach* range from no change during instruction when warranted to lessons in which the teacher modified a lesson based on a student's statement or inquiry. Two members of the research staff independently rated each piece of evidence according to the levels in each index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. The indices for *teachers' interactive decisions*, however, differ from the others in a significant way. In order to describe variation in the data, the categories in each index for interactive decisions were further classified into decisions that were (a) least aligned with teaching for understanding, (b) reflective of standard pedagogy, or (c) most aligned with teaching for understanding. Patterns of variation among the teachers became more apparent with this additional categorization of *teachers' interactive decisions*. A profile of each *teacher's interactive decisions* was constructed. In the initial phase of this process, the percent of coded items in each category was calculated. *Explanation-oriented decisions, task-oriented decisions*, and *shifts in pedagogical approaches* were set in juxtaposition along a continuum from least aligned with teaching for understanding to most aligned with teaching for understanding in order to look for patterns among the teachers. The percent of coded items in the three subcategories (decisions least aligned with teaching for understanding, reflective of good standard pedagogy, and most aligned with teaching for understanding) were calculated. The profiles formed the basis of the composite index for *teachers' interactive decisions*. Five levels of interactive decisions were identified in this process. (See Shafer (2000a) for a detailed description of this index.)

This index measured the extent to which a teacher's interactive decisions were aligned with teaching mathematics for understanding:
1. *Least aligned with teaching for understanding*. Teachers' interactive decisions were least likely to support students' learning mathematics with understanding. At least 60% of the coded items represented decisions least aligned with teaching mathematics for understanding.
2. *More emphasis on standard pedagogy, but decisions predominantly coded as least aligned with teaching for understanding*. More interactive decisions were classified as reflective of good standard pedagogy and were more aligned with teaching for understanding than at Level 1, although 40–55% of the decisions were still coded as least aligned with teaching for understanding.
3. *Stronger emphasis on standard pedagogy*. Teachers' interactive decisions in general were more reflective of good standard pedagogy and more supportive of teaching for understanding than at Level 2, and decisions least aligned with teaching for understanding were less pronounced (around 30% of the coded interactive decisions).
4. *More emphasis on standard pedagogy and teaching for understanding*. Teachers' interactive decisions were characterized by greater attention to standard pedagogy and teaching for understanding than at Level 3, and decisions least aligned with teaching for understanding were less likely to occur (generally around 20% of the coded decisions).
5. *Most aligned with teaching for understanding.* Teachers' interactive decisions were predominantly aligned with teaching for understanding and were least likely to be coded as least aligned with teaching for understanding (around 10%).

The Nature of Students' Explanations. The index for the *nature of students' explanations* was included in the classroom observation instrument. (See Shafer (2000b) for a detailed description of this index.) The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only*. Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures*. Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes*. Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Elicitation of Multiple Strategies. The index for the *elicitation of multiple strategies* was included in the classroom observation instrument. This index measured the extent to which students were asked to consider different perspectives in approaching the solution to a problem:

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies as a substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Lesson Reflection or Summary. Data gathered through the teacher log regarding student activities that provided opportunities for students to reflect on or summarize the mathematics in a lesson or series of lessons were used to support data gathered from observer comments on the observation reports. Two members of the research staff independently rated each piece of evidence according to the levels in the index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. An overall rating was then assigned for each teacher. Although variability did occur in the multiple pieces of evidence for each teacher over the course of the school year, each teacher maintained basic elements of pedagogy that permitted the characterization of lesson reflection or summary at one particular level. This index measured the extent to which the teacher included reflection on or summary of lesson concepts:

1. *Limited opportunities.* Few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. The lesson or series of lessons concluded at the end of the class period without discussion of the primary mathematics content or connections among mathematical ideas.
2. *Some opportunities.* Some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. These opportunities may have included completion of reflection questions in section summaries.
3. *Frequent opportunities.* Students were frequently provided with opportunities to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. These opportunities included teacher-designed writing prompts or questions posed during whole-class discussions.

Classroom Assessment Practice

Three subcategories characterized *classroom assessment practice*: *evidence sought; purpose and coherence of feedback*; and *content of feedback*. Two members of the research staff independently rated each piece of evidence according to the levels in each index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers.

Evidence Sought. This index measured the evidence teacher regarded as indicative of student achievement and understanding:

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.
2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.
3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations were often void of mathematical substance and often promoted without purpose (i.e., to generate some form of communication rather than assess student understanding). Teachers at this level valued student explanations but might have had difficulty eliciting student responses and orchestrating substantive whole class discussions.
4. *Conservative process orientation.* Teachers at this level value were somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures. The teacher might have required students to justify solutions with explanations, but often forced their own strategy and restricted the responses that were shared. Student misconceptions were rarely utilized as opportunities for instruction.
5. *Principled process orientation.* Teacher viewed student explanations as evidence of student learning. While the teacher sought both process and product as evidence, s/he sought demonstration of student learning through verbal or written communication of process. Answers, solutions, and procedures were recognized as inadequate forms of demonstrating understanding. Instruction wais used as an opportunity to gather evidence of student learning. Teachers at this level regularly elicited student (mis)conceptions to guide instruction.

Feedback Coherence and Purpose. This index measured the method and goal orientation of feedback that the teacher makes available to students.

1. *No feedback.* Teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.
2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through teacher-directed practices that involved “more of the same,” such as additional instruction and practice sets. Ineffective attempts to promote student-to-student interaction often resulted in reteaching or additional practice. Feedback might have also occurred as individualized responses to specific student questions and procedural errors where the teacher moved from desk to desk responding to student requests for assistance during seatwork. Feedback provided by other students was minimal or non-existent.
3. *Emerging shared responsibility.* Students received feedback from peers through student–student discussions in pairs or groups and through sharing examples of their responses to assigned work. However, student–student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making. Students’ role or purpose in sharing their work with others might have been unclear.

4. *Purposeful shared responsibility.* Student interactions were used to promote sense making of tasks, student responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work in progress and examples of refined responses.
5. *Toward student self-assessment.* In addition to the practices described at Level 4, the process and criteria used by the teacher to evaluate mathematical work is revealed to students, and they are invited to assess their own and other students' work. The teacher provides opportunities for students to participate in the creation or modification of performance standards, reflect upon and judge their own work, and revise more complex assignments that require elaborated communication such as projects, reports, and written explanations.

Content of Feedback. This index measured the degree of substantive feedback provided to students, from teachers, students, and available resources:
1. *Feedback withheld and/or misleading.* Teacher feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance. Teacher did not attend to student misconceptions and often promoted or sustains student misconceptions.
2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions. Feedback might have been limited to praise, criticism or student work habits.
3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer (such as simplifying fractions) rather than clarifying explanations or developing student understanding.
4. *Mixed, superficial feedback.* An emerging blend of feedback addressed skills, procedures, and concepts. Feedback was directed toward mathematics although, at times, feedback might have favored problem context over mathematical content. Feedback and guidance given in response to student work was clear and mathematically sound.
5. *Concept-directed feedback.* Feedback was directed towards conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall. The process and criteria used by the teacher to evaluate mathematical work was revealed to students, and they were invited to assess their own and other students' work. The teacher provided opportunities for students to participate in the creation or modification of performance standards, reflect upon and judge their own work, and revise more complex assignments that require elaborated communication such as projects, reports, and written explanations.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students*; and *level of student engagement*. The indices for these subcategories were included in the classroom observation instrument. (See Shafer (2000b) for a detailed description of the index for the *nature of student–student conversation*.)

Nature of Student–Student Conversation. The index for student–student conversation measured the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:
1. *Conversation not encouraged*. Conversation among students was not permitted or was social in nature.
2. *Limited conversation*. Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature*. Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.

4. *Substantive conversation*. Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Student Collaborative Working Relationships. This index measured the extent to which students collaborated with one another during the lesson:

N/A. *Independent work.* The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. *No collaboration among students.* None of the students worked together in small-group or large-group settings. If students worked in small groups, then one student typically gave answers to other members of the group without explanation for the use of certain procedures.
2. *Limited exchange of ideas.* Few students shared ideas or discussed how a problem should be solved in small-groups or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems at different paces.
3. *Uneven participation.* Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.
4. *Substantive collaboration.* Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Student Engagement during the Lesson. This index measured the extent to which students remained on task during the lesson:

1. *Disruptive disengagement*. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. *Passive disengagement*. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. *Sporadic or episodic engagement*. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. *Widespread engagement*. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

The Composite Index *Instruction*

Although teachers in all four research sites completed interviews, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index for instruction, therefore, was created only for teachers in Districts 1 and 2 for whom there was a complete set of ratings on all 19 indices. Thirty-four teachers were involved in the analysis for the first year of data collection. One teacher was not included in the analysis because she did not teach a full semester during the study.

The composite index *Instruction* was created in a multiple-step process. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the Instruction Total.7 Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subcategories and the Instruction Total (see Table E5).

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7 The sum of the weighted results was taken as a measure of the quality of instruction. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore,

Table E5.

Correlation between the Instruction Total and the Subcategories of Instruction

Subcategory	Unit Planning			Lesson Planning				Mathematical Interaction						Classroom Assessment			Student Pursuits		
	SPK	US	PI	SPPL	PL	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE
SPK																			
US	0.092																		
PI	0.321**	0.017																	
SPPL	0.164	0.295**	0.051																
PL	0.136	-0.026	0.147	0.409***															
FIFD	0.038	-0.077	0.229*	0.177	0.326**														
SAPD	0.016	-0.181	0.134	0.118	0.474**	0.640***													
LPD	-0.023	0.104	0.176	0.157	0.312**	0.537***	0.546***												
NI	-0.026	-0.116	0.208	0.217*	0.506**	0.665***	0.549***	0.647***											
ID	-0.033	-0.015	0.285**	0.099	0.309**	0.568***	0.527***	0.676***	0.702***										
SE	-0.008	-0.072	0.141	0.175	0.260*	0.497***	0.468***	0.559***	0.692***	0.660***									
MS	-0.088	-0.060	0.048	0.025	0.227*	0.523***	0.426***	0.552***	0.635***	0.619***	0.788***								
LCS	-0.032	0.000	0.084	0.172	0.291**	0.476***	0.534***	0.556***	0.476***	0.495***	0.445***	0.471***							
ES	0.005	0.000	0.015	0.152	0.257*	0.408***	0.457***	0.650***	0.573***	0.602***	0.635***	0.599***	0.537***						
FCP	-0.036	0.055	0.108	0.079	0.266*	0.442***	0.423***	0.570***	0.520***	0.532***	0.561***	0.523***	0.480***	0.791***					
FC	-0.136	0.030	0.142	0.159	0.228*	0.398***	0.401***	0.624***	0.547***	0.613***	0.551***	0.469***	0.464***	0.695***	0.753***				
SC	0.015	-0.002	0.101	0.189	0.375**	0.493***	0.425***	0.436***	0.624***	0.532***	0.506***	0.475***	0.409***	0.483***	0.503***	0.578***			
SWR	0.073	0.008	0.063	0.109	0.336**	0.437***	0.422***	0.369***	0.532***	0.487***	0.521***	0.480***	0.385***	0.450***	0.486***	0.492***	0.845***		
OSE	-0.049	0.015	0.082	0.220	0.272*	0.322**	0.336**	0.525***	0.562***	0.512***	0.623***	0.572***	0.364***	0.548***	0.489***	0.518***	0.592***	0.593***	
Instr. Total	0.089	0.081	0.272*	0.309**	0.501**	0.697***	0.665***	0.801***	0.833***	0.803***	0.781***	0.721***	0.654***	0.787***	0.748***	0.751***	0.723***	0.667***	0.688***

*p<.05
**p<.01
***p<.001

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction
Instr. Total--Instruction Total

measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of instruction.

Five subcategories were not well correlated with the Instruction Total and other subcategories: consideration of students' prior knowledge; unit sequence; pace of instruction; students' performance in the previous lesson; and the purpose of the lesson. To verify these results, a principle component factor analysis was completed using SAS. Factors 1 and 2 accounted for a significant amount of the variance among the subcategories. Fourteen subcategories were included in Factors 1 and 2 (see Table 6). The five subcategories that had weak correlations to the Instruction Total were not influential in Factors 1 and 2 and were important only in the composition of other factors. Consequently, these subcategories were excluded from the analysis. The Instruction Total for each teacher was then recalculated.

Table E6.
Contribution of Subcategories to Principle Component Factors

Subcategory	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
SPK	-18	11	16	72*	9
US	4	-5	7	18	83*
PI	21	-5	-4	85*	7
SPPL	5	9	75*	0	46*
PL	19	21	78*	11	-12
FIFD	60*	15	33	23	-32
SAPD	59*	9	44*	9	-41*
LPD	83*	10	17	7	6
NI	65*	39	33	12	-23
ID	76*	28	8	19	-11
SE	68*	45*	5	5	-10
MS	68*	39	-2	-3	-17
LCS	68*	6	29	-4	-5
ES	79*	30	4	-10	12
FCP	75*	332	-1	-4	15
FC	74*	35	0	-8	17
SC	36	80*	22	6	-6
SWR	29	85*	15	8	-8
OSE	46*	65*	8	-5	9

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

Key

SPK--Consideration of Students' Prior Knowledge
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FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
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ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

Using the revised Instruction Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into six groups. For each group of teachers, common characteristics from the subcategories of instruction were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each subcategory of instruction. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. Similar to the index for each subcategory, the underlying single dimension of the composite index was teaching was teaching mathematics for understanding. The levels of the composite index *Instruction* were on a continuum from least to most reflective of teaching mathematics for understanding. The six levels are summarized in Table 7.

Level 6: Most reflective of teaching for understanding
Level 5: Reflective of teaching for understanding
Level 4: Attempted to teach mathematics for understanding
Level 3: Limited attention to conceptual understanding
Level 2: Focus on procedures
Level 1: Underdeveloped lessons.

Level 6: Most Reflective of Teaching Mathematics for Understanding

Lessons emphasized conceptual understanding, and students actively participated in lessons with the support of their teachers. Lesson presentations set the stage for students to explore the mathematical content of the lessons on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher. Mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge. Students were encouraged to make generalizations about mathematical ideas, and connections between mathematics and students' life experiences were discussed. Students' explanations varied in attention to mathematical reasoning. During some lessons, students explained the processes used in finding solutions; during other lessons, explanations focused on procedures used to determine answers. Interactive decisions were predominantly aligned with teaching for understanding. Teachers frequently asked students to articulate their thinking, discuss mathematical reasoning, and talk about the reasonableness of solutions. Teachers shifted students' attention from small-group work to whole-class discussion, introduced or encouraged students to introduce alternative strategies, or modified a lesson in response to a student's statement or inquiry.

Teachers' classroom assessment practices were characterized by attention to mathematical processes such as justifying approaches to problems, explanations of reasoning, and supporting conclusions. Teachers used students' misconceptions to guide instruction through the use of probing questions, counterexamples, and alternative representations. Feedback promoted making sense of mathematical tasks, students' solutions, and mathematical conventions. Criteria for judging student work were openly discussed, and students were given opportunities to create or modify performance standards. Students assessed their own work and the work of others and revised assignments that required elaborate communication, such as projects and reports.

Conversation among students frequently was characterized by students discussing procedures or asking each other for clarification of a procedure presented in class. Occasionally, conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend the ideas to a new level. During collaborative group work, students exchanged ideas or provided assistance to their classmates, but all students did not make equal contributions to problem solving. Most students were on task, seriously pursuing the substance of the lesson.

Level 5: Reflective of Teaching Mathematics for Understanding

Lessons emphasized conceptual understanding of the mathematical content, and students and their teachers actively participated in lessons. Student conjectures were characterized by investigating the veracity of particular statements. Teachers and students discussed connections among mathematical ideas, or teachers clearly explained such connections. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Teachers' interactive decisions were characterized by less attention to teaching for understanding than at Level 6. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools such as fraction bars and focused students' attention on pertinent elements of tasks. Their questioning techniques focused on articulation of student thinking, understanding mathematics, or reasonableness of solutions. They added a different context, a review lesson, or explanations that promoted connections among mathematical ideas or between mathematics and students' lives. Teachers were more likely to modify a lesson based on a student's statement or inquiry than at Level 4.

Teachers' classroom assessment practices were characterized by accepting students' explanations as evidence of their learning. Some teachers orchestrated discussions around students' solutions to learn about their procedural competence. Other teachers sought evidence of student learning through students' verbal or written communication of mathematical processes. These teachers used students' misconceptions to guide instruction through the use of probing questions, counterexamples, and alternative representations. Feedback was an ongoing, purposeful, and shared responsibility of both teachers and students. Feedback promoted making sense of mathematical tasks, students' solutions, and mathematical conventions.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Occasionally, students discussed procedures or asked each other for clarification of procedures demonstrated by their teachers. Although seated in groups, students generally worked at different paces on the assigned work. Most students were on task, seriously pursuing the substance of the lesson.

Level 4: Attempt to Teach Mathematics for Understanding

Teachers attempted to teach for conceptual understanding, but the primary focus of lessons was on building students' procedural understanding. On some occasions, students were asked if different strategies were used in solving particular problems, but this was not a primary goal of instruction. Although students were allowed to find their own solution strategies, they generally used a procedure or strategy presented by the teacher. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Interactive decisions were characterized by greater attention to standard pedagogy and teaching for understanding than at Level 3. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools, included additional exercises based on student interest or need for practice, or focused students' attention on pertinent elements of tasks. They added minilessons on algorithms or procedures, a different context, or a review lesson. Teachers also asked students about the reasonableness of their solutions. At times, interactive decisions involved the introduction of alternative strategies by teachers or students.

Teachers’ classroom assessment practices were characterized by gathering evidence from student explanations as evidence of their skills and procedures. Teachers elicited students’ solution strategies and orchestrated whole-class discussions around them, but the primary goal of their assessment practice remained demonstration of procedural competence. Student misconceptions were rarely used as opportunities for instruction. Feedback students received from other students rarely went beyond sharing answers and procedures. Teachers' feedback included attention to mathematical concepts or the contexts in which problems were situated.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Although seated in groups, students worked at different paces on the assigned work. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Teachers’ planning for teaching instructional units or chapters was consistent with Level 5.

Level 3: Limited Attention to Conceptual Understanding

Inquiry during class provided limited attention to conceptual understanding of the mathematical content. The main focus of lessons was on building students’ procedural understanding. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students’ explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Interactive decisions were more reflective of good standard pedagogy than at Level 2. Teachers emphasized understanding of mathematical vocabulary and at times used questioning techniques that focused on articulation of thinking, understanding of mathematics, or the reasonableness of solutions. Occasionally, interactive decisions involved shifting students’ attention from small-group work to whole-class discussion. Teachers at times added a different context or a review lesson, or modified a lesson based on a student’s statement or inquiry.

Teachers’ classroom assessment practices were characterized by gathering evidence from homework and classwork to substantiate students’ procedural competence. Occasionally, teachers included student explanations as evidence of their skills and procedures, but explanations were generally elicited to generate communication in class rather than as an avenue for assessment of student understanding or as a basis for substantive mathematical conversation. Teacher-directed feedback was indirectly responsive to student needs in that it involved additional whole-class instruction using the same presentation method or the assignment of additional exercises of the same type. For some teachers, feedback was directed toward correct procedures and the format of the answers such as simplifying fractions. The feedback of other teachers, however, included attention to mathematical concepts or the contexts in which problems were situated. Their feedback was mathematically sound and clear to students.

Level 2: Focus on Procedures

The focus of lessons was on particular procedures or strategies, and students were expected to use the methods presented by their teachers. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor

did they explore connections between mathematics and students' life experiences. Teachers' interactive decisions were predominantly less aligned with teaching for understanding. Limited changes were made in instruction even when teachers or students experienced confusion or misunderstanding.

Teachers' classroom assessment practices were characterized as gathering evidence from homework and classwork to determine students' procedural competence. Teacher-directed feedback was indirectly responsive to student needs and was directed toward correct procedures and the format of the answers. Student–student conversation, students' working relationships, students' engagement in lessons, and teachers' planning for teaching instructional units or chapters were generally consistent with Level 3.

Level 1: Underdeveloped Lessons

Mathematics was presented in ways that gave students only a surface treatment of the content. Inquiry during class was limited to lower order thinking, and lessons did not promote conceptual understanding. Some teachers devoted a major portion of the class period to review of a previous lesson or homework. The subsequent lesson presentation was not well developed; consequently, students began independent work or small-group work with little direction. Teachers assisted individuals or groups of students on a one-to-one basis during independent work or small-group work. Teachers presented particular procedures or strategies, and students were expected to use those methods. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences. Teachers' interactive decisions were least likely to support students' learning mathematics with understanding. Teachers' questions focused on following particular step-by-step procedures, and their own explanations were preferred over student explanations. No changes were made during instruction to address student questions, difficulties, or unexpected strategies.

Teachers' classroom assessment practices were characterized by gathering evidence from homework and classwork to determine students' procedural competence. Teacher-directed feedback was indirectly responsive to student needs. Individualized feedback occurred as teachers responded to questions from specific students during independent seatwork. Feedback provided by students was minimal or nonexistent. For some teachers, feedback was not attentive to student misconceptions, was misleading, or lacked mathematical substance. For other teachers, feedback was directed toward correct procedures and the format of answers rather than to the development of mathematical understanding.

Conversation among students was not encouraged. When students did talk with one another, the conversation was social in nature. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Planning for teaching instructional units or chapters was consistent with Levels 2 and 3.

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Appendix F

Instruction

Grade 6

Grade 6

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the 19 indices, grouped by major categories, and the composite index for instruction, are described and illustrated with evidence from classroom observations, teacher interviews, teaching logs, and teacher journal entries. The ratings assigned to each teacher in Districts 3 and 4 on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in Districts 3 and 4. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 1

In District 1, six Grade 6 teachers participated in the study. Five teachers used MiC, and one teacher used the conventional curriculum already in place in her school.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*. When planning to teach a MiC unit, Gollen obtained an overall view of the unit by reading through it entirely, identifying the goals and vocabulary, and studying the Hints and Comments section. Then she looked through each section to get an overall idea of pacing. She then planned one section at a time: "That's how I do everything. I take a day at a time and see how [the students] are doing with it. Then I go from there. Either I keep going on or I have to go backwards. It's been a lot of trial and error" (Gollen, Interview 5/10/99). Gollen thought it was important to be aware of students' prior knowledge. However, because her students came from several schools and she had no information about their prior educational experiences until mid-year, Gollen planned to adjust unit plans according to how students performed on their daily work. Gollen planned to use other resources to provide skill practice and practice for the district standardized tests. She often referred to the state standards for guidance. Gollen thought planning to teach a MiC unit was very different and more difficult for her than planning to teach a unit

from a conventional textbook because the MiC approach was so new to her: “It’s really hard for me as a teacher to look at this material and try to figure out how not to talk too much, but instead how to lead the kids through exploration to discovery” (Gollen, Interview 5/10/99).

Table F1
Subcategories of Unit Planning: Gollen, Addams Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students’ Prior Knowledge	1	Little or no consideration of students’ prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Weatherspoon followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the unit, identifying the development of the mathematical content, the major concepts presented, and the skills needed. She sorted the problems into two groups, those that required her to do direct teaching and those that students could solve more independently when working with partners. Weatherspoon divided the unit into teachable sections and determined problems that could be eliminated or used as homework assignments. She also identified the skills with which her students lacked proficiency and planned warm-up activities for extra practice. She noted places suitable for students to use calculators. Weatherspoon planned to use MiC ancillary materials (*Number Tools*) and student activity sheets that accompanied MiC units. She tried to follow the suggested pace for instruction, but anticipated that her students would need more time. Weatherspoon recorded each state standard the unit addressed as expected by the district. Even though Weatherspoon thought the district’s standardized test somewhat reflected MiC content, she used specific resource material to prepare the students for the test (Weatherspoon, Interview 4/13/99).

Table F2
Subcategories of Unit Planning: Weatherspoon, Fernwood Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students’ Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Brown generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, he looked through the entire unit, identified the major skills and competencies presented, chose the competencies he wanted to emphasize or provide extra drill and practice, and decided activities from the unit to be used as homework. Brown then examined his other resources for games and projects that would complement the skills and competencies in the MiC unit. He explained:

> I look in my bag of tricks, my catalog of things and activities that I like to do with the children. I find out which ones will fly or fit best with that [unit] and plan my activities for around that time. For example, when we were in a unit that involved the order of

> operations, I threw in activities like working with the 24 Game. I also pulled out some old worksheets for reinforcement, for drill and practice. (Brown, Interview 5/6/99)

Brown also added activities that gave students experience in answering multiple-choice questions in preparation for district standardized tests. Although he noted the suggested pace for instruction, Brown did not follow it because of the supplementary activities he added to the unit (Brown, Interview 5/6/99).

Table F3
Subcategories of Unit Planning: Brown, Von Humboldt Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Parsons followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read carefully through the unit, especially the overview which identified unit objectives and the suggested pace for instruction. Although Parsons planned to follow the suggested pace provided in the teacher guide, she anticipated more time because she was not aware of students' prior knowledge of the unit content and could not foresee any problems they might have. She planned to use supplementary materials to address any problems students experienced with concepts or skills. Parsons planned homework assignments from the MiC unit or from other resources, depending on the unit. She recorded each state standard addressed in the unit but did not consider district standardized tests as she planned. Parsons thought planning to teach a MiC unit was no different from planning to teach a unit from a conventional textbook (Parsons, Interview 5/6/99).

Table F4
Subcategories of Unit Planning: Parsons, Von Humboldt Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Lovell followed the recommended sequence of MiC units. When planning to teach a MiC unit, she became familiar with the unit by reading the entire unit including the introduction and then working each problem. Through this process, she tried to anticipate the difficulties students might encounter and developed strategies she could use to help students overcome the problems. Because this was her first year teaching MiC, Lovell planned to teach the unit exactly as the teacher guide suggested: "I don't change anything right now. I never change anything beforehand because it's my first year teaching the book and I may not see what might be of value the first year through" (Lovell, Interview 5/3/99). A pretest administered at the beginning of the school year alerted Lovell to the fact that her students were weak in basic skills. She planned to use other

resources to provide extra skill practice before teaching the unit. Lovell tried to follow the suggested pace of instruction, but found she could not keep up. She noted the state standards addressed in each unit. Because district standardized tests were new, she was not aware if they reflected MiC content. Lovell thought planning to teach a MiC unit took a lot more time than planning to teach a unit from a conventional textbook (Lovell, Interview 5/3/99).

Table F5
Subcategories of Unit Planning: Lovell, Wacker Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	1B	Little or no consideration of pacing

Harrison considered the content and dates of district standardized tests and the linkages across units of the same content strand when she sequenced chapters in the conventional textbook that she used. When planning to teach a chapter, she skimmed through it to get a general impression of the content and worked through the example problems. She eliminated word problems that she felt would confuse students. Using information gained from the school placement test administered at the beginning of the school year, she anticipated problems her students might have. Harrison did not outline a specific pace for instruction when she planned for the entire chapter. Rather, she adjusted the pace based on student performance: "Basically I go with how well the students are doing. That's why I don't like to plan too far in advance because I know I'll never meet whatever [pace] I plan. If the kids are having a really hard time with it, I might spend a little more time on it than I had originally planned" (Harrison, Interview 4/26/99). Although Harrison thought most textbooks met all state standards, she noted the standards addressed in each chapter. She planned additional problem-solving activities because the new district standardized tests required problem-solving skills:

"[Students] can give you a 100 % on a quiz, because they can do it. But they can't explain it. They just can't explain it. And that's what so much of the district testing is now. That's what we're trying to do more and more" (Harrison, Interview 4/26/99).

Table F6
Subcategories of Unit Planning: Harrison, Fernwood Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

In summary, the six sixth-grade teachers in District 1 varied in all three subcategories of *unit planning* (see Figure F1). With respect to *students' prior knowledge* when planning to teach a unit, two teachers assessed students' understanding of skills or concepts needed in a particular unit. One teacher based unit planning on her perceptions of students' prior knowledge in mathematics or reading without informal or formal assessment. Three teachers followed the unit development in the teacher guide with little or no consideration of students' prior knowledge.

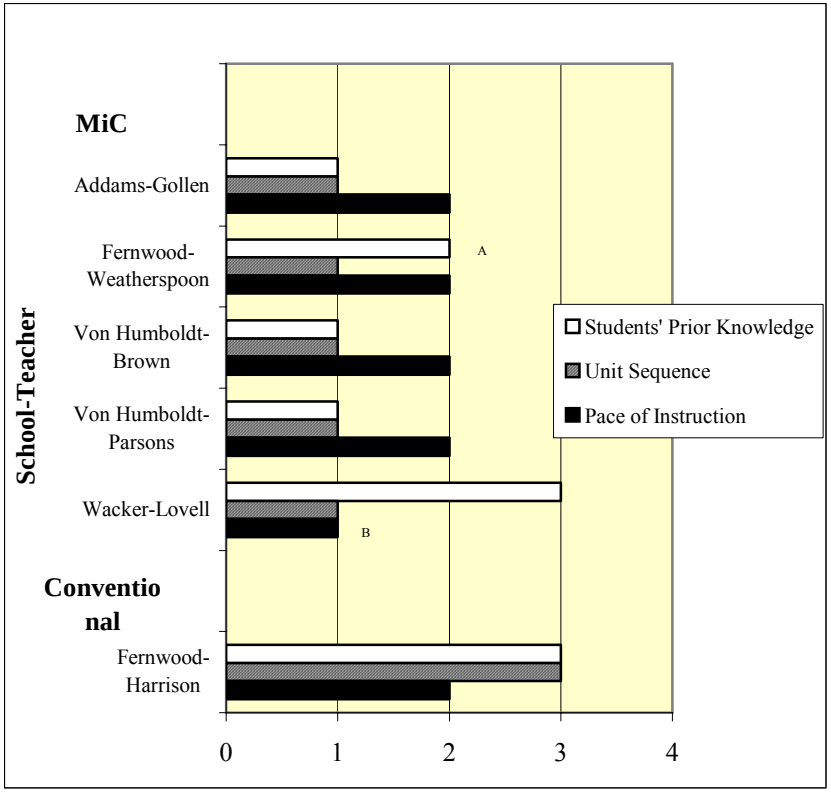


Figure F1. Unit planning, Grade 6, District 1.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table F63 in this appendix.

Only one teacher varied from the other teachers in determining the *sequence of instructional units* by considering linkages across units of the same content strand. The other five teachers considered external factors such as the content of the statewide testing program when sequencing instructional units. Five teachers anticipated that various factors such as needed prerequisite skills or difficulty of the content would necessitate adjusting the recommended pacing for instruction. One teacher did not plan unit pacing because the curriculum was unfamiliar to her.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

When planning to teach individual MiC lessons, Gollen read through the section to determine its main goal, but did not work each problem. She determined reasonable expectations for each day and anticipated modifying her plans depending on how students performed each day. Deciding when to move on was difficult for Gollen:

> I tend not to want to budge until I feel like everybody understands [the material]. This is very unrealistic. But if I feel like the majority of the kids are not getting something then I have to back track. I don't just move plodding on one step at a time. I'm also going to learn, maybe by experience, what I can actually leave out of a unit. I am the type of person who is very detail-oriented and drive myself crazy. I want to make sure every kid gets the concept. I like to cover every little nook and cranny, which is totally unrealistic. But I don't know enough about MiC to know what to leave out. (Gollen, Interview 5/10/99)

Gollen decided the form of instruction that would be most effective for the lesson. She thought large-group instruction was difficult for sixth-grade students, but direct teaching was needed for some concepts. When an activity was open-ended or dependent on discussion, Gollen thought small-group work was most effective. She valued small-group instruction because "I think it makes things casual and it makes things less threatening. It makes things easier for a lot of different kinds of kids, especially the ones who are real unsure, need a little bit of extra help, or need more self-confidence" (Gollen, Interview 5/10/99). Gollen thought the disadvantages of small-group work were that some kids became very bossy and others became very lazy. She had tried several ways to group students and thought the most effective way was to have them work in pairs. She also allowed students to work alone if that was their preference (Gollen, Interview 5/10/99).

Gollen frequently used two instructional formats, small-group work (on 75% of the reported days) and teacher presentation (68%), and two instructional formats on approximately half of the reported days, review of previous material (55%) and whole-class discussion (43%; see Tables F65-F70 in this appendix). Each form was given equal emphasis with other instructional forms on most of the reported days. Independent practice was used on only 16% of the reported days and was frequently given equal emphasis with other instructional forms. Gollen rarely used warm-up activities (1%; Gollen, Teacher Log 1998-99).

Three student activities were important elements in Gollen's instruction: investigation of problems (on 77% of the reported days), discussion of answers and solution strategies (74%), and participation in whole-class discussions (74%; see Tables F71-F78 in this appendix). On most of the reported days, each activity was given equal emphasis with other student activities. On about one third of the reported days, students practiced computation (38%) and listened to the teacher or took notes (37%). Both were given equal emphasis with other activities. Students took a test or quiz on only 15% of the reported days but for a significant amount of time. Test taking was generally given more that half of the class time Students rarely reflected on or summarized lesson concepts (4%) or began homework (1%; Gollen, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 supported the information Gollen reported in her teacher logs. As an example of a lesson observed in Gollen's class, the lesson on 1/22/99 included: housekeeping duties, including taking roll and checking homework (11 minutes); teacher presentation (11 minutes); and small-group activity (25 minutes; Gollen, Observation 1/22/99).

Table F7
Subcategories of Lesson Planning: Gollen, Addams Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Weatherspoon worked through each problem to become familiar with the lesson and to decide the problems that students could complete by themselves. She explained:

> It depends on what we're doing and what the activities are. If the problems are like the ones today with exchanging, I think it was good that they worked in partners so [students] could see another way to approach the problem. It also allowed one student to explain [the problem] to someone else. I'll just decide beforehand if I want a problem to be a small-group problem, an individual problem, or a whole-class problem. (Weatherspoon, Interview 4/13/99)

Weatherspoon chose a daily warm-up problem that was related to the previous day's lesson as a way to assess how well the students understood the concepts or skills. Student performance on the warm-up problem determined whether she went on with the next lesson or reviewed concepts presented the previous day. She felt that whole-class instruction that included student discussion of their ideas and solutions was more effective for some classes (Weatherspoon, Interview 4/13/99).

As an example of a lesson observed in Weatherspoon's class, the lesson on 12/7/98 included: whole-class warm-up activity; (4 minutes); individual activity, 10 problems on the board (23 minutes); teacher presentation (7 minutes); small-group measurement activity (10 minutes);

whole-class activity, check measurements (3 minutes); small-group measurement activity (35 minutes); and closure (3 minutes; Weatherspoon, Observation 12/7/98). Teacher Log data for Weatherspoon were unavailable.

Table F8
Subcategories of Lesson Planning: Weatherspoon, Fernwood Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Brown read through the lesson to identify the skills. He did not work through the problems because "I have been comfortable going through the unit with the children and as issues come up, where surprises come up, I deal with them at that time" (Brown, Interview 5/6/99). If students did not complete the previous lesson, he continued it the next day. Brown always planned for small-group work except for assessments. He said, "The groups will automatically just turn their desks around and start facing each other when it's clear that it's a MiC working day. There are some activities that are more group-oriented, but I like to use [groups] all the time" (Brown, Interview 5/6/99).

As an example of a lesson observed in Brown's class, the lesson on 2/22/99 included: individual worksheet (10 minutes); whole-class activity, correct first part of worksheet (8 minutes); whole class, teacher gave directions (5 minutes); individual activity, complete worksheets and work on

fraction assignment (35 minutes); and teacher assigned homework (5 minutes; Brown, Observation 2/22/99). Teacher Log data for Brown were unavailable.

Table F9
Subcategories of Lesson Planning: Brown, Von Humboldt Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Parsons worked through all problems to determine if there were any difficult areas and, if there were, ways to clarify the activity for the students. She built alternatives into her plans to accommodate for students' performance on daily lessons. Parsons valued small-group work because:

> They like to talk and so that's an opportunity [for them] to talk. A lot of kids are anxious to help others and so it makes them feel good to help. It is not always the same kids helping. Different kids are strong in different units. [Small-group work] gives [students] a chance to discuss a problem. They don't discuss well with each other in a large group. (Parsons, Interview 5/6/99)

Parsons did not plan to use small-group work very often because as a beginning teacher she was not comfortable with her ability to ensure students stayed on task. Students worked in pairs for open-ended activities and other activities that lent themselves to small-group work (Parsons, Interview 5/6/99).

Parsons began most days with warm-up activities (95% of the reported days), but usually for less than 15% of class time (see Tables F65-F70 in this appendix). Two instructional forms were used on approximately two thirds of the reported days, independent practice (65%) and review of previous material (61%), but they were given very different amounts of class time. Independent practice was given at least half of the class time on 76% of the days and equal emphasis with other instructional forms on 19% of the days. Review of previous material was given at least half of the class time on only 9% of the days, equal emphasis with other instructional forms on 23% of the days, and less than 15% of class time on 69% of the days. On approximately half of the reported days, Parsons used teacher presentations (54%). This instructional format was given less than 15% of class time on 58% of the days and equal emphasis with other instructional forms on 35% of the days. Small-group work was seldom used (12%

of the reported days), but it was given significant amounts of class time. Parsons did not use whole-class discussions (Parsons, Teacher Log 1998-99).

Parsons' students most frequently listened to the teacher or took notes (84% of the reported days; see Tables F71-F78 in this appendix). This student activity was given equal emphasis with other student activities on 48% of the days and less than 15% of class time on 44% of the days. On more than half of the reported days, students investigated problems (56%) for significant amounts of time. This student activity was given at least half of the class time on 59% of the days, equal emphasis with other student activities on 25% of the days, and less than 15% of class time on 16% of the days. On approximately one third of the reported days, students discussed answers and solution strategies (32%), and on approximately one fourth of the days, they participated in whole-class discussions (26%), practiced computation (23%), and took a test or quiz (23%). The amount of class time given to each of these student activities varied. Less time was given to discussing answers and solution strategies and to participating in whole-class discussions than was given to practicing computation and to taking a test or quiz. Students seldom reflected on or summarized lesson concepts (12% of reported days) and began homework (5%; Parsons, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Parsons reported in her teacher logs. As an example of a lesson observed in Parsons' class, the lesson on 12/7/98 included: individual activity, problem of the day (8 minutes); whole class, teacher talked about the problem of the day (12 minutes); individual activity, six questions from the state math league (24 minutes); individual activity, page 24 (10 minutes); and whole-class teacher-led discussion of page 24 (10 minutes; Parsons, Observation 12/7/98).

Table F10
Subcategories of Lesson Planning: Parsons, Von Humboldt Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Lovell felt that working through all of the problems gave her a broad understanding of the lesson. She first used her own strategy for solving a particular problem and then used each strategy suggested in the teacher guide. She tried to anticipate places in which students might run into trouble and ways to redirect their thinking when they did. Lovell planned for a week at a time, but altered her plans to accommodate students' daily performance. Lovell valued small-group work because it provided students with opportunities to exchange ideas, opinions, and problem-solving strategies. She moved among the groups to hear the quality of their discussion, to redirect their

thinking if they were going too far astray from the desired direction, and to ask probing questions if they seemed to be at an impasse. Each member of the group was expected to hand in his/her work. Lovell also valued large-group instruction. She often held whole-class discussions after small-group work. She had two concerns about large-group discussions, however: the quality of students' large-group discussion might not be as high as it was when they worked in small groups; and her own role in large-group discussions. She explained, "I tend to be a dominant personality. I was afraid I would dominate large-group discussion too much. Now I'm confident enough that I won't dominate [the large group], so I'm going to move a little more toward large-group discussion. I'm going to always maintain small-group instruction too" (Lovell, Interview 5/3/99).

The three instructional forms that Lovell incorporated into her teaching most often were whole-class discussion (54% of the reported days), small-group work (47%), and teacher presentation (42%; see Tables F65-F70 in this appendix). Small-group work was given the most class time: at least half of the class time on 50% of the days, equal emphasis with other instructional forms on 39% of the days, and less than 15% of class time on 11% of the days. Whole-class discussion was given at least half of the class time on 19% of the days, equal emphasis with other student activities on 38% of the days, and less than 15% of class time on 44% of the days. Teacher presentation was given at least half of the class time on 16% of the days, equal emphasis with other student activities on 16% of the days, and less than 15% of class time on 68% of the days. On approximately one fourth of the reported days, Lovell used warm-up activities (24%) and review of previous material (22%). Neither instructional form was given significant amounts of class time. Lovell rarely used independent practice (5%, Lovell, Teacher Log 1998-99).

Lovell's students most often discussed answers and solution strategies (58% of the reported days), participated in whole-class discussions (42%), and investigated problems (42%; see Tables F71-F78 in this appendix). The amount of class time given each student activity varied. Students discussed answers and solution strategies for at least half of the class period on 30% of the days, but for less than 15% of class time on 47% of the days. On the remaining days the activity was given equal emphasis with other student activities. Students participated in whole-class discussions for at least half of the class period on 28% of the days, but for less than 15% of class time on 52% of the days. On the remaining days, the activity was given equal emphasis with other student activities. Students investigated problems for at least half of the class period on 48% of the reported days, but for less than 15% of class time on 40% of the days. On the remaining days, this activity was given equal emphasis with other student activities. On approximately one third of the reported days, students practiced computation (31%) and listened to the teacher or took notes (29%). More class time was given to practicing computation than listening to the teacher or taking notes. Students seldom began homework (8% of the reported days), reflected on or summarized lesson concepts (7%), or took a test or quiz (5%; Lovell, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Lovell reported in her teacher logs. As an example of a lesson observed in Lovell's class, the lesson on 3/25/99 included: whole class, teacher assigned homework and returned tests (5 minutes); individual activity, skills practice (7 minutes); whole class, read page 6, problem 12 (2 minutes); small-group work, do problem 12 (5 minutes); whole class, share solutions for problem 12 (5 minutes); whole class, teacher worked through problem 13 (4 minutes); and small-group work, problems 14 a-f (13 minutes; Lovell, Observation 3/25/99).

Table F11
Subcategories of Lesson Planning: Lovell, Wacker Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual lessons from a conventional textbook, Harrison worked through all of the word problems and each example problem. She eliminated any word problems she thought would be too difficult for her students. She also looked through the practice problems to get an idea of the lesson's complexity. Harrison planned a warm-up activity that would give her some idea of how well students understood the previous lesson or if they had the prerequisite skills needed to be successful in the new lesson. If she found that students were not ready for the new lesson, she substituted a review lesson in its place. Harrison was most comfortable when students sat in rows and she did direct teaching. If the lesson involved practice, she planned for students to work individually. Harrison thought it was difficult to hold everyone's attention during large-group discussions. Some students often dominated the discussion and others often did not participate even as listeners. She thought working in small groups was valuable in the right situation. She explained:

> I think it is valuable for [students] to work together. Maybe a stronger student can help out a weaker student in certain areas. [Some students] don't want to raise their hands in the middle of class and call out questions. Every once in a while we might have an activity where I'll have [students] push their desks together and have a small-group activity, especially if it's a new activity. (Harrison, Interview 4/26/99)

For the few times Harrison planned for small-group work, she made sure that she had all of the needed materials together, that there was enough time for students to get into groups, and that students were aware of the rules (Harrison, Interview 4/26/99).

Harrison usually began each day with a warm-up activity (93% of the reported days; see Tables F65-F70 in this appendix). However, this activity was always given less than 15% of class time. Students often participated in independent practice (67% of the reported days) for a significant amount of class time, at least half of the class period on 50% of the days and equal emphasis with other instructional forms on 40% of the days. Two instructional forms were used on 40% of the reported days, teacher presentation and review of previous material. On half of the reported days, teacher presentation was given equal emphasis with other instructional forms. On the other half of the days, it was given less than 15% of

class time. Review of previous material was given at least half of the class period on a third of the days, equal emphasis with other instructional forms on a third of the days, and less than 15% of class time on the remaining days. Harrison seldom used small-group work (7% of the reported days), but when she did, it was always given at least half of the class time (Harrison, Teacher Log 1998-99).

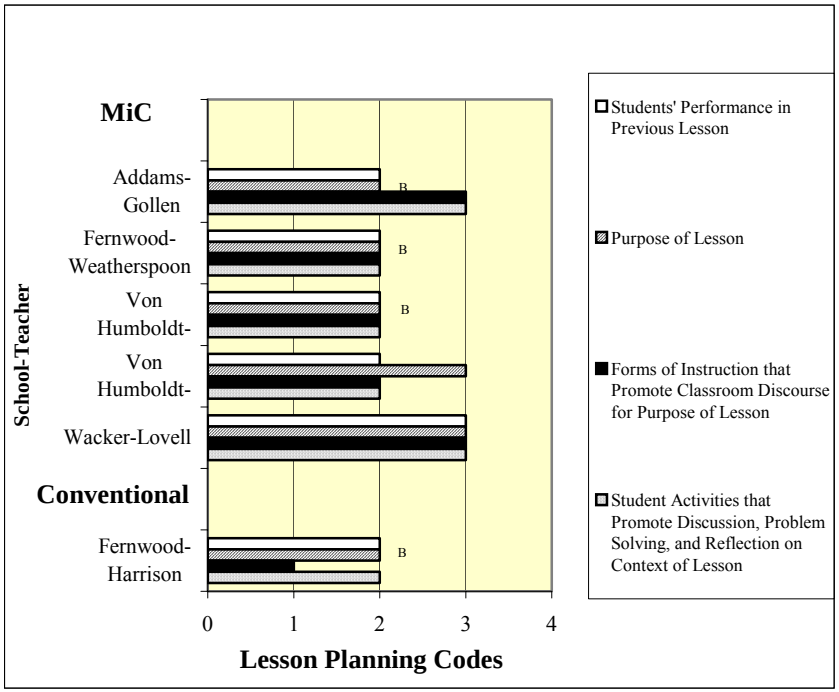
Two student activities were important elements in Harrison's instruction, practicing computation, used on 73% of the reported days, and listening to the teacher or taking notes, used on 60% of the reported days (see Tables F71-F78 in this appendix). Practicing computation was given at least half of the class time on 27% of the days, equal emphasis with other instructional forms on 36% of the days, and less than 15% of class time on 36% of the days. Listening to the teacher or taking notes was given at least half of the class time on 11% of the days, equal emphasis with other student activities on 33% of the days, and less than 15% of class time on 56% of the days. Students often discussed answers and solution strategies (53% of the reported days) and reflected on or summarized lesson concepts (40%). Discussing answers and solution strategies was given equal emphasis with other student activities on 63% of the days and less than 15% of class time on 38% of the days. Reflecting on or summarizing lesson concepts was given at least half of the class period on 17% of the days, equal emphasis with other student activities on 50% of the days, and less than 15% of class time on 33% of the days. On approximately one fourth of the reported days, students investigated problems (27%) and participated in whole-class discussions (27%). Investigating problems was always given equal emphasis with other student activities. Participating in whole-class discussions was given at least half of the class time on 25% of the days, equal emphasis with other student activities on 25% of the days, and less than 15% of class time on 50% of the days. Students seldom began homework (13% of the days) or took a test or quiz (7%; Harrison, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Harrison reported in her teacher logs. As an example of a lesson observed in Harrison's class, the lesson on 11/16/98 included: whole class, housekeeping duties (10 minutes); individual work, 5 warm-up problems (15 minutes); teacher presentation (10 minutes); whole-class teacher-led discussion of warm-up problems (5 minutes); teacher presentation, line graphs (29 minutes); whole-class discussion (16 minutes); and individual activity (4 minutes; Harrison, Observation 12/15/98).

Table F12
Subcategories of Lesson Planning: Harrison, Fernwood Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	1	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the six sixth-grade teachers in District 1 varied in all four subcategories of *lesson planning* (see Figure F2). With respect to *students' performance in the previous lesson*, five teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The other teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. Little variation was seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Four teachers planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.



*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F64 in this appendix.

Figure F2. Lesson planning, Grade 6, District 1.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Two teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. Three teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. One teacher seldom planned forms of instruction that encouraged classroom discourse. The teachers also varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For two of the teachers

investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan. Four teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*. The description for each teacher includes discussion of all six subcategories.

Gollen. The evidence gathered for lesson presentation and development for Gollen ranged from Level 2A to 6. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table F79 in this appendix). To illustrate this rating, a lesson presented by Gollen (Observation 2/4/99) from a class using the MiC sixth-grade number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole & Brendefur, 1998) is described. In this lesson (pp. 13–14), students use segmented bars and non-segmented bars to compare sets of two or three objects. The lesson began with a whole-class discussion of the homework assignment, page 13 question 12, parts a and b. In part a, students were asked to make segmented bars to show the amounts of apple juice in two drinks, one containing 1/4 apple juice and the other containing 3/10 apple juice. In part b), they were to find the difference of the amounts. Students explained their solutions. Some strategies were not questioned to help students clarify their thinking, however. For example, to find the number of shaded segments for 3/10, a student said, "You use 10 divided by 40 and get 4. Then multiply that by 3, and you get 12 segments." Another student said, "Since 40 equals 10, I broke the bar into 40 pieces. I put three segments together and got 12." For part a, some students used 20 segments whereas other students used 40 segments. Gollen emphasized that either one could be used. When a student remarked that he couldn't use 20 as the number of segments, another student immediately said, "But I just did it." Gollen interjected, "Can you draw a picture to show how you did it?" The student went to the board and drew a segmented bar to demonstrate his procedure. The observer noted: "The students did fairly well with the lesson. [Ms. Gollen] struggled. She didn't know how to steer the students in the right direction without telling them. She tended to jump from question to question without closure and with little direction. Sometimes she skipped questions that were relevant to the development of concepts like question 12, part b" (Gollen, Observation 2/4/99). For example, Gollen wanted the students to say that 1/4, 5/20, and 10/40 were equivalent expressions for the same amount of juice. On the board a student had drawn a bar with 20 segments with five segments shaded. Rather than divide each segment to show an equivalent representation of 10/40, she told the students that the fractions represented the same amount. Furthermore, Part b, finding the difference of the amounts, was never discussed with the class. In the next part of the lesson, pairs of students worked to complete p. 14, question 13, in which students were asked to think of three numbers of segments that were useful in representing both 1/4 and 1/6; to select one of those numbers and make segmented bars representing 1/4 and 1/6; and to find the difference between the fraction of students in Canada (1/4) and in the U. S. (1/6) whose favorite fruit was apples. They discussed part a at the end of the class period.

This lesson illustrates a rating of 2 on the composite index for the nature of inquiry during instruction, although an overall rating of 3 for nature of inquiry was assigned for Gollen (see Table F80). In this lesson, Gollen attempted to develop a conceptual basis for comparing and subtracting

fractions. Although the students seemed to make sense of comparing fractions by using segmented bars, Gollen did not attempt to restate or encourage students to restate their strategies in mathematically sound ways. Nor did she use a representation of 5/20 to demonstrate how it could be used to show the equivalent representation for 10/40. Gollen did, however, encourage students to talk about the procedures they used and demonstrate them to the class. In this lesson, students used segmented bars to develop a conceptual basis for comparing fractions, although differences in the amounts were not explicitly discussed. Students' conjectures consisted of making connections between the new problem and ones they had already seen. Connections among mathematical ideas were limited to comparison of fractional amounts, and connections between the mathematics and students' lives were not discussed.

Gollen was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure F10). For Gollen, 19% of the decisions were coded as reflective of good standard pedagogy, 49% were most aligned with teaching for understanding, and 34% were least aligned with teaching for understanding.

Across all observations for Gollen, the mean rating for the index on students' explanations was 2.22, indicating that their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F81). The mean rating for elicitation of multiple strategies was 3.00, indicating that students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction (see Table F82). A rating of 2 was assigned for Gollen on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F13
Overall Ratings for Mathematical Interaction for Gollen, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to develop conceptual understanding
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	2.22	Focus on procedures
Elicitation of Multiple Strategies	3.00	Strategies not primary emphasis
Lesson Reflection, Summary, or Closure	2	Some opportunities

Weatherspoon. The evidence gathered for lesson presentation and development for Weatherspoon ranged from Level 3B to 6. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table F79). To illustrate this rating, a lesson presented by Weatherspoon (Observation 2/22/99) from a class using the MiC sixth-grade number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole & Brendefur, 1998) is described. In this lesson (pp. 36–38), students are introduced to an informal method for calculating a fraction of a fraction or to multiply two common fractions. Using this method, the given fractions are first related to whole numbers in the context. For example, for 1/5 of 2/3 of $30, 2/3 of $30 = $20, and 1/5 of $20 = $4. The class period began with a warm-up activity composed of five ratios for which students were to write a simplified fraction, a decimal, and a percent. When checking the answers, students gave answers and the procedures they used to find their answers. Weatherspoon then introduced the lesson with an activity that she designed. She distributed 20 bags to the students and instructed students not to look into the bags. She told the students that half of the bags had play money in them and 1/5 of those bags had more than $5 in them. She asked what fraction of the bags had more than $5 in them and how the students could find the answer. Students gave suggestions for solving the problem:

- I would count them.
- Take 1/5 of 20.
- First figure out how many have money in them by taking 1/2 of 20 to get 10.
- Only 1/5 of the 10 have more than $5 in them or 1/5 of the 1/2.
- I think it's 2 because 1/5 of 10 is 2.

Weatherspoon then asked students to name the fraction of the whole number of bags that had more than $5 in them. A student answered, "2 out of 20 or 1/10 or 10%." Weatherspoon queried, "What did you just figure out?" A student answered, "1/5 of 1/2 is 1/10." Students then checked the bags to verify this result. Students read p. 36 and worked together to understand an expression in the context of recycling aluminum cans in a park. As a class, they worked through p. 37, question 4, parts a-d, in which they calculated 9/10 of 4/5 of 250 kg of aluminum cans that would be recycled. Weatherspoon led the class through each part of question 4 by asking them what they would do to solve the problem and why they chose that method. As they worked through the question, their conjectures consisted of making connections between the new problem and the teacher-designed activity. Weatherspoon emphasized equivalent representations of numbers, and students expressed them in their answers. For example,

when students changed a decimal to a percent, they stated that percent involved hundredths, and they looked for a number out of 100. Weatherspoon also emphasized the informal method for multiplying fractions introduced in the lesson. By doing so, she provided a conceptual foundation for taking a fraction of a fraction. Multiple strategies were encouraged and valued. For example, when students calculated 4/5 of 250, strategies included using a calculator to divide 5 into 250, then multiply by 4; using a ratio table; or using a bar to represent 250, dividing it into five segments, and finding the amount for four segments. When students worked in groups, Weatherspoon began asking students probing questions, but she had little success. She then used direct questions to specific students.

This lesson illustrates the overall rating of 4 on the composite index for the nature of inquiry during instruction (see Table F80). In this lesson, Weatherspoon continually pressed students to develop a conceptual basis for the mathematical content, and she shared the mathematical work with her students. She also emphasized connections among mathematical ideas in both the warm-up activity and the lesson. Although students were interested in the contexts, particularly the activity with money in bags, connections were not apparent between the mathematics and students' life experiences.

Weatherspoon was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure F11 at the end of this appendix). For Weatherspoon, 65% of the interactive decisions were coded as most aligned with teaching for understanding, 12% were reflective of good standard pedagogy, and 24% were least aligned with teaching for understanding.

Across all observations for Weatherspoon, the mean rating for the index on students' explanations was 2.33, indicating that their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F81). The mean rating for elicitation of multiple strategies was 2.25, indicating that on many occasions, students were not asked if alternate strategies were used in solving particular problems (see Table F82). A rating of 1 was assigned for Weatherspoon on the index for reflection or summary, indicating that limited opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F14
Overall Ratings for Mathematical Interaction for Weatherspoon, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.33	Focus on procedures
Elicitation of Multiple Strategies	2.25	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Brown. The evidence gathered for lesson presentation and development for Brown ranged from Level 1 to 3B. An overall rating of Level 2A was assigned, indicating that a major portion of the lesson was devoted to review of a previous lesson, homework, or warm-up activity (see Table F79 in this appendix). To illustrate this rating, a lesson presented by Brown (Observation 11/9/98) from a class using the MiC sixth-grade algebra unit *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole & Burrill, 1998) is described. During this class period, students were given the option of independently completing pages 7 and 8 from the unit (investigating ways to give change at a checkout counter in a store) or worksheets for practicing division with one- or two-digit divisors. Neither lesson option was discussed at length. Most students chose to complete the worksheets for which they used either a "short division" algorithm or the standard division algorithm. Little interaction occurred between the teacher and students or among students.

In contrast, a lesson presented by Brown (Observation, 10/19/98) from a class using the MiC sixth-grade geometry unit *Reallotment* (Gravemeijer, Clarke, & Pligge, 1998) is used to illustrate a rating of 3B, which indicates that they practiced the procedure or strategy presented by their teacher in a rote fashion. In this lesson (p. 43), students were to investigate the relationship between volume and surface area, beginning with an activity to find as many different sized boxes that could hold 24 cubes. For each box, students were to record the subcategories and surface area of the box. The class began with a five-minute warm-up activity in which students used the formula base × height to calculate the area of parallelograms. Brown then gave students the answers to the questions. He introduced the activity on p. 43, distributed cubes, and asked students to copy the chart of p. 43 into their notebooks. Brown changed the number of cubes in the problem from 24 to 80 and proceeded to present a lesson on calculating volume by using formulas. Surface area was not addressed. Even though students had cubes on their desks, the lesson as presented did not allow for students to use the blocks to think about volume. Brown went to the abstract immediately, telling students the formula length × width × height for calculating volume. Students were not given the opportunity to conjecture about the meaning of the formula or why it worked. Nor were they given the opportunity to explain any alternative methods they might have used. Brown provided all the explanations. The lesson presentation was interspersed with questions directed to particular students, which were phrased to generate very specific responses. Brown had clear expectations for students to use formulas, and most students followed his directions. During the last five minutes of class, Brown asked students for the subcategories of the boxes that had a volume of 80 cubic units. He wrote their responses on the board: 10 × 4 x 2; 2 × 4 × 10; 1 × 2 × 40; 1 × 4 × 20; and 20 × 4 ×1. At his point, Brown asked if anyone saw a pattern in the combinations. A student replied that the subcategories could be multiplied together to determine the volume. Brown accepted the answer by saying, "Right." No further elaboration occurred. For homework, Brown assigned a worksheet from a conventional textbook for which students were to calculate volumes of various rectangular prisms.

This lesson illustrates the overall rating of 1 on the composite index for the nature of inquiry during instruction (see Table F80). In this example, the lesson was changed from an investigation of the relationship between volume and surface area to one in which students simply used a formula for calculating the volumes of rectangular prisms. The lesson did not promote conceptual understanding. Brown presented a procedure and the students followed it in a rote manner. Students were not given opportunities to make conjectures about mathematical ideas. The change in the lesson eliminated opportunities for students to think about relationships between volume and surface area. The lesson did not promote connections between mathematics and students' life experiences.

Brown was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F12 at the end of this appendix). For Brown, 75% of the interactive decisions were coded as least aligned with teaching for understanding, 16% were reflective of good standard pedagogy, and 8% were most aligned with teaching for understanding.

Across all observations for Brown, the mean rating for the index on students' explanations was 1.44, indicating that at times students stated answers and were not expected to elaborate on their reasoning or solution strategies, and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or solution strategies (see Table F81). The mean rating for elicitation of multiple strategies was 1.33, indicating that multiple strategies were generally not elicited from students (see Table F82). A rating of 1 was assigned for Brown on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F15
Overall Ratings for Mathematical Interaction for Brown, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	2A	Emphasis on review
Nature of Inquiry	1	Limited to lower order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.44	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.33	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Parsons. The evidence gathered for lesson presentation and development for Parsons were all coded at 2B. An overall rating of Level 2B was assigned, indicating that there was an emphasis on review and that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table F79). To illustrate this rating, a lesson presented by Parsons (Observation 11/11/98) from a class using the MiC sixth-grade algebra unit *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole & Burrill, 1998) is described. In this lesson (pp. 4–5), students were to describe calculations used to complete a table using arrow language and to calculate the change over several years in the area of an island. The class period began with a warm-up activity in which students used arrow language to solve the following: 392 + 46; 34 × 6; and 5 × 2 × 8. The class then reviewed homework, which was question 10 on p. 3. Students used arrow language to find the depth of snow after several days of melting and snowing. A student wrote an arrow string for this calculation on the board. Parsons, however, completely ignored the student's work and wrote her own arrow string at the overhead projector. Parsons then assigned the day's homework assignment, p. 6 questions 19–22. Parsons introduced the context on p. 4, airline reservations and the need for waiting lists based on seats requested and cancellations. Parsons did not seem to understand the problem. Rather than focus on the mathematics and the reasons the context was used, she searched for a procedure for students to use. As the students began to solve the problems on p. 4, they stated answers or wrote them on the board. Parsons gave the explanations for the answers or she ignored student answers and provided her own. She wanted students to use arrow language only when she directed them to

do so. Students were confused, but they did not ask meaningful questions to help clarify the situation. Parsons changed the homework assignment to p. 5–6, questions 15–18.

This lesson illustrates the rating of 2 on the composite index for the nature of inquiry during lessons (see Table F80). In this lesson, Parsons preferred to provide students with a procedure to use, but because she did not understand the questions, students were often left in a state of confusion. Even so, their conjectures consisted of connecting new problems with ones they already solved. For example, they solved the snowfall problem in the same way they solved previous problems. Conceptual understanding was promoted in the lesson as students needed refer to the context to keep track of their calculations. Connections between mathematical ideas were not explored in the lesson, but connections between the mathematics and students' life experiences were apparent.

Parsons was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F13 at the end of this appendix). For Parsons, 79% of the interactive decisions were coded as least aligned with teaching for understanding, 14% were reflective of good standard pedagogy, and 7% were most aligned with teaching for understanding.

Across all observations for Parsons, the mean rating for the index on students' explanations was 1.33, indicating that their explanations were focused on answers only rather than on procedures or elaboration of reasoning (see Table F81). The mean rating for elicitation of multiple strategies was 1.22, indicating that multiple strategies were not elicited from students (see Table F82). A rating of 1 was assigned for Parsons on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F16
Overall Ratings for Mathematical Interaction for Parsons, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.33	Answers only
Elicitation of Multiple Strategies	1.22	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Lovell. All evidence gathered for lesson presentation and development for Lovell was rated at Level 3B, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table F79). To illustrate this rating, a lesson presented by Lovell (Observation 3/25/99) from a class using the MiC sixth-grade number unit *More or Less* (Keijzer et al., 1998) is described. In this lesson (pp. 6–7, students were asked to check the accuracy of given total prices using estimation and to describe ways to calculate total prices for produce

purchases with and without a calculator. The class period began with a warm-up called Sharp Skills, which consisted of finding answers for 2/4 of 100; 1/2 + 2/8; and 4 × 0.02. When reviewing the answers, Lovell said, "For the first one, I was hoping you could do this one in your head. 2/4 is 1/2 and 1/2 of 100 is 50. How many people got it right?" On the board she wrote: 2/4 of 100 = 50% or 1/2, and continued to state the answers for the other two items. As a class, the students read p. 6. Lovell talked about the lesson context and relayed a story about an incident in which she had estimated a total that the cashier calculated incorrectly. Question 12 was assigned for work in small groups: Estimate to determine which of five prices was correct, given the price per kilogram, the number of kilograms, and a total price. Lovell said, "I had trouble with this. Get in groups and work on it together. Don't multiply. Estimate. Make sure you tell why you think the prices are right or not." During subsequent whole-class discussion, students explained their responses. For example, given $1.50/kg, 1.250 kg, and $1.88 total price, one student said, "$1.50 rounds to $2.00 and 1.250 kg rounds to 1, so I get $2.00. $1.88 [the given price] is close to $2.00. That price is OK." Given $1.85/kg, 0.930 kg, and $17.21, another student said, "You are not even buying a whole kilogram, which is $1.85. They charged too much." Students then read p. 7. Lovell worked question 13 with the class because it involved using arrow language, which she assumed they had forgotten. Students worked on question 14, parts a–f in small groups. Students calculated the prices of different amounts of apples that cost $1.20/kg without a calculator. Some students noticed a relationship with decimals that if 12 kg cost $14.40, then 1.2 kg cost $1.44 just by moving the decimal point. Other students always used the multiplication algorithm. The observer noted, however, that Lovell was not comfortable with students using different strategies for the same question and that connections among the problems were not discussed or emphasized, which could have been done, for example with 4 kg, 0.4 kg, and 0.04 kg of apples. Instead, each question was treated as a separate entity. The observer commented: "[Ms. Lovell] was very directive. She explained to the students how she did the problems before they had a chance to try them or to make conjectures about how to solve them." Although [Ms. Lovell] told the students that they could use any strategy they wanted, she directed them to use her own: "[Ms. Lovell] tended to tell and not discuss. A student would start an explanation, and she usually finished it using her own strategy. [Ms. Lovell] really doesn't listen to the students." The homework assignment was to complete a worksheet from a supplemental resource that was unrelated to the lesson.

This lesson illustrates the overall rating of 4 on the composite index for the nature of inquiry during instruction (see Table F80). The lesson in the unit promoted the development of conceptual understanding of multiplication with decimals and fractions. During this lesson, however, Lovell presented a procedure and encouraged students to use it rather than determine their own strategies. Opportunities for students to develop their own strategies and to conjecture were limited by her tendency to direct them back to her own strategy. The connections between decimals and fractions and between the change in the decimal point in the multiplicand and the product were not made explicit. Each question was treated as a special case. Nevertheless, student conjectures consisted of making connections between the new problem and ones previously completed, and students did come up with solution strategies that differed from their teacher's strategy. Connections between the mathematics and students' life experiences were apparent in the lesson.

Lovell was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F14 at the end of this appendix). For Lovell, 60% of the interactive decisions were coded as least aligned with teaching for understanding, 27% were reflective of good standard pedagogy, and 14% were most aligned with teaching for understanding.

Across all observations for Lovell, the mean rating for the index on students' explanations was 2.20, indicating that their explanations were focused on procedures rather than on elaboration of reasoning or solution strategies (see Table F81). The mean rating for elicitation of multiple strategies was 1.40, indicating that multiple strategies were rarely elicited from students (see Table F82). A rating of 1 was assigned for Lovell on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F17
Overall Ratings for Mathematical Interaction for Lovell, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of procedure or strategy
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	2.20	Focus on procedures
Elicitation of Multiple Strategies	1.40	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Harrison. Evidence gathered for lesson presentation and development for Harrison ranged from ratings of Level 2A to Level 3B. An overall rating of 2B was assigned, indicating that there was an emphasis on review and that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table F79). In this lesson, students reviewed how to convert mixed numbers into improper fractions and studied multiplication of mixed numbers. The class period began with a 30-minute session on completing and checking a warm-up called the Daily Review. Harrison then asked students for answers to the homework assignment. Students stated only the answers. If an answer was incorrect, Harrison asked another student or she provided the correct answer, but no further elaboration occurred. Students were never asked if their answers were reasonable. After reviewing changing a mixed number into an improper fraction, a student inquired whether 4 4/3 is changed to 4/4. Harrison replied that she would come back to that question later in the class period. However, she did not follow up on the student's question. Harrison then introduced the lesson to the class by stating all the rules and procedures for multiplying mixed numbers. Occasionally she asked students to provide the next step in a particular procedure. Harrison encouraged students to simplify improper fractions before multiplying, but she did suggest that students could multiply first and then simplify. Half the points for an answer were reserved for simplifying the answer. Students were asked to complete 38 problems in which they multiplied mixed numbers.

This lesson illustrates the overall rating of 1 on the composite index for the nature of inquiry during instruction (see Table F80). In this lesson, students practiced the procedure presented by their teacher in a rote fashion. Harrison sought correct answers, with an occasional question about steps in a procedure. Students were not given opportunities to conjecture about mathematical ideas, nor did the lesson promote conceptual understanding. Connections among mathematical ideas were not explored, and connections between the mathematics and students' lives were not made apparent.

Harrison was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F15 at the end of this appendix). For Harrison, 78% of the interactive decisions were coded as least aligned with teaching for understanding, and 24% were reflective of good standard pedagogy.

Across all observations for Harrison, the mean rating for the index on students' explanations was 1.22, indicating that their explanations were focused on answers only rather than on procedures or elaboration of reasoning (see Table F81). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table F82). A rating of 1 was assigned for Harrison on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F18
Overall Ratings for Mathematical Interaction for Harrison, District 1, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	1	Limited to lower order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.22	Answers only
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the six Grade 6 teachers in District 1 varied from Level 2A to Level 5 in *lesson presentation and development* (see Table F19). One MiC teacher was assigned Level 5, indicating that the lesson featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. One MiC teacher was assigned Level 4, indicating that attempts were made to develop a conceptual basis for the mathematical content. One MiC teacher was assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. One MiC teacher was assigned Level 2A, indicating that a major portion of the lesson was devoted to review of a previous lesson, homework, or warm-up activity. The remaining two teachers one MiC) were assigned Level 2B, indicating that there was an emphasis on review and that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident.

Table F19
Nature of Mathematical Interaction, Grade 6 Teachers in District 1, 1998-1999

School-Teacher	Lesson Presentation and Development	Nature of Inquiry	Teachers' Interactive Decisions	Nature of Student Explanations	Elicitation of Multiple Strategies	Lesson Closure, Reflection, or Summary	Total
			— MiC —				
Addams-Gollen	4	3	3	2.22	3.00	2	17.22
Fernwood-Weatherspoon	5	4	5	2.38	2.25	1	19.63
Von Humboldt-Brown	2A	1	1	1.44	1.33	1	7.77
Von Humboldt-Parsons	2B	2	1	1.33	1.22	1	8.55
Wacker-Lovell	3B	4	1	2.20	1.40	1	12.60
			— Conventional —				
Fernwood-Harrison	2B	1	1	1.22	1.00	1	7.22

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 4. Two MiC teachers were assigned Level 4, indicating that in-depth exploration of mathematics occurred during lessons. One MiC teacher was assigned Level 3, indicating that attention was given to conceptual understanding during instruction. One MiC teacher was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction. The remaining two teachers (one MiC) were assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to *teachers' interactive decisions*, the teachers varied from Level 1 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding. One MiC teacher was assigned Level 3, indicating that her interactive decisions provided greater attention to good standard pedagogy. The four remaining teachers (three MiC) were assigned Level 1, indicating that their interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.22 to 2.38. Student explanations in the classes of three MiC teachers were focused on procedures. Student explanations in the classes of three teachers (two MiC) were limited to answers only. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 3.00. In the classes of one MiC teacher, students were asked if alternate strategies were used in solving particular problems, but this was not a primary emphasis during lessons. One MiC teacher rarely elicited multiple strategies. The remaining four teachers (three MiC) did not elicit multiple strategies from their students. With respect to *lesson reflection, summary, or closure*, one MiC teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons. The remaining five teachers (four MiC) provided few, if any, opportunities for students to reflect on the mathematics they were learning.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The sixth-grade teachers in District 1 varied in all three subcategories of *classroom assessment* (see Tables F20-F23). With respect to the *evidence sought* during classroom assessment, One MiC teacher maintained a conservative process orientation, and one MiC teacher had an underdeveloped process orientation. One MiC teacher sought procedural competence of student learning. Four teachers (two MiC) sought little evidence of student learning. *Feedback* in the classes of two MiC teachers was characterized as emerging shared responsibility for feedback among teacher and students. Feedback in the classes of four teachers (three MiC) was very teacher-directed. Two MiC teachers provided mixed, superficial feedback to their students. Two MiC teachers provided low-level closed feedback. The remaining teachers (one MiC) provided feedback that allowed students to know whether answers were correct or incorrect.

Table F20
Evidence Sought through Classroom Assessment Practice, Grade 6 Teachers, District 1

	Rating	Description
MiC		
Addams-Gollen	3	Underdeveloped process orientation
Fernwood-Weatherspoon	4	Conservative process orientation
Von Humboldt-Brown	1	Limited evidence
Von Humboldt-Parsons	1	Limited evidence
Wacker-Loevll	2	Procedural competence
Conventional		
Fernwood-Harrison	1	Limited evidence

Table F21
Feedback Coherence and Purpose, Grade 6 Teachers, District 1

	Rating	Description
MiC		
Addams-Gollen	3	Emerging shared responsibility
Fernwood-Weatherspoon	3	Emerging shared responsibility
Von Humboldt-Brown	2	Teacher-directed feedback
Von Humboldt-Parsons	2	Teacher-directed feedback
Wacker-Loevll	2	Teacher-directed feedback
Conventional		
Fernwood-Harrison	2	Teacher-directed feedback

Table F22
Feedback Content, Grade 6 Teachers, District 1

	Rating	Description
MiC		
Addams-Gollen	4	Mixed, superficial feedback
Fernwood-Weatherspoon	4	Mixed, superficial feedback
Von Humboldt-Brown	3	Low-level closed feedback
Von Humboldt-Parsons	2	Answer-only feedback
Wacker-Loevll	3	Low-level closed feedback
Conventional		
Fernwood-Harrison	2	Answer-only feedback

Table F23
Classroom Assessment, Grade 6 Teachers, District 1

School-Teacher	Evidence Sought	Feedback Coherence and Purpose	Feedback Content	Total
		— MiC —		
Addams-Gollen	3	3	4	10
Fernwood-Weatherspoon	4	3	4	11
Von Humboldt-Brown	1	2	3	6
Von Humboldt-Parsons	1	2	2	5
Wacker-Lovell	2	2	3	7
		— Conventional —		
Fernwood-Harrison	1	2	2	5

See index next page.

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Gollen ranged from Level 1 to Level 3 (see Table F83 in this appendix). While the mean rating across observations was 2.44, during four of the nine observations, student–student conversation was rated at Level 4, indicating that student–student conversation was substantive. During four other observations, however, student–student conversation was rated at Level 1, indicating that student–student conversation was not encouraged. To illustrate a rating at Level 4, student–student conversations that occurred during a lesson by Gollen are described. The observer noted, "When the students worked with a partner, the conversation was mathematical and substantive. Most students discussed how they arrived at a solution and then discussed alternative strategies that they could have used" (Gollen, Observation 5/10/99). In this example, student–student conversation was substantive and was characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand them and build discussion around them. To illustrate a rating at Level 1, student–student conversations that occurred during a different lesson by Gollen are described. The observer noted, "There were no exchanges between peers in small groups or as a part of the large group. Students talked only with the teacher" (Gollen, Observation 4/27/99). In this example, students conversed with the teacher; student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Weatherspoon ranged from Level 1 to Level 4 (see Table F83 in this appendix). While the mean rating across observations was 1.78, on five of the nine observations student–student conversation was rated at Level 1, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Weatherspoon are described. The observer noted, "The students do not talk to each other about mathematics. They talk to the teacher" (Weatherspoon, Observation 5/24/99). In this example, students conversed with the teacher; student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Brown ranged from Level 1 to Level 3 (see Table F83 in this appendix). The mean rating across observations was 1.22, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Brown are described. The observer noted, "Students did not discuss the mathematics with their peers even though they were seated in pairs" (Brown, Observation 12/10/98). In this example, student–student conversation did not exist.

The index ratings about the nature of student–student conversation for Parsons were all Level 1 (see Table F83 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Parsons are described. The observer noted, "Students did not share mathematical ideas with their peers. All interaction between

peers was social" (Parsons, Observation 3/26/99). In this example, student–student conversations were not about mathematics, but were social in nature.

The index ratings about the nature of student–student conversation for Lovell ranged from Level 1 to Level 4 (see Table F83 in this appendix). The mean rating across observations was 2.60, indicating that student–student conversation was frequent but was not substantive in nature. To illustrate a rating at Level 3, student–student conversations that occurred during a lesson by Lovell are described. The observer noted, "Most students did not discuss the mathematical strategies or content although they worked on the questions sitting in groups. They usually accepted one member's answer" (Lovell, Observation 3/25/99). In this example, student–student conversation was not substantive in nature. Students accepted another student's answer without an explanation of how it was found.

The index ratings about the nature of student–student conversation for Harrison were all at Level 1 (see Table F83 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Harrison are described. The observer noted, "Students frequently interact socially but never talk about mathematics" (Harrison, Observation 12/15/98). In this example, student–student conversations were not about mathematics, but were social in nature.

In summary, the mean ratings for *student–student conversation* for the six Grade 6 teachers in District 1 varied from 1.00 to 2.60. One MiC teacher had a mean rating near 3.00, indicating that student–student conversation occurred frequently, but was not substantive in nature. Although two MiC teachers had mean ratings close to 2.00, the majority of observations for one of them were rated Level 1, indicating that conversation among students was not encouraged or did not exist. The mean rating for the other MiC teacher represented an equal number of observations rated Level 1 and Level 4, indicating that on some occasions student exchanges were substantive, reciprocal, and extended the discussion to a new level, and on other occasions, conversation among students was not encouraged or did not exist. Mean ratings for three teachers (two MiC) were at or close to 1.00, indicating that conversation among students was not encouraged or did not occur.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Gollen ranged from Level 1 to Level 4 (see Table F84 in this appendix). Although the mean rating across observations was 2.11, on five of the nine observations students' collaboration was rated at Level 1, indicating that none of the students worked together in small-group or large-group settings, or if they worked in small groups, one student typically gave answers to other members of the group without explanation. On the remaining observations, students' collaboration was rated at Levels 3 or 4, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Gollen is described. The observer noted, "None of the students worked in small groups or with a partner. Students did not share strategies or solutions with each other" (Gollen, Observation 4/27/99). In this example, none of the students worked together in small groups or in a large-group setting; they did not work collaboratively to solve the problems. To illustrate a rating for students' collaboration at Level 3, student collaboration that occurred during another lesson by Gollen is described. The observer notes, "Some students depended on others for answers to problems but many worked on the

solutions together. Usually the contributions to solving the problems was not made equally by each student" (Gollen, Observation 5/10/99). In this example, some students exchanged ideas, but the contributions to solving problems were not made equally by all students.

The index ratings about the nature of students' collaboration in the classroom for Weatherspoon ranged from Level 1 to Level 3 (see Table F84 in this appendix). The mean rating across observations was 1.89, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Weatherspoon is described. The observer noted, "Even though most students were seated in pairs, they did not really discuss how to solve the problem or share any ideas about the problems" (Weatherspoon, Observation 10/19/98). In this example, although students physically sat together, they did not work collaboratively to solve the problems.

The index ratings about the nature of students' collaboration in the classroom for Brown ranged from Level 1 to Level 2 (see Table F84 in this appendix). The mean rating across observations was 1.13, indicating that none of the students worked together in small or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Brown is described. The observer noted, "Occasionally one student would give another student an answer, but they were not working together collaboratively by sharing strategies or solution paths" (Brown, Observation 2/22/99). In this example, students shared only answers; they did not work collaboratively to solve the problems.

The index ratings about the nature of students' collaboration in the classroom for Parsons were all Level 1 (see Table F84 in this appendix), indicating that none of the students worked together in small or large-group settings, or if they worked in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Parsons is described. The observer noted, "Students were seated in groups but they did not work with each other on assigned tasks" (Parsons, Observation 1/20/99). In this example, students sat in groups, but they did not work collaboratively to solve the problems.

The index ratings about the nature of students' collaboration in the classroom for Lovell ranged from Level 1 to Level 4 (see Table F84 in this appendix). The mean rating across observations was 3.00, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. To illustrate a rating for students' collaboration at Level 3, student collaboration that occurred during a lesson by Lovell is described. The observer noted, "When students are forced to talk to each other, they will discuss the solutions and strategies. Otherwise, they do not. Most students just rely on others in the groups to answer the questions" (Lovell, Observation 3/25/99). In this example, some students exchanged ideas, but the contributions to solving problems were not made equally by all students.

The index ratings about the nature of students' collaboration in the classroom for Harrison were all Level 1 (see Table F84 in this appendix), indicating that none of the students worked together in small or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Harrison is described. The observer noted, "Students were seated in groups but no one worked with anyone else.

Some students copied answers given to them by another member of the group" (Harrison, Observation 1/12/99). In this example, students shared only answers; they did not work collaboratively to solve the problems.

In summary, the mean ratings for *student collaboration* for the six Grade 6 teachers in District 1 varied from 1.00 to 3.00. One MiC teacher had a mean rating at 3.00, indicating that some students exchanged ideas or providing peer assistance. One MiC teacher had a mean rating greater than 2.00, but on the majority of observations students' collaboration was rated at Level 1, indicating that none of the students worked together in small or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation. On the remaining observations for this teacher, students' collaboration was rated at Level 3 and 4, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. One MiC teacher had a mean rating near 2.00, indicating that few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems at different paces. Three teachers (two MiC) had mean ratings at or near 1.00, indicating that none of the students worked together in small-group or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Gollen ranged from Level 2 to Level 4 (see Table F85 in this appendix). The mean rating across observations was 3.67, indicating that most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Gollen is described. The observer noted, "Most students worked on the tasks diligently the entire time. They were serious about the work and were trying hard" (Gollen, Observation 5/10/99). In this example, most students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Weatherspoon ranged from Level 2 to Level 4 (see Table F85 in this appendix). The mean rating across observations was 3.22, indicating that student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Weatherspoon is described. The observer noted, "Most students were working on the lesson seriously for about 30 minutes. The rest of the time [Weatherspoon] had to keep pushing them to stay on task" (Weatherspoon, Observation 1/19/99). In this example, most students were engaged in class activities, but this engagement was inconsistent and depended on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Brown ranged from Level 2 to Level 4 (see Table F85 in this appendix). The mean rating across observations was 2.89, indicating that student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Brown is described. The observer noted, "Most students did the division problems on the worksheet the whole period. Some students did very little work, but they were quiet and well behaved. One student threw eraser pieces several times during the period" (Brown, Observation 11/9/98). In this example, most students engaged in class activities, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Parsons ranged from Level 1 to Level 3 (see Table F85 in this appendix). Although the mean rating across observations was 1.56, on six of the nine observations student engagement during instruction was rated at Level 1, indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. To illustrate a rating for student engagement at Level 1, students' on-task behavior that occurred during a lesson by Parsons is described. The observer noted, "Most students were disruptive and off task for most of the period. A couple of students were very quiet and passive" (Parsons, Observation 11/11/98). In this example, students were frequently off task, as evidenced by serious disruptions.

The index ratings about the extent to which students were engaged during the lesson for Lovell ranged from Level 3 to Level 4 (see Table F85 in this appendix). The mean rating across observations was 3.80, indicating that most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Lovell is described. The observer noted, "Most students during the class were on the task assigned most of the time. They took their work seriously and tried hard to complete the problems and to understand strategies that others developed" (Lovell, Observation 4/15/99). In this example, most students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Harrison were all Level 1 (see Table F85 in this appendix), indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. To illustrate a rating for student engagement at Level 1, students' on-task behavior that occurred during a lesson by Harrison is described. The observer noted, "Students were very restless or bored. Some had their heads down. Some were talking loudly, walking around the room, and making noise. No one was doing much work" (Harrison, Observation 9/29/98). In this example, students were frequently off task during the lesson.

In summary, the mean ratings for *student engagement in lessons* for the six Grade 6 teachers in District 1 varied from 1.00 to 3.80. Two MiC teachers had mean ratings near 4.00, indicating that student engagement was widespread with students on task pursuing the substance of the lesson most of the time. Two MiC teachers had mean ratings near 3.00, indicating that most students were engaged in class activities some of the time, but this engagement was inconsistent and dependent on prodding from the teacher. One MiC teacher had a mean rating greater than 1.00, but on the majority of observations, student engagement was rated at Level 1, indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. The remaining teacher, using a conventional curriculum, also had a mean rating of 1.00.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table F24
Teacher level of Instruction, Grade 6, District 1

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE		
— MiC —																
Addams-Gollen	3.75	3.75	4	3	3	3.70	3.75	3.33	3	3	4	3.05	2.64	4.59	48.56	**5**
Fernwood-Weatherspoon	2.5	2.5	5	4	5	3.97	2.81	1.67	4	3	4	2.23	2.36	4.03	47.07	**5**
Von Humboldt-Brown	2.5	2.5	2	1	1	2.40	1.66	1.67	1	2	3	1.53	1.41	3.61	27.28	**1**
Von Humboldt-Parsons	2.5	2.5	2	2	1	2.22	1.53	1.67	1	2	2	1.25	1.25	1.95	24.87	**1**
Wacker-Lovell	3.75	3.75	3	4	1	3.67	1.75	1.67	2	2	3	3.25	3.75	4.75	41.34	**4**
— Conventional —																
Fernwood-Harrison	1.25	2.5	2	1	1	2.03	1.25	1.67	1	2	2	1.25	1.25	1.25	21.45	**1**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

District 2

In District 2, six Grade 6 teachers participated in the study. Four teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Unit Planning

In this study, three subcategories characterized *unit planning*: consideration of *students' prior knowledge, unit sequence*, and *pace of instruction*.

Broughton generally followed the sequence of MiC units as suggested by the mathematics department in her school. The dates and content of district standardized tests as well as the department's assessment of unit difficulty were factors that influenced the sequence of units. When planning to teach a MiC unit, Broughton worked through the first two sections, making sure she understood the problems: "I know that if I don't understand it very well, when [the students] need help, I won't be able to help them" (Broughton, Interview 4/12/99). Broughton relied on her own judgment when setting the pace for instruction. She anticipated that she would have to adjust the pace many times during a unit based on student performance. When planning to teach a MiC unit, Broughton was more influenced by the students' prior experience with MiC than with their prior knowledge of mathematical concepts or skills. However, as the unit progressed and students' lack of conceptual understanding or skill development emerged, Broughton added minilessons, supplemental worksheets for skill practice, and homework. Broughton realized that students would have multiple opportunities to learn difficult concepts in MiC by revisiting the concepts throughout the curriculum. Although she thought the state standardized tests reflected MiC content, Broughton (and the mathematics department at her school) emphasized test preparation in her plans (Broughton, Interview 4/12/99).

Table F25
Subcategories of Unit Planning: Broughton, Guggenheim Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Redling generally followed the sequence of MiC units as suggested by the mathematics department in her school. The dates and content of district standardized tests as well as the department's assessment of unit difficulty were factors that influenced the sequence of units. When planning to teach a MiC unit, Redling followed the suggestions in the teacher guide. She looked through the entire unit to get a general impression of the content and the suggested pace for instruction. Redling used her best judgment in determining the number of pages students might complete in a day and problems that could be assigned for homework: "I try and cover at least three to four pages a day. I try to push myself because I find that if we don't push ourselves, it takes a long time to complete the unit" (Redling, Interview 4/12/99). She planned self-correcting worksheets for times she had to be out of the classroom. Redling thought that planning to teach a

MiC unit took more time than planning to teach a unit from a conventional textbook: "I have to really look through the questions. I have to work out the questions myself. It's not one little objective a day like in the old textbook, one skill per day. This is high-level work using thinking skills. So it takes a lot more planning for a teacher" (Redling, Interview 4/12/99). Redling thought MiC was well aligned with the state and district guidelines. However, she thought MiC was not aligned with district standardized tests, so she planned to review basic skills for about two weeks before the test was administered (Redling, Interview 4/12/99).

Table F26
Subcategories of Unit Planning: Redling, Guggenheim Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Ferguson followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read the objective of the unit, identified materials she needed to prepare, and worked out the problems, determining those to include when teaching and those to skip. She commented:

> I have to go through and do the problems, read the questions, and make sure I understand what they're asking. Half of the questions I don't understand what they're saying. So I have to ask someone else, "If my kids ask this, what am I supposed to do?" I have to go through and make sure that for every possible question they could ask, I know the answer. (Ferguson, Interview 5/10/99)

Whenever Ferguson noticed that students did not have the needed prior knowledge, either before she planned the unit or while she was teaching the unit, she taught a minilesson on the missing concepts or skills. Ferguson thought MiC was well aligned with the state standards and the district curriculum guide. However, she thought she needed to teach specific skills and concepts, other than those in MiC, to prepare students for the district standardized tests (Ferguson, Interview 5/10/99).

Table F27
Subcategories of Unit Planning: Ferguson, Weir Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from text sequence
Pace of Instruction	1B	Little or no consideration of pacing

Kellner followed the recommended sequence of MiC units. When planning to teach a MiC unit, she first determined her students' prior knowledge by looking at the mathematics content students had studied the previous year, the content students had studied during the present year, and/or by giving a quiz or some type of informal evaluation to see what students had already mastered. This information helped Kellner plan for the kind of instruction, the amount of direct teaching, the length of unit introduction, and membership of small groups. As Kellner identified the concepts presented in the unit, she compared them with those listed in the district curriculum guidelines, finding that they usually matched. Kellner also listed all of the materials she needed to have on hand for successful implementation of the unit. She set the pace for instruction as she taught the unit: "It varies. It depends on what we're doing. I try to cover a section of the unit in a week. That can be anywhere from three to five pages in a week" (Kellner, Interview 5/10/99). For supplementary material, Kellner used a general mathematics textbook. She thought the new state testing reflected MiC content (Kellner, Interview 5/10/99).

Table F28
Subcategories of Unit Planning: Kellner, Weir Middle School, District 1, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from text sequence
Pace of Instruction	2	Adjustment anticipated

Renlund chose and sequenced the chapters in the two conventional mathematics textbooks she used. She chose the topics sixth-grade students would find on the state standardized test: whole numbers, decimals, fractions, measurement and geometry. When planning to teach a unit from the conventional mathematics textbooks, Renlund did not consider students' prior knowledge because "as a whole, most of the kids come from elementary school. They've had very little exposure to those chapters or sections any way" (Renlund, Interview 5/11/99). Renlund estimated the pace for instruction, adjusting it weekly, depending on how well students comprehended the material. She planned to use chapters from both books for each topic, along with teacher support materials from the publishers (Renlund, Interview 5/11/99).

Table F29
Subcategories of Unit Planning: Renlund, Newberry Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Friedman chose and sequenced chapters in the conventional mathematics textbook she used in order to coincide with the district curriculum guide. She commented:

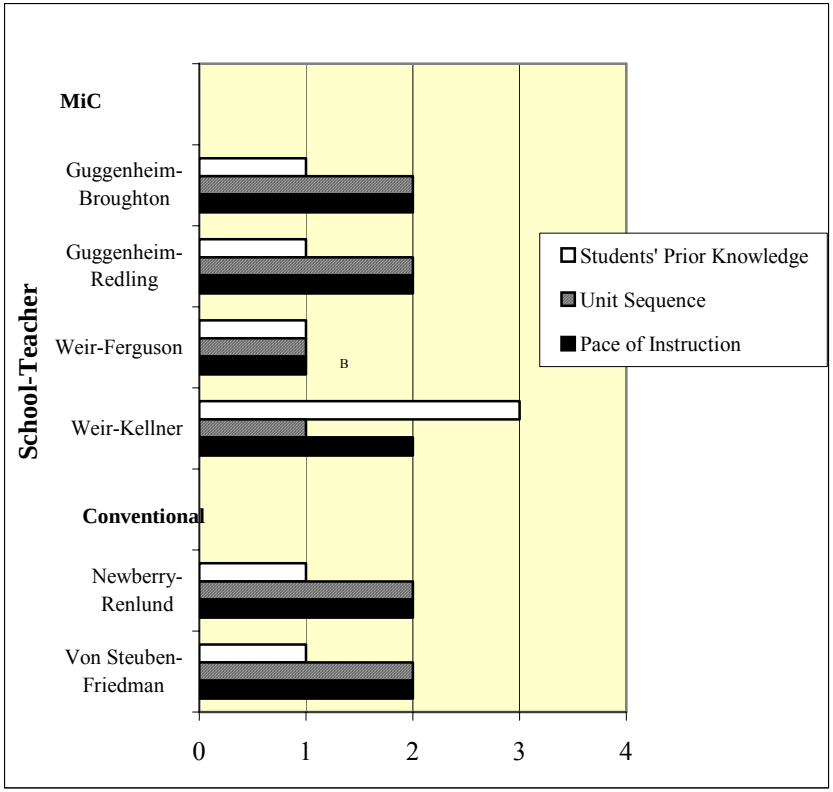
> I plan according to the curriculum guide. For example, I'm going to spend a month doing decimals. I'm going to spend a month doing fractions. I'm going to spend a month doing percents. So I do not necessarily go chapter by chapter in the book. I decide what [students] need to know from beginning to end, and whatever chapter the content is in, then I use that chapter. (Friedman, Interview 6/15/99)

Friedman planned to use a variety of resources about 60% of the time and the adopted textbook for the remaining time. She planned to adjust the pace of instruction according to the needs of the students (Friedman, Interview 6/15/99).

Table F30
Subcategories of Unit Planning: Friedman, Von Steuben Elementary School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the six sixth-grade teachers in District 2 varied minimally in all three subcategories of *unit planning* (see Figure F3). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. The other five teachers followed the unit development in the teacher guide with little or no consideration of students' prior knowledge when planning to teach a unit.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure F3. Unit planning, Grade 6, District 2.

Four teachers considered external factors such as the content of the statewide testing program when *sequencing instructional units*. Two teachers used the sequence of units recommended in teacher support materials. Most of the teachers, five, anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of instruction characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

When planning to teach individual MiC lessons, Broughton usually worked through all of the problems in the lesson and read the teacher guide to check whether she had given the intended meaning to the problems. When there was a discrepancy in interpretation, she planned to include hints from the teacher guide during instruction. Broughton altered her daily lesson plans to accommodate students' performance in the previous lesson, slowing the pace if students needed more time to work through an activity, speeding up the pace if students were ready to move on, or reviewing concepts or skills that were difficult for students. Broughton valued small-group work when students listened to each other, stayed on topic, and persistently worked through the problem. Some students did not want to work in groups because they felt distracted. Consequently, Broughton often chose whole-class direct teaching and problem solving or whole-class direct teaching for the introduction followed by students working independently to solve the problems (Broughton, Interview 4/12/99).

Broughton used the instructional format small-group work on 43% of the reported days; (see Tables F88-F93 in this appendix). Small-group work was given at least half of the class time on 49% of the days and equal emphasis with other instructional forms on 49% of the days. Broughton used three forms of instruction on approximately one third of the reported days: teacher presentation (35%), whole-class discussion (35%), and warm-up activities (34%). The class time given to each varied, however. Teacher presentation was given equal emphasis with other instructional forms on 53% of the days and less than 15% of class time on 47% of the days. Whole-class discussion was given equal emphasis with other instructional forms on 37% of the days and less than 15% of class time on 63% of the days. Warm-up activities were given less than 15% of class time on 97% of the days. On approximately one fourth of the days, Broughton used review of previous material (26%) and independent practice (23%). Independent practice was frequently given equal emphasis with other instructional forms (75% of the days). Review of previous material was generally given less than 15% of class time (Broughton, Teacher Log 1998-99).

Broughton's students most often investigated problems (44% of the reported days; see Tables F94-F101 in this appendix) and engaged in this activity for significant amounts of time. Investigation of problems was given at least half of the class time on 37% of the days, equal emphasis with other student activities on 45% of the days, and less than 15% of class time on 18% of the days. Students in Broughton's classes discussed answers and solution strategies (31% of the reported days), practiced computation (23% of the reported days), listened to the teacher or took notes (21% of the reported days), and participated in whole-class discussions (20% of the reported days). The amount of class time given these activities varied widely. Practicing computation was frequently given equal emphasis with other student activities. Discussion of answers and solution strategies was given equal emphasis with other student activities on 56% of the days and less than 15% of class time on 41% of the days. Participation in whole-class discussions was given equal emphasis with other student activities on 53% of the days and less than 15% of class time on 47% of the days. Listening to the teacher or taking notes was given equal emphasis with other student activities on 39% of the days and less than 15% of class time on 61% of the days. Broughton's students seldom took tests or quizzes (10% of the days) or reflected on or summarized lesson concepts (1%). Students did not begin homework during class time (Broughton, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Broughton reported in her teacher logs. As an example of a lesson observed in Broughton's class, the lesson on 1/11/99 included: whole-class housekeeping duties, including students handing in homework (16 minutes); whole-class teacher-led review of fractions as decimals, multiplying and reading decimals, decimals and money (19 minutes); teacher presentation, page 8 (6 minutes); individual seatwork, page 8 (30 minutes); teacher presentation, page 11 (14 minutes); and individual seatwork, page 11 (12 minutes; Broughton, Observation 1/11/99).

Table F31
Subcategories of Lesson Planning: Broughton, Guggenheim Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Redling became familiar with the lesson by working through all of the problems and studying the Hints and Comments section in the teacher guide. She made note of the problems that were difficult for her and of problems she thought would be difficult for students. Redling thought the lessons at the beginning of the units were easier for students, but lessons became progressively more difficult as students moved toward the end of the unit because they required higher level thinking skills. She anticipated that the lessons near the end of the unit might take more time, that students might need probing questions to push their thinking to a higher level, and that students might need more review of the concepts. Redling was especially aware of students' performance on previous lessons and adjusted her lesson plans to reflect the need to slow the pace and add more questioning and review. Redling valued small-group work because "I think [students] gain a lot from each other. They especially gain self-confidence. [Students] feel they can ask each other things and not be afraid. I think it's good for the kids" (Redling, Interview 4/12/99). Students self-selected their small groups unless behavior was problematic. Redling planned whole-class discussions to follow small-group work. Each group reported to the whole class about their approach to problem solving and thinking strategies. Other groups could question the findings and/or processes presented by a particular group (Redling, Interview 4/12/99).

Redling used four instructional forms on approximately half of the reported days, whole-class discussion (54%), small-group work (53%), review of previous material (49%), and independent practice (44%; see Tables F88-F93 in this appendix). Each of these instructional forms was given equal emphasis with other instructional forms on a significant number of days. Teacher presentation, reported on 38% of the days, was given equal emphasis with other instructional forms on 70% of the days and less than 15% of the class time on the remaining days. Redling rarely used warm-up activities (2%; Redling, Teacher Log 1998-99).

On approximately half of the reported days, Redling's students were engaged in three student activities: discussing answers and solution strategies (57%), participating in whole-class discussions (55%), and beginning homework (49%; see Tables F94-F101 in this appendix). These student activities were usually given equal emphasis with other activities. On approximately one third of the reported days, students reflected on or summarized lesson concepts (38%), and this activity was given equal emphasis with other student activities on all of the reported days. Redling's students seldom listened to the teacher or took notes (16% of the days), practiced computation (11%), took a test or quiz (9%), or investigated problems (3%; Redling, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Redling reported in her teacher logs. As an example of a lesson observed in Redling's class, the lesson on 3/23/99 included: whole-class housekeeping duties (14 minutes); whole-class discussion of the Dear Student Letter (4 minutes); small-group activity, read page 1, discuss how to solve problem, write solution on chart paper, be ready to share with class (47 minutes); small-group presentations to the whole class (47 minutes); large-group discussion, different strategies used by small groups to solve problem and different modes used to present solution (6 minutes); and whole class, homework assigned and started (19 minutes; Redling, Observation 3/23/99).

Table F32
Subcategories of Lesson Planning: Redling, Guggenheim Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with the content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Ferguson usually worked through all of the problems in the lesson. She planned one day at a time in order to consider students' performance in the previous lesson. However, she thought she had to move on before all of the students in the heterogeneous class of 40 students had mastered the concepts and/or skills in each lesson. While Ferguson valued small-group work, some students did not. She assigned partners, a capable student with a less capable student, for all unit problems that she felt at least one of the partners could solve. Some pairs worked well together, whereas some students did not want to help each other, were very critical of others, or preferred to work alone. Ferguson changed partners about four times a year. When the lesson involved more difficult or complex problems, Ferguson planned whole-class direct instruction. Students worked individually as she guided them through the lesson. She explained:

> I was doing graphing with them, and I actually had [the students] do it individually as a whole class, because they paid attention to me. I had it on the overhead projector and I was also walking around. When I want to talk to the whole class, when I need to get through a section, I don't want them to do it with their partner. I have them sit facing me, not in groups, so they don't get distracted. We go through it together. Students do the work individually with me. (Ferguson, Interview 5/10/99)

Ferguson tried whole-class discussions, but thought it was difficult to get more than just simple answers from most students even though she asked questions and encouraged explanations of solution strategies (Ferguson, Interview 5/10/99).

As an example of a lesson observed in Ferguson's class, the lesson on 2/9/99 included: whole-class housekeeping duties (20 minutes); whole-class activity, checking homework (10 minutes); whole-class activity, MiC lesson, including correcting previous lesson and discussing new lesson (52 minutes); and teacher presentation of skills lesson (20 minutes; Ferguson, Observation 2/9/99). Teacher Log data for Ferguson were unavailable.

Table F33
Subcategories of Lesson Planning: Ferguson, Weir Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Kellner worked through all of the problems in the lesson so she would be familiar with the purpose of the lesson; to ensure that her presentation of the lesson would be compatible with the teacher guide; to identify needed prerequisite concepts and skills; and to incorporate a review of prerequisites in the lesson introduction. Kellner began each class period with a short quiz on the previous day's lesson and/or the homework assignment. She explained, "I need to know what they are comfortable with and how much introduction of the lesson I have to do. I need to know to what degree I should lecture or to what degree I should just put them to work in pairs or a small group" (Kellner, Interview 5/10/99). Kellner valued small-group work, but felt she had mixed success using it with students:

> Sometimes you have kids who work well together. They've come up with the same or basically the same answers, but they've come up with them differently. If one person didn't understand, other members of the group helped. They're helping each other. [Some students] even want to help other groups. Other times, you have some kids who are lazy and want to piggyback off someone else who's doing the work in the group. They want to copy or cheat or whatever. They have no idea how they got the answer. All they know is they got an answer. So I do think that to a certain degree it's not beneficial for some students to work in

> groups. The teacher must have really good classroom management [skills]. You have to be careful the way you group [students] because children argue. Kids don't know how to debate in a constructive way. Sometimes they tear each other down. [Students] ridicule, and that's not the point. (Kellner, Interview 5/10/99)

Kellner developed a system to impartially choose a member of each group to present the group's finding to the whole class. She explained, "Each student in each group pulls a number when they are working in their groups. When it's time for groups to share, I'll say that I want to hear from the number four's in each group. Then I go down the line and each number four explains how the group solved the problem" (Kellner, Interview 5/10/99). However, Kellner reported that basically the type of explanations the students gave were computational and procedural. Kellner felt more successful with teacher-directed whole-class discussion (Kellner, Interview 5/10/99).

Kellner used four instructional forms on at least half of the reported days: warm-up activities (73%), teacher presentation (67%), review of previous material (60%), and independent practice (50%; see Tables F88-F93 in this appendix). Of these four instructional forms, independent practice was given the most class time: at least half of the class time on 73% of the days and equal emphasis with other instructional forms on the remaining days. Teacher presentation was given equal emphasis with other instructional forms on 80% of the days. Review of previous material was given at least half of the class time on 22% of the days, equal emphasis with other instructional forms on 50% of the days, and less than 15% of the class time on 28% of the days. Warm-up activities were given the least amount of class time, frequently less than 15% of class time. Kellner chose small-group work less often (40% of the reported days), but gave it considerable amounts of class time: at least half of the class time on 50% of the days and equal emphasis with other instructional form on the remaining days. Kellner seldom planned whole-class discussion (13% of the reported days; Kellner, Teacher Log 1998-99).

Investigation of problems was an important element in Kellner's instruction and was used on 93% of the reported days (see Table F94-F101 in this appendix). This student activity was given at least half of the class time on 36% of the days and equal emphasis with other student activities on 61% of the days. Two other activities in which students were engaged on more than half of the reported days were discussing answers and solution strategies (67%) and listening to the teacher or taking notes (53%). Students discussed answers and solution strategies on more than half of the class time on 55% of the days and with equal emphasis with other student activities on the remaining days. Listening to the teacher or taking notes was given equal emphasis with other student activities on 63% of the days and less than 15% of class time on 25% of the days. Students participated in some activities on fewer days: reflecting on or summarizing lesson concepts (33% of the reported days), participating in whole-class discussion (30%), practicing computation (23%), and taking a test or quiz (23%). Each of these student activities, however, was given more than half of the class time on more than half of the reported days. Students did not begin homework on any of the reported days (Kellner, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Kellner reported in her teacher logs. As an example of a lesson in Kellner's class, the lesson observed on 3/12/99 included: individual work on problem of the day, review of multiplication of fractions, and whole-class discussion of solutions (26 minute); whole class, Dear Student Letter and directions for riddle on page 1 (3 minutes); small-group work, riddle on page 1 (5 minutes); lunch and silent reading, school wide project (68 minutes); whole-class discussion, riddle on page 1 (10 minutes); small-group work, page 2 (10 minutes); whole-class discussion about small-group work and teacher presentation on arrow language

(11 minutes); small-group work, page 3 (12 minutes); and whole-class discussion about small-group work and homework assigned (14 minutes; Kellner, Observation 3/12/99).

Table F34
Subcategories of Lesson Planning: Kellner, Weir Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual lessons from a conventional textbook, Renlund looked over the section to be taught during the week. She modified her weekly plans each day to accommodate students' performance on the previous lesson. Renlund began each lesson with whole-class direct teaching. Whole-class discussions were teacher led and consisted mainly of students asking her questions and occasionally of students answering her questions. Renlund valued students working in small groups at specific times. She explained:

> There is a time when I think [students] should work in small groups. I think that small groups should be used after we've gone over the concepts, [students] have worked independently, and I've addressed any questions or concerns they may have had. And then I think the small-group work should be like reinforcement. Small-group instruction can also happen during class work time, when the kids who have mastered or understand the skill work with students who are having problems. And then we do have planned lessons where we're working in small groups. So it just depends on the day and the situation. (Renlund, Interview 5/11/99)

Renlund had students sit in groups so that any time during the class period, some students were available to assist others (Renlund, Interview 5/11/99).

Renlund used each instructional format for similar amounts of time: independent practice (51% of the reported days), review of previous material (49%), warm-up activities (47%), teacher presentation (42%), whole-class discussions (36%), and small-group work (36%; see Tables F88-F93 in this appendix). Each of the instructional forms was given equal emphasis with other instructional forms on most days (Renlund, Teacher Log 1998-99).

On 40–50% of the reported days, Renlund's students engaged in four activities: discussing answers and solution strategies (49%), investigating problems (46%), participating in whole-class discussions (40%), and practicing computation (40%; see Tables F94-F101 in this appendix). Each of these student activities was given equal emphasis with other activities on at least 90% of the days. Students were engaged on fewer days in listening to the teacher or taking notes (27%), reflecting on or summarizing lesson concepts (26%), and beginning homework (17%). Each of these activities was usually given equal emphasis with other student activities. Students took tests less frequently (Renlund, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Renlund reported in her teacher logs. As an example of a lesson observed in Renlund's class, the lesson on 1/14/99 included: problem of the day, individual activity followed by teacher-led, whole-class discussion about the solution (13 minutes); whole-class teacher-led activity, correcting of previously taken multiple-choice test with no discussion (7 minutes); teacher presentation of the lesson on fractions, finding the greatest common factor, writing fractions, and reducing fractions (15 minutes); individual work on assignment, problems 1-5, and checked by teacher (18 minutes); individual seatwork, problems 6-35 with some discussion about the problems among a few students (27 minutes; Renlund, Observation 1/14/99).

Table f35
Subcategories of Lesson Planning: Renlund, Newberry Middle School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	1	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in lesson plan

When planning to teach individual lessons from a conventional textbook, Friedman referred to her chapter plans in which she had included supplementary activities, such as games, small projects, and manipulatives. Because Friedman was so familiar with the curriculum she was using, she did not work through the problems. Depending where the lesson was in the chapter, Friedman might include some of the supplementary activities in the lesson plan. At the end of each unit, Friedman gave a test that included questions from the previous test. She used this information to determine the amount of review she needed to add to the next lesson. Friedman valued teacher-led whole-class discussions because through the discussion process she determined what students were thinking, and what they understood. She encouraged students to explain their solution strategies. She commented:

> I love when I hear new ways or new methods of solving a problem. I usually send [students] to the board for them to explain [their thinking] to the class. Sometimes the way students explain [their thinking], the class doesn't understand so I have to help a little bit. It's really interesting. (Friedman, Interview 6/15/99)

Friedman valued small groups for projects. She explained, "I think it's extremely valuable, because they have a chance to do something different than just answering questions for the teacher. So when I tell them we're going to the library to work in groups, or to get into groups of four, or that we'll have a project, they love it" (Friedman, Interview 6/15/99). Friedman introduced lessons to the whole class using direct teaching. Students usually did their class work individually. On the few days that Friedman planned for small-group activities, students worked in groups of three or four. She changed the group membership occasionally.

The instructional format that Friedman used most frequently was review of previous material (51% of the reported days; see Tables F88-F93 in this appendix) and it generally was given equal emphasis with other instructional forms. On approximately one fourth of the reported days, Friedman used teacher presentation (27%), independent practice (26%), and warm-up activities (24%). Each of these instructional forms was given equal emphasis with other instructional forms on approximately half of the days. Friedman seldom used whole-class discussion (17% of the reported days) and small-group work (10%). However, when used, each of them was given significant amounts of time. Small-group work was given at least half of the class time on 57% of the days and equal emphasis with other instructional forms on the remaining days. Whole-class discussions was given at least half of the class time on 48% of the days and equal emphasis with other instructional forms on 52% of the days (Friedman, Teacher Log 1998-99).

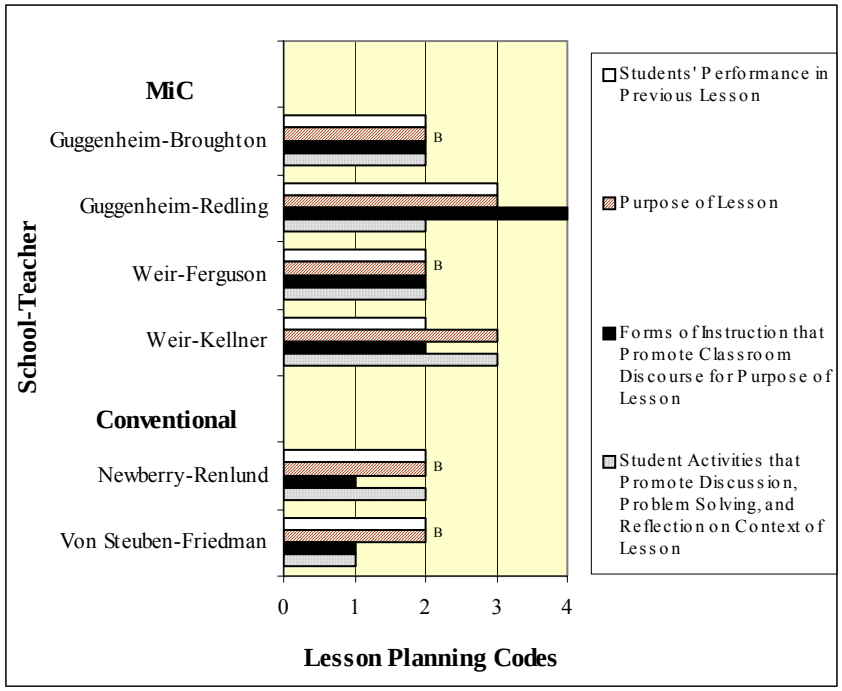
On approximately one fourth of the reported days, Friedman's students listened to the teacher or took notes (27%), participated in whole-class discussions (25%), and discussed answers and solution strategies (22%; see Tables F94-F101 in this appendix). Each of these student activities was given significant amounts of class time. Several student activities were used on a similar number of days but were given varying amounts of class time: beginning homework (16% of the reported days), taking tests or quizzes (16%), reflecting on or summarizing lesson concepts (15%), and practicing computation (10%). Except for taking a test or quiz, each was given equal emphasis with other student activities on at least half of the days (Friedman, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Friedman reported in her teacher logs. As an example of a lesson observed in Friedman's class, the lesson on 2/24/99 included: housekeeping duties, including collecting homework (11 minutes); teacher presentation with discussion and practice on angles (40 minutes); and clean up and homework assigned (3 minutes; Friedman, Observation 2/24/99).

Table F36
Subcategories of Lesson Planning: Friedman, Von Steuben Elementary School, District 2, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	1	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	1	Investigation of problems and discussion of mathematical ideas seldom planned for the lesson

In summary, the six sixth-grade teachers in District 2 varied in all four subcategories of *lesson planning* (see Figure F4). With respect to *students' performance in the previous lesson*, one teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. Five teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Little variation was also seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Four teachers planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F87 in this appendix.

Figure 4. Lesson planning, Grade 6, District 2.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned forms of instruction that promoted substantive conversation such as classroom activities that encouraged students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. Three teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Two teachers seldom planned forms of instruction that encouraged classroom discourse. Teachers also varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution strategies were dominant in the lesson plans and reflection on or summarization of the lesson was an important element in instruction. Four teachers included whole-class discussions and small-group work in their lesson planning, but the significance of these student activities in learning mathematics with understandings was not considered. The remaining teacher seldom planned for investigation of problems and discussion of mathematical ideas.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies;* and *lesson reflection, summary, or closure*. The description for each teacher includes discussion of all six subcategories.

Boughton. The evidence gathered for lesson presentation and development for Broughton ranged from Level 2A to 4. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table F102 in this appendix). To illustrate this rating, a lesson presented by Broughton (Observation 11/16/98) from a class using the MiC sixth-grade number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole & Brendefur, 1998) is described. In this lesson (pp. 1–4), students collected data on their favorite colors and represented the data with a segmented bar and a pie chart. The lesson began with reading the Letter to the Student and pp. 1–2. Broughton led a whole-class discussion of the content and noted things on the board. In interpreting the bar chart on p. 1, many students estimated that the shaded part of the bar was five or six times the size of the unshaded part of the bar. Students were not asked to explain the reasons for their responses. Instead, Broughton drew a bar on the board that was similar to the unit and stated that the shaded part of the bar was four times the size of the unshaded bar. She guided students to a generalization about the complement of a fraction relative to one whole (e.g., 3/5 is the complement of 2/5 with respect to one whole). Broughton was very interested in students' participation in the discussion but did not attempt to find out why students gave particular responses. For example, a student volunteered that 2/5 was a possible fraction that could represent a large majority of the people surveyed. Although Broughton wrote 2/5 on the board, she explained that 3/5 would be a better choice to represent a majority. Broughton conducted the survey of the students' favorite colors. Students then worked independently to construct a segmented bar for the results and use the bar to make a pie chart. Broughton skipped the third question, which involved conducting another survey and making a segmented bar and a pie chart to represent the survey results.

This lesson illustrates an overall rating of 3 on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, Broughton attempted to develop a conceptual basis for the mathematical content. At times, however, she provided the explanations for the students or simply gave her interpretation of the situation. When talking about the complements of simple fractions, student conjectures consisted of connecting new problems to ones they had already experienced. Connections between the segmented bar and the pie chart were apparent, although they were not explicitly discussed. The class survey and other points in the whole-class discussion connected the mathematics with students' experiences.

Broughton was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure F16). For Broughton, 57% of the interactive decisions were coded as reflective of good standard pedagogy, 25% were most aligned with teaching for understanding, and 19% were least aligned for teaching for understanding.

Across all observations for Broughton, the mean rating for the index on students' explanations was 1.50, indicating that at times students stated answers and were not expected to elaborate on their reasoning or solution strategies, and on other occasions their explanations were focused on

procedures rather than on elaboration of reasoning or solution strategies (see Table F104). The mean rating for elicitation of multiple strategies was 1.67, indicating that different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher (see Table G105). A rating of 1 was assigned for Broughton on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F37
Overall Ratings for Mathematical Interaction for Broughton, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to develop conceptual understanding
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.50	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.67	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Redling. The evidence gathered for lesson presentation and development for Redling ranged from Level 4 to 6. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table F102). To illustrate this rating, a lesson presented by Redling (Observation 11/16/98) from a class using the MiC sixth-grade algebra unit *Operations* (Abels, Wijers, Burrill, Simon, & Cole, 1998) is described. In this lesson (pp. 19–22), students added and subtracted integers in the context of keeping score in a game. Chips with + or - signs on them were used to represent positive and negative integers. The lesson began with a review of previous homework, which involved using number lines on a world map to solve problems about time zones (informal addition and subtraction of integers). Redling asked a series of questions that allowed her to assess students' understanding of the mathematics. She then distributed "chips" to each student, paper squares with a + sign on one side and a - sign on the other side, and split the class into two groups, Teams A and B. As Redling asked questions to each group (e.g., What is 2^2? What is $\sqrt{4}$? What is 1/2 of 1/2?), students kept score using a + for every correct answer and a - for every incorrect answer for four rounds of the game. Redling also wrote the points earned in each round on the board: Team A + + + + and Team B - - + +. Redling asked students to calculate the score for each team. When students readily used the chips to give accurate scores, Redling asked students to come up with various ways to determine the score for Team B. Students then began working in small groups on pp. 19–22. Students, however, had difficulty determining how one team in the unit got a score of $^{-}2$ after four rounds of the game. Redling shifted students' attention to a whole-class discussion in which the situation was clarified. Students continued work in small groups until they reached question 4, which asked them to find all possible scores a team could have after six rounds of the game. Again, Redling shifted students' attention to whole-class discussion of the question. Students then continued to complete the work on their own. They generalized that "a + and a - cancel," which Redling rephrased as "a + and a - make 0."

This lesson illustrates an overall rating of 4 on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, the presentation featured a conceptual basis for the mathematical content, and both Redling and her students actively participated in the lesson. The physical representations of positive and negative integers created a conceptual basis for working with integers, and students generalized that a + score point and a - score point combined to 0. By using chips in the students' own game, Redling promoted connections between the mathematics and their own experiences.

Redling was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure F17). For Redling, 80% of the interactive decisions were coded as most aligned with teaching for understanding, and 20 % were reflective of good standard pedagogy.

Across all observations for Redling, the mean rating for the index on students' explanations was 2.33, indicating that their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F104). The mean rating for elicitation of multiple strategies was 3.00, indicating that students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction (see Table F105). A rating of 2 was assigned for Redling on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F38
Overall Ratings for Mathematical Interaction for Redling, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.33	Focus on procedures
Elicitation of Multiple Strategies	3.00	Strategies not primary emphasis
Lesson Reflection, Summary, or Closure	2	Some opportunities

Ferguson. The evidence gathered for lesson presentation and development for Ferguson ranged from Level 1 to 4. An overall rating of Level 3B was assigned, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion (see Table F102). The lesson described here was assigned Level 2A, indicating that lessons were mainly devoted to review. The lesson was presented by Ferguson (Observation 12/1/98) from a class using the MiC sixth-grade number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole & Brendefur, 1998) is described. In this lesson (pp. 23–24), students were to investigate the results of a survey and estimate the fraction of respondents in each category. Students were to explain why a given newspaper headline related to survey results was inappropriate and to suggest appropriate headlines. The lesson began with a 20-minute period in which students organized graded papers for their folders. Then they checked homework, which consisted

of a worksheet on converting fractions to decimals and decimals to fractions, as Ferguson read the answers (10 minutes). Students' graded work from p. 22 of the unit was distributed, and students corrected any work that was marked as incorrect (25 minutes). Ferguson led a whole-class discussion of pp. 23–24 in the unit (27 minutes) and introduced a skills lesson unrelated to the unit on finding a percent of a number using the proportion method (20 minutes).

An overall rating of 3 was assigned for Ferguson on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, a major portion of the class period was devoted to review of previously completed work. In other lessons, attention was given to conceptual understanding of the mathematical content, and students investigated the veracity of statements. As in this lesson, connections among survey results, fractions, and a representation of the data on a segmented bar were explored. In the case of the survey, connections were made between mathematics and students' lives.

Ferguson was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F18). For Ferguson, 77% of the interactive decisions were coded as least aligned with teaching for understanding, and 22% were most aligned with teaching for understanding.

Across all observations for Ferguson, the mean rating for the index on students' explanations was 1.00, indicating that students stated answers and were not expected to elaborate on their reasoning or solution strategies (see Table F104). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table F105). A rating of 1 was assigned for Ferguson on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F39
Overall Ratings for Mathematical Interaction for Ferguson, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of procedure or strategy
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Kellner. The evidence gathered for lesson presentation and development for Kellner ranged from Level 2A to 5. An overall rating of Level 3A was assigned, indicating that students were unable to solve problems using the procedure or strategy presented by their teacher (see Table F102). To illustrate this rating, a lesson presented by Kellner (Observation 5/13/99) from a class using the MiC sixth-grade algebra unit *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole & Burrill, 1998) is described. In the lesson (pp. 41–42), students investigated calculations in

which the order of operations was used. The lesson began with a warm-up activity in which students completed four exercises on addition, subtraction, and multiplication of signed numbers and a written explanation of the rules for addition, subtraction, and multiplication of integers. After students completed the warm-up activity, Kellner provided the explanation for the fifth question. On the overhead projector she wrote (-) + (-) = (-) and (+) + (+) = (+). For both (-) + (+) and (+) + (-) she wrote "Subtract, take the sign of the bigger." Two students volunteered to explain their responses to questions 1 and 2 at the overhead projector. Kellner helped the second student complete the explanation. She then split the class into three groups, which alternated among three assignments. The group with Kellner read p. 41 and reviewed an arrow string that involved the order of operations. They talked about the reasons that different answers were displayed on different calculators. Students completed question 1 independently and discussed it with the group. They then completed question 2 parts a to d and question 3 independently. The second group completed two worksheets on representing expressions with positive or negative integers (e.g., 3 feet above sea level as +3) and 16 exercises on integer addition and subtraction, while the third group completed worksheets on plotting points on coordinate axes. After Kellner met with each group, she led a whole-class discussion of the answers to questions 2 and 3, and students described how they found their answers. However, no one, not even the teacher, appeared to understand question 3, which asked why the following way of writing this calculation was inadequate: $2 \times 37 = 74 + 25 = 99 \times 3 = 297$. Questions 4 and 5 on p. 42 were assigned for homework.

An overall rating of 3 was assigned for Kellner on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, Kellner presented particular procedures for integer operations during discussion of the warm-up activity and expected students to use these procedures in rote ways. During the discussion of the MiC portion of the lesson, the discussion fell short of conceptual understanding. The connections between the order of operations and arrow strings were not clearly demonstrated or explained. Students' conjectures were based on their use of arrow strings earlier in the unit and previous talk about the order of operations, but they did not have firm conjectures about how to write arrow strings in order to account for the order of operations. Connections between mathematics and students' life experiences were briefly mentioned as students investigated the order of operations, but the teacher did not emphasize these connections. The discussion between the two students in the unit was not referred to again after its initial reading.

Kellner was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure F19). For Kellner, 47% of the interactive decisions were coded as least aligned with teaching for understanding, 12% were reflective of good standard pedagogy, and 42% were most aligned with teaching for understanding.

Across all observations for Kellner, the mean rating for the index on students' explanations was 1.71, indicating that at times students stated answers and were not expected to elaborate on their reasoning or solution strategies, and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or solution strategies (see Table F104). The mean rating for elicitation of multiple strategies was 1.86, indicating that different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher (see Table F105). A rating of 1 was assigned for Kellner on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F40
Overall Ratings for Mathematical Interaction for Kellner, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	3A	Demonstration of procedure or strategy
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	2	Limited attention to conceptual understanding
Nature of Student Explanations	1.71	Answers only or focused on procedures
Elicitation of Multiple Strategies	1.86	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Renlund. The evidence gathered for lesson presentation and development for Renlund ranged from Level 3B to 4. An overall rating of Level 3B was assigned, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table F102). To illustrate this rating, a lesson presented by Renlund (Observation 1/14/99) from a class using a conventional textbook is described. In the lesson, students were introduced to Greatest Common Factor (GCF) and used the GCF to simplify fractions. They then practiced both procedures. The lesson began with students independently solving the Problem of the Day in which they drew a stack of four cubes according to three statements about the positions of the cubes in relation to their color. One student explained her solution, including the fact that one statement said that the green cube was *on* the pink cube whereas the other clues noted that the position of a particular cube was *above* another one. Renlund expounded on the detail noticed by the student. Preparation booklets for the state standardized test were distributed. Students checked multiple-choice answers for 25 exercises on whole number operations as Renlund read them, and they recorded the number correct. The answers were not discussed. Rendlund then introduced the lesson by writing five pairs of numbers on the whiteboard and asking students to find the GCF for each pair. Only one method to find the GCF was introduced: Students were told to write out all factors of each of the two given numbers and then select the largest common factor. Even though many concepts were involved, they were seldom connected to why the process worked, for example, that factors of a given number were divisors of that number. Students worked independently but relayed their answers during subsequent whole-class discussion. Renlund then asked students to form a fraction with each pair of numbers and use the GCF of the numbers to simplify the fraction. A student asked why Renlund wanted them to write a fraction for each pair of numbers. Renlund's response was that the goal of the lesson was to work with whole numbers and fractions. Again, only one method for simplifying fractions was introduced and practiced: Divide the GCF from both the numerator and denominator. Although Renlund mentioned that other methods could be used to simplify fractions, no other methods were explained. Particularly with the fraction 18/42, multiple steps by dividing the numerator and denominator consecutively by factors of 2 and 3, yielding the same result as dividing them by 6, could have been discussed. Also, it was not apparent that students had discussed fractions that are equivalent to one whole (e.g., 6/6) so that connections could be made to the property that dividing any number, including an unsimplified fraction, by one whole (in any form) does not change its value. Students then completed five textbook exercises independently. Students provided answers orally, and Renlund asked the class if the answers were correct. No further discussion occurred. At one point, a student stated that the simplified form of 36/42 was 1/2. Renlund asked if 36 was 1/2 of 42. After the student responded "No," Renlund said, "Then 36/42 cannot be 1/2." No further explanation clarified a procedure for determining the simplest form. During the last part of the class period, students completed 30 textbook exercises using the same methods introduced in the lesson. Students did not have the opportunity to make conjectures about mathematical ideas.

They practiced algorithms for finding the GCF and using it to simplify fractions by dividing both the numerator and denominator by the GCF. During the postobservation interview, Renlund commented that her goals for the lesson were met and that students understood the relationship between GCF and simplifying fractions, although she noted that some students needed to be reminded of the procedure for using the GCF for simplifying fractions, and others were "thrown off" when given improper fractions to simplify.

An overall rating of 2 was assigned for Renlund on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, Renlund presented particular procedures for finding the GCF of a pair of numbers and using the GCF to simplify fractions, and students used these procedures in rote ways. Alternative methods were mentioned in passing but not explained. The lesson did not promote conceptual understanding, connections among mathematical ideas, or connections between mathematics and students' life experiences.

Renlund was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F20). For Renlund, 62% of the interactive decisions were coded as least aligned with teaching for understanding, 29% were reflective of good standard pedagogy, and 10% were most aligned with teaching for understanding.

Across all observations for Renlund, the mean rating for the index on students' explanations was 1.20, indicating that students stated answers and were not expected to elaborate on their reasoning or solution strategies (see Table F104). The mean rating for elicitation of multiple strategies was 1.20, indicating that multiple strategies were not elicited from students (see Table F105). A rating of 1 was assigned for Renlund on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F41
Overall Ratings for Mathematical Interaction for Renlund, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of procedure or strategy
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.20	Answers only
Elicitation of Multiple Strategies	1.20	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Friedman. The evidence gathered for lesson presentation and development for Friedman ranged from Level 2A to 3B. An overall rating of Level 3B was assigned, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table F102). To illustrate a rating of 2A, indicating that lessons were mainly devoted to review, a lesson presented by Friedman (Observation 2/2/99) from a class using a conventional textbook is described. In the lesson, students reviewed multiplication of fractions and practiced writing reciprocals of fractions. The lesson began with a 33-minute session devoted to checking the homework assignment that consisted of two worksheets with three types of exercises: multiplication of fractions, multiplication of fractions and mixed numbers, and multiplication of mixed numbers. On the overhead projector, Friedman worked out any problems students requested and then collected student papers. For the remainder of the class period (6 minutes), Friedman introduced reciprocals, which was presented on a single-sided worksheet. Limited attention was given to conceptual understanding of the mathematical content. The process for finding the reciprocal was to "flip the fraction over," even though Friedman asked students to say, "Write the reciprocal." There were no opportunities for students to conjecture about reciprocals. Although Friedman did mention that the product of a fraction and its reciprocal was one whole, students were only asked to answer whether two fractions were reciprocals. No proofs were requested. Students were expected to follow the procedure provided by Friedman. When students did explain their answers during the homework review, their answers focused on procedures. No connections were made between the mathematics and students' life experiences in either the homework assignment or the lesson presentation. The homework assignment was another worksheet on multiplication of mixed numbers.

An overall rating of 1 was assigned for Friedman on the composite index for the nature of inquiry during instruction (see Table F103). In this lesson, a major portion of the class period was devoted to review of previously completed work. Little, if any, attention was given to conceptual understanding of the mathematical content, and no opportunities occurred for students to make conjectures. Connections among mathematical ideas were talked about, but not emphasized or used in subsequent practice, and connections between the mathematics and students' life experiences were not discussed.

Friedman was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F21). For Friedman, 70% of the interactive decisions were coded as least aligned with teaching for understanding, and 30% were reflective of good standard pedagogy.

Across all observations for Friedman, the mean rating for the index on students' explanations was 1.63, indicating that at times students stated answers and were not expected to elaborate on their reasoning or solution strategies, and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or solution strategies (see Table F104). The mean rating for elicitation of multiple strategies was 1.25, indicating that multiple strategies were not elicited from students (see Table F105). A rating of 1 was assigned for Friedman on the index for reflection or summary, indicating that few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F42
Overall Ratings for Mathematical Interaction for Friedman, District 2, Grade 6

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of procedure or strategy
Nature of Inquiry	1	Limited to lower order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.20	Answers only
Elicitation of Multiple Strategies	1.20	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the six Grade 6 teachers in District 2 varied from Level 3A to Level 5 in lesson presentation and development (see Table F43). One MiC teacher was assigned Level 5, indicating that the lesson featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. One MiC teacher was assigned Level 4, indicating that attempts were made to develop a conceptual basis for the mathematical content. Three teachers (one MiC) were assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. One MiC teacher was assigned Level 3A, indicating that students were unable to solve problems using the procedure or strategy presented by their teacher.

Table F43
Nature of Mathematical Interaction, Grade 6 Teachers in District 2, 1998-1999

School-Teacher	**Lesson Presentation and Development**	**Nature of Inquiry**	**Teachers' Interactive Decisions**	**Nature of Student Explanations**	**Elicitation of Multiple Strategies**	**Lesson Closure, Reflection, or Summary**	**Total**
			— MiC —				
Guggenheim-Broughton	4	3	4	1.50	1.67	1	15.17
Guggenheim-Redling	5	4	5	2.33	3.00	2	21.33
Weir-Ferguson	3B	3	1	1.00	1.00	1	10.00
Weir-Kellner	3A	3	2	1.71	1.86	1	12.57
			— Conventional —				
Newberry-Renlund	3B	2	1	1.20	1.20	1	9.40
Von Steuben-Friedman	3B	1	1	1.63	1.25	1	8.88

With respect to the nature of inquiry during instruction, teachers varied from Level 1 to Level 4. One MiC teacher was assigned Level 4, indicating that in-depth exploration of mathematics occurred during lessons. Three MiC teachers were assigned Level 3, indicating that attention was given to conceptual understanding during instruction. One teacher using a conventional curriculum was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction. The remaining teacher using a conventional curriculum was assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to teachers' interactive decisions, the teachers varied from Level 1 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding. One MiC teacher was assigned Level 4, indicating that more emphasis was placed on standard pedagogy and teaching for understanding. One MiC teacher was assigned Level 2, indicating that more attention was given to good standard pedagogy. The three remaining teachers (one MiC) were assigned Level 1, indicating that their interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the nature of student explanations ranged from 1.00 to 2.33. Student explanations in the classes of one MiC teacher were focused on procedures. Student explanations in the classes of three teachers (two MiC) were limited to answers only on some occasions and focused on procedures on other occasions. For the classes of two teachers (one MiC), student explanations were limited to answers only. The overall means for the elicitation of multiple strategies ranged from 1.00 to 3.00. In the classes of one MiC teacher, students were asked if alternate strategies were used in solving particular problems, but this was not a primary emphasis during lessons. Two MiC teachers rarely elicited multiple strategies. The remaining three teachers (one MiC) did not elicit multiple strategies from their students. With respect to lesson reflection, summary, or closure, one MiC teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons. The remaining five teachers (four MiC) provided few, if any, opportunities for students to reflect on the mathematics they were learning.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The sixth-grade teachers in District 2 varied in all three subcategories of classroom assessment (see Tables F44-F47). With respect to the evidence sought during classroom assessment, one MiC teacher maintained an underdeveloped process orientation. The remaining teachers (three MiC) sought procedural competence of student learning. Feedback in the classes of one MiC teacher was characterized as an emerging shared responsibility among students and between students and their teachers. The feedback of the remaining five teachers (three MiC) was very teacher-directed. One MiC teacher provided mixed, superficial feedback, and two teachers (one MiC) provided low-level closed feedback. Three teachers (two MiC) provided feedback that allowed students to know whether answers were correct or incorrect.

Table F44

Evidence Sought through Classroom Assessment Practice, Grade 6 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	2	Procedural competence
Guggenheim-Redling	3	Underdeveloped process orientation
Weir-Furguson	2	Procedural competence
Weir-Kellner	2	Procedural competence
Conventional		
Newberry-Renlund	2	Procedural competence
Von Steuben-Friedman	2	Procedural competence

Table F45

Feedback Coherence and Purpose, Grade 6 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	2	Teacher-directed feedback
Guggenheim-Redling	3	Emerging shared responsibility
Weir-Furguson	2	Teacher-directed feedback
Weir-Kellner	2	Teacher-directed feedback
Conventional		
Newberry-Renlund	2	Teacher-directed feedback
Von Steuben-Friedman	2	Teacher-directed feedback

Table F46

Feedback Content, Grade 6 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	3	Low-level, closed feedback
Guggenheim-Redling	4	Mixed, superficial feedback
Weir-Furguson	2	Answer-only feedback
Weir-Kellner	2	Answer-only feedback
Conventional		
Newberry-Renlund	3	Low-level, closed feedback
Von Steuben-Friedman	2	Answer-only feedback

Table F47
Classroom Assessment, Grade 6, District 2

School-Teacher	Evidence Sought	Feedback Coherence and Purpose	Feedback Content	Total
		— MiC —		
Guggenheim-Broughton	2	2	3	7
Guggenheim-Redling	3	3	4	10
Weir-Ferguson	2	2	2	6
Weir-Kellner	2	2	2	6
		— Conventional —		
Newberry-Renlund	2	2	3	7
Von Steuben-Friedman	2	2	2	6

See index next page.

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Broughton ranged from Level 1 to Level 2 (see Table F106 in this appendix). The mean rating across observations was 1.67, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Broughton are described. The observer noted, "Some students shared answers and several helped others" (Broughton, Observation 2/10/99). In this example, student–student conversations were limited and usually consisted of sharing answers.

The index ratings about the nature of student–student conversation for Redling ranged from Level 1 to Level 4 (see Table F106 in this appendix). The mean rating across observations was 2.67, indicating that student–student conversation was frequent but not substantive in nature. To illustrate a rating at Level 3, student–student conversations that occurred during a lesson by Redling are described. The observer noted, "It was noticed that some students were too complacent in their groups (pairs) and not questioning partner's responses. They appeared to be too willing to go along with their partner's first responses to questions in the unit" (Redling, Observation 11/16/98). In this example, student–student conversations occurred frequently during class time, was characterized by students asking for answers, and did not promote shared understanding of mathematical ideas.

The index ratings about the nature of student–student conversation for Ferguson ranged from Level 1 to Level 3 (see Table F106 in this appendix). The mean rating across observations was 1.86, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Ferguson are described. The observer noted, "Very little conversation about the activity was observed" (Ferguson, Observation 3/8/99). In this example, student–student conversations were limited.

The index ratings about the nature of student–student conversation for Kellner ranged from Level 1 to Level 3 (see Table F106 in this appendix). The mean rating across observations was 2.00, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Kellner are described. The observer noted, "Some students compared notes on MiC problems, but most conversation was between teacher and student" (Kellner, Observation 5/13/99). In this example, student–student conversations were limited and consisted of sharing answers.

The index ratings about the nature of student–student conversation for Renlund ranged from Level 1 to Level 3 (see Table F106 in this appendix). While the mean rating across observations was 1.60, on three of the five observations student–student conversation was rated at Level 1, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred

during a lesson by Renlund are described. The observer noted, “No student–student discourses occurred during the large-group lesson. Discussions during the individual seatwork were mainly to check answers” (Renlund, Observation 3/10/99). In this example, student–student conversation was limited and usually consisted of reporting answers.

The index ratings about the nature of student–student conversation for Friedman ranged from Level 1 to Level 3 (see Table F106 in this appendix). The mean rating across observations was 1.25, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Friedman are described. The observer noted, “Students didn’t converse with each other about the math lesson” (Friedman, Observation 2/24/99). In this example, student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the six Grade 6 teachers in District 2 varied from 1.25 to 2.67. One MiC teacher had a mean rating of 2.67, indicating that student–student conversation occurred frequently during class time, but was not substantive in nature. Three MiC teachers had mean ratings at or near 2.00, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. Two teachers using conventional curricula had mean ratings (or the majority of ratings) near 1.00, indicating that student–student conversation was not encouraged or did not exist.

Collaborative Working Relationships Among Students

The index ratings about the nature of students’ collaboration in the classroom for Broughton ranged from Level 1 to Level 2 (see Table F107 in this appendix). The mean rating across observations was 1.75, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students’ collaboration at Level 2, student collaboration that occurred during a lesson by Broughton is described. The observer noted, “Students who were seated in clusters didn’t necessarily support each other or provide unsolicited assistance” (Broughton, Observation 2/10/99). In this example, although students physically sat together, there was little exchange of ideas or assistance.

The index ratings about the nature of students’ collaboration in the classroom for Redling ranged from Level 1 to Level 4 (see Table F107 in this appendix). The mean rating across observations was 2.33, indicating that usually a few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students’ collaboration at Level 2, student collaboration that occurred during a lesson by Redling is described. The observer noted, “Students did share but didn’t take responsibility for others’ work. Students didn’t work on the same problems at the same time” (Redling, Observation 2/10/99). In this example, some students did share, but they worked on different problems at different paces.

The index ratings about the nature of students’ collaboration in the classroom for Ferguson ranged from Level 1 to Level 2 (see Table F107 in this appendix). The mean rating across observations was 1.29, indicating that usually none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students’ collaboration at Level 1, student collaboration that occurred during a lesson by Ferguson is described. The observer noted, “No small-group work was observed. Students worked independently of each other” (Ferguson, Observation 2/22/99). In this example, none of the students work together in small groups or in a large-group setting.

The index ratings about the nature of students' collaboration in the classroom for Kellner ranged from Level 1 to Level 4 (see Table F107 in this appendix). The mean rating across observations was 2.14, indicating that usually a few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Kellner is described. The observer noted, "Students were seated so that they could work in pairs and attempted to work together, at first, with geoboards" (Kellner, Observation 11/11/98). In this example, some students exchanged ideas or gave assistance to their peers.

The index ratings about the nature of students' collaboration in the classroom for Renlund ranged from Level 1 to Level 3 (see Table F107 in this appendix). Although the mean rating across observations was 1.50, on three of the four observations, students' collaboration was rated at Level 1, indicating that none of the students worked together in small-group or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Renlund is described. The observer noted, "Students didn't work in small groups" (Renlund, Observation 11/4/98). In this example, none of the students worked together in small groups or in a large-group setting.

The index ratings about the nature of students' collaboration in the classroom for Friedman ranged from Level 1 to Level 3 (see Table F107 in this appendix). The mean rating across observations was 1.29, indicating that often none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Friedman is described. The observer noted, "Students were not seated in groups and didn't work together on their seatwork" (Friedman, Observation 12/15/98). In this example, none of the students worked together in small groups or in a large-group setting.

In summary, the mean ratings for *student collaboration* for the six Grade 6 teachers in District 2 varied from 1.29 to 2.33. Three MiC teachers had mean ratings near 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. Three teachers (two MiC) had mean ratings near 1.00 (or the majority of the ratings were 1.00), indicating that on many occasions none of the students were working collaboratively.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Broughton ranged from Level 2 to Level 4 (see Table F108 in this appendix). The mean rating across observations was 3.17, indicating that student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Broughton is described. The observer noted, "[Broughton] had to prod students. When students were to start on page 8, it was noted that several students were totally off task" (Broughton, Observation 1/11/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and depended on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Redling ranged from Level 3 to Level 4 (see Table F108 in this appendix). The mean rating across observations was 3.67, indicating that most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Redling is described. The observer noted, "[Students exhibited] a high level of interest and involvement. They related to the Team A [versus] Team B competition" (Redling, Observation 11/16/98). In this example, most students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Ferguson ranged from Level 3 to Level 4 (see Table F108 in this appendix). The mean rating across observations was 3.29, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Ferguson is described. The observer noted, "Some students tended to be off task more than on task" (Ferguson, Observation 2/22/99). In this example, some students were engaged in class activities some of he time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Kellner ranged from Level 3 to Level 4 (see Table F108 in this appendix). The mean rating across observations was 3.43, indicating that on some occasions most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort, but on other occasions, student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Kellner is described. The observer noted, "Students were very actively involved and interested in the class' activities" (Kellner, Observation 3/22/99). In this example, most students were on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during another lesson by Kellner is described. The observer noted, "Before the lunch break, students were mostly attentive to classmates explaining their surveys. After the lunch break, students were often socializing" (Kellner, Observation 12/9/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Renlund ranged from Level 3 to Level 4 (see Table F108 in this appendix). The mean rating across observations was 3.40, indicating that on some occasions most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort, but on other occasions, student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Renlund is described. The observer noted, "Students were attentive during the lesson discussions and seatwork time" (Renlund, Observation 3/10/99). In this example, students were on task pursuing the substance of the lesson. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during another lesson by Renlund is described. The observer noted, "Most students were working during the cub activity. During the other parts of the lesson, students were passive yet most were on task" (Relund, Observation 12/10/98). In this example, most students were engaged in class activities some of he time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Friedman ranged from Level 3 to Level 4 (see Table F108 F108 in this appendix). The mean rating across observations was 3.75, indicating that most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort. To illustrate a rating for student engagement at Level 4,

students' on-task behavior that occurred during a lesson by Friedman is described. The observer noted, "Students were on task and productive. Everyone worked" (Friedman, Observation 4/15/99). In this example, most students were on task pursuing the substance of the lesson most of the time.

In summary, the mean ratings for student engagement in lessons for the six Grade 6 teachers in District 2 varied from 3.17 to 3.75. Two teachers (one MiC) had mean ratings near 4.00, indicating that on most occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time. Four teachers (three MiC) had mean ratings greater than 3.00, indicating that on occasion student engagement was widespread and on other occasions sporadic or episodic.

Instruction **Composite Variable**

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table F48
Teacher level of Instruction, Grade 6, District 2

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE		
— MiC —																
Guggenheim-Broughton	2.5	2.5	4	3	4	2.50	2.09	1.67	2	2	3	2.09	2.19	3.96	37.50	**3**
Guggenheim-Redling	5	2.5	5	5	5	3.88	3.75	3.33	3	3	4	3.34	2.91	4.59	54.30	**5**
Weir-Ferguson	2.5	2.5	3	3	1	1.67	1.25	1.67	2	2	2	2.33	1.61	4.11	30.64	**2**
Weir-Kellner	2.5	3.75	3	3	2	2.85	2.33	1.67	2	2	2	2.50	2.78	4.29	36.67	**3**
— Conventional —																
Newberry-Renlund	1.25	2.5	3	2	1	2.00	1.50	1.67	2	2	3	2.00	1.88	4.25	30.05	**2**
Von Steuben-Friedman	1.25	1.25	3	1	1	2.72	1.56	1.67	2	2	2	1.56	1.61	4.69	27.31	**2**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

The ratings assigned to each teacher in this district on each of the indices related to *unit and lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 3

In District 3, four Grade 6 teachers participated in the study. All four teachers used MiC. Classroom observation and teacher log data were unavailable for these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*. Data on unit planning for Schlueter for 1998–1999 were unavailable. The following is a summary of her comments with respect to unit planning

from the 1997–1998 school year. Schleuter thought planning to teach a MiC unit took longer than planning to teach a unit from a conventional textbook. However, she felt that having taught MiC the previous year made planning somewhat easier during the current year. Schlueter planned to teach a MiC unit by reading through the section summaries and working through each problem. She gained a general overview of the content, determined prerequisite knowledge, anticipated student difficulties, and planned supplemental activities. Schleuter considered both low- and high-ability students when planning the pace for instruction. Because the state curriculum guidelines had been fluctuating, Schleuter found it difficult to consider these guidelines as she planned: "We just think we have it [state standards]. . . and then they change it again. But MiC seems to fall into it anyhow real comfortably, so we don't seem to have a problem. It's just what the state wants" (Schleuter, Interview 5/5/98). Schleuter often planned with a team of mathematics teachers at Calhoun North Middle School (Schleuter, Interview 5/5/98).

Table F49
Subcategories of Unit Planning: Schlueter, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Solomon used three criteria to determine the sequence of MiC units: recommendations in the teacher guides, external factors such as various seasonal events and dates of district standardized tests, and the logical order of mathematical reasoning. When planning to teach a MiC unit, she referred to her notes from previous years for pacing suggestions, vocabulary to emphasize, prerequisite skills, possible problem areas, essential manipulatives, and supplementary activities. Solomon commented:

> I base [unit planning] on notes that I've left to myself in the past. For example, my notes might say: use manipulatives with this lesson; need to teach some basic skills from our textbook before this unit; lay a foundation in basic skills before this individual lesson or after that lesson. It's kind of like a weaving act. I never just flat out follow what I wrote to myself last year. I always try new things too. (Solomon, Interview 5/7/99)

Students' prior experience with MiC influenced Solomon's unit planning more than their prior knowledge of mathematics. The sixth-grade MiC program was developed with the understanding that students would be taught MiC in Grade 5. Because Solomon taught science and mathematics, she integrated the two content areas whenever possible, using various computer software programs. She thought MiC was well aligned with the new state and district standards. However, she planned intensive review for two or three weeks before the standardized tests (Solomon, Interview 5/7/99).

Table F50
Subcategories of Unit Planning: Solomon, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Tierney generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, he first assessed, formally or informally, students' knowledge of the unit content. Using this information, he determined the activities to include, concepts or skills that needed to be pre-taught (if any) and the pace of instruction. He commented: "I do a little bit of pre-assessment. Sometimes it's just in terms of discussing prior knowledge with the class and getting an overall feel. Sometimes I actually use a pre-assessment instrument of some sort that I put together to try to see what they need to be learning and what they may already know" (Tierney, Interview 3/23/99). Tierney used supplementary materials to provide needed skill practice, either before teaching a specific unit or before district testing. He thought MiC was well aligned with the district curriculum guidelines and the state standards (Tierney, Interview 3/23/99).

Table F51
Subcategories of Unit Planning: Tierney, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Vetter, a teacher of students in special education, generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the unit, looked at individual lessons, and noted the needed prerequisite skills for her students. Vetter looked for supplementary materials that would complement the MiC unit, develop skill facility, and serve as homework. Vetter did not determine the pace of instruction prior to instruction. Students' ability to grasp the concepts and skills in the daily lessons and their attitudes toward the lessons determined the pace of instruction. She commented: "The kids set the pace in a lot of the ways. How fast we can move depends on how they're doing. And you can tell when they're losing interest and it's kind of bogging down. Then you have to fast forward a little bit and move them on even though you don't think they're getting it. You just need to move on" (Vetter, Interview 5/4/99). Vetter thought MiC was aligned with the state standards (Vetter, Interview 5/4/99).

Table F52
Subcategories of Unit Planning: Vetter, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the four sixth-grade teachers in District 3 varied in two of the subcategories of *unit planning*, *students' prior knowledge* and *unit sequence* (see Figure F6). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Two teachers based unit planning on their perceptions of students' prior knowledge in mathematics or reading without informal or formal assessment. The remaining teacher followed the unit development in the teacher guide with little or no consideration of students' prior knowledge.

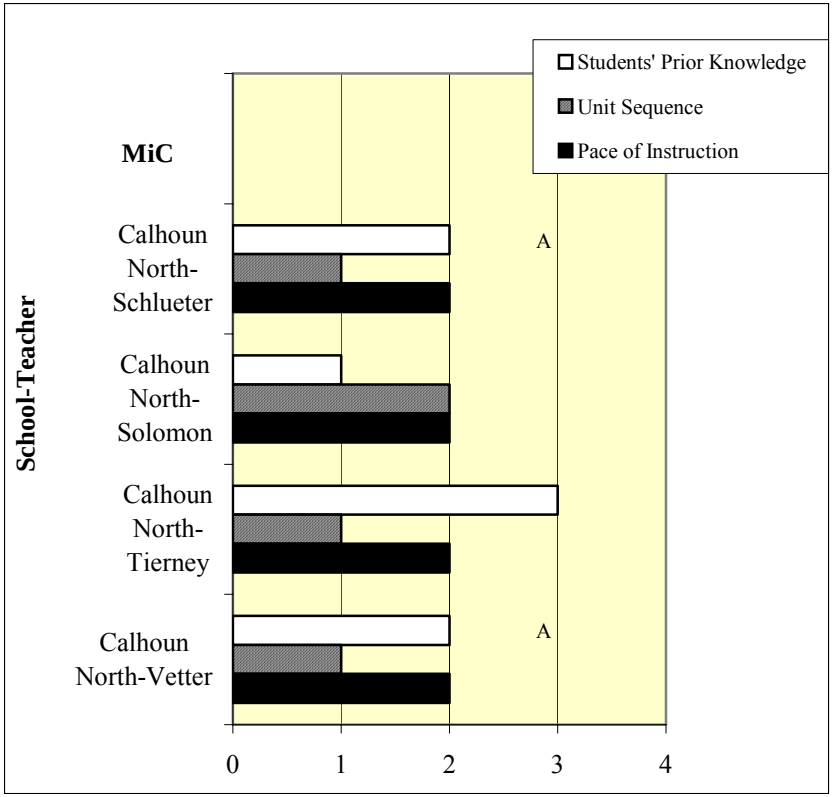


Figure F6. Unit planning, Grade 6, District 3.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Three teachers used the *sequence of units* recommended in teacher support materials when planning to teach a unit. The other teacher considered external factors such as the content of the statewide testing program when sequencing instructional units. All of the teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended *pace for instruction*.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Data on unit planning for Schlueter for 1998-1999 were unavailable. The following is a summary of her 1997-1998 comments with respect to lesson planning. When Schlueter planned to teach a MiC lesson, she considerably lengthened the pacing suggested in the teacher guide. She found that students often took a long time to finish a MiC lesson because they had to

physically move to another classroom and had to change perspective from one context to another as they completed lesson problems. Schlueter planned for small-group instruction when the problems in the lesson had more than one solution. She believed that cooperative groups gave students an opportunity to discuss different answers to a problem and work together to solve a problem that they might not have been able to answer on their own. Schlueter also planned many supplementary activities such as games and projects (Schleuter, Interview 5/5/98).

Table F53
Subcategories of Lesson Planning: Schlueter, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	2A	Lesson content selected to reflect continuity of mathematical content, integrating lesson materials from various resources
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan

When planning to teach individual MiC lessons, Solomon worked through all of the problems to acquaint herself with the purpose of the lesson, to identify the needed vocabulary, and to anticipate where students might have difficulties. She studied the notes made when she previously taught the lesson for suggestions about pacing, needed prerequisite skills, appropriate manipulatives to use, and when to add basic skills practice or use other supplementary materials. Solomon considered the basic abilities of the students in the class. She explained, "I have one math class that takes a lot longer and struggles a lot more. So I plan a little differently for that class" (Solomon, Interview 5/7/99). Solomon valued small-group work but thought it created some disadvantages for gifted and talented students. She explained:

> I think if you weigh it out, it is better for the kids to work in groups. I have concern for my [gifted and talented] kids. They sometimes pull the load and the kids that are lower just kind of coast. I have done some ability grouping, placing the [gifted and talented] kids in one group. But that doesn't work out for the low kids. They just sit there. They have no clue. So it's not good for all of the kids all of the time. You have to do a juggling act. (Solomon, Interview 5/7/99)

Most students frequently contributed to small-group discussions. But during whole-class discussions, the leaders of the small groups explained their solution strategies and discussed how different strategies could lead to the same answer (Solomon, Interview 5/7/99).

Table F54
Subcategories of Lesson Planning: Solomon, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Tierney worked through all of the problems to identify possible difficulties with the format. He explained: "Just visually a lot of the kids who have any kind of visual processing problems have trouble with MiC. Partly because it's not always clear where the example is or where the problem is set up and its actual question. Some of my kids get very confused with it" (Tierney, Interview 3/23/99). Working through the problems also helped him identify the problems related to the same concepts and skills. He wanted to stay alert to any redundancy: "I see a lot of frustration, especially among the brighter, faster students who ask, 'Why are we doing this again?' I make notes and adjust as well as I can" (Tierney, Interview 3/23/99). Tierney planned each lesson the night before to account for students' performance in the previous lesson. Tierney valued small-group work when students were grouped homogeneously. He commented:

> [Students] can bounce ideas off each other and really grow in understanding that way. But I believe that the group should be fairly homogeneous rather then heterogeneous. I've observed that when students are not assigned to specific groups, they tend to divide themselves basically by skill level. I let them do that at several points this year and observed what happened. There seemed to be faster gains in my lower group. I think because they were forced to rely on themselves a little more and not just get answers from the faster kids. And my higher-level groups went farther than I had expected. (Tierney, Interview 3/23/99)

Tierney thought students often participated in discussions, although some students contributed more frequently in small-group discussions than in whole-class discussions. Students' contributions were about mathematics, but they were more procedural than substantive in nature (Tierney, Interview 3/23/99).

Table F55
Subcategories of Lesson Planning: Tierney, Calhoun North Middle School, District 3, Grade 6

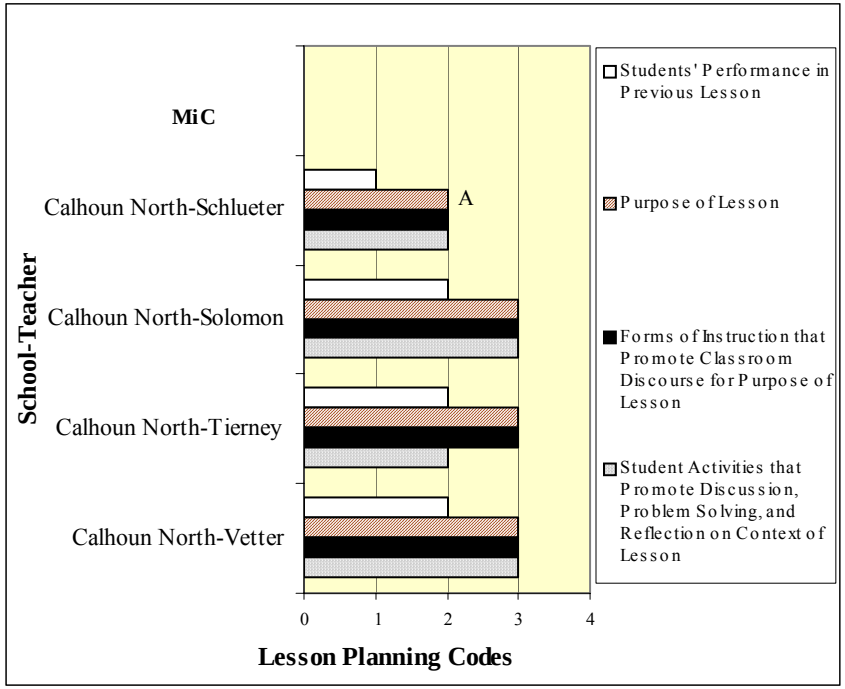
Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Vetter did not work through all of the problems when she had previously taught the lesson. This familiarity allowed her to anticipate student questions and difficulties and modify the lesson when necessary. Vetter's approach to lesson planning was similar to her approach to unit planning. As a teacher of students in special education, she was always aware of their skill levels and special needs. Vetter identified the basic skills needed for the lesson and planned to include a quick review before beginning the lesson. She gathered supplementary materials that would complement the MiC lesson, develop skill facility, and serve as homework. The students' abilities to grasp the concepts and skills in the lesson and their attitudes toward the lessons determined the pace of instruction. Vetter valued small-group work and planned for it on a fairly regular basis, especially when students were starting a new activity. After presenting the lesson to the whole group, Vetter had students work as pairs or in groups of three. Group membership was determined by behavior and skill level. Vetter explained: "I try to move them around so that everybody has to work with somebody different. I try to, sometimes depending on what the lesson is, group one who is strong with one who is weak. Other times I let them pick their groups. It changes depending on the task" (Vetter, Interview 5/4/99). Following the small-group or pair work, Vetter brought the whole class together to discuss each group's work. Through sharing how each group solved each problem, all of the students worked through the problems again in a large group. When students gave their answers they were expected to explain their solution strategies as best they could (Vetter, Interview 5/4/99).

Table F56
Subcategories of Lesson Planning: Vetter, Calhoun North Middle School, District 3, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

In summary, the four sixth-grade teachers in District 3 varied in all four subcategories of *lesson planning* (see Figure F7). With respect to *students' performance in the previous lesson*, three teachers made decisions about extending the lesson to complete a task, adding a review, or accounting for individual differences. The other teacher planned in the same manner daily without considering students' previous performances. Little variation was seen in teachers' attention to the *purpose of the lesson*. Three teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. The other teacher selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure F7. Lesson planning, Grade 6, District 3.

Little variation was also seen in the teachers' choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Three teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The other teacher included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. The teachers also varied in the fourth subcategory of *lesson planning*, *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For two teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plans. However, questions or activities that encouraged students to reflect on or summarize lesson concepts

were not included in the lesson plans. Two teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

The ratings assigned to each teacher in this district on each of the indices related to *unit and lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable *Instruction* was not calculated for these teachers.

District 4

In District 4, three Grade 6 teachers participated in the study. All three teachers used MiC. Classroom observation and teacher log data were unavailable for these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*. Becker followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the unit and noted the vocabulary listed in the glossary in the teacher guide. Prior to teaching the unit, Becker gave each student a copy of the vocabulary words, which they were to define as they worked through the lessons. With the students, she talked about unit topics and elicited the content knowledge students brought to the unit. Becker tried to follow the suggested pace for instruction outlined in the teacher guide, but "it depends on how much the kids know. Sometimes I think it shouldn't take that long, but it does because the kids really don't have any prior knowledge of it" (Becker, Interview 9/23/99). Becker thought that in one sense planning to teach a MiC unit was harder than planning to teach a unit in a conventional textbook because "there are little gaps in the content that the kids really don't know. I have to try to fill those in sometimes and that takes a little bit longer" (Becker, Interview 9/23/99). On the other hand, she thought planning to teach a MiC unit was easier because "the activities are really there. It's a little easier in that sense because you have it right in front of you" (Becker, Interview 9/23/99). Becker thought that the set of sixth-grade MiC units met the district mathematics guidelines (Becker, Interview 9/23/99).

Table F57
Subcategories of Unit Planning: Becker, Kelvyn Park Middle School, District 4, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Downer reorganized the recommended sequence of MiC units in order to emphasize concepts needed for district standardized tests. When planning to teach a MiC unit, she read through the unit, noting the activities, concepts, and skills. Downer assumed students had mastered most of the concepts and skills included in the fifth-grade mathematics curriculum until students showed her otherwise. From previous experience teaching MiC, Downer knew that some lessons were problematic for students. She planned to do some review before teaching the unit. Downer determined the pace of instruction according to the difficulty of concepts: "For example, students usually have a difficult time with adding fractions with different denominators. That might take a little longer than adding fractions with the same denominator. It also depends on the level of the class" (Downer, Interview 7/14/99).

Table F58
Subcategories of Unit Planning: Downer, Kelvyn Park Middle School, District 4, Grade 6

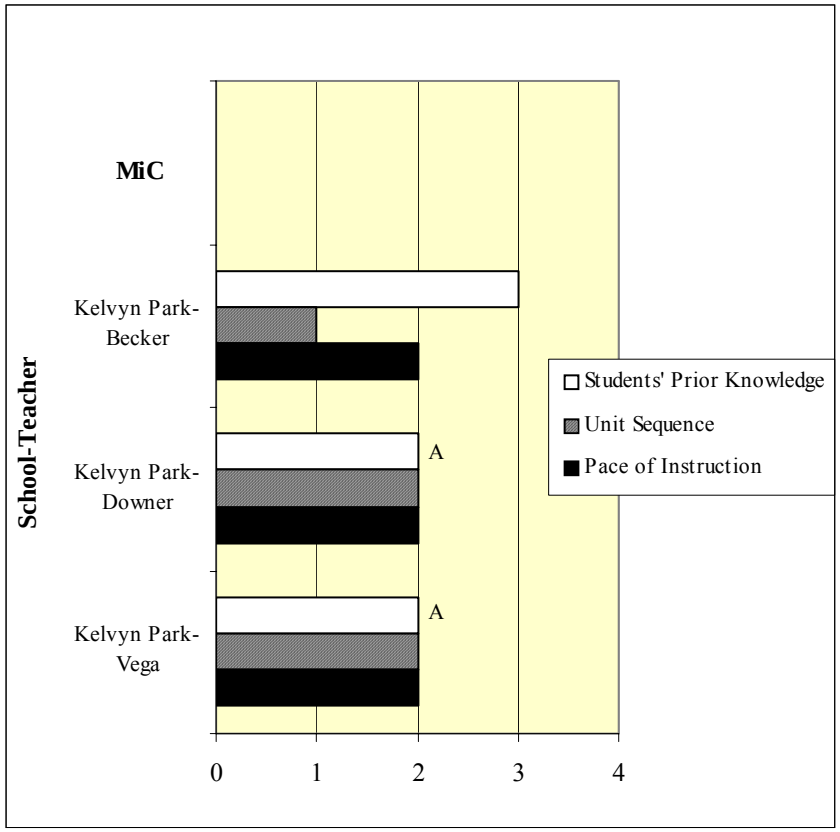
Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Vega generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the information in the teacher guide. She planned to teach many of the prerequisite skills and concepts identified in the teacher guide because "[students] come in from elementary school knowing next to nothing. We pretty much have to start from scratch" (Vega, Interview 6/14/99). She was aware of the suggested pace for instruction, but the pace was influenced by the limited amount of class time: "We work with 45-minute periods. But when the kids come in and get settled, you're already losing 10-15 minutes. Then of course there's homework that has to be gone over and that's usually another 10 minutes at least. So we're talking about 20-25 minutes at the most for actual class time. Many of the lessons require a lot of time" (Vega, Interview 6/14/99). The pace of instruction was also influenced by the need to cover many concepts and skills before district standardized tests, which were administered at the end of April.

Table F59
Subcategories of Unit Planning: Vega, Kelvyn Park Middle School, District 4, Grade 6

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the three sixth-grade teachers in District 4 varied in two of the subcategories of *unit planning, students' prior knowledge* and *unit sequence* (see Figure F8). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Two teachers based unit planning on their perceptions of students' prior knowledge in mathematics or reading without informal or formal assessment.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure F8. Unit planning, Grade 6, District 4.

Two teachers considered external factors such as the content of the statewide testing program when *sequencing instructional units*. The other teacher used the sequence of units recommended in teacher support materials. All of the teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended *pace for instruction*.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Becker worked through all of the problems so the thought processes and strategies involved were clear in her mind. She adjusted her plans to reflect students' performance on the previous lesson. If students exhibited a lack of understanding, she provided a similar example for more practice. If students picked up the concepts easily, she eliminated unnecessary review or moved ahead at a faster pace. Becker felt that the students had substantive discussions about mathematics. She explained:

> [Students are] always discussing. They're always sharing their ideas with their partners, with their groups. They like to come up to the board and actually write out what they did. They want to show other students how they got their answers, all the different ways to solve the problem. (Becker, Interview 9/23/99)

Becker valued small-group work: "I find that a lot of times [students] learn so much from the their partners. They also learn because when [students are] explaining [their thinking] to their partners, they often realize other insights" (Becker, Interview 9/23/99). Becker liked to pair students who had different mathematical abilities. Sometimes when two students of similar ability were productive working together, she made them partners. She explained further:

> I have [students] paired pretty much every day. Every once in a while they'll do a question on their own. But basically they're paired when we do bigger activities, it all depends on the activity. [Students are] arranged so they can just quickly turn their chairs around and form groups of four. (Becker, Interview 9/23/99)

Table F60
Subcategories of Lesson Planning: Becker, Kelvyn Park Middle School, District 4, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Downer worked through all of the problems, identifying several solution strategies for each problem. This process enabled her to be free of the teacher guide when she taught the lesson, to anticipate students' questions and answers, and to determine the best form of instruction. Downer thought her lesson planning was similar to her unit planning. She explained, "I see what activities I'm going to need, what I have to do first, and how I have to organize my class" (Downer, Interview 7/14/99). Downer valued small-group work, especially for students who were struggling:

> They realize that together as a group they can come up with the solution. One person might start the answer or the solution. Then if that person can't finish, somebody else might say, "Oh, but you could do this afterwards." Then another person might say, "But look, continue like this." This is what I hear them say as I move around the room. (Downer, Interview 7/1/99)

When groups had finished the activity, Downer brought all of them together to share their solution strategies. The leader for that day from each group presented the group's work and answered students' questions about their process. Downer also brought the groups together as a whole class when none of the groups were successful in working through the problem. She helped students work through the process by asking probing questions to guide their thinking. Downer provided extra practice for students to work on individually when she felt students did not have a firm understanding of the concept (Downer, Interview 7/14/99).

Table F61
Subcategories of Lesson Planning: Downer, Kelvyn Park Middle School, District 4, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard or students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Vega worked through all of the problems so that she would know the expected outcomes and typical solutions, the materials she needed to organize, and possible students' responses. She valued small-group work:

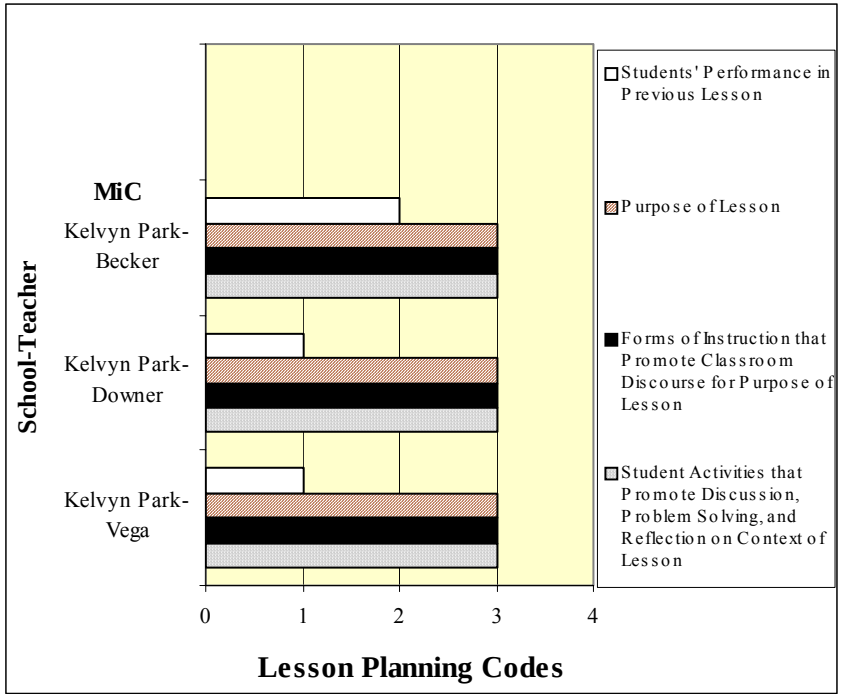
> It allows students to talk through what they're thinking and share ideas. Sometimes you'll have a kid who's completely lost in a group. Another kid can explain it. Even if the first student doesn't understand it completely, he/she winds up understanding something. And [small-group work] lets all of the students talk a little bit. You know they like to talk. (Vega, Interview 6/14/99)

Small groups were determined randomly unless members in a group could not work productively together, in which case Vega regrouped them. During small-group work, Vega walked among the groups and listened to their problem solving, answered questions, and redirected students if necessary. Vega found that she had to plan lessons differently for each class because the classes were so different. Kelvyn Park Middle School grouped students homogeneously, so a class of capable students could do more difficult problems at a faster rate than a class of less capable students. For the latter group, Vega omitted difficult problems, moved at a slower pace, and provided more teacher support (Vega, Interview 6/14/99).

Table F62
Subcategories of Lesson Planning: Vega, Kelvyn Park Middle School, District 4, Grade 6

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

In summary, the three sixth-grade teachers in District 4 varied in only one of the four subcategories of *lesson planning, students' performance in the previous lesson* (see Figure F9). With respect to students' performance in the previous lesson, one teacher made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The other two teachers planned in the same manner daily without considering students' previous performance. No variation was seen in teachers' attention to the *purpose of the lesson*. All teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure F9. Lesson planning, Grade 6, District 4.

No variation was seen in teachers' choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. All teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. Teachers also did not vary in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson.* For all teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect or summarize lesson concepts were not included in the lesson plan.

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Table F63
Unit Planning, District 1, Grade 6, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Addams-Gollen	1	1	2	4
Fernwood-Weatherspoon	2A	1	2	5
Von Humboldt-Brown	1	1	2	4
Von Humboldt-Parsons	1	1	2	4
Wacker-Lovell	3	1	1B	5
		— Conventional —		
Fernwood-Harrison	3	3	2	8

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table F64

Lesson Planning, District 1, Grade 6, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Addams-Gollen	2	2B	3	3	10
Fernwood-Weatherspoon	2	2B	2	2	8
Von Humboldt-Brown	2	2B	2	2	8
Von Humboldt-Parsons	2	3	2	2	9
Wacker-Lovell	3	3	3	3	12
			— Conventional —		
Fernwood-Harrison	2	2B	1	2	7

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.

2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.

3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.

4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.

Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.

2. *Overall curriculum plan in mind.*

A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.

B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).

3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.

2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)

3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.

4. *Forms of instruction that promote substantive conversation planned.* The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.

2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.

3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.

4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table F65

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	1	0	100	0	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	95	94	4	2	0
Wacker-Lovell	59	24	93	7	0	0
Conventional						
Fernwood-Harrison	15	93	100	0	0	0

Table F66

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	55	3	93	3	1
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	61	69	23	9	0
Wacker-Lovell	59	22	77	15	8	0
Conventional						
Fernwood-Harrison	15	40	33	33	33	0

Table F67

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	68	1	98	1	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	54	58	35	6	0
Wacker-Lovell	59	42	68	16	16	0
Conventional						
Fernwood-Harrison	15	40	50	50	0	0

Table F68

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	43	0	97	3	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	0				
Wacker-Lovell	59	54	44	38	13	6
Conventional						
Fernwood-Harrison	15	0				

Table F69

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	75	0	97	3	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	12	0	14	43	43
Wacker-Lovell						
Conventional						
Fernwood-Harrison	15	7	0	0	100	0

Table F70

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	16	0	73	9	18
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	65	5	19	38	38
Wacker-Lovell	59	5	33	0	33	33
Conventional						
Fernwood-Harrison	15	67	10	40	20	30

Table F71

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	37	8	92	0	0
Fernwood-Weatherspoon						
Von Humboldt-Brown						
Von Humboldt-Parsons	57	84	44	48	8	0
Wacker-Lovell	59	29	82	12	6	0
Conventional						
Fernwood-Harrison	15	60	56	33	11	0

Table F72

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	77	0	99	0	1
Fernwood-Weatherspoon						
Von Humboldt-Brown						
Von Humboldt-Parsons	57	56	16	25	25	34
Wacker-Lovell	59	42	40	12	28	20
Conventional						
Fernwood-Harrison	15	27	0	100	0	0

Table F73
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	74	1	97	2	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	32	50	33	17	0
Wacker-Lovell	59	58	47	24	15	15
Conventional						
Fernwood-Harrison	15	53	38	63	0	0

Table F74
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 1, Grade 6, 1987-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	74	0	98	2	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	26	67	33	0	0
Wacker-Lovell	59	42	52	20	16	12
Conventional						
Fernwood-Harrison	15	27	50	25	25	0

Table F75
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	38	0	100	0	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	23	15	31	38	15
Wacker-Lovell	59	31	56	28	17	0
Conventional						
Fernwood-Harrison	15	73	36	36	18	9

Table F76
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 1, Grade 6, 1987-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	15	0	5	40	55
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	23	0	15	0	85
Wacker-Lovell	59	5	33	0	0	67
Conventional						
Fernwood-Harrison	15	7	100	0	0	0

Table F77

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	4	0	100	0	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	12	71	29	0	0
Wacker-Lovell	59	7	50	25	25	0
Conventional						
Fernwood-Harrison	15	40	33	50	0	17

Table F78

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 1, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Addams-Gollen	136	1	0	100	0	0
Fernwood-Weatherspoon	NA	NA				
Von Humboldt-Brown	NA	NA				
Von Humboldt-Parsons	57	5	33	67	0	0
Wacker-Lovell	59	8	100	0	0	0
Conventional						
Fernwood-Harrison	15	13	100	0	0	0

Table F79

Teacher Level of Lesson Presentation and Development, District 1, Grade 6

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Addams-Gollen	9	0	1	0	1	2	3	1	1	4
Fernwood-Weatherspoon	9	0	0	0	0	1	2	5	1	5
Von Humboldt-Brown	9	1	4	1	1	2	0	0	0	2A
Von Humboldt-Parsons	8	0	0	7	0	1	0	0	0	2B
Wacker-Lovell	4	0	0	0	0	4	0	0	0	3B
Conventional										
Fernwood-Harrison	9	0	4	4	0	1	0	0	0	2B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation.* Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy.* A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding.* During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher.* The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding.* Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding.* Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics.* The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table F80

Nature of Mathematical Inquiry, Grade 6, District 1

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation										
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	*Mean*	***Rating***
MiC																																							
Addams-Gollen	4	2	2	2	1	4	1	2	4	2	2	2	2	2	2	1	1	4	3	2	2	1	1	1	1	2	4	1	1	3	3	1	1	1	2	4	*2.06*	***3***
Fernwood-Weatherspoon	4	4	2	4	1	4	4	4	4	2	2	2	4	1	2	2	2	2	2	1	1	2	2	4	2	4	3	2	1	3	1	1	1	3	1	2	*2.39*	***4***
Von Humboldt-Brown	3	1	1	1	1	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	1	1	1	1	1	1	1	2	1	1	1	3	1	2	1	1	*1.31*	***1***
Von Humboldt-Parsons	2	1	2	2	1	2	1	1	1	1	1	2	2	1	2	1	1	1	2	1	1	1	3	3	1	1	1	2	1	2	1	1	3	1	1	1	*1.44*	***2***
Wacker-Lovell	1	1	4	4	4					1	1	2	4	2					2	1	1	4	4					1	1	3	2	3					*2.30*	***4***
Conventional																																							
Fernwood-Harrison	1	1	1	1	2	1	1	1	1	1	2	1	1	2	1	1	1	1	1	1	2	2	1	1	1	2	1	1	1	2	1	1	3	1	1	1	*1.25*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table F81

Nature of Students' Explanations, Grade 6, District 1

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-Gollen	3	3	2	2	1	1	2	3	3	*2.22*
Fernwood-Weatherspoon	3	2	2	3	1	2	3	3	2	*2.33*
Von Humboldt-Brown	2	1	1	2	1	1	3	1	1	*1.44*
Von Humboldt-Parsons	2	1	1	1	2	2	1	1	1	*1.33*
Wacker-Lovell	2	1	3	2	3					*2.20*
Conventional										
Fernwood-Harrison	1	1	1	2	2	1	1	1	1	*1.22*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table F82
Elicitation of Multiple Strategies, Grade 6, District 1

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-Gollen	4	4	4	2	1	1	3	4	4	*3.00*
Fernwood-Weatherspoon	3	3	2	3	1	3	1	2	1	*2.11*
Von Humboldt-Brown	3	1	1	1	1	1	2	1	1	*1.33*
Von Humboldt-Parsons	1	1	1	1	1	3	1	1	1	*1.22*
Wacker-Lovell	1	1	2	2	1					*1.40*
Conventional										
Fernwood-Harrison	1	1	1	1	1	1	1	1	1	*1.00*

Elicitation of Multiple Strategies
This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.
1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. Strategies substantive element of instruction. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table F83

Nature of Student–Student Conversation, Grade 6, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-Gollen	4	1	2	4	1	1	4	1	4	*2.44*
Fernwood-Weatherspoon	2	2	1	4	1	1	1	3	1	*1.78*
Von Humboldt-Brown	1	1	1	1	1	1	1	3	1	*1.22*
Von Humboldt-Parsons	1	1	1	1	1	1	1	1	1	*1.00*
Wacker-Lovell	4	1	3	4	1					*2.60*
Conventional										
Fernwood-Harrison	1	1	1	1	1	1	1	1	1	*1.00*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table F84

Students' Collaborative Working Relationships, District 1, Grade 6

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-Gollen	4	1	1	4	1	1	3	1	3	*2.11*
Fernwood-Weatherspoon	2	2	1	3	1	1	3	3	1	*1.89*
Von Humboldt-Brown	1	1	1	1	1	1	1	2	N/A	*1.13*
Von Humboldt-Parsons	1	N/A	1	N/A	1	N/A	N/A	N/A	N/A	*1.00*
Wacker-Lovell	4	N/A	3	4	1					*3.00*
Conventional										
Fernwood-Harrison	N/A	1	1	N/A	1	1	N/A	1	N/A	*1.00*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table F85

Student Engagement during Instruction, Grade 6, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-Gollen	4	4	4	4	2	4	3	4	4	*3.67*
Fernwood-Weatherspoon	4	3	3	4	3	3	3	4	2	*3.22*
Von Humboldt-Brown	4	2	3	2	4	2	4	3	2	*2.89*
Von Humboldt-Parsons	1	1	1	2	1	3	1	1	3	*1.56*
Wacker-Lovell	4	4	4	4	3					*3.80*
Conventional										
Fernwood-Harrison	1	1	1	1	1	1	1	1	1	*1.00*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table F86
Unit Planning, District 2, Grade 6, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Guggenheim-Broughton	1	2	2	5
Guggenheim-Redling	1	2	2	5
Weir-Ferguson	1	1	1B	3
Weir-Kellner	3	1	2	6
		— Conventional —		
Newberry-Renlund	1	2	2	5
Von Steuben-Friedman	1	2	2	5

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
   A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
   B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
   A. The teacher follows the recommendations for pacing in teacher support materials.
   B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table F87

Lesson Planning, District 2, Grade 6, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Guggenheim-Broughton	2	2B	2	2	8
Guggenheim-Redling	3	3	4	2	12
Weir-Ferguson	2	2B	2	2	8
Weir-Kellner	2	3	2	3	10
			— Conventional —		
Newberry-Renlund	2	2B	1	2	7
Von Steuben-Friedman	2	2B	1	1	6

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.

2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.

3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.

4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.

Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.

2. *Overall curriculum plan in mind.*

A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.

B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).

3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.

2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)

3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.

4. *Forms of instruction that promote substantive conversation planned.* The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.

2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.

3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.

4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table F88

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	34	97	3	0	0
Guggenheim-Redling	87	2	0	100	0	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	73	77	18	5	0
Conventional						
Newberry-Renlund	162	47	1	97	1	0
Von Steuben-Friedman	146	24	37	54	9	0

Table F89

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	26	82	18	0	0
Guggenheim-Redling	87	49	9	86	2	2
Weir-Ferguson	NA	NA				
Weir-Kellner	30	60	28	50	22	0
Conventional						
Newberry-Renlund	87	49	9	86	2	2
Von Steuben-Friedman	162	51	1	96	4	0

Table F90

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	35	47	53	0	0
Guggenheim-Redling	87	38	30	70	0	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	67	10	80	10	0
Conventional						
Newberry-Renlund	162	42	4	94	1	0
Von Steuben-Friedman	146	27	3	41	38	18

Table F91

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	35	63	37	0	0
Guggenheim-Redling	87	54	0	96	4	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	13	0	25	50	25
Conventional						
Newberry-Renlund	162	36	0	98	2	0
Von Steuben-Friedman	146	17	0	52	16	32

Table F92

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	43	3	49	49	0
Guggenheim-Redling	87	52	0	93	7	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	40	0	50	50	0
Conventional						
Newberry-Renlund	162	36	0	98	2	0
Von Steuben-Friedman	146	10	0	43	21	36

Table F93

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	23	15	75	10	0
Guggenheim-Redling	87	44	3	68	13	16
Weir-Ferguson	NA	NA				
Weir-Kellner	30	50	0	27	60	13
Conventional						
Newberry-Renlund	162	51	0	95	2	2
Von Steuben-Friedman	146	26	11	61	16	13

Table F94

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	21	61	39	0	0
Guggenheim-Redling	87	16	36	50	7	7
Weir-Ferguson	NA	NA				
Weir-Kellner	30	53	25	63	13	0
Conventional						
Newberry-Renlund	162	27	2	98	0	0
Von Steuben-Friedman	146	27	3	50	30	18

Table F95

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	44	18	45	34	3
Guggenheim-Redling	87	3	0	67	33	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	93	4	61	32	4
Conventional						
Newberry-Renlund	162	46	1	95	4	0
Von Steuben-Friedman	146	3	0	60	20	20

Table F96

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	31	41	56	4	0
Guggenheim-Redling	87	57	2	90	8	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	67	0	45	45	10
Conventional						
Newberry-Renlund	162	49	1	97	1	0
Von Steuben-Friedman	146	22	0	75	19	6

Table F97

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	20	47	53	0	0
Guggenheim-Redling	87	55	0	94	4	2
Weir-Ferguson	NA	NA				
Weir-Kellner	30	30	0	33	67	0
Conventional						
Newberry-Renlund	162	40	0	92	6	2
Von Steuben-Friedman	146	25	3	61	19	17

Table F98

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	23	10	75	15	0
Guggenheim-Redling	87	11	0	50	40	10
Weir-Ferguson	NA	NA				
Weir-Kellner	30	23	0	29	57	14
Conventional						
Newberry-Renlund	162	40	0	92	6	2
Von Steuben-Friedman	146	10	7	60	20	13

Table F99

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 2, Grade 6, 1997-1998

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	10	11	56	33	0
Guggenheim-Redling	87	9	0	38	25	38
Weir-Ferguson	NA	NA				
Weir-Kellner	30	23	0	14	71	14
Conventional						
Newberry-Renlund	162	9	0	67	20	13
Von Steuben-Friedman	146	16	0	4	21	75

Table F100

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	1	0	100	0	0
Guggenheim-Redling	87	38	0	100	0	0
Weir-Ferguson	NA	NA				
Weir-Kellner	30	33	0	30	70	0
Conventional						
Newberry-Renlund	162	27	2	98	0	0
Von Steuben-Friedman	146	15	14	55	27	5

Table F101

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 2, Grade 6, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Broughton	86	0				
Guggenheim-Redling	87	49	0	98	0	2
Weir-Ferguson	NA	NA				
Weir-Kellner	30	0				
Conventional						
Newberry-Renlund	162	17	0	100	0	0
Von Steuben-Friedman	146	16	22	52	13	13

Table F102

Teacher Level of Lesson Presentation and Development, District 2, Grade 6

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Broughton	5	0	1	0	0	1	3	0	0	4
Guggenheim-Redling	6	0	0	0	0	0	1	4	1	5
Weir-Ferguson	7	1	1	1	0	1	3	0	0	3B
Weir-Kellner	7	0	2	0	2	0	2	1	0	3A
Conventional										
Newberry-Renlund	5	0	0	0	0	4	1	0	0	3B
Von Steuben-Friedman	8	0	5	0	0	3	0	0	0	3B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation.* Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy.* A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding.* During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher.* The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding.* Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding.* Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics.* The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table F103

Nature of Mathematical Inquiry, Grade 6, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																						
Guggenheim-Broughton	3	1	3	4	1	1				2	1	2	2	1	1				2	1	3	3	1	1				3	1	3	1	3	1				*1.88*	***3***
Guggenheim-Redling	4	3	3	3	4	3				4	2	2	4	3	2				3	2	2	3	2	1				3	3	1	3	3	3				*2.75*	***4***
Weir-Ferguson	1	2	3	3	3	3	1			1	1	2	2	2	2	1			1	2	2	1	3	2	1			2	3	2	1	3	1	1			*1.86*	***3***
Weir-Kellner	1	3	1	4	4	3	2			1	2	1	2	2	2	2			1	2	1	2	3	2	1			1	3	1	3	3	2	2			*2.04*	***3***
Conventional																																						
Newberry-Renlund	2	2	2	3	1					1	2	1	2	1					1	2	1	3	1					1	1	1	1	1					*1.50*	***2***
Von Steuben-Friedman	2	1	1	2	2	3	1	3		1	1	1	1	1	1	1	1		1	1	2	1	1	1	1	2		1	1	1	1	1	1	1	1		*1.28*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table F104

Nature of Students' Explanations, Grade 6, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	1	2	1	2	2				*1.50*
Guggenheim-Redling	2	2	1	3	3	3				*2.33*
Weir-Ferguson	1	1	1	1	1	1	1			*1.00*
Weir-Kellner	1	2	1	2	2	2	2			*1.71*
Conventional										
Newberry-Renlund	1	1	2	1	1					*1.20*
Von Steuben-Friedman	1	2	2	2	1	2	1	2		*1.63*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table F105

Elicitation of Multiple Strategies, Grade 6, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	2	1	1	4	1	1				*1.67*
Guggenheim-Redling	3	4	2	2	3	4				*3.00*
Weir-Ferguson	1	1	1	1	1	1	1			*1.00*
Weir-Kellner	1	2	1	2	4	2	1			*1.86*
Conventional										
Newberry-Renlund	1	1	1	1	2					*1.20*
Von Steuben-Friedman	1	2	1	1	2	1	1	1		*1.25*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table F106

Nature of Student–Student Conversation, Grade 6, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	2	2	2	2	1	1				*1.67*
Guggenheim-Redling	3	1	2	3	4	3				*2.67*
Weir-Ferguson	2	1	2	2	2	3	1			*1.86*
Weir-Kellner	2	2	1	3	3	2	1			*2.00*
Conventional										
Newberry-Renlund	1	3	2	1	1					*1.60*
Von Steuben-Friedman	1	1	1	1	1	1	3	1		*1.25*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table F107

Students' Collaborative Working Relationships, Grade 6, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	2	N/A	2	2	N/A	1				*1.75*
Guggenheim-Redling	3	1	1	2	4	3				*2.33*
Weir-Ferguson	1	1	1	2	2	1	1			*1.29*
Weir-Kellner	2	2	1	3	4	2	1			*2.14*
Conventional										
Newberry-Renlund	1	3	1	N/A	1					*1.50*
Von Steuben-Friedman	N/A	1	1	1	1	1	3	1		*1.29*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table F108

Student Engagement during Instruction, Grade 6, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	4	2	3	4	3	3				*3.17*
Guggenheim-Redling	4	4	3	4	4	3				*3.67*
Weir-Ferguson	3	4	3	3	4	3	3			*3.29*
Weir-Kellner	3	3	3	4	4	4	3			*3.43*
Conventional										
Newberry-Renlund	3	3	4	3	4					*3.40*
Von Steuben-Friedman	2	4	4	4	4	4	4	4		*3.75*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Explanation-Oriented	1 *4%*	2 *4%*	3	4 *7%*	5 *11%*		6 *4%*	7 *19%*	8	
Task-Oriented					1	2 *4%*	3	4		
Shift in Pedagogical Approach	1 *4%*	2 *15%*			3	4 *4%*	5	6 *22%*	7 *4%*	8
(27 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F10. Interactive Decisions, Gollen, Addams Middle School.

Explanation-Oriented	1	2	3 *18%*	4	5		6	7 *29%*	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1 *6%*	2			3 *6%*	4 *6%*	5 *6%*	6 *6%*	7 *24%*	8
(17 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F11. Interactive Decisions, Weatherspoon, Fernwood Middle School.

Explanation-Oriented	1	2 *42%*	3	4 *8%*	5 *8%*		6	7	8	
Task-Oriented					1	2 *8%*	3	4		
Shift in Pedagogical Approach	1 *25%*	2			3	4	5	6 *8%*	7	8
(12 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F12. Interactive Decisions, Brown, Von Humboldt Middle School.

Explanation-Oriented	1 *13%*	2 *13%*	3	4 *27%*	5		6	7	8	
Task-Oriented					1 *7%*	2 *7%*	3 *7%*	4		
Shift in Pedagogical Approach	1 *13%*	2 *13%*			3	4	5	6	7	8
(15 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F13. Interactive Decisions, Parsons, Von Humboldt Middle School.

Explanation-Oriented	1 *13%*	2 *20%*	3 *13%*	4 *7%*	5		6	7	8	
Task-Oriented					1	2 *20%*	3	4		
Shift in Pedagogical Approach	1	2 *7%*			3 *7%*	4	5 *7%*	6 *7%*	7	8
(15 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F14. Interactive Decisions, Lovell, Wacker Middle School.

Explanation-Oriented	1 *18%*	2 *6%*	3 *6%*	4 *18%*	5		6	7	8	
Task-Oriented					1 *6%*	2	3	4		
Shift in Pedagogical Approach	1 *18%*	2 *12%*			3 *18%*	4	5	6	7	8
(14 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F15. Interactive Decisions, Harrison, Fernwood Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		6%		*13%*	*13%*		6%	*13%*		
Task-Oriented					1	2	3	4		
						25%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
					13%	*6%*		*6%*		
(16 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F16. Interactive Decisions, Broughton, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
					10%		*10%*	*30%*		
Task-Oriented					1	2	3	4		
						10%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
							30%	*10%*		
(10 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F17. Interactive Decisions, Redling, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	11%	*11%*		*33%*			*11%*			
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	11%	*11%*					*11%*			
(9 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F18. Interactive Decisions, Ferguson, Weir Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	6%			*12%*				*12%*	*6%*	
Task-Oriented					1	2	3	4		
						6%		6%		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
		29%				*6%*		*6%*	*12%*	
(17 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F19. Interactive Decisions, Kellner, Weir Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	5%	*14%*		*29%*	*24%*		*5%*	*5%*		
Task-Oriented					1	2	3	4		
					5%					
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
		14%								
(21 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F20. Interactive Decisions, Renlund, Newberry Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		5%	*5%*	*40%*	*10%*					
Task-Oriented					1	2	3	4		
					5%					
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	5%	*15%*			*5%*	*10%*				
(20 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F21. Interactive Decisions, Friedman, Von Steuben Middle School.

Appendix G

Instruction

Grade 7

Grade 7

The composite variable instruction includes five major categories: unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice, and student pursuits during instruction. These categories were further subdivided into 19 subcategories, and an index was created for each. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the 19 indices, grouped by major categories, and the composite index for instruction, are described and illustrated with evidence from classroom observations, teacher interviews, teaching logs, and teacher journal entries. The ratings assigned to each teacher in Districts 3 and 4 on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in Districts 3 and 4. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 1

In District 1, seven Grade 7 teachers participated in the study. Four teachers used MiC, and three teachers used the conventional curricula already in place in their schools.

Unit Planning

In this study, three subcategories characterized unit planning: consideration of students' prior knowledge, unit sequence, and pace of instruction. Heath generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she skimmed the entire unit to get a general impression of the content and studied each section of the unit, sorting the concepts that might be entirely new to the students from those that were more familiar. Heath identified concepts or skills for which students needed additional practice and created the needed supplementary materials: "I don't feel this book gives enough examples over and over on the same mathematical concept in different contexts for [students] to be able to then realize that skill" (Heath, Interview 5/11/99). She planned to use MiC ancillary materials (*Number Tools*) and student activity sheets that accompanied MiC units when she thought they were appropriate. Heath selected activities and instructional groupings to provide variety and motivation for students. Because this was only the second year Heath had taught MiC, she did not feel confident in determining unit pacing prior to instruction, although she estimated that most units needed about a month to complete. Heath thought MiC was well aligned with the state standards and that district standardized tests reflected MiC content (Heath, Interview 5/11/99).

Table G1
Subcategories of Unit Planning: Heath, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Bartlett generally followed the recommended sequence of MiC units, but he sometimes selected units based on student interest or his desire to expose students to another content strand. When planning to teach a MiC unit, Bartlett became familiar with the unit by reading through it and doing some of the activities and problems. He identified the state standards that the unit addressed. He did not consider his students' prior knowledge when he planned the unit. He explained:

> It makes me nervous because I don't do as well as I should with it. I have difficulty knowing what they've gotten last year because our school is just really getting going with MiC. So I will ask the kids frequently, "How much of this did you see last year? Did you go over arrow language?" Kids who had Teacher A will say, "Yeah, we had lots of that." And kids who had Teacher B will say, "What's arrow language?" So that's a difficulty for me. (Bartlett, Interview 4/15/99)

Bartlett made adjustments to accommodate for students' lack of knowledge or skills when he started teaching a unit and used supplementary materials to address any problems. Because MiC was a new program for Bartlett, he did not feel confident in determining unit pacing prior to instruction. Bartlett thought unit planning with MiC was more rigid than planning to teach a chapter from a conventional mathematics textbook (Bartlett, Interview 4/15/99).

Table G2
Subcategories of Unit Planning: Bartlett, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	1B	Little or not consideration of pacing

Muldoon followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the unit to become familiar with the content and to know the goal "so I know what actually the thrust of the book is going to be about. I don't want any surprises in the middle" (Muldoon, Interview 4/15/99). Muldoon, who taught students with average or below average skills, identified anticipated problems students might have with the unit and made appropriate adjustments for them including minilessons on key concepts, extra skill practice, whole-class discussions, and additional time for the unit. Previous experience teaching MiC led Muldoon to determine the pace for instruction by multiplying the suggested number of days for completing a unit by three. However, this year she found she needed to modify the rule to multiply the number of days by four. Muldoon did not think more planning was involved in teaching MiC than a conventional textbook because: "Your kids are at a certain place, and you're using a certain material, and you have to fill in wherever they need it filled in" (Muldoon, Interview 4/15/99). She considered the state standards and district standardized tests as she planned to teach a unit. Muldoon emphasized the major standards addressed by the unit and provided assessments similar in format to the district tests (Muldoon, Interview 4/15/99).

Table G3
Subcategories of Unit Planning: Muldoon, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Burton generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she became familiar with the goals of the unit and the concepts to be presented by working through the entire unit section by section. She then divided the unit into manageable class periods and determined how to present the lessons. In the beginning of the year, Burton identified and adjusted for students' prior knowledge as she introduced the unit. As the year progressed and she become more aware of students' conceptual understandings and skill levels, she incorporated review, practice, or compacting activities (more concise lessons for capable students) in her unit plans. Occasionally she used conventional mathematics textbooks for resource material. Burton tried to follow the recommended pacing of MiC units, but made adjustments when needed: "If they say that a section should take three to five days, I try not to give [students] much past that five days, because otherwise I think that I would just get stuck in a section maybe and never get past it" (Burton, Interview 5/5/99). Burton thought that more time was needed to plan to teach MiC than a conventional textbook because: "It's a whole new way for me to look at math. This isn't how I learned math. So I really look through it a lot more than I probably would if it was just, okay, I have to teach this skill and [students are] going to do all these problems" (Burton, Interview 5/5/99). Burton thought MiC was well aligned with the state standards and that the district's new standardized tests reflected MiC content (Burton, Interview 5/5/99).

Table G4
Subcategories of Unit Planning: Burton, Wacker Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

St. James generally followed the recommended sequence of chapters in the conventional mathematics textbook he used. His primary objective was "to try and give [students] the broadest knowledge and exposure I can give them before they go into eighth grade" (St. James, Interview 5/14/99). When planning to teach a chapter, St. James browsed through the chapter and determined for each class concepts that would need only a brief review, concepts that could be combined, and parts of the chapter that would need to be expanded because of level of difficulty. This initial overview, along with his experience teaching the curriculum in the previous year, provided the basis for determining the pace of instruction. St. James thought the conventional mathematics textbook he used was well aligned with the district mathematics curriculum and the state standards. He considered it a complete mathematics program (St. James, Interview 5/14/99).

Table G5
Subcategories of Unit Planning: St. James, Addams Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of students abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Hodge used a holistic approach when he planned to teach a chapter from a conventional mathematics textbook. He knew the curriculum students had studied during the previous year and assumed the students were proficient with it. He also used the NCTM *Addenda Series* and the state standards as references. He commented:

> There's really no set plan. There is some kind of order that I came up with. I wanted the whole beginning of the year just to be getting [students] to think about numbers. We started off with number sense—just thinking about whether answers are reasonable and stuff like that. That's a good one to start off with because we've got to apply that all the way throughout the year actually. So we began on number sense. Then we applied it to fractions, decimals, and percents. We took whatever we needed from there. Then ratios came in and we started talking about algebra, equations. There's really not a set path. (Hodge, Interview 4/26/99)

Hodge planned to keep to a pace that he thought was reasonable, but left time for adjustments to facilitate student learning (Hodge, Interview 4/26/99).

Table G6
Subcategories of Unit Planning: Hodge, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of students abilities
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

Rubin generally followed the recommended sequence of chapters in the conventional mathematics textbook she used. When planning to teach a chapter, Rubin read through the unit to develop an understanding of major objectives and specific problems before she outlined lessons, assessments, and an anticipated pace. Rubin adjusted her initial plan after an informal assessment on the first day teaching the chapter. She explained:

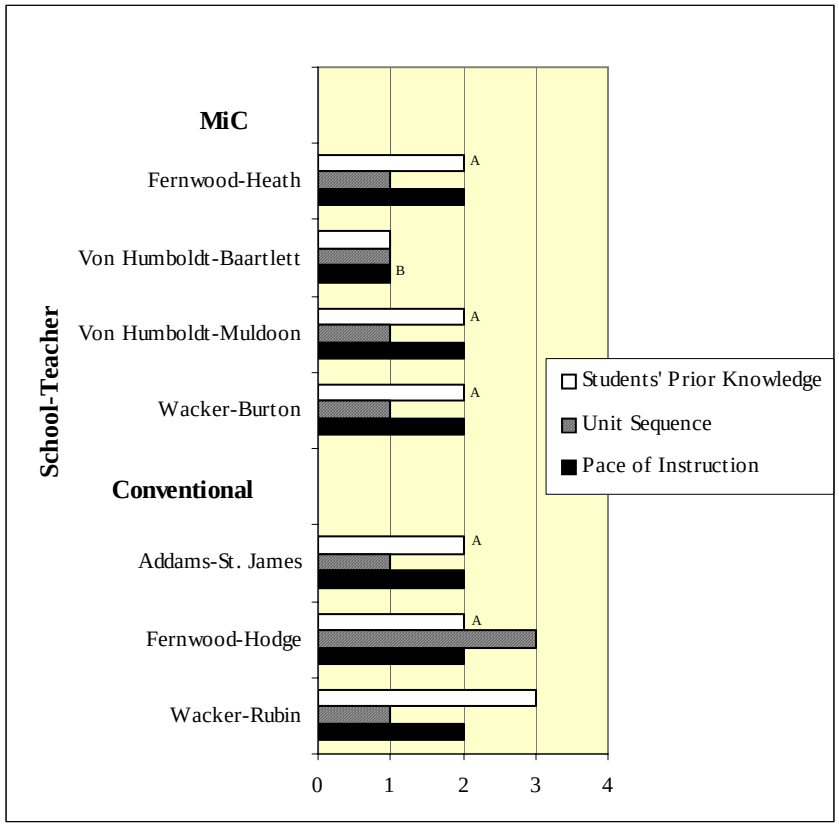
> I'll go ahead and do some planning. Then on the first day of the new chapter, I'll do an informal assessment just with questioning and maybe do some problems on the board and see if the kids can pick it up. From that I'll go into different areas. If they need help in one thing, I would go further into that. It's very informal at first and then I build on that information. (Rubin, Interview 5/3/99)

Rubin planned to use supplementary materials with any students who experienced difficulty with the chapter as it was presented in the textbook. Rubin thought the textbook was well aligned with the state standards. She felt, however, that the format of part of the district test was incongruent with the textbook's assessment format (Rubin 5/3/99).

Table G7
Subcategories of Unit Planning: Rubin, Wacker Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the seven seventh-grade teachers in District 1 varied in all three subcategories of unit planning (see Figure G1). With respect to students' prior knowledge when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Five teachers based unit planning on their perceptions of students' prior knowledge in mathematics or in reading without formal or informal assessment. One teacher followed the unit development in the teacher guide with little or no consideration of students' prior knowledge when planning to teach a unit.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table G58 in this appendix.

Figure G1. Unit planning, Grade 7, District 1.

One teacher varied from the other teachers in determining the sequence of instructional units by considering linkages across units of the same content strand. Most of the teachers used the sequence of units/chapters recommended in teacher support materials. Similarly, most teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction. One teacher did not plan unit pacing because the curriculum was unfamiliar to him.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Heath worked through all of the problems and read the teacher guide to check whether she had given the intended meaning to the problems and whether she understood the purpose and direction of the lesson. When there was a discrepancy in interpretation, she planned to include hints from the teacher guide during instruction. She also planned to emphasize portions of the lesson that would clarify its purpose and direction. Heath planned daily for individual lessons, taking into consideration students' performance on the previous lesson, group dynamics, and the conceptual complexity of the lesson. The form of instruction she chose was determined by the lesson and accompanying activities. Heath valued small-group work:

> Some students won't want to ask a question when it's with the whole class. When they're working with their small group, they'll ask more or they'll explain more to each other. They'll talk about it. Some [students] might have thought about it one way and another thinks about it another way. They can share that. (Heath, Interview 5/11/99)

Heath thought small groups of three or four worked best in her classroom. When the group was larger than four or when students worked in pairs, participation was uneven. She explained: "There's nothing wrong with pairs, but I just think it's better for me to get three or four in a group because then there will be more interaction. If somebody gets stuck there will be more than just one other to help" (Heath, Interview 5/11/99). Following small-group interactions, Heath wanted students to work individually so that each student could think through and apply what the group had done (Heath, Interview 5/11/99).

Heath used teacher presentation on 59% of the reported days (see Tables G60-G65 in this appendix). Most of the time (59%) it was given equal emphasis with other instructional forms and less than 15% of class time on most of the remaining days. Heath used small-group work on 48% of the reported days and it was given significant amounts of class time. At least half of the class time on 57% of the days and equal emphasis with other instructional formats on the remaining days was devoted to small-group work. Heath used warm-up activities 41% of the reported days, but these activities involved less than 15% of class time. Review of previous material, used on 34% of the reported days, received varying amounts of class time: at least half of the class time on 30% of the days, equal emphasis with other instructional formats on 40% of the days, and less than 15% of class time on 30% of the days. When Heath used independent practice (on 21% of the reported days) it was given at least half of the class time on 50% of the days and equal emphasis with other instructional forms on the remaining days (Heath, Teacher Log 1998-99).

Four student activities were important elements in Heath's instruction: listening to the teacher or taking notes (59% of the reported days), investigating problems (55%), discussing answers and solution strategies (52%), and participating in whole-class discussions (41%; see Tables G66-G73 in this appendix). Investigating problems was given the most class time: at least half of the class time on 50% of the days and equal emphasis with other student activities on the remaining days. Discussing answers and solution strategies was also given a considerable amount of class time: at least half of the class period on 26% of the days and equal emphasis with other student activities on most of the remaining days. Listening to the teacher or taking notes was given equal emphasis with other student activities on 47% of the days and less than 15% of class time

on 41% of the days. Participating in whole-class discussions was given equal emphasis with other student activities on 50% of the days and less than 15% of class time on the remaining days. Students reflected on or summarized lesson concepts on fewer days (24%) and for less class time. Heath's students did not begin homework during class time (Heath, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Heath reported in her teacher logs. As an example of a lesson observed in Heath's class, the lesson on 3/16/99 included: individual warm-up activity (12 minutes); whole-class discussion of warm-up answers (5 minutes); whole-class discussion of homework (15 minutes); whole class, teacher introduction of lesson (8 minutes); small-group activity, make cube (23 minutes); teacher presentation (4 minutes); small-group activity, volume (14 minutes); and whole-class discussion of answers and close (8 minutes; Heath, Observation 3/16/99).

Table G8
Subcategories of Lesson Planning: Heath, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Bartlett worked through the problems to determine the activities and problems students would do each day for a week, anticipating that he would modify his plans to adjust for student performance on each lesson. Bartlett thought direct teaching to the whole group followed by individual work was the most effective form of instruction for him. However, he explained:

> There are times when we're introducing the concept or when we're early on into the piece that I'd rather have [students] working together because they can help each other in feedback. A lot of times I'll have a good reader with a struggling reader because the biggest problem that I'm having is the reading. So they can help each other with what the lesson says. I was trying groups of four, but I have them now in groups of two. I have them working with each other when they need help discussing. (Bartlett, Interview 4/15/99)

As an example of a lesson observed in Bartlett's class, the lesson on 9/22/98 included: teacher presentation (8 minutes); individual activity (5 minutes); small-group activity, construction (8 minutes); whole-class discussion (17 minutes); individual activity, construction (7 minutes) whole-

class discussion (6 minutes); individual activity, pyramid (4 minutes); whole-class discussion, stability (2 minutes); and whole class, assignment of homework and closure (6 minutes; Bartlett, Observation 9/22/98). Teacher Log data for Bartlett were unavailable.

Table G9
Subcategories of Lesson Planning: Bartlett, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Muldoon looked over the notes she made during unit planning, which included the thrust of the lesson, possible problems with format or language, sections with unclear directions, and concepts that needed to be further developed. She considered the students' performance on the previous day's lesson, noting whether she needed to reteach or review a particular concept or skill or skip repetitious activities. Muldoon thought it was valuable for students to work in small groups, especially when the lesson required physical activity. However, she thought small groups were not valuable when "the work is over [students'] heads. They'll start fooling around. They can't get it. They're just not interested. Then the groups are useless at that point" (Muldoon, Interview 4/15/99). When Muldoon planned for group work, she determined the group membership, the directions and expectations, and consequences for not following directions. Muldoon recognized that in order to have successful group work, each group needed a leader, an initiator. She explained, "Then I could put one kid in each group and there would be somebody that could at least initiate an idea" (Muldoon, Interview 4/15/99). Because she taught students with average or below average math skills, she felt that it was difficult to find enough group leaders (Muldoon, Interview 4/15/99).

Four instructional forms were important elements in Muldoon's classes: review of previous material (on 70% of the reported days), whole-class discussion (66%), warm-up activities (63%), and teacher presentation (55%; see Tables G60-G65 in this appendix). Whole-class discussion was given the most class time: at least half of the class period on 44% of the days, equal emphasis with other instructional forms on 22% of the days, and less than 15% of class time on 34% of the days. Teacher presentation and review of previous material received similar amounts of class time. Teacher presentation was given at least half of the class period on 17% of the days, equal emphasis with other instructional forms on 33% of the days, and less than 15% of class time on the remaining days. Review of previous material was given at least half of the class period on 10% of the days, equal emphasis with other instructional forms on 30% of the days, and less than 15% of class time on the remaining days. Warm-up activities were always given less than 15% of class time. Muldoon often used independent practice (on 41% of the reported days) for a significant

amount of time: at least half of the class period on 52% of the days, equal emphasis with other instructional forms on 26% of the days, and less that 15% of class time on 23% of the days. Muldoon chose small-group work on fewer days (22%; Muldoon, Teacher Log 1998-99).

Five student activities were important elements in Muldoon's instruction: investigating problems (on 75% of the reported days), discussing answers and solution strategies (72%), participating in whole-class discussions (72%), listening to the teacher or taking notes (67%), and reflecting on or summarizing lesson concepts (63%; see Tables G66-G73 in this appendix). Of the five activities, investigating problems was given the most class time: at least half of the class period on 42% of the days, equal emphasis with other student activities on 44% of the days, and less that 15% of class time on the remaining days. Listening to the teacher or taking notes and reflecting on or summarizing lesson concepts were frequently given less than 15% of class time. The other two student activities were given similar amounts of class time. Participating in whole-class discussions was given at least half of the class period on 32% of the days, equal emphasis with other student activities on 33% of the days and less than 15% of class time on 35% of the days. Discussing answers and solution strategies was given at least half of the class period on 26% of the days, equal emphasis with other student activities on 40% of the days, and less than 15% of class time on 35% of the days. On approximately one third of the days, students practiced computation (37%). This activity was given at least half of the class period on 14% of the days, equal emphasis with other student activities on 29% of the days, and less than 15% of class time on 57% of the days. Students seldom began homework during class time (12%) and frequently were given less than 15% of class time for this activity. Students took a test or quiz on only 5% of the reported days (Muldoon, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Muldoon reported in her teacher logs. As an example of a lesson observed in Muldoon's class, the lesson on 9/24/98 included: housekeeping duties (6 minutes); whole-class discussion (24 minutes); partner activity (23 minutes); and clean up (1 minute; Muldoon Observation 9/24/98).

Table G10
Subcategories of Lesson Planning: Muldoon, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with the content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach a MiC lesson, Burton read through each problem and estimated the answer. Then she checked her estimate against the answer in the teacher guide. If there was a discrepancy, she worked through the problem and identified the misunderstanding. She indicated in her

plans the places in which students might have misunderstandings and how she might direct their thinking in order to avoid misinterpretation. Burton considered students' performance on previous lessons. She explained:

> I usually look at what we did during the past week. If we didn't get where I had planned to get, then I had to readjust that. Usually at the end of every day, I evaluate if we got through what I wanted them to get through, if I needed to go back. Maybe they didn't understand it as well as they would need to later on in the book. So sometimes I have to readjust a little bit. So I do a little bit every day on that. (Burton, Interview 5/5/99)

Burton valued small-group instruction:

> A lot of times [students] might not understand what I said. Maybe the person sitting next to them can tell them. Or maybe a [student] is embarrassed to raise his/her hand and ask me. But [that student] won't hesitate to ask somebody sitting next to him/her. It makes a student feel a little bit more confident to know he/she can talk answers over with the persons sitting nearby before having to share them with anybody else or share them with the teacher. {Students] seem to like that better than when they're on their own. They actually seem to do more work when they're working with partners than when they are separated. (Burton, Interview 5/5/99)

This year MiC was a new mathematics program for students in Burton's class. Students were unsure about how to complete the activities. Consequently, Burton planned for large-group instruction until students were more confident with MiC. Burton then planned to have large-group instruction for half of the class period and small-group or partner instruction for the other half of the class period. Burton facilitated discussions by making a seating plan that placed students who were capable in math next to students who would benefit from capable partners (Burton, Interview 5/5/99).

As an example of a lesson observed in Burton's class, the lesson on 2/23/99 included: housekeeping duties (4 minutes); whole-class activity, teacher and students correct homework (7 minutes); individual activity (4 minutes); teacher presentation (19 minutes); individual activity (7 minutes); and whole-class, teacher-led discussion and homework assignment (2 minutes; Burton, Observation 2/23/99). Teacher Log data for Burton were unavailable.

Table G11
Subcategories of Lesson Planning: Burton, Wacker Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with the content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included n the lesson plan

When planning to teach individual lessons using a conventional textbook, St. James worked all of the word problems and looked to see how the problems related to the following lesson. He focused on the mathematics vocabulary and on the difficulties he anticipated individual students might have. St. James began each lesson with a quick review of the previous lesson. He explained:

> With a five-minute check, I'll rehearse or restate what we did yesterday. And instead of spending the time I spent the first day on it, I'll fire questions at them. It's like building blocks. You have to make sure [students] are with you on this page before you take them to the next page. So I will give them a capsulated version before we jump into the next lesson everyday. (St. James, Interview 5/14/99)

St. James thought direct teaching to the whole class, with limited time for discussion, was usually the most effective form of instruction. Occasionally he planned for students to work in pairs or small groups. The sizes of groups varied by class, depending on work habits, mathematics skills, learning styles, and tolerance for noise. He believed there were times students could learn from each other. But because he was concerned that students working in groups could learn misinformation, he always followed group work with a check for concept and/or skill comprehension (St. James, Interview 5/14/99).

Three instructional forms were important elements in St. James' classes: review of previous material (on 72% of the reported days), warm-up activities (66%), and teacher presentation (61%; see Tables G60-G65 in this appendix). Each of these instructional forms was frequently given equal emphasis with other instructional forms. When St. James used whole-class discussion (36% of the reported days), it was generally given equal emphasis with other instructional forms. The two instructional forms least used by St. James were independent practice (13%) and small-group work (5%; St. James, Teacher Log, 1998-99).

Four student activities were used frequently by St. James: listening to the teacher or taking notes (on 65% of the reported days), discussing answers and solution strategies (63%), whole-class discussion (58%), and investigating problems (57%; see Tables G66-G73 in this appendix). Each activity was given equal emphasis with other student activities on most of the reported days. Students reflected on or summarized lesson concepts (44%) and practiced computation (28%). These activities were given equal emphasis with other activities on most occasions. Students rarely took a test or quiz (12%) or began homework (5%; St. James, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes did not support the information St. James reported in his teacher logs. As an example of a lesson observed in St. James' class, the lesson on 10/21/98 included: housekeeping duties (6 minutes); teacher presentation (6 minutes); whole-class, teacher-led activity, check true-and-false items (7 minutes); whole-class, teacher-led vocabulary review (5 minutes); whole class, correct assignment, teacher worked problems when many students had errors (8 minutes); whole-class discussion of order of operations (7 minutes); whole class, complete correcting assignment (4 minutes), and assign homework (2 minutes; St. James, Observation 1021/98).

Table G12
Subcategories of Lesson Planning: St. James, Addams Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	1	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	1	Investigation of problems and discussion of mathematical ideas seldom planned for the lesson

When planning to teach individual lessons from a conventional textbook, Hodge looked at specific examples, skills, and applications in contrast to the general topics he reviewed during unit planning. Hodge looked at the problems in the lesson and sequenced them from easier to more difficult. He explained:

> I'll look at the example [in the book] and see what they did, step by step. If it's a homework problem then I'll ask myself if I think [students] will be able to figure it out in their heads. There were a couple problems that I thought [students] might struggle with, but I just wanted them to do the problems anyway just to see who knows what they are doing. (Hodge, Interview 4/26/99)

If students' struggled during the previous lesson, Hodge built in more review. In most situations, Hodge preferred whole-class direct teaching as the primary form of instruction for his classes and whole-class discussions were generally open. When Hodge chose small-group instruction, the

size of the groups depended on the activity. If students were to compare answers, they most likely worked with the person sitting next to them. Activities that involved gathering information or doing a statistical analysis needed more structured heterogeneous groups (Hodge, Interview 4/26/99).

Hodge began virtually every class with a warm-up activity (on 98% of the reported days; see Tables G60-G65 in this appendix). Warm-up activities were given less than 15% of class time on 55% of the days, but on other occasions they were given equal emphasis with other instructional forms (30%), or at least half of the class period (14%). Hodge often reviewed previous material (on 81% of the reported days) and used teacher presentation as the primary instructional form (78%). Each of these was given significant amounts of class time. Review of previous material was given at least half of the class period on 42% of the days, equal emphasis with other instructional forms on 50% of the days, and less than 15% of class time on the remaining days. Teacher presentation was given at least half of the class period on 84% of the days and equal emphasis with other instructional forms on 13% of the days. On 44% of the reported days, Hodge chose independent practice, usually giving it equal emphasis with other instructional forms (65%). When Hodge used whole-class discussion (on 36% of the reported days) it was most often given equal emphasis with other instructional forms. The instructional form used least by Hodge was small-group work (28%), but it was given considerable amounts of class time when used: at least half of the class period on 38% of the days and equal emphasis with other instructional forms on 53% of the days (Hodge, Teacher Log 1998-99).

Students in Hodge's class often listened to the teacher or took notes (on 79% of the reported days), discussed answers and solution strategies (76%), participated in whole-class discussions (74%), investigated problems (66%), and reflected on or summarized lesson concepts (59%; see Tables G66-G73 in this appendix). Each activity was given a significant amount of class time, frequently at least half of the class period or equal emphasis with other student activities. Students seldom practiced computation (15%), took a test or quiz (14%), and began homework (1%; Hodge, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Hodge reported in his teacher logs. As an example of a lesson observed in Hodge's class, the lesson on 2/2/99 included: housekeeping duties (5 minutes) individual activity, problem of the day (7 minutes); whole class, teacher went over answers to problem of the day (8 minutes); whole class, teacher checked homework (14 minutes); teacher presentation of new activity (17 minutes); small-group activity (14 minutes); and whole class, teacher reviewed for quiz (24 minutes; Hodge, Observation 2/2/99).

Table G13
Subcategories of Lesson Planning: Hodge, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual lessons from a conventional textbook, Rubin worked through the example problems. If students struggled with the previous lesson, she built in review or tried to present the concept or skill differently. If students exhibited facility with the previous lesson, she eliminated some of the practice pages from her plans. Rubin thought whole-group direct instruction was the most effective form of instruction. She tried to engage students in whole-class discussions, but thought generally the discussions were student–teacher not student–student. Rubin valued small-group work: "Some kids may not be comfortable talking in front of a full class, but they'll talk with a group of two to six. They won't feel as though I might find out if they're wrong" (Rubin, Interview 5/3/99). Rubin planned for students to practice concepts and skills in small groups, using supplementary materials provided by the textbook after she had presented the lesson to the whole group. She determined the group membership to avoid dysfunctional groups and usually required each student to hand in completed work to insure each student's participation (Rubin, Interview 5/3/99).

Two important elements in Rubin's instruction were warm-up activities, reported on 79% of the days, and review of previous material, reported on 74% of the days (see Tables G60-G65 in this appendix). However, less class time was given to warm-up activities, generally less than 15% of class time, in contrast to equal emphasis with other instructional forms on 57% of the days for review of previous material. Rubin chose teacher presentation (53%) and independent practice (47%) on approximately half of the reported days. Each was given a considerable amount of class time. Teacher presentation was given at least half of the class period on 40% of the days, equal emphasis with other instructional forms on 50% of the days, and less than 15% of class time on the remaining days. Independent practice was given at least half of the class period on 17% of the days, equal emphasis with other instructional forms on 72% of the days, and less than 15% of class time on the remaining days. On approximately one fourth of the days, Rubin used small-group work and gave it a significant amount of class time, at least half of the class period on 67% of the days and equal emphasis with other instructional forms on the remaining days (Rubin, Teacher Log 1998-99).

Rubin's students often discussed answers and solution strategies (on 71% of the reported days), practiced computation (66%), listened to the teacher or took notes (61%), participated in whole-class discussions (58%), and reflected on or summarized lesson concepts (47%; see Tables G66-G73 in this appendix). Discussion of answers and solution strategies, computation practice, and listening to their teacher or taking notes were

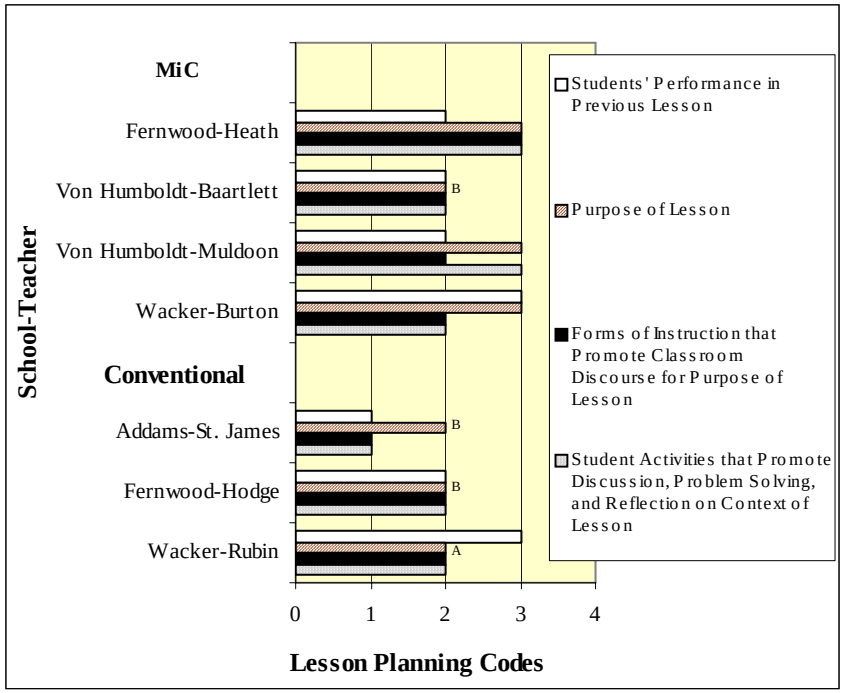
given at least half of the class period on about half of the reported days. Students' participation in whole-class discussions was given at least half of the class period on 36% of the days, equal emphasis with other student activities on 55% of the days, and less than 15% of class time on the remaining days. Students spent less time reflecting on or summarizing lesson concepts: less than 15% of class time on 61% of the days, equal emphasis with other student activities on 22% of the days, and at least half of the class period on 17% of the days. Students investigated problems on 39% of the days and began homework, on 34% of the days. Investigating problems was given a significant amount of class time, at least half of the class period on 47% of the days and equal emphasis with other activities on the remaining days. In contrast, beginning homework was usually given less than 15% of class time. Students took tests or quizzes less frequently (13%; Rubin, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Rubin reported in her teacher logs. As an example a lesson observed in Rubin's class, the lesson on 2/23/99 included: housekeeping duties (2 minutes); whole class, correct homework and teacher presentation of new lesson (26 minutes); and partner work, worksheet (13 minutes; Rubin, Observation 2/23/99).

Table G14
Subcategories of Lesson Planning: Rubin, Wacker Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the seven seventh-grade teachers in District 1 varied in all four subcategories of *lesson planning* (see Figure G2). With respect to *students' performance in the previous lesson*, two teachers' decisions focused on students' understanding of the mathematical content of the lesson, using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. Four teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The remaining teacher planned in the same manner daily without considering students' previous performance. Variation was also seen in teachers' attention to the *purpose of the lesson*. Three teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Three teachers planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context. The other teacher selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.

Figure G2. Lesson planning, Grade 7, District 1.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. Five teachers included the use of whole-class discussion and small-group work; but the focus was on completing tasks rather than on developing substantive conversation. The remaining teacher seldom planned forms of instruction that encouraged classroom discourse. The teachers also varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For two teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. Four teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered. The remaining teacher seldom planned for investigation of problems and discussion of mathematical ideas.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Heath. The evidence gathered for lesson presentation and development for Heath ranged from Level 2A to 5. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table G74 in this appendix). An overall rating of 3 was assigned for Heath for the nature of inquiry that transpired during instruction (see Table G75). Heath was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure G8). For Heath, 66% of the decisions were coded as reflective of good standard pedagogy, 11% were most aligned with teaching for understanding, and 22% were least aligned with teaching for understanding.

Across all observations for Heath, the mean rating for the index on students' explanations was 1.38, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Heath on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G15
Overall Ratings for Mathematical Interaction for Heath, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to develop conceptual understanding
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.38	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Bartlett. The evidence gathered for lesson presentation and development for Bartlett ranged from Level 2A to 5. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table G74 in this appendix). An overall rating of 2 was assigned for Bartlett for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G75). Bartlett was assigned Level 1 on the composite index for interactive decision-making,

indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G9). For Bartlett, 63% of the interactive decisions were coded as least aligned with teaching for understanding, and 37% were most aligned with teaching for understanding.

Across all observations for Bartlett, the mean rating for the index on students' explanations was 1.67, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.67, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Bartlett on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G16
Overall Ratings for Mathematical Interaction for Bartlett, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to teach for understanding
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.67	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.67	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Muldoon. The evidence gathered for lesson presentation and development for Muldoon ranged from Level 2A to 3B. An overall rating of Level 3B was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table G74 in this appendix). An overall rating of 2 was assigned for Muldoon for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G75). Muldoon was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G10). For Muldoon, all of the interactive decisions were coded as least aligned with teaching for understanding.

Across all observations for Muldoon, the mean rating for the index on students' explanations was 1.33, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.11, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Muldoon on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G17
Overall Ratings for Mathematical Interaction for Muldoon, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	2	Limited attention to teaching for understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.33	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.11	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Burton. The evidence gathered for lesson presentation and development for Burton were all at Level 3B, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table G74 in this appendix). An overall rating of 4 was assigned for Burton for the nature of inquiry that transpired during instruction, indicating in-depth exploration of mathematics occurred during lessons (see Table G75). Burton was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G11). For Burton, 60% of the interactive decisions were coded as least aligned with teaching for understanding, 20% were reflective of good standard pedagogy, and 20% were most aligned with teaching for understanding.

Across all observations for Burton, the mean rating for the index on students' explanations was 1.80, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Burton on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G18
Overall Ratings for Mathematical Interaction for Burton, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Emphasis on review
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.80	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

St. James. The evidence gathered for lesson presentation and development for St. James ranged from Level 2A to 3B. An overall rating of Level 2A was assigned, indicating that a major portion of the class period was devoted to review (see Table G74 in this appendix). An overall rating of 2 was assigned for St. James for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G75). St. James was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G12). For St. James, 77% of the interactive decisions were coded as least aligned with teaching for understanding, 12% were reflective of good standard pedagogy, and 12% were most aligned with teaching for understanding.

Across all observations for St. James, the mean rating for the index on students' explanations was 1.22, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.22, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for St. James on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G19
Overall Ratings for Mathematical Interaction for St. James, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2A	Emphasis on review
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.22	Answers only
Elicitation of Multiple Strategies	1.22	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Hodge. The evidence gathered for lesson presentation and development for Hodge ranged from Level 3B to 5. An overall rating of Level 3B was assigned, indicating that strategies or procedures were demonstrated and students practiced them in a rote fashion (see Table G74 in this appendix). An overall rating of 3 was assigned for Hodge for the nature of inquiry that transpired during instruction, indicating attention to teaching mathematics for understanding (see Table G75). Hodge was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure G13). For Hodge, 55% of the interactive decisions were coded as least aligned with teaching for understanding, 12% were reflective of good standard pedagogy, and 33% were most aligned with teaching for understanding.

Across all observations for Hodge, the mean rating for the index on students' explanations was 1.50, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Hodge on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G20
Overall Ratings for Mathematical Interaction for Hodge, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	2	More emphasis on standard pedagogy
Nature of Student Explanations	1.50	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Rubin. The evidence gathered for lesson presentation and development for Rubin was all at Level 3B, indicating that a major portion of the class period was devoted to review (see Table G74 in this appendix). An overall rating of 1 was assigned for Rubin for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G75). Rubin was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G14). For Rubin, 84% of the interactive decisions were coded as least aligned with teaching for understanding, 11% were reflective of good standard pedagogy, and 6% were most aligned with teaching for understanding.

Across all observations for Rubin, the mean rating for the index on students' explanations was 1.11, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G76). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were generally not elicited from students (see Table G77). A rating of 1 was assigned for Rubin on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G21
Overall Ratings for Mathematical Interaction for Rubin, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.11	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.00	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the six Grade 7 teachers in District 1 varied from Level 2A to Level 4 in *lesson presentation and development* (see Table G22). Two MiC teachers were assigned Level 4, indicating that attempts were made to develop a conceptual basis for the mathematical content. Four teachers (two MiC) were assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. The remaining teacher using a conventional curriculum was assigned Level 2A, indicating that a major portion of the lesson was devoted to review of a previous lesson, homework, or warm-up activity.

Table G22
Nature of Mathematical Interaction, Grade 7 Teachers in District 1, 1998-1999

School-Teacher	**Lesson Presentation and Development**	**Nature of Inquiry**	**Teachers' Interactive Decisions**	**Nature of Student Explanations**	**Elicitation of Multiple Strategies**	**Lesson Closure, Reflection, or Summary**	**Total**
			— MiC —				
Fernwood-Heath	4	3	3	1.38	1.50	1	13.88
Von Humboldt-Bartlett	4	2	1	1.67	1.67	1	11.34
Von Humboldt-Muldoon	3B	2	1	1.33	1.11	1	9.44
Wacker-Burton	3B	4	1	1.80	1.00	1	11.80
			— Conventional —				
Addams-St. James	2A	1	1	1.22	1.22	1	7.44
Fernwood-Hodge	3B	3	2	1.50	1.50	1	12.00
Wacker-Rubin	3B	1	1	1.11	1.00	1	8.11

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 4. One MiC teacher was assigned Level 4, indicating that in-depth exploration of mathematics occurred during lessons. Two teachers (one MiC) were assigned Level 3, indicating that attention was given to conceptual understanding during instruction. Two MiC teachers were assigned Level 2, indicating that limited attention was

given to conceptual understanding during instruction. The remaining two teachers using a conventional curriculum were assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to *teachers' interactive decisions*, teachers varied from Level 1 to Level 3. One MiC teacher was assigned Level 3, indicating that her interactive decisions provided greater attention to good standard pedagogy. One teacher using a conventional curriculum was assigned Level 2, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding. The remaining teachers (three MiC and two using conventional curricula) were assigned Level 1, indicating that their interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.11 to 1.80, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 1.67. In genera, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, teachers offered few, if any, opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The seventh-grade teachers in District 1 varied in two of the three subcategories of classroom assessment (see Tables G23-G26). With respect to the evidence sought during classroom assessment, one MiC teacher maintained an underdeveloped process orientation. Five teachers (two MiC) sought procedural competence of student learning. The remaining MiC teacher sought little evidence of student learning. Feedback in the classes of all these teachers was very teacher-directed. Two MiC teachers provided low-level closed feedback. The remaining teachers (two MiC) provided feedback that allowed students to know whether answers were correct or incorrect.

Table G23
Evidence Sought through Classroom Assessment Practice, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Fernwood-Heath	2	Procedural competence
Von Humboldt-Bartlett	3	Underdeveloped process orientation
Von Humboldt-Muldoon	2	Procedural competence
Wacker-Burton	1	Limited evidence
Conventional		
Addams-St. James	2	Procedural competence
Fernwood-Hodge	2	Procedural competence
Wacker-Rubin	2	Procedural competence

Table G24
Feedback Coherence and Purpose, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Fernwood-Heath	2	Teacher-directed feedback
Von Humboldt-Bartlett	2	Teacher-directed feedback
Von Humboldt- Muldoon	2	Teacher-directed feedback
Wacker- Burton	2	Teacher-directed feedback
Conventional		
Addams-St. James	2	Teacher-directed feedback
Fernwood-Hodge	2	Teacher-directed feedback
Wacker-Rubin	2	Teacher-directed feedback

Table G25
Feedback Content, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Fernwood-Heath	3	Low-level closed feedback
Von Humboldt-Bartlett	3	Low-level closed feedback
Von Humboldt- Muldoon	2	Answer-only feedback
Wacker- Burton	2	Answer-only feedback
Conventional		
Addams-St. James	2	Answer-only feedback
Fernwood-Hodge	2	Answer-only feedback
Wacker-Rubin	2	Answer-only feedback

Table G26
Classroom Assessment, Grade 7, District 1

School-Teacher	Evidence Sought	Feedback Coherence and Purpose	Feedback Content	Total
		— MiC —		
Fernwood-Heath	2	2	3	7
Von Humboldt-Bartlett	3	2	3	8
Von Humboldt-Muldoon	2	2	2	6
Wacker-Burton	1	2	2	5
		— Conventional —		
Addams-St. James	2	2	2	6
Fernwood-Hodge	2	2	2	6
Wacker-Rubin	2	2	2	6

See index next page.

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Heath ranged from Level 1 to Level 4 (see Table G78 in this appendix). Although the mean rating across observations was 1.63, on five of the eight observations student–student conversation was rated at Level 1, indicating that student–student conversation on most occasions was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Heath are described. The observer noted, "There was no exchange between peers in small groups or as a part of the large-group discussions. All conversation was between the teacher and one student at a time" (Heath, Observation 10/26/98). In this example, students conversed with the teacher, but student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Bartlett ranged from Level 1 to Level 2 (see Table G78 in this appendix). The mean rating across observations was 1.11, indicating that student–student conversation on most occasions was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Bartlett are described. The observer noted, "There were no exchanges between peers in small groups or in the large group. All interchange was between the teacher and one student at a time" (Bartlett, Observation 5/24/99). In this example, students conversed with the teacher, but student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Muldoon ranged from Level 1 to Level 4 (see Table G78 in this appendix). While the mean rating across observations was 1.56, on seven of the nine observations student–student conversation was rated at Level 1, indicating that student–student conversation on most occasions was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Muldoon are described. The observer noted, "There were no exchanges between peers in small groups or as a formal part of the large group. Students did not discuss mathematics with each other" (Muldoon, Observation 5/27/99). In this example, student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Burton were all at Level 1 (see Table G78 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Burton are described. The observer noted, "The students did not discuss mathematics with their peers. They only responded to the teacher's questions" (Burton, Observation 10/26/98). In this example, students conversed with their teacher, but student–student conversation did not take place.

The index ratings about the nature of student–student conversation for St. James were all at Level 1 (see Table G78 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by St. James are described. The observer noted, "The only conversations were between the teacher and the students. There was no

discourse between students" (St. James, Observation 10/21/98). In this example, students conversed with the teacher, but student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Hodge were all at Level 1 (see Table G78 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Hodge are described. The observer noted, "Students do not interact with each other. They only interact with the teacher" (Hodge, Observation 11/16/98). In this example, students conversed with the teacher, but student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Rubin ranged from Level 1 to Level 2 (see Table G78 in this appendix). The mean rating across observations was 1.11, indicating that student–student conversation on most occasions was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Rubin are described. The observer noted, "Students did not discuss the lesson with each other. They only gave answers to the teacher" (Rubin, Observation 10/21/98). In this example, students conversed with the teacher, but student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the seven Grade 7 teachers in District 1 varied from 1.00 to 1.63 (see Figure X). However, all teachers (four MiC) had mean ratings at or near 1.00, or the majority of ratings were at Level 1, indicating that students conversed with the teacher but student–student conversation was not encouraged or did not occur.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Heath ranged from Level 1 to Level 3 (see Table G79 in this appendix). Although the mean rating across observations was 2.25, on four of the eight observations students' collaboration was rated at Level 3, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. To illustrate a rating for students' collaboration at Level 3, student collaboration that occurred during a lesson by Heath is described. The observer noted, "When students were constructing the cube on page 2, they worked well together, but they did not do much mathematics collaboratively at other times during the class. Contributions to solving problems were not made equally by all students while collaborating" (Heath, Observation 3/16/99). In this example, some students exchanged ideas, but the contributions to solving problems were not made equally by all students.

The index ratings about the nature of students' collaboration in the classroom for Bartlett ranged from Level 1 to Level 2 (see Table G79 in this appendix). The mean rating across observations was 1.33, indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. However, on six of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Bartlett is described. The observer noted, "Students did not collaborate on strategies or solutions in the large group even though they were seated in pairs" (Bartlett, Observation 2/22/99). In this example, students did not work collaboratively to solve problems.

The index ratings about the nature of students' collaboration in the classroom for Muldoon ranged from Level 1 to Level 4 (see Table G79 in this appendix). Although the mean rating across observations was 1.86, on two of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. On four of the remaining seven observations, students' collaboration was rated at Level 1, indicating that often none of the students worked together in small or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Muldoon is described. The observer noted, "Students did not work together in small or large groups. There was no individual practice. Students only listened to the teacher" (Muldoon, Observation 3/25/99). In this example, students did not work collaboratively to solve problems.

The only index rating about the nature of students' collaboration in the classroom for Burton was at Level 1 (see Table G79 in this appendix), indicating that none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. However, on four of the five observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed practice or independent work. During the one class in which a rating for this index was assigned, the observer noted, "Students were seated in pairs, but they did not collaborate with each other. They did not exchange ideas about the problems" (Burton, Observation 2/23/99). In this example, although students physically sat together, few students shared ideas or discussed how a problem should be solved.

The index ratings about the nature of students' collaboration in the classroom for St. James were all Level 1 (see Table G79 in this appendix), indicating that none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. However, on three of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by St. James is described. The observer noted, "Students worked alone and with the teacher in a large-group situation. They did not work collaboratively" (St. James, Observation 11/11/98). In this example, students did not work collaboratively to solve problems.

The index ratings about the nature of students' collaboration in the classroom for Hodge were all Level 1 (see Table G79 in this appendix), indicating that none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Hodge is described. The observer noted, "Students gave answers to each other but did not work together to solve problems" (Hodge, Observation 9/28/98). In this example, students did not work collaboratively to solve problems.

The index ratings about the nature of students' collaboration in the classroom for Rubin ranged from Level 1 to Level 2 (see Table G79 in this appendix). The mean rating across observations was 1.50, indicating that on many occasions none of the students worked together in small or large-group settings, or if students did work in small groups, one student typically gave answers to other members of the group without explanation. However, on six of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to

give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Rubin is described. The observer noted, "Students spent most of the time listening to the teacher and copying down what she wrote on the overhead" (Rubin, Observation 1/20/99). In this example, students did not work collaboratively to solve problems.

In summary, the mean ratings for *student collaboration* for the seven Grade 7 teachers in District 1 varied from 1.00 to 2.25. One MiC teacher had a mean rating of more than 2.00, but on four of the eight observations, students' collaboration was rated at 3.00, indicating that some students exchanged ideas or provided peer assistance. Three teachers (two MiC) had mean ratings of more than 1.00, or the majority of observations were rated at Level 1, indicating that on many occasions, none of the students worked collaboratively. However, on six of the nine observations for two of the teachers (one MiC) no rating was given because the main purpose of the lesson was to give students individual practice or independent work. Three teachers (one MiC) had mean ratings of 1.00, indicating that none of the students worked collaboratively. However, on three of the nine observations for the MiC teacher and on four of the five observations for the teacher using a conventional curriculum no rating was given because the main purpose of the lesson was to give students individual practice or independent work.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Heath ranged from Level 2 to Level 4 (see Table G80 in this appendix). The mean rating across observations was 3.25, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Heath is described. The observer noted, "Students were interested once they knew how to start the problems. Some were off task and needed prodding to get back to what they were supposed to be doing" (Heath, Observation 10/26/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and dependent on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Bartlett ranged from Level 1 to Level 4 (see Table G80 in this appendix). The mean rating across observations was 2.89, indicating that on most occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Bartlett is described. The observer noted, "Most students were engaged for the whole period, but some needed the teacher to prod them at times. They were not listening to their peers but were doing the activities" (Bartlett, Observation 9/22/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and depended on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Muldoon ranged from Level 2 to Level 4 (see Table G80 in this appendix). Although the mean rating across observations was 2.56, on six of the nine observations student engagement was rated at Level 2, indicating that on many occasions students appeared lethargic and were only occasionally on task carrying out assigned activities, were clearly off task or nominally on task for substantial portions of time, or did not seem to put forth much effort. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by Muldoon is described. The observer noted, "The students are listless and bored. They get confused easily and find the long division problems tedious. They were not trying very hard" (Muldoon, Observation 4/15/99). In this example, students appeared lethargic and did not seem to put forth much effort.

The index ratings about the extent to which students were engaged during the lesson for Burton ranged from Level 3 to Level 4 (see Table G80 in this appendix). The mean rating across observations was 3.60, indicating that usually most students were on task pursuing the substance of the lesson most of the time, seemed to take work seriously, and put forth much effort. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Burton is described. The observer noted, "The students were very involved in the lesson for the whole period. They were prepared for class and worked well on the activities" (Burton, Observation 1/13/99). In this example, most students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for St. James ranged from Level 2 to Level 4 (see Table G80 in this appendix). The mean rating across observations was 2.89, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by St. James is described. The observer noted, "Most students were paying attention but were not very enthusiastic about the material" (St. James, Observation 9/23/98). In this example, most students were engaged in class activities some of the time, but this engagement was only mildly enthusiastic.

The index ratings about the extent to which students were engaged during the lesson for Hodge ranged from Level 1 to Level 3 (see Table G80 in this appendix). The mean rating across observations was 2.50, indicating that on some occasions student engagement was sporadic or episodic. But on other occasions, students appeared lethargic and were only occasionally on task carrying out assigned activities, were clearly off task or nominally on task for substantial portions of time, or did not seem to put forth much effort. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Hodge is described. The observer noted, "Students were on task most of the time but needed prodding. Other students were just off task some of the time. They tended to waste time" (Hodge, Observation 9/29/99). In this example, student engagement was inconsistent and depended on frequent prodding from the teacher. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during another lesson by Hodge is described. The observer noted, "Students waste a lot of time and seem disinterested most of the time. They are not engaged in much activity during the period" (Hodge, Observation 1/12/99). In this example, students appeared lethargic, were only occasionally on task, and did not seem to put forth much effort.

The index ratings about the extent to which students were engaged during the lesson for Rubin ranged from Level 2 to Level 4 (see Table G80 in this appendix). The mean rating across observations was 3.00, indicating that student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Rubin is described. The observer noted, "Most students were interested in the lesson. Some students were very passive and just mildly enthusiastic" (Rubin, Observation 2/23/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and students exhibited only mild enthusiasm.

In summary, the mean ratings for *student engagement in lessons* for the seven Grade 7 teachers in District 1 varied from 2.50 to 3.60. Three teachers (two MiC) had mean ratings greater than 3.00, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time, but on other occasions student engagement was sporadic or episodic with most students engaged in class activities some of the time. Four teachers (two MiC) had mean ratings greater than 2.00, indicating that on some occasions

student engagement was sporadic or episodic with most students engaged in class activities some of the time, and on other occasions, students were passively disengaged, appeared lethargic, were off task for substantial portions of time, and seemed not to put forth much effort.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table G27
Teacher Level of Instruction, Grade 7, District 1

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Fernwood-Heath	3.75	3.75	4	3	3	2.30	1.89	1.67	2	2	3	2.94	2.81	4.06	**40.17**	**4**
Von Humboldt-Bartlett	2.5	2.5	4	2	1	2.78	2.09	1.67	3	2	3	1.39	1.66	3.61	**33.20**	**2**
Von Humboldt-Muldoon	2.5	3.75	3	2	1	2.22	1.39	1.67	2	2	2	1.95	2.33	3.20	**31.01**	**2**
Wacker-Burton	2.5	2.5	3	4	1	3.00	1.25	1.67	1	2	2	1.25	1.25	4.50	**30.92**	**3**
— Conventional —																
Addams-St. James	1.25	1.25	2	1	1	2.03	1.53	1.67	2	2	2	1.25	1.25	3.61	**23.84**	1
Fernwood-Hodge	2.5	2.5	3	3	2	2.50	1.89	1.67	2	2	2	1.25	1.25	3.13	**30.69**	3
Wacker-Rubin	2.5	2.5	3	1	1	1.85	1.25	1.67	2	2	2	1.39	1.88	3.75	**27.79**	**2**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

District 2

In District 2, five Grade 7 teachers participated in the study. Four teachers used MiC, and one teacher used the conventional curriculum already in place in her school.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*. Carlson followed the recommended sequence of MiC units. She planned to teach a MiC unit by reading through the unit, working out the problems, determining where students might have difficulty, and deciding the amount of the unit students might do in a day. Because Carlson taught students identified as at risk, she knew she would need to slow the pace of instruction suggested in the MiC teacher guide: "I try and figure out what it is I want them to do at the beginning of a class, and then we go over it. But if it seems like everybody is having difficulty, I'll knock a couple problems off, and I'll ask them how they're doing. And then they let me know" (Carlson, Interview 4/13/99). Carlson identified the prerequisite skills, concepts, and vocabulary she would need to teach in order for students to be successful with the unit: "I teach the at risk community so my kids are missing large chunks of material that they should know. There's a lot of fill in for me" (Carlson, Interview 4/13/99). Carlson planned to use supplementary materials to help her students develop conceptual understanding and to practice skills. She thought MiC was well aligned with the new state comprehensive assessment, but that students needed more skill practice before taking the SAT: "When we get close to the district test, we stop doing a lot of MiC and we do a lot more drill and kill just simply because that's how the SAT is. Once we switch to the state test it won't matter. That [test] is perfect for MiC" (Carlson, Interview 4/13/99). Carlson thought it was more difficult to plan to teach a MiC unit than to plan to teach a unit from a conventional textbook because "with MiC you have to read through it, you have to work it out, you have to figure out where [students] are going to have problems and be ready to help them with the prerequisite knowledge they need. The [conventional] book goes in order" (Carlson, Interview 4/13/99).

Table G28
Subcategories of Unit Planning: Carlson, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Dillard generally followed the recommended sequence of MiC units, varying the sequence for the advanced students he taught. Dillard completed all of the units in the algebra strand first because "[these students] will be taking honors algebra next year for high school credit" (Dillard, Interview 4/12/99). When planning to teach a MiC unit, he read over the unit, noted the different concepts that were to be studied and the materials needed in each section, and worked through each section. Dillard noted the suggested pace for instruction, but knew he would need to adjust the pace because MiC was a new mathematics program for his students. He considered the amount of time it took him to work through each section

when he made the adjustment. Dillard planned activities from supplementary materials to help students with prerequisite concepts and skills. He thought MiC was well aligned with the state standards (Dillard, Interview 4/12/99).

Table G29
Subcategories of Unit Planning: Dillard, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Caputo followed the recommended sequence of MiC units. When planning to teach a MiC unit, he read through the unit to become familiar with the problems and activities, keeping in mind the district's curriculum guidelines. Caputo planned to follow the pace of instruction as suggested in the teacher guide; however, he anticipated having to adjust the pace to accommodate students' learning. Caputo did not plan to use other resources. He thought the main difference in planning to teach a MiC unit was the amount of time a teacher needed to understand the unit: "When teaching [a chapter] in a regular textbook, you just know what you're going to do. [With MiC] you definitely have to know what you're doing. If you don't, you're going to be lost" (Caputo, Interview 5/13/99). Caputo thought MiC prepared students for the district standardized tests, but he did take time to review specific skills prior to testing (Caputo, Interview 5/13/99).

Table G30
Subcategories of Unit Planning: Caputo, Weir Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Gallardo generally followed the recommended sequence of MiC units. In planning to teach a MiC unit, Gallardo read through the unit, determined unit goals, and aligned his thinking with those goals. Keeping in mind the state standards and requirements for district standardized tests, he identified the skills and concepts his students needed. Gallardo did not plan for a specific pace of instruction; instead, student performance determined the pace: "I just went on the needs of my students. When I determined that the students had grasped the concept, I moved on. I know sometimes [the unit] gives you a guide, three lessons, four lessons per section. I didn't pay any attention to it" (Gallardo, Interview 5/26/99). Gallardo often supplemented the unit with materials he had created. He thought it took more time to plan to teach a MiC unit than it did to plan to teach a chapter from a conventional textbook because of the presentation of the mathematics and the increased thinking needed to understand the unit (Gallardo, Interview 5/26/99).

Table G31
Subcategories of Unit Planning: Gallardo, Weir Middle School, District 2, Grade 7

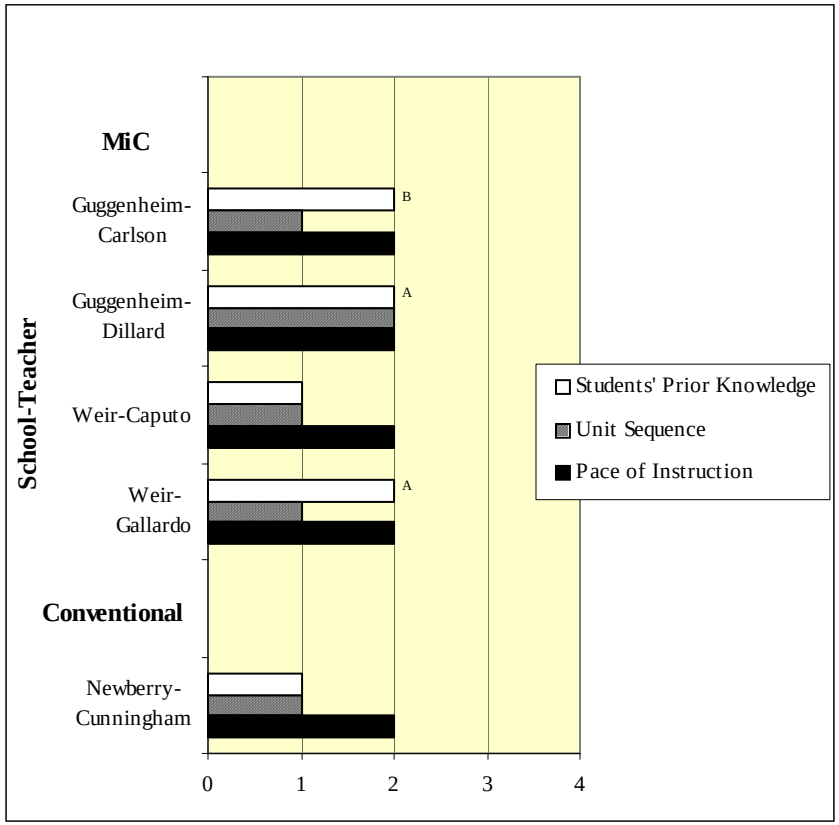
Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Cunningham generally followed the recommended sequence of chapters in the conventional textbook she used but modified the sequence when she thought it would clarify conceptual development for the students. Cunningham included supplementary materials that involved manipulatives or problem-solving activities. Although she noted the suggested pace for instruction, her plan was flexible, allowing her to adjust the pace based on how well students understood the concept or skill. Cunningham considered the district curriculum guidelines, state standards, and district standardized tests as she planned (Cunningham, Interview 4/14/99).

Table G32
Subcategories of Unit Planning: Cunningham, Newberry Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the five seventh-grade teachers in District 2 varied in two of the subcategories of *unit planning, students' prior knowledge* and *unit sequence* (see Figure X). Two teachers considered students' prior knowledge in mathematics or reading without informal or formal assessment. One teacher considered students' needs related to the development of concepts and procedures. Two other teachers followed the unit development in the teacher guide with little or no consideration to students' prior knowledge.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table G81 in this appendix.

Figure G3. Unit planning, Grade 7, District 2.

Most of the teachers, four, used the *sequence of units*/chapters recommended in teacher support materials. One teacher considered external factors such as the content of the statewide testing program when sequencing instructional units. All of the teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Carlson read through the lesson and worked each problem, trying to anticipate students' difficulties with the content, concepts, or language. Students' performance in previous lessons helped her identify parts of the lesson in which students might experience conceptual difficulties. Knowledge of students' weak language skills helped Carlson identify problems and/or questions that needed to be restated. Carlson also determined the problems best solved individually, in pairs, in small groups, or in whole-class instruction, and the portions that could reasonably be completed in a class period. Carlson preferred to present the lesson using whole-class direct instruction followed by a small-group activity. But she was not confident in her ability to have substantive work going on in the groups. She explained: "This is the first year that I'm really using a lot of group work and I'm struggling with it. I think I'm getting better at it, but it's difficult to control [students] when they're at a table and they're talking. It's hard to tell if they're really working or if they're goofing off" (Carlson, Interview 4/13/99). The small groups were not static. She said, "My groups are of six or seven depending on how big my class is. But the [students] don't always have to work together. They can work in groups of two or three within the group or they can work as a whole group" (Carlson, Interview 4/13/99).

As an example of a lesson observed in Carlson's class, the lesson on 1/19/99 included: whole class, problem of the day (9 minutes) whole-class review (6 minutes); teacher presentation of new lesson (22 minutes); small-group rotation activities, answering questions/problems in new lesson, working on a scale drawing, working on a division skills sheet, and computer activity (39 minutes) lunch (38 minutes); and small-group activities continued (30 minutes; Carlson, Observation 1/19/99). Teacher Log data for Carlson were unavailable.

Table G33
Subcategories of Lesson Planning: Carlson, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Dillard worked through the problems. Students' performance on the previous lesson helped him decide the amount of review and practice and the number of examples needed during the new lesson. Dillard preferred to use small-group work because he thought the quality of discussions in small groups was higher than it was with the whole class. Dillard explained:

> It gets students thinking and discussing. They can generate more ideas among themselves. They can actually say why something is wrong or right and give supporting details. Any time you get the kids explaining to other kids, it's beneficial. One gives an explanation. The others get a more in-depth understanding of it as he/she thinks about the explanation. (Dillard, Interview 4/12/99)

Dillard's small groups changed over time. The number in each group and the method for determining group membership varied (Dillard, Interview 4/12/99).

Dillard used two instructional forms frequently, small-group work (on 70% of the reported days) and whole-class discussion (65%; see Tables G83-G88 in this appendix). Small-group work was given more class time: at least half of the class period on 47% of the days and equal emphasis with other instructional forms on 50% of the days. Whole-class discussion was given equal emphasis with other instructional forms on 70% of the days and less than 15% of class time on 28% of the days. On approximately half of the reported days, Dillard reviewed previous material (45%) and used teacher presentations (42%). Review of previous material was given equal emphasis with other instructional forms on 29% of the days and less than 15% of class time on 69% of the days. Teacher presentation was given equal emphasis with other instructional forms on 16% of the days and less than 15% of class time on the remaining days. On 32% of the reported days, Dillard started class with a warm-up activity, but this activity was given less than 15% of class time. On 27% of the reported days, Dillard used independent practice. This form of instruction was given at least half of the class period on 33% of the days, equal emphasis with other instructional forms on 43% of the days, and less than 15% of class time on 24% of the days (Dillard, Teacher Log 1998-99).

Investigation of problems, an important element in Dillard's instruction, was used on 75% of the reported days (see Tables G89-G96 in this appendix). This student activity was also given a significant amount of class time: at least half of the class period on 68% of the days and equal emphasis with other student activities on 31% of the days. Discussing answers and solution strategies and participating in whole-class discussions were also important elements in Dillard's instruction. Each activity was used on 73% of the reported days, but for less class time than investigation of problems. Dillard's students often listened to the teacher or took notes (64%), but usually for less than 15% of class time. Practicing computation was used on 32% of the days, but for a small amount of class time. Taking a test or quiz (8%), reflecting on or summarizing lesson concepts (1%), and beginning homework (1%) were rarely used (Dillard, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Dillard reported in his teacher logs. As an example of a lesson observed in Dillard's class, the lesson on 11/16/98 included: housekeeping duties (12 minutes); whole-class instruction alternating with small-group work, including review of vocabulary, the assignment, and whole-class discussion (65 minutes); lunch (30 minutes); and whole-class/small-group work continued (37 minutes; Dillard, Observation 11/16/98).

Table G34
Subcategories of Lesson Planning: Dillard, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Caputo familiarized himself with the lesson by reading through it and working each problem. The amount of review or reteaching of particular concepts or skills was determined by students' performance on the previous lesson. He explained: "If there's any concept that they don't know, then I have to go back to the basics and explain things" (Caputo, Interview 5/13/99). Caputo varied the form of instruction:

> I want to have different settings. I don't want to get into a routine. I want [students] to experience different situations. So I have had students work individually, in pairs, and in groups of three or four. The best grouping is pairs. When [students] are paired you get less noise, and the teacher can have more control. (Caputo, Interview 5/13/99)

Caputo thought the group membership influenced the group's success. He explained, "If you are able to pair a kid that is knowledgeable with a kid that is less knowledgeable, it's great. It is good if the kids help each other" (Caputo, Interview 5/13/99). Caputo also encouraged large-group discussions. He found the large-group discussions were more fluent and substantive when students understood the activity and concepts (Caputo, Interview 5/13/99).

As an example of a lesson observed in Caputo's class, the lesson on 5/13/99 included: housekeeping duties (13 minutes); whole class, introduction of lesson (13 minutes); small-group work, problem 1 (10 minutes); whole-class discussion of problem 1 (5 minutes); small-group work, problems 4–7 (36 minutes); whole-class discussion of problems 4–7 (18 minutes); and small-group work continued, problems 4–7, 8, and 9 (8 minutes; Caputo, Observation 5/13/99). Teacher Log data for Caputo were unavailable.

Table G35
Subcategories of Lesson Planning: Caputo, Weir Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Gallardo worked through the problems to become familiar with the content and the purpose of the lesson. He encouraged large-group discussion, but found students often discussed the context more than the mathematics. He explained, "The hardest thing that I need to do is to get the students to see the math part of what we're trying to learn" (Gallardo, Interview 5/26/99). Gallardo valued small-group work because "it helps those who might not have an idea. Usually somebody in the group is going to have an idea. Then the rest of them can build on it" (Gallardo, Interview 5/26/99).

As an example of a lesson observed in Gallardo's class, the lesson on 12/8/98 included: whole class, introduction of measurement (10 minutes); small-group measuring activity (5 minutes); whole-class discussion (17 minutes); small-group or individual work on SAS #1 and SAS #2 (55 minutes); clean up (5 minutes); and whole-class discussion, closure of the lesson (10 minutes; Gallardo, Observation 12/8/98). Teacher Log data for Gallardo were unavailable.

Table G36
Subcategories of Lesson Planning: Gallardo, Weir Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

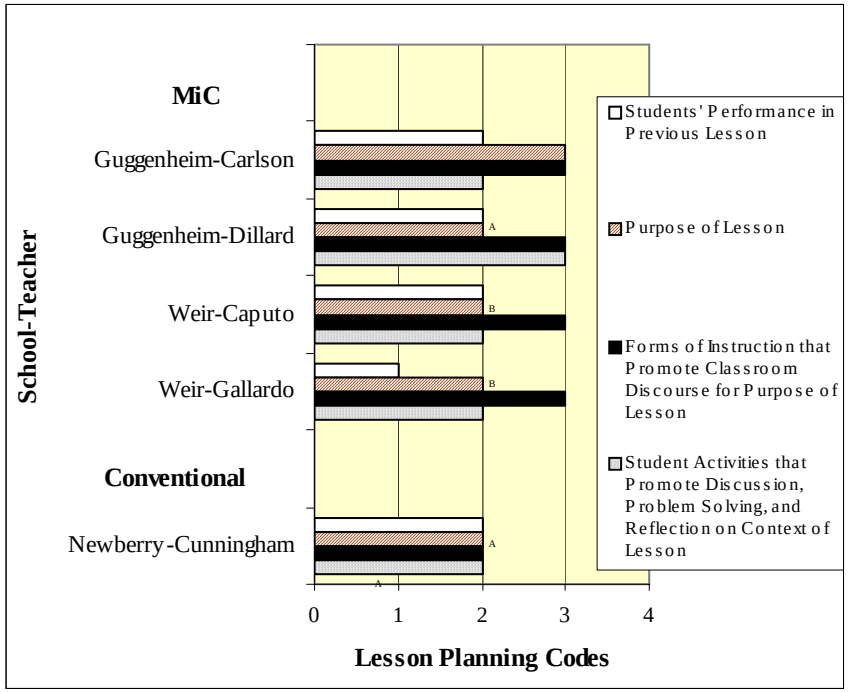
When planning to teach individual lessons from a conventional textbook, Cunnningham worked through one or two of the problems. If the content of the new lesson was dependent on the students' understanding of the previous lesson, Cunningham moved on to the new lesson when the majority of the students exhibited understanding of the previous lesson. Cunningham used supplementary materials to find complex problems for the weekly dilemma. She encouraged students to collaborate to figure out how to solve these problems. Cunningham initiated whole-class discussion of substantive ideas by asking probing questions. She also valued students working in heterogeneous small groups: "It allows [students] to collaborate. What one person doesn't see, another person may see. [Students] can help each other. It gives them a support system that you as a teacher can't" (Cunningham, Interview 4/14/99).

As an example of a lesson observed in Cunningham's class, the lesson on 3/10/99 included: housekeeping duties (15 minutes); individual seat work (11 minutes); whole-class discussion of the seat work (7 minutes); and teacher presentation (42 minutes; Cunningham, Observation 3/10/99). Teacher Log data for Cunningham were unavailable.

Table G37
Subcategories of Lesson Planning: Cunningham, Newberry Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the five seventh-grade teachers in District 2 varied in all four subcategories of *lesson planning* (see Figure G4). With respect to *students' performance in the previous lesson*, four teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The other teacher planned in the same manner daily without considering students' previous performance. More variation was seen in teachers' attention to the *purpose of the lesson*. One teacher went beyond checking her own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Two teachers planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context. The other two teachers selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table G82 in this appendix.

Figure G4. Lesson planning, Grade 7, District 2.

Teachers varied minimally in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Four teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The remaining teacher included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Teachers also varied minimally in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution

strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. The remaining four teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Carlson. The evidence gathered for lesson presentation and development for Carlson ranged from Level 2A to 4. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table G97 in this appendix). An overall rating of 4 was assigned for Carlson for the nature of inquiry that transpired during instruction (see Table G98). Carlson was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure G15). For Carlson, 40% of the interactive decisions were coded as reflective of good standard pedagogy, 40% were most aligned with teaching for understanding, and 20% were least aligned for teaching for understanding.

Across all observations for Carlson, the mean rating for the index on students' explanations was 1.83, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G99). The mean rating for elicitation of multiple strategies was 1.33, indicating that multiple strategies were generally not elicited from students (see Table G100). A rating of 1 was assigned for Carlson on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G38
Overall Ratings for Mathematical Interaction for Carlson, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to develop conceptual understanding
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.83	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.33	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Dillard. The evidence gathered for lesson presentation and development for Dillard ranged from Level 2A to 5. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table G97 in this appendix). An overall rating of 4 was assigned for Dillard for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G98). Dillard was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure G16). For Dillard, 75% of the interactive decisions were coded as most aligned with teaching for understanding, and 25% were least aligned with teaching for understanding.

Across all observations for Dillard, the mean rating for the index on students' explanations was 2.17, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G99). The mean rating for elicitation of multiple strategies was 2.83, indicating that multiple strategies were generally not a primary emphasis of instruction (see Table G100). A rating of 1 was assigned for Dillard on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G39
Overall Ratings for Mathematical Interaction for Dillard, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.17	Focus on procedures
Elicitation of Multiple Strategies	2.83	Strategies not primary emphasis
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Caputo. The evidence gathered for lesson presentation and development for Caputo ranged from Level 1 to 2B. An overall rating of Level 2B was assigned, indicating that limited introduction to the lesson, vague direction, or lack of appropriate planning was evident during lessons (see Table G97 in this appendix). An overall rating of 2 was assigned for Caputo for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G98). Caputo was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure G17). For Caputo, 58% of the interactive decisions were coded as least aligned with teaching for understanding, 8% were reflective of good standard pedagogy, and 33% were most aligned with teaching for understanding.

Across all observations for Caputo, the mean rating for the index on students' explanations was 1.43, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies

(see Table G99). The mean rating for elicitation of multiple strategies was 1.43, indicating that multiple strategies were rarely elicited from students (see Table G100). A rating of 1 was assigned for Caputo on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G40
Overall Ratings for Mathematical Interaction for Caputo, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	2	Limited attention to teaching for understanding
Interactive Decisions	2	More emphasis on standard pedagogy
Nature of Student Explanations	1.43	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.43	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Gallardo. The evidence gathered for lesson presentation and development for Gallardo ranged from Level 2A to 5. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table G97 in this appendix). An overall rating of 4 was assigned for Gallardo for the nature of inquiry that transpired during instruction, indicating in-depth exploration of mathematics occurred during lessons (see Table G98). Gallardo was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure G18). For Gallardo, 26% of the interactive decisions were coded as reflective of good standard pedagogy, 51% were most aligned with teaching for understanding, and 26% were least aligned for teaching for understanding.

Across all observations for Gallardo, the mean rating for the index on students' explanations was 1.83, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G99). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were rarely elicited from students (see Table G100). A rating of 2 was assigned for Gallardo on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G41
Overall Ratings for Mathematical Interaction for Gallardo, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.83	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Cunningham. The evidence gathered for lesson presentation and development for Cunningham ranged from Level 2A to 5. An overall rating of at Level 3B, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table G97 in this appendix). An overall rating of 3 was assigned for Cunningham for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G98). Cunningham was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure G19). For Cunningham, 22% of the interactive decisions were coded as reflective of good standard pedagogy, 55% were most aligned with teaching for understanding, and 22% were least aligned for teaching for understanding.

Across all observations for Cunningham, the mean rating for the index on students' explanations was 1.50, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table G99). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were generally not elicited from students (see Table G100). A rating of 1 was assigned for Cunningham on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G42
Overall Ratings for Mathematical Interaction for Cunningham, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.50	Answers only
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the five Grade 7 teachers in District 2 varied from Level 2B to Level 5 in *lesson presentation and development* (see Table G43). Two MiC teaches were assigned Level 5, indicating that their lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. The teacher using a conventional curriculum was assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. The remaining MiC teacher was assigned Level 2A, indicating that limited introduction to the lesson, vague direction, or lack of appropriate planning was evident during lessons.

Table G43
Nature of Mathematical Interaction, Grade 7 Teachers in District 1, 1998-1999

School-Teacher	**Lesson Presentation and Development**	**Nature of Inquiry**	**Teachers' Interactive Decisions**	**Nature of Student Explanations**	**Elicitation of Multiple Strategies**	**Lesson Closure, Reflection, or Summary**	**Total**
			— MiC —				
Guggenheim-Carlson	4	4	4	1.83	1.33	1	16.16
Guggenheim-Dillard	5	4	5	2.17	2.83	1	20.00
Weir-Caputo	2B	2	2	1.43	1.43	1	9.86
Weir-Gallardo	5	4	4	1.83	1.50	2	18.33
			— Conventional —				
Newberry-Cunningham	3B	3	4	1.50	1.50	1	14.00

With respect to the *nature of inquiry during instruction*, teachers varied from Level 2 to Level 4. Three MiC teachers were assigned Level 4, indicating that in-depth exploration of mathematics occurred during lessons. The one teacher using a conventional curriculum was assigned Level

3, indicating that attention was given to conceptual understanding during instruction. The remaining MiC teacher was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction.

With respect to *teachers' interactive decisions*, teachers varied from Level 2 to Level 5. One MiC teacher was assigned Level 5, indicating that his interactive decisions were most aligned with teaching for understanding. Three teachers (two MiC) were assigned Level 4, indicating that their interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding. One MiC teacher was assigned Level 2, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding.

The overall means for the *nature of student explanations* ranged from 1.43 to 2.17, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.43 to 1.83. In genera, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, One MiC teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons. The remaining teachers offered few, if any, opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.

Classroom Assessment

Three subcategories of instruction characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The seventh-grade teachers in District 2 varied in all three subcategories of classroom assessment (see Tables G44-G47). With respect to the evidence sought during classroom assessment, three MiC teachers maintained an underdeveloped process orientation. The teacher using a conventional curriculum sought procedural competence of student learning. The remaining MiC teacher gathered limited evidence of student learning. Feedback in the classes of one MiC teacher was characterized as purposeful shared responsibility from teacher and students. The feedback of the remaining four teachers (three MiC) was very teacher-directed. Four teachers (three MiC) provided low-level closed feedback. The remaining MiC teacher provided feedback that allowed students to know whether answers were correct or incorrect.

Table G44
Evidence Sought through Classroom Assessment Practice, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	3	Underdeveloped process orientation
Guggenheim-Dillard	3	Underdeveloped process orientation
Weir-Caputo	1	Limited evidence
Weir-Gallardo	3	Underdeveloped process orientation
Conventional		
Newberry-Cunningham	2	Procedural competence

Table G45
Feedback Coherence and Purpose, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	2	Teacher-directed feedback
Guggenheim-Dillard	2	Teacher-directed feedback
Weir-Caputo	2	Teacher-directed feedback
Weir-Gallardo	4	Purposeful shared responsibility
Conventional		
Newberry-Cunningham	2	Teacher-directed feedback

Table G46
Feedback Content, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	3	Low-level, closed feedback
Guggenheim-Dillard	3	Low-level, closed feedback
Weir-Caputo	2	Answer-only feedback
Weir-Gallardo	3	Low-level, closed feedback
Conventional		
Newberry-Cunningham	3	Low-level, closed feedback

Table G47
Classroom Assessment, Grade 7, District 2

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Guggenheim-Carlson	3	2	3	8
Guggenheim-Dillard	3	2	3	8
Weir-Caputo	1	2	2	5
Weir-Gallardo	3	4	3	10
		— Conventional —		
Newberry-Cunningham	2	2	3	7

See index next page.

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Carlson ranged from Level 1 to Level 3 (see Table G101 in this appendix). The mean rating across observations was 1.83, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Carlson are described. The observer noted, "Some students shared answers. If they were exchanging details of their work, it wasn't observed" (Carlson, Observation 1/19/99). In this example, student–student conversation occurred on a limited basis and usually consisted of sharing answers.

The index ratings about the nature of student–student conversation for Dillard ranged from Level 1 to Level 3 (see Table G101 in this appendix). The mean rating across observations was 2.17, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Dillard are described. The observer noted, "In small groups, most students were not discussing mathematics, although in one group they did discuss the creation of and drawing of graphs to represent the high/low tides as recorded in a table" (Dillard, Observation 3/23/99). In this example, student–student conversation was limited and focused on procedures.

The index ratings about the nature of student–student conversation for Caputo ranged from Level 1 to Level 2 (see Table G101 in this appendix). The mean rating across observations was 1.29, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Caputo are described. The observer noted, "Students did not talk to each other about math" (Caputo, Observation 2/22/99). In this example, student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Gallardo ranged from Level 1 to Level 3 (see Table G101 in this appendix). The mean rating across observations was 2.17, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Gallardo are described. The observer noted, "Not most, but a large number of students participated in the large group, or with those seated nearby, to discuss mathematics" (Gallardo, Observation 11/12/98). In this example, student–student conversation was limited and focused on procedures.

The index ratings about the nature of student–student conversation for Cunningham ranged from Level 1 to Level 3 (see Table G101 in this appendix). Although the mean rating across observations was 1.50, on four of the six observations student–student conversation was rated at Level 1, indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Cunningham are described. The observer noted, "Students didn't discuss any of the mathematics with the teacher or with their teammates" (Cunningham, Observation 2/18/99). In this example, student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the five Grade 7 teachers in District 2 varied from 1.29 to 2.17. Three MiC teachers had mean ratings near 2.00, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. Two teachers (one MiC) had mean ratings (or the majority of ratings) near 1.00, indicating that usually student–student conversation was not encouraged or did not exist.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Carlson ranged from Level 1 to Level 2 (see Table G102 in this appendix). The mean rating across observations was 1.67, indicating that usually few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Carlson is described. The observer noted, "Most students in groups were working on different problems" (Carlson, Observation 1/19/99). In this example, many of the students in a group were working on different problems at different paces.

The index ratings about the nature of students' collaboration in the classroom for Dillard ranged from Level 1 to Level 3 (see Table G102 in this appendix). The mean rating across observations was 2.00, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Dillard is described. The observer noted, "Students in groups worked at different paces" (Dillard, Observation 2/17/99). In this example, many of the students in a group worked on different problems and at different paces.

The index ratings about the nature of students' collaboration in the classroom for Caputo ranged from Level 1 to Level 2 (see Table G102 in this appendix). The mean rating across observations was 1.57, indicating that usually few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Caputo is described. The observer noted, "Students sat in pairs but didn't take any responsibility for the partner. Very few pairs were working on the same part of a problem at the same time" (Caputo, Observation 5/13/99). In this example, most partners worked on different problems at different paces.

The index ratings about the nature of students' collaboration in the classroom for Gallardo ranged from Level 1 to Level 3 (see Table G102 in this appendix). The mean rating across observations was 1.83, indicating that usually few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Gallardo is described. The observer noted, "Students were not seated in small groups, but a large number of students participated in the large group or with those seated nearby to discuss mathematics" (Gallardo, Observation 11/12/98). In this example, some students exchanged ideas or gave assistance to their peers.

The index ratings about the nature of students' collaboration in the classroom for Cunningham ranged from Level 1 to Level 3 (see Table G102 in this appendix). The mean rating across observations was 1.33, indicating that usually none of the students worked together in small or large-group settings, or if they did work in small groups, one student typically gave answers to other members of the group without explanation.. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Cunningham is described. The observer noted, "Students were not working in groups today" (Cunningham, Observation 4/14/99). In this example, students did not work together in small or large-group settings.

In summary, the mean ratings for *student collaboration* for the five Grade 7 teachers in District 2 varied from 1.33 to 2.00. Four MiC teachers had mean ratings at or near 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. The remaining teacher using a conventional curriculum had a mean rating near 1.00, indicating that on most occasions, none of the students worked collaboratively.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Carlson ranged from Level 2 to Level 3 (see Table G103 in this appendix). The mean rating across observations was 2.83, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Carlson is described. The observer noted, "The lesson dealt with many large numbers. Even though there was a tendency by the students to get lost in the use of these many large numbers, [Carlson] kept drawing students into the classroom discussion" (Carlson, Observation 11/19/98). In this example, most students were engaged in class activities some of the time, but this engagement was dependent on frequent encouragement from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Dillard ranged from Level 3 to Level 4 (see Table G103 in this appendix). The mean rating across observations was 3.50, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time, and on other occasions sporadic or episodic. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Dillard is described. The observer noted, "Most students were on task. The teacher helped the pacing by periodically checking answers in the large-group setting" (Dillard, Observation 11/16/98). In this example, student engagement was widespread and most students were on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during another lesson by Dillard is described. The observer noted, "[Dillard], through multiple questioning, leading, and calling upon many students by name kept students very involved during the large-group activity" (Dillard, Observation 3/23/99). In this example, most students were engaged in class activities some of the time, but this engagement was dependent on frequent encouragement from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Caputo ranged from Level 1 to Level 3 (see Table G103 in this appendix). The mean rating across observations was 1.29, indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. To illustrate a rating for student engagement at Level 1, students' on-task behavior that occurred during a lesson by Caputo is described. The observer noted, "Throughout most of the class period, a lot of off-task behavior was observed, e.g., drumming on

desks, chanting rap talk, and social talking" (Caputo, Observation 12/1/98). In this example, students were frequently off task, as evidenced by their inattentive behaviors.

The index ratings about the extent to which students were engaged during the lesson for Gallardo ranged from Level 3 to Level 4 (see Table G103 in this appendix). The mean rating across observations was 3.50, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time, and on other occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Gallardo is described. The observer noted, "Students were very interested in the lesson activities" (Gallardo, Observation 3/25/99). In this example, student engagement was widespread as most students were on task pursuing the substance of the lesson. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during another lesson by Gallardo is described. The observer noted, "Students were very interested in today's activity but were hindered by a lack of sufficient equipment. One student said she was bored, referring to just sitting while she waited for the availability of a pair of scissors" (Gallardo, Observation 12/8/98). In this example, most students were engaged in class activities some of the time, but this engagement inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Cunningham ranged from Level 2 to Level 4 (see Table G103 in this appendix). The mean rating across observations was 3.00, indicating that on some occasions student engagement was sporadic or episodic and on other occasions students appeared lethargic and were only occasionally on task. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Cunningham is described. The observer noted, "Students were well behaved and usually on task. When the class moved on to the next activity, it was observed that several students had not begun the first activity" (Cunningham, Observation 11/4/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

In summary, the mean ratings for *student engagement in lessons* for the five Grade 7 teachers in District 2 varied from 1.29 to 3.50. Two MiC teachers had mean ratings greater than 3.00, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson and on other occasions sporadic or episodic. Two teachers (one MiC) had mean ratings greater than 2.00, indicating that on some occasions student engagement was sporadic or episodic and on other occasions passive. One MiC teacher had a mean rating greater than 1.00, indicating students were frequently off task as evidenced by lethargy, gross inattention, or serious disruption.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table G48
Teacher Level of Instruction, Grade 7, District 2

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE		
— MiC —																
Guggenheim-Carlson	3.75	2.5	4	4	4	2.22	1.66	1.67	3	2	3	2.29	2.09	3.54	**39.72**	**4**
Guggenheim-Dillard	3.75	3.75	5	4	5	3.62	3.50	1.67	3	2	3	2.71	2.50	4.38	**47.88**	**5**
Weir-Caputo	3.75	2.5	2	2	2	2.38	1.79	1.67	1	2	2	1.61	1.96	1.61	**28.27**	**2**
Weir-Gallardo	3.75	2.5	5	5	4	3.05	1.88	3	3	4	3	2.71	2.29	4.38	**47.89**	**5**
— Conventional —																
Newberry-Cunningham	2.5	2.5	3	3	4	2.50	1.88	1.67	2	2	3	1.88	1.66	3.75	**35.34**	**3**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

The ratings assigned to each teacher in this district on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 3

In District 3, two Grade 7 teachers participated in the study. Both teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*. Perry generally followed the recommended sequence of MiC units. However, she chose to begin the year with a unit that had been very successful for students the previous year. When planning to teach a MiC unit, Perry read the teacher guide and tried to follow the suggested pace for instruction. She noted the prerequisite concepts and skills students needed to have. When she introduced the unit she informally checked to see if the students had the needed prior knowledge. If they didn't, she planned supplementary activities: "I'll take a little side trip and do some other things with it. I know MiC does expect the kids to come with a certain amount of knowledge" (Perry, Interview 4/29/99). Perry did not take into consideration district mathematics guidelines or state standards when planning to teach a unit because they were in the process of being revised (Perry, Interview 4/29/99).

Table G49
Subcategories of Unit Planning: Perry, Calhoun North Middle School, District 3, Grade 7

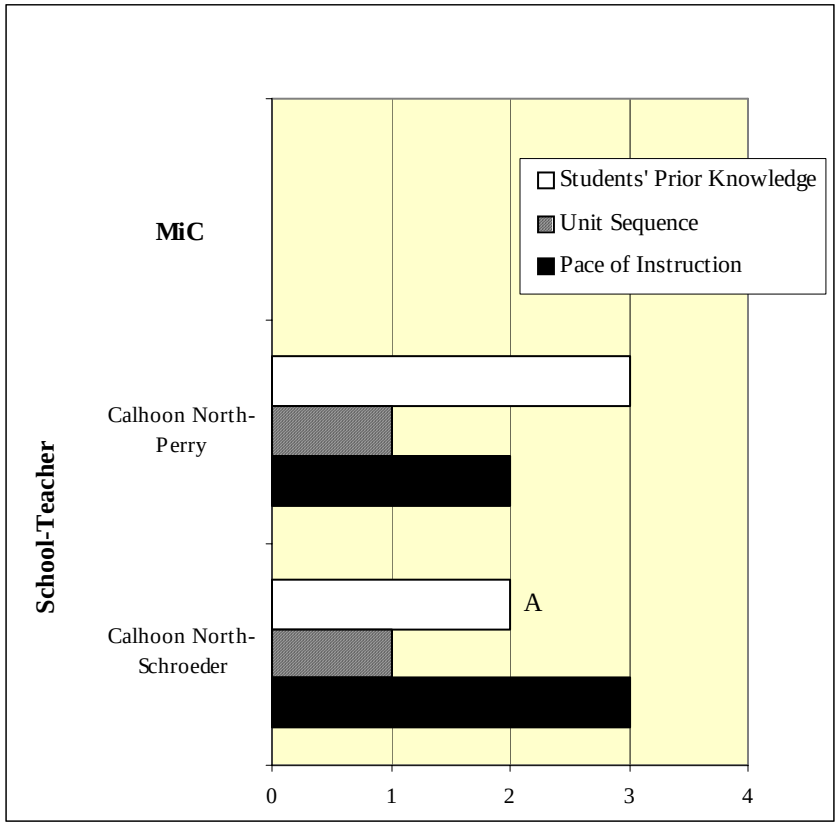
Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Schroeder, a teacher of students in special education, tried to follow the recommended sequence of MiC units as written for teachers of students in regular education. She modified the sequence when a unit was too difficult for her students. For example: "My students just could not do *Cereal Numbers*. That unit was just way beyond them, so I skipped that" (Schroeder, Interview 6/9/99). Schroeder added more practice activities by creating parallel problems or bringing in other resources. She did not preplan a pace for instruction, but made the decision daily based on the students' daily performance: "We do more than what's in the unit because just going through the unit doesn't give enough practice. Sometimes I find some steps are left out for my students" (Schroeder, Interview 6/9/99). Schroeder did not consider district guidelines and state standards because they were in the process of being revised (Schroeder, Interview 6/9/99).

Table G50
Subcategories of Unit Planning: Schroeder, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	3	Consideration of the needs of current students

In summary, the two seventh-grade teachers in District 3 varied in two of the subcategories of *unit planning, students' prior knowledge* and *pace of instruction*. With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. The other teacher based unit planning on her perceptions of her students' (in special education) prior knowledge in mathematics or reading without informal or formal assessment.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure G4. Unit planning, Grade 7, District 3.

Both teachers used the *sequence of units* recommended in the teacher support materials. One teacher considered the learning styles and reasoning skills of her current students when planning the pace of instruction. The other teacher anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended *pace for instruction*.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Perry worked through all of the problems, even though she had previously taught the lesson. She explained:

> I do solve them again. I try to get an idea, because it's been a whole year. I'll do [the problems] myself, even the assessments and the test. I like to see how I would solve the problems, and I want to show the kids. I tell them, "This is what I had to do." I hold up a paper and say, "This is all the work I had to do so solve this problem." That gives them an idea that it's not always done in your head. It's hard to get them to write down things. I also make notes. (Perry, Interview 4/29/99)

Perry studied her notes from the previous year and reviewed students' performance on the assessment or summary questions from the previous section in the unit. The form of instruction she chose was determined by the lesson and accompanying activities. She facilitated large-group discussions by asking probing questions and used several methods to ensure each student's participation in the discussion. Perry explained: "Often I will call on the same people if I'm not careful. Sometimes I use 3 x 5 cards to make sure I'm calling on every one. Sometimes I'll just go up and down each row, telling them it's their turn now. Sometimes it is pretty open. I don't make them raise their hands" (Perry, Interview 4/29/99).

Table G50
Subcategories of Lesson Planning: Perry, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Schroeder worked through the problems to identify the needed prerequisite skills and modifications so that the lesson would be accessible to her students in special education. She explained: "We do more than what is in the book because just going through the lessons doesn't give enough practice. Sometimes some steps are left out for us. Sometimes the questions lead into the next one, but sometimes they don't. I need to prepare the students for that or give them more information (Schroeder, Interview 6/9/99). Schroeder also considered students' performance on the previous lesson, making adjustments to allow for continuity and maximum understanding.

She explained: “It isn’t nice to have three students in a class. One year I had just two and you don’t get any discussion or any enthusiasm or motivation at all. So very few [students] are not good, but seven is nice and ten is nice. After that it’s a little unmanageable” (Schroeder, Interview 6/9/99). Students worked through the activity in groups of three, as pairs, or individually and came back together to discuss their work (Schroeder, Interview 6/9/99).

Table G51
Subcategories of Lesson Planning: Schroeder, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students’ Performance in Previous Lesson	3	Changes focuses on students’ understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

In summary, the two seventh-grade teachers in District 3 varied on only one of the four subcategories of *lesson planning, students' performance in the previous lesson* (see Figure G5). With respect to *students' performance in the previous lesson*, one teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. The other teacher made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. No variation between teachers was seen in their attention to the *purpose of the lesson*. Both teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels.

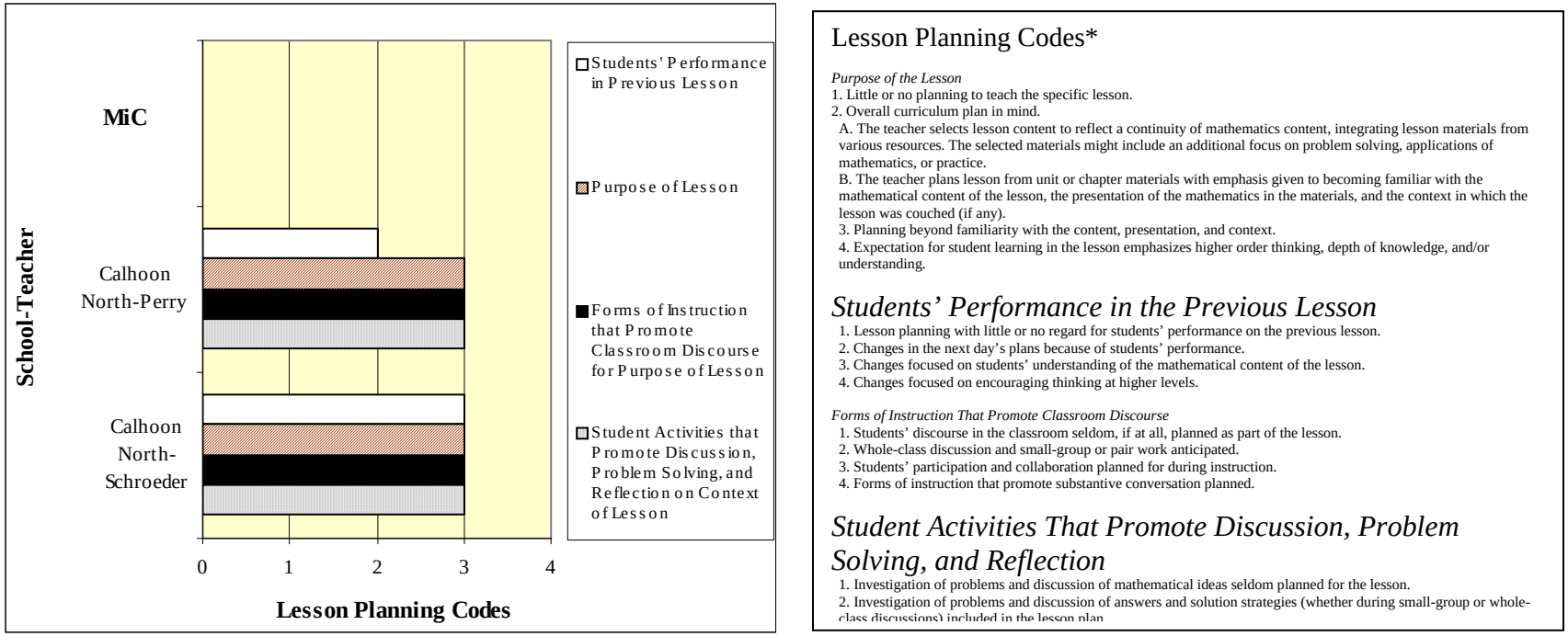


Figure G5. Lesson planning, Grade 7, District 3.

No variation between teachers was seen in their choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Both teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. No variation between teachers was seen in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For both teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan.

The ratings assigned to each teacher in this district on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 4

In District 4, three Grade 7 teachers participated in the study. All three teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*. Finn and the assistant principal for mathematics and science identified the specific topics students needed for the district standardized tests and sequenced the MiC units accordingly. When planning to teach a MiC unit, Finn read the unit overview and goals. She then worked through each section: "It's most important to do the units first on your own to see how you can go ahead and do it with the kids" (Finn, Interview 5/4/99). When Finn recognized concepts, skills, or topics that her students didn't know, she planned an introduction or review of the problem areas using supplementary materials. Because this was only the second year Finn had used MiC, she did not feel confident in determining unit pacing prior to instruction. She commented that she never had finished a unit in the amount of time the teacher guide suggested (Finn, Interview 5/4/99).

Table G52
Subcategories of Unit Planning: Finn, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	1B	Little or no consideration of pacing

Kane sequenced MiC units by looking for connections among them: "I tried to look at how the units are connected so that I could emphasize connections in class. For example, in *Operations* when we compared the areas of the small figures to the enlarged figures, I told the students to look at it as a ratio. That way when we got into *Ratios and Rates*, they would have heard the language, they would be familiar with it" (Kane, Interview 5/7/99). When planning to teach a MiC unit, Kane worked through the unit the way a student would, noted the amount of time it took, divided the unit into portions appropriate for a class period, looked for appropriate links to children's literature, and chose a pre-unit warm-up activity that would help identify the prior knowledge students brought to the topic. Kane thought that MiC was well aligned with the state standards and district mathematics guidelines (Kane, Interview 5/7/99).

Table G53
Subcategories of Unit Planning: Kane, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

Woodward generally followed the recommended sequence of MiC units, but modified the sequence to teach content needed for district standardized tests. When planning to teach a MiC unit, Woodward studied the unit in order to determine unit objectives or goals. If he found a section did not include an investigation or contained an investigation that did not provide enough practice, he created an additional investigation. Woodward used supplementary materials in preparation for district standardized texts. He did not consider students' prior knowledge when planning, but hoped to react to students' needs during instruction. Woodward found it difficult to follow the suggested pace for instruction:

> We couldn't meet the pacing schedule. So we tried to do as much as we could during class, as far as the investigations and certain things were concerned, so that [the students] would have an idea of what they were to do, making sure it was clear in their minds. [Then we] gave them certain questions for homework to keep up the pacing. That seemed to work, but it was still at a slower pace based on what the unit was asking us to do. (Woodward, Interview 6/17/99)

Table G54
Subcategories of Unit Planning: Woodward, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the three seventh-grade teachers in District 4 varied in all three subcategories of *unit planning* (see Figure G6). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Two teachers followed the unit development in the teacher guide with little or no consideration of students' prior knowledge.

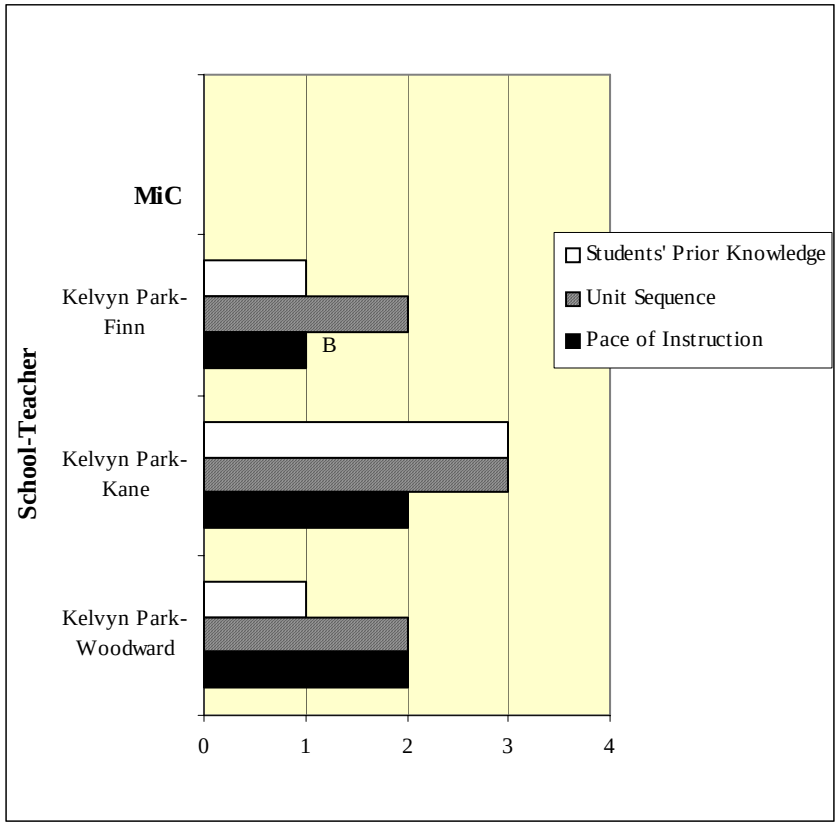


Figure G6. Unit planning, Grade 7, District 4.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

In determining the *sequence of instructional units*, one teacher considered linkages across units of the same content strand. The other two teachers considered external factors such as the content of the statewide testing program when sequencing instructional units. In determining the *pace for instruction*, two teachers anticipated that various factors such as needed prerequisite skills of the difficulty of content would necessitate adjusting the planned pace for instruction. The other teacher did not plan unit pacing because the curriculum was unfamiliar to her.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* When planning to teach individual MiC lessons, Finn first considered the ability level of the class, which influenced the amount of review and practice she included in her plans, the pace of instruction, and the kinds of thinking she expected from the students. Finn also worked through the problems and activities: "I think it is important that I go through it first. Otherwise it will be a total failure. Whatever I struggle with, I assume the kids will probably struggle with too. I think of what they might need before I start doing it with them" (Finn, Interview 5/4/99). The form of instruction she chose was determined by the lesson and accompanying activities. She explained:

> Some sections in the book require a lot of individual work when [students] are actually working on their own. And then we share responses. Some of the sections in the book require a lot of group activity. If you don't do the [activities] in groups, you'll never finish them. Some of the activities require a large-group discussion first. It depends on the difficulty. If I see that a question is really difficult to understand, we discuss it before they attempt to do it on their own. (Finn, Interview 5/4/99)

When students shared their responses, they sometimes discussed the problems from their seats or explained their solution strategies at the overhead projector. Finn valued small-group work for two reasons: Students could learn from each other and students could encourage or pressure others to participate. However, she thought that grouping students with different learning styles and work skills was critical to the success of the group (Finn, Interview 5/4/99).

Table G55
Subcategories of Lesson Planning: Finn, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focuses on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Kane worked through all of the problems. This enabled her to develop probing questions that would facilitate student learning and to divide the lesson into reasonable amounts for a class period. She considered how students performed in the

previous lesson. If there were difficulties, she planned to use a new approach or to break the lesson down into smaller pieces. She planned for opposite ends of the student-ability spectrum. On the low end, she planned fewer activities at a slower pace. She planned to use the advanced students who might complete the investigation quickly as peer tutors. She explained: “I find that they truly learn the material. It’s interesting to see how they model my questioning. They don’t tell the other kids how to do it. They question them to get them to think” (Kane, Interview 5/7/99). Kane valued small-group work because of the effectiveness of peer tutoring. She commented, “I love to watch [peer tutors] because I learn a lot from the way they see things. Peers will ask the right questions or say things that help others understand” (Kane, Interview 5/7/99). Kane made notes in her lesson plans when small groups were especially effective. She found small groups of four worked best in her class, with three the minimum and six the maximum. She grouped students heterogeneously and changed the groups at least four times a year:

> I don’t want the [students] to be too comfortable. And since we each have our strengths and weaknesses in different areas, [changing groups] helps students build up their weaknesses and get stronger in other areas. I don’t want them becoming dependent. I want them to start thinking for themselves. (Kane, Interview 5/7/99)

Kane expected students to participate in large-group discussions. After each investigation, students shared their observations, discussed their actions, and explained the conclusions they drew from their investigations (Kane, Interview 5/7/99).

Table G56
Subcategories of Lesson Planning: Kane, Kelvyn Park Middle School, District 4, Grade 7

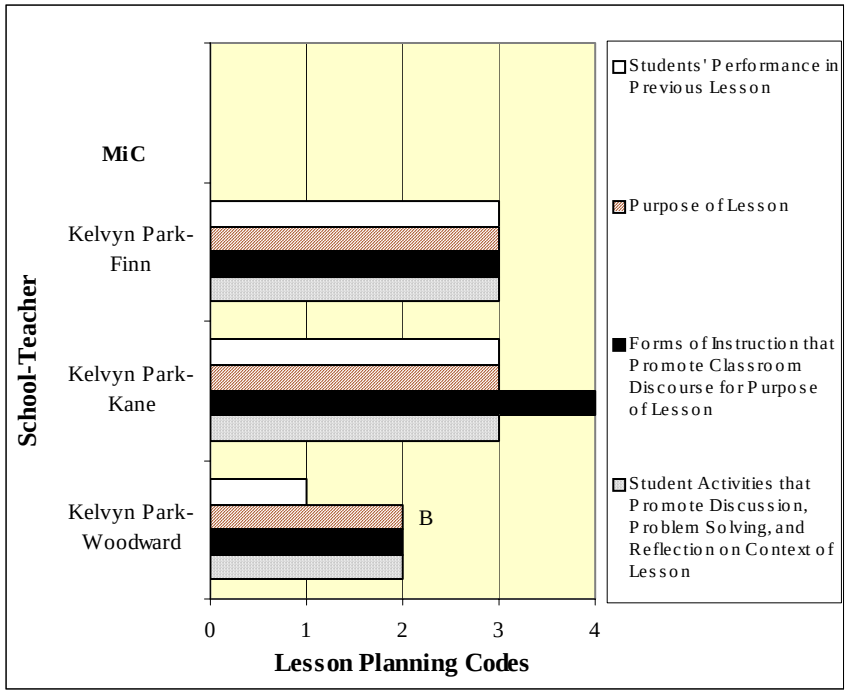
Subcategory	Rating	Description of Rating
Students’ Performance in Previous Lesson	3	Changes focuses on students’ understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Woodward read through the lesson to become familiar with the content, unit goals, and expected student outcomes. He also worked each problem to get an idea of the questions and solutions students might have. The form of instruction he chose, whether large-group direct teaching, large-group discussion, or small-group work, was determined by the lesson and the activities. Woodward valued small-group work and physically arranged the room to facilitate group activities. Students were heterogeneously grouped at tables (Woodward, Interview 6/17/99).

Table G57
Subcategories of Lesson Planning: Woodward, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the three seventh-grade teachers in District 4 varied in all four subcategories of *lesson planning* (see Figure G7). With respect to *students' performance in the previous lesson*, two teachers' decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. The other teacher planned in the same manner daily without considering students' previous performance. Little variation was seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. The other teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.

Figure G7. Lesson planning, Grade 7, District 4.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned forms of instruction that promoted substantive conversation such as classroom activities that encouraged students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The remaining teacher included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Little variation was seen in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For two teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan. The remaining teacher included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Table G58
Unit Planning, District 1, Grade 7, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Fernwood-Heath	2A	1	2	5
Von Humboldt-Bartlett	1	1	1B	3
Von Humboldt-Muldoon	2A	1	2	5
Wacker-Burton	2A	1	2	5
		— Conventional —		
Addams-St. James	2A	1	2	5
Fernwood-Hodge	2A	3	2	7
Wacker-Rubin	3	1	2	6

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table G59
Lesson Planning, District 1, Grade 7, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Fernwood-Heath	2	3	3	3	11
Von Humboldt-Bartlett	2	2B	2	2	8
Von Humboldt-Muldoon	2	3	2	3	10
Wacker-Burton	3	3	2	2	10
			— Conventional —		
Addams-St. James	1	2B	1	1	5
Fernwood-Hodge	2	2B	2	2	8
Wacker-Rubin	3	2A	2	2	9

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:
1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.
Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:
1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.
2. *Overall curriculum plan in mind.*
A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.
Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:
1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned*. The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.
Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:
1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table G60

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	41	100	0	0	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	63	94	6	0	0
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	66	18	77	3	2
Fernwood-Hodge	125	98	55	30	12	2
Wacker-Rubin	38	79	83	17	0	0

Table G61

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	34	30	40	20	10
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	70	60	30	6	4
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	72	10	74	10	7
Fernwood-Hodge	125	81	8	50	31	11
Wacker-Rubin	38	74	29	57	4	11

Table G62

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	59	35	59	6	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	55	50	33	10	7
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	61	6	82	11	1
Fernwood-Hodge	125	78	3	13	83	1
Wacker-Rubin	38	53	10	50	30	10

Table G63

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	0				
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	66	34	22	38	6
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	36	2	96	2	0
Fernwood-Hodge	125	35	0	2	98	0
Wacker-Rubin	38	0				

Table G64

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	48	0	43	43	14
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	22	12	12	59	18
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	5	0	71	14	0
Fernwood-Hodge	125	27	9	53	35	3
Wacker-Rubin	38	24	0	33	67	0

Table G65

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	21	0	50	50	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	41	23	26	23	29
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	143	12	18	76	6	6
Fernwood-Hodge	125	44	20	65	13	2
Wacker-Rubin	38	47	11	72	17	0

Table G66

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	59	41	47	12	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	67	71	24	6	0
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	65	6	80	7	6
Fernwood-Hodge	125	79	5	28	66	1
Wacker-Rubin	38	61	13	39	39	9

Table G67

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	55	0	50	44	6
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	75	14	44	37	5
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	57	5	91	4	0
Fernwood-Hodge	125	66	8	35	43	13
Wacker-Rubin	38	39	0	53	47	0

Table G68

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	52	7	67	27	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	72	35	40	24	2
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	63	4	90	4	1
Fernwood-Hodge	125	76	1	12	79	8
Wacker-Rubin	38	71	0	44	44	11

Table G69

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	41	50	50	0	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	72	35	33	29	4
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	58	5	94	1	0
Fernwood-Hodge	125	74	2	5	85	8
Wacker-Rubin	38	58	9	55	27	9

Table G70

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	14	0	100	0	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	37	57	29	14	0
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	28	10	85	2	2
Fernwood-Hodge	125	15	0	47	42	11
Wacker-Rubin	38	66	12	40	40	8

Table G71

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	7	0	50	0	50
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	5	25	0	50	25
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	13	0	50	11	39
Fernwood-Hodge	125	14	0	0	11	89
Wacker-Rubin	38	13	0	60	20	20

Table G72

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	24	57	43	0	0
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	63	60	33	6	0
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	44	6	92	2	0
Fernwood-Hodge	125	59	26	61	14	0
Wacker-Rubin	38	47	61	22	6	11

Table G73

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 1, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Heath	29	0				
Von Humboldt-Bartlett	NA	NA				
Von Humboldt-Muldoon	76	12	78	22	0	0
Wacker-Burton	NA	NA				
Conventional						
Addams-St. James	144	5	43	57	0	0
Fernwood-Hodge	125	1	0	100	0	0
Wacker-Rubin	38	34	92	8	0	0

Table G74

Teacher Level of Lesson Presentation and Development, District 1, Grade 7

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Fernwood-Heath	7	0	1	0	0	2	3	1	0	4
Von Humboldt-Bartlett	8	0	2	0	0	2	3	1	0	4
Von Humboldt-Muldoon	8	0	3	2	0	3	0	0	0	3B
Wacker-Burton	3	0	0	0	0	3	0	0	0	3B
Conventional										
Addams-St. James	9	0	6	1	0	2	0	0	0	2A
Fernwood-Hodge	7	0	0	0	0	6	0	1	0	3B
Wacker-Rubin	9	0	0	0	0	9	0	0	0	3B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support*. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table G75

Nature of Mathematical Inquiry, Grade 7, District 1

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																							
Fernwood-Heath	2	2	2	2	3	1	1	2		2	2	3	1	3	1	1	4		2	1	2	2	2	1	1	1		1	3	3	2	1	1	2	1		*1.81*	***3***
Von Humboldt-Bartlett	2	2	1	1	1	2	4	1	2	3	1	1	2	1	1	2	1	1	2	2	1	1	3	2	1	1	2	3	2	1	2	1	1	3	1	2	*1.67*	***2***
Von Humboldt-Muldoon	1	1	1	1	4	2	2	1	3	3	2	1	1	2	2	1	1	1	3	1	1	2	1	1	1	2	1	2	1	1	1	3	1	3	2	2	*1.64*	***2***
Wacker-Burton	3	1	1	4	4					2	2	1	4	4					2	1	1	4	4					3	2	1	3	2					*2.45*	***4***
Conventional																																							
Addams-St. James	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	*1.11*	***1***
Fernwood-Hodge	3	1	3	3	3	2	1	1		2	2	2	2	2	1	2	2		2	2	3	3	3	2	1	3		2	1	1	2	1	1	1	1		*1.91*	***3***
Wacker-Rubin	2	1	1	1	1	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	1	1	*1.14*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table G76

Nature of Students' Explanations, Grade 7, District 1

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Heath	1	3	1	2	1	1	1	1		*1.38*
Von Humboldt-Bartlett	2	2	1	1	2	2	2	1	2	*1.67*
Von Humboldt-Muldoon	1	1	1	1	2	1	2	2	1	*1.33*
Wacker-Burton	1	1	1	3	3					*1.80*
Conventional										
Addams-St. James	2	1	1	1	1	1	1	2	1	*1.22*
Fernwood-Hodge	2	1	2	1	1	2	2	1		*1.50*
Wacker-Rubin	2	1	1	1	1	1	1	1	1	*1.11*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table G77

Elicitation of Multiple Strategies, Grade 7, District 1

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Heath	1	4	2	1	1	1	1	1		*1.50*
Von Humboldt-Bartlett	1	1	1	1	1	4	2	1	3	*1.67*
Von Humboldt-Muldoon	1	1	1	1	1	1	1	1	2	*1.11*
Wacker-Burton	1	1	1	1	1					*1.00*
Conventional										
Addams-St. James	1	1	1	1	1	2	1	2	1	*1.22*
Fernwood-Hodge	1	1	3	1	1	2	2	1		*1.50*
Wacker-Rubin	1	1	1	1	1	1	1	1	1	*1.00*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table G78

Nature of Student–Student Conversation, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Heath	1	1	4	2	2	1	1	1		*1.63*
Von Humboldt-Bartlett	2	1	1	1	1	1	1	1	1	*1.11*
Von Humboldt-Muldoon	3	1	1	1	4	1	1	1	1	*1.56*
Wacker-Burton	1	1	1	1	1					*1.00*
Conventional										
Addams-St. James	1	1	1	1	1	1	1	1	1	*1.00*
Fernwood-Hodge	1	1	1	1	1	1	1	1		*1.00*
Wacker-Rubin	1	1	1	1	1	2	1	1	1	*1.11*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table G79

Students' Collaborative Working Relationships, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Heath	2	2	3	3	3	1	3	1		*2.25*
Von Humboldt-Bartlett	2	N/A	N/A	N/A	N/A	1	1	N/A	N/A	*1.33*
Von Humboldt-Muldoon	3	N/A	1	N/A	4	2	1	1	1	*1.86*
Wacker-Burton	N/A	1	N/A	N/A	N/A					*1.00*
Conventional										
Addams-St. James	1	N/A	1	N/A	N/A	1	1	1	1	*1.00*
Fernwood-Hodge	1	1	1	N/A	1	1	1	1		*1.00*
Wacker-Rubin	1	N/A	N/A	N/A	N/A	2	N/A	N/A	N/A	*1.50*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table G80

Student Engagement during Instruction, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Heath	4	3	4	3	3	2	3	4		*3.25*
Von Humboldt-Bartlett	3	3	3	4	2	4	4	1	2	*2.89*
Von Humboldt-Muldoon	2	4	2	2	3	4	2	2	2	*2.56*
Wacker-Burton	4	3	3	4	4					*3.60*
Conventional										
Addams-St. James	3	4	4	2	2	2	3	4	2	*2.89*
Fernwood-Hodge	3	3	3	3	2	3	2	1		*2.50*
Wacker-Rubin	2	4	2	3	2	3	3	4	4	*3.00*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table G81
Unit Planning, District 2, Grade 7, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Guggenheim-Carlson	2B	1	2	5
Guggenheim-Dillard	2A	2	2	6
Weir-Caputo	1	1	2	4
Weir-Gallardo	2A	1	2	5
		— Conventional —		
Newberry-Cunningham	1	1	2	4

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table G82
Lesson Planning, District 2, Grade 7, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Guggenheim-Carlson	2	3	3	2	10
Guggenheim-Dillard	2	2A	3	3	10
Weir-Caputo	2	2B	3	2	9
Weir-Gallardo	1	2B	3	2	8
			— Conventional —		
Newberry-Cunningham	2	2A	2	2	8

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:
1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.
Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:
1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.
2. *Overall curriculum plan in mind.*
A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.* The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.
Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:
1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction.* The teacher encourages such participation, but it is still not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned.* The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.
Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:
1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.* Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table G83

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	32	100	0	0	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G84

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	45	69	29	3	
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G85

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	42	84	16	0	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G86

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	65	28	70	2	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G87

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 2, Grade 7, 1998-1999

Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	70	4	50	43	4
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G88

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	27	24	43	14	19
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G89

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	64	88	10	2	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G90

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	75	2	31	59	9
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G91

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	73	34	63	4	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G92

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	73	45	52	4	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G93

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	32	96	4	0	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G94

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	8	17	50	17	17
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G95

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	1	100	0	0	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G96

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 2, Grade 7, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Carlson	NA	NA				
Guggenheim-Dillard	77	1	100	0	0	0
Weir-Caputo	NA	NA				
Weir-Gallardo	NA	NA				
Conventional						
Newberry-Cunningham	NA	NA				

Table G97

Teacher Level of Lesson Presentation and Development, District 2, Grade 7

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Carlson	6	0	2	0	1	0	3	0	0	4
Guggenheim-Dillard	6	0	2	0	0	0	2	5	0	5
Weir-Caputo	7	2	0	5	0	0	0	0	0	2B
Weir-Gallardo	4	0	0	0	0	0	0	3	1	5
Conventional										
Newberry-Cunningham	6	0	1	0	0	3	1	1	0	3B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table G98

Nature of Mathematical Inquiry, Grade 7, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation										
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	*Mean*	***Rating***
MiC																																							
Guggenheim-Carlson	4	3	2	3	3	2				2	2	2	2	1	2				3	3	2	1	2	4				2	3	1	2	3	3				*2.38*	***4***
Guggenheim-Dillard	3	4	4	1	4	3				2	2	2	1	2	1				2	3	2	2	3	2				1	2	3	2	3	2				*2.33*	***4***
Weir-Caputo	2	2	1	1	1	2	2			1	1	1	1	1	1	2			1	2	1	1	1	2	1			1	2	2	2	3	2	3			*1.54*	***2***
Weir-Gallardo	3	3	3	3	1	4				4	4	3	4	1	3				3	2	4	2	1	2				2	3	2	3	2	3				*2.71*	***4***
Conventional																																							
Newberry-Cunningham	4	2	3	1	3	2				3	1	3	1	1	1				1	2	2	1	1	1				3	3	3	1	1	1				*1.88*	***3***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table G99

Nature of Students' Explanations, Grade 7, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	2	2	2	2	1	2				*1.83*
Guggenheim-Dillard	2	3	2	1	3	2				*2.17*
Weir-Caputo	2	1	1	2	1	1	2			*1.43*
Weir-Gallardo	2	2	2	2	1	2				*1.83*
Conventional										
Newberry-Cunningham	2	1	2	1	1	2				*1.50*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table G100

Elicitation of Multiple Strategies, Grade 7, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	2	1	1	1	1	2				*1.33*
Guggenheim-Dillard	3	4	4	2	3	1				*2.83*
Weir-Caputo	1	2	1	1	1	2	2			*1.43*
Weir-Gallardo	2	2	2	1	1	1				*1.50*
Conventional										
Newberry-Cunningham	3	1	2	1	1	1				*1.50*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table G101

Nature of Student–Student Conversation, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	1	3	2	3	1	1				*1.83*
Guggenheim-Dillard	3	3	1	2	2	2				*2.17*
Weir-Caputo	1	1	1	1	2	1	2			*1.29*
Weir-Gallardo	2	3	2	3	2	1				*2.17*
Conventional										
Newberry-Cunningham	3	1	1	1	2	1				*1.50*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table G102

Students' Collaborative Working Relationships, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	1	2	2	2	1	2				*1.67*
Guggenheim-Dillard	2	3	2	2	1	2				*2.00*
Weir-Caputo	1	2	1	1	2	2	2			*1.57*
Weir-Gallardo	2	2	1	3	2	1				*1.83*
Conventional										
Newberry-Cunningham	3	1	1	1	1	1				*1.33*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table G103

Student Engagement during Instruction, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	3	3	2	3	3	3				*2.83*
Guggenheim-Dillard	4	4	3	3	3	4				*3.50*
Weir-Caputo	1	1	1	1	1	1	3			*1.29*
Weir-Gallardo	4	3	3	4	3	4				*3.50*
Conventional										
Newberry-Cunningham	3	2	4	4	2	3				*3.00*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Explanation-Oriented	1	2	3	4	5		6	7	8	
				22%	*22%*				*11%*	
Task-Oriented					1	2	3	4		
						44%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(9 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G8. Interactive Decisions, Heath, Fernwood Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	13%		*6%*	*25%*				*25%*		
Task-Oriented					1	2	3	4		
							6%			
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	6%	*13%*					*6%*			
(16 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G9. Interactive Decisions, Bartlett, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		14%	*14%*	*29%*						
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	19%	*14%*								
(7 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G10. Interactive Decisions, Muldoon, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		20%		*40%*				*20%*		
Task-Oriented					1	2	3	4		
						20%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(5 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G11. Interactive Decisions, Burton, Wacker Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	24%			*41%*				*12%*		
Task-Oriented					1	2	3	4		
						6%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	6%	*6%*				*6%*				
(17 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G12. Interactive Decisions, St. James, Addams Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		21%		*13%*	*4%*		*4%*	*17%*		
Task-Oriented					1	2	3	4		
						4%	*8%*			
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	8%	*13%*			*4%*				*4%*	
(24 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G13. Interactive Decisions, Hodge, Fernwood Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	17%	*11%*	*22%*	*17%*				*6%*		
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	11%	*6%*				*11%*				
(18 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G14. Interactive Decisions, Rubin, Wacker Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
				20%	*20%*			*40%*		
Task-Oriented					1	2	3	4		
					20%					
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(5 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G15. Interactive Decisions, Carlson, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		8%		*17%*				*17%*	*25%*	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
							8%		*25%*	
(12 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G16. Interactive Decisions, Dillard, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		8%	*8%*	*25%*						
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
		17%			*8%*		*8%*	*8%*	*17%*	
(12 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G17. Interactive Decisions, Caputo, Weir Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	13%			*13%*			*13%*		*25%*	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
					13%	*13%*			*13%*	
(8 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G18. Interactive Decisions, Gallardo, Weir Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	11%		*11%*		*11%*			*44%*		
Task-Oriented					1	2	3	4		
					11%					
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
									11%	
(9 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G19. Interactive Decisions, Cunningham, Newberry Middle School.

Appendix H

Instruction

Grade 8

Grade 8

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the 19 indices, grouped by major categories, and the composite index for instruction, are described and illustrated with evidence from classroom observations, teacher interviews, teaching logs, and teacher journal entries. The ratings assigned to each teacher in Districts 3 and 4 on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in Districts 3 and 4. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 1

In District 1, six Grade 8 teachers participated in the study. Two teachers used MiC, and four teachers used the conventional curricula already in place in their schools. One study class that used a conventional curriculum had three teachers over the course of the school year (Marin, Kendall, and DiMatteo, respectively).

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Reichers generally followed the recommended sequence of MiC units. However, she taught *Great Expectations*, a unit on probability, before district standardized tests. When planning to teach a MiC unit, Reichers read the unit overview in the teacher guide, noting the concepts to be taught, concepts she expected students to master, the outline of conceptual development, the prerequisites in the MiC curriculum, and the suggested pace for instruction. She modified lessons because her students did not have MiC in Grades 5 and 6. These modifications included slowing the suggested pace of instruction, using supplementary materials to practice skills, and eliminating some problems she considered too difficult for the class. She commented:

> I've tried to follow the teacher guide. Sometimes I just felt like I needed to do more. I needed to insert other activities to reinforce things. I had to do supplementary things so that the kids would even have a clue about what was going on. There were [other] times when I'd skip a couple pages if the kids were already struggling with a concept. I knew that this wasn't the end of the road, it wasn't the last time they were going to see it. I would just skip those two pages because I didn't want [students] to shut down. (Reichers, Interview 5/11/99)

Reichers believed that MiC was based on the NTCM *Standards*, so if she completed all of the eighth-grade units, she felt that she would meet those standards. However, Reichers found it difficult to teach all recommended units. She also felt that planning to teach MiC took more time than planning to teach from a conventional textbook because MiC made her think about mathematics in a different way (Reichers, Interview 5/11/99).

Table H1
Subcategories of Unit Planning: Reichers, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Waters generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she worked through the whole unit as a student would. This allowed her to identify troublesome parts and to develop strategies for dealing with them. Waters felt that a problem inherent with most of the eighth-grade units was the lack of program continuity. Because MiC was designed with a spiral format (mastery over time), the development of the concepts studied in eighth-grade units had begun years earlier. Students whose mathematics instruction was not consistently MiC or had not included all of the units for each year from Grades 5–7 might not have prerequisite conceptual understanding or skill in procedures unique to MiC such as using a ratio table for the division of fractions or decimals. Waters anticipated problems and planned to use supplementary materials such as reinforcement sheets from a conventional textbook or seventh-grade MiC units to help breach the conceptual gaps. Waters was aware of the suggested pace of instruction, but felt she was still learning how to teach MiC and was not yet successful in determining the pace prior to instruction. She commented:

> I'm never sure what's going to happen. Sometimes things take me twice as long, and sometimes they take me half the time than I think it's going to take. For me, it has been a lot of feeling my way through it as far as pacing. I think that by next year it will be easier. My sense is that I'll be able to move a little faster because I'll know what some of the pitfalls are and hopefully will be able to catch those. (Waters, Interview 4/15/99)

Waters thought it took more time to plan to teach a MiC unit than it did to teach a unit using a conventional textbook because the presentation of mathematics in MiC was different and she needed time to preview the lessons. She also needed time to organize the materials needed for each MiC unit (Waters, Interview 4/15/99).

Table H2
Subcategories of Unit Planning: Waters, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	1B	Little or no consideration of pacing

Wolfe followed the recommended sequence of chapters in the conventional textbook she used. When planning to teach a chapter, she reviewed it, made the overhead transparencies she wanted to use, and developed ways she could connect the mathematical ideas in the chapter. Wolfe talked with the students' different teachers in the previous year to find out how students had performed in his class, what topics were covered, and any other information that would help her when planning chapters. She learned about students' prior knowledge when she noted their reaction to the topic during the chapter introduction. She explained, "I kind of bring up the topic and give a brief little explanation and maybe kind of go on their reaction to it. I don't really give them a test or anything to find out where they are. I guess I kind of go from their verbal expressions or reactions" (Wolfe, Interview 4/29/99).

Table H3
Subcategories of Unit Planning: Wolfe, Addams Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	1A	Little or no consideration of pacing

Data on unit planning for Marin were unavailable. Kendall replaced Marin during the second quarter of the school year. Data on unit planning for Kendall were unavailable. DiMatteo replaced Kendall at the beginning of the second semester. DiMatteo generally followed the recommended sequence of chapters in the conventional textbook she used. When planning to teach a chapter, she scanned the sections in the chapter to get a general impression of the content. She outlined how she would relate the topic to something students already knew. She explained, "No matter what chapter it is or what section it is, I'll always relate it to something they already have concrete knowledge about so it's prior knowledge that I hit first" (DiMatteo, Interview 5/5/99). DiMatteo planned only three sections of a chapter at a time because the topics included in a chapter were too broad. She did not determine the pace for instruction prior to instruction, but made daily decisions based on student performance (DiMatteo, Interview 5/5/99).

Table H4

Subcategories of Unit Planning: DiMatteo, Wacker Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the six eighth-grade teachers in District 1 varied minimally in all three subcategories of *unit planning* (see Figure H1). Data on lesson planning for two teachers (Kendall and Marin) were unavailable. Two teachers based their plans on their perceptions of students' needs related to the development of concepts and procedures. One teacher based her plan on her perceptions of students' prior knowledge in mathematics or reading without informal or formal assessment. The remaining teacher planned the unit with little or no understanding of the students' prior knowledge.

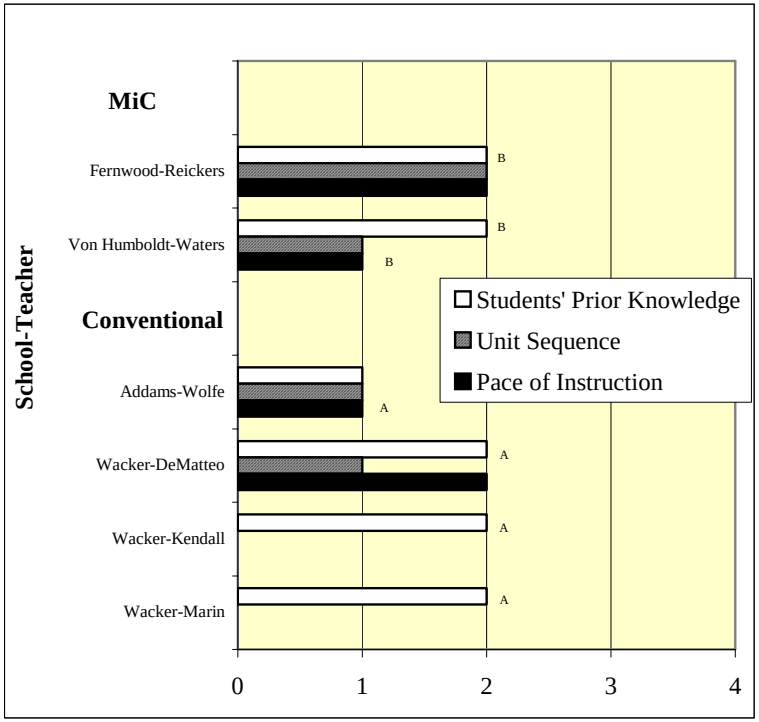


Figure H1. Unit planning, Grade 8, District 1.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table H45 in this appendix.

In determining the *sequence of instructional units*, one teacher considered external factors such as the content of statewide testing program when sequencing instructional units. Three teachers followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, two teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction. The other two teachers followed the recommendations for pacing in teacher support materials.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Reichers worked through the problems to become aware of the level of difficulty and some possible solution strategies. She also considered the students' performance in the previous lesson. She explained:

> I could tell if students had difficulties by looking at their work, by the number of questions [students] asked during the activity, or by the number of [students] who were really frustrated. If I had to help a lot of people get through the lesson from the day before, then I'll go over those problems again or pick some supplemental problems to do to get [students] started for the day. (Reichers, Interview 5/11/99)

Reichers valued small-group work. She believed the saying: "The more they talk, the better they learn." She felt, however, that classroom management was problematic. She explained: "How do I keep kids on task and not have them fooling around? How do I deal with students who just want to work by themselves? I finally ended up direct teaching more" (Reichers, Interview 5/11/99). When bright students who had worked independently handed in solutions with no depth of understanding, she tried having students work with a partner some of the time. Students began to have more success solving the problems, and the quality of their answers improved. Reichers commented:

> When [students] are working in pairs, there have been some days when I would hear such exciting talk between the two people and that would be so exciting to me. I try to go around and say, "Wow! That is great math talk!" and pat them on the shoulder. I remember some days when people were reading the questions to each other and pointing out things in the book to each other. I thought, "This is wonderful!" (Reichers, Interview 5/11/99)

Reichers chose three instructional forms approximately half of the reported days, small-group work (54%), review of previous material (49%), and teacher presentation (47%; see Tables H47-H52 in this appendix). Of the three forms, small-group work was given the most class time: at least half of the class period on 84% of the days. Review of previous material and teacher presentation were often given equal emphasis with other instructional forms. Three other instructional forms were chosen approximately one third of the reported days, warm-up activities, (35%), whole-class discussion (30%), and independent practice (27%). Whole-class discussion and independent practice were given similar amounts of class

time. Both were given at least half of the class period on about 60% of the days and equal emphasis with other instructional forms on about 30% of the days. Warm-up activities were frequently given equal emphasis with other instructional forms (Reichers, Teacher Log 1998-99).

Three student activities were important elements in Reichers' instruction, discussing answers and solution strategies (on 63% of the reported days), listening to teacher or taking notes (55%), and participating in whole-class discussions (49%; see Tables H53-H60 in this appendix). Each activity was given significant amounts of class time. Discussing answers and solutions strategies was given at least half of the class period on 74% of the days, equal emphasis with other activities on 24% of the days, and less than 15% of class time on the remaining days. Participating in whole-class discussions was given at least half of the class period on 52% of the days, equal emphasis with other activities on 36% of the days, and less than 15% of class time on the remaining days. Listening to the teacher or taking notes was given equal emphasis with other activities on 72% of the days. On approximately one third of the reported days, Reichers' students reflected on or summarized lesson concepts (37%) and investigated problems (34%) and each activity was given considerable class time. Investigating problems was given at least half of the class period on 66% of the days and equal emphasis with other activities on 34% of the days. Reflecting on or summarizing a lesson was given at least half of the class period on 21% of the days and equal emphasis with other activities on 79% of the days. Students practiced computation on 23% of the reported days for considerable amounts of class time: at least half the class period on 64% of the days and equal emphasis with other activities on 33% of the days. Students less frequently took a test or quiz (12%) and began homework (7%; Reichers, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Reichers reported in her teacher logs. The lesson observed on 11/4/98, for example, included: housekeeping duties (5 minutes); individual activity (5 minutes); large-group discussion (8 minutes); work in pairs (4 minutes); teacher-led discussion of work done in pairs (6 minutes); work in pairs (7 minutes); teacher-led discussion of work done in pairs (7 minutes); large-group discussion (9 minutes); work in pairs (8 minutes); large-group discussion (10 minutes); teacher presentation (7 minutes); and individual activity (12 minutes; Reichers, Observation 11/4/98).

Table H5
Subcategories of Lesson Planning: Reichers, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Waters worked through all of the problems in the lesson, trying to think of various solution strategies so that she would not emphasize just one strategy when presenting the lesson. She also made note of skills students would need in order to be successful with the lesson. Students' performance in the previous lesson influenced the homework assignment. Problems not completed during class time became homework. When the homework was corrected in a whole-class setting, Waters used that time to reteach if necessary. Waters valued small-group work because students could learn from each other. She commented: "I do think that this curriculum appeals to more than just mathematical logical reasoners. I see students, who I wouldn't normally think were very bright, who can catch on to something and can explain it to kids who I thought would have gotten it right away" (Waters, Interview 4/15/99). Waters used whole-group direct teaching, whole-group discussion, pairs, and small groups of four, depending on the unit and the activity. She preferred small groups that changed over time, but did not think that the behavior in the small groups was always productive. Activities that interested the students and were discovery-based helped the groups stay focused. Waters thought that as she became more comfortable with MiC, she would be more comfortable with students working in small groups (Waters, Interview 4/15/99).

Waters chose three instructional forms most often: teacher presentation (on 42% of the reported days), small-group work (35%), and review of previous material (34%; see Tables H47-H52 in this appendix) and similar amounts of class time were devoted to them. Teacher presentation was given at least half of the class period on 31% of the days, equal emphasis with other instructional forms on 57% of the days, and less than 15% of class time on 12% of the days. Small-group work was given at least half of the class time on 37% of the days, equal emphasis with other instructional forms on 58% of the days, and less than 15% of class time on the remaining days. Review of previous material was given at least half of the class period on 44% of the days, equal emphasis with other instructional forms on 38% of the days, and less than 15% of class time on 18% of the days. Warm-up activities were used on 26% of the days generally for less than 15% of class time. Whole-class discussion (15%) and independent practice (8%) were used less frequently (Waters, Teachers Log, 1998-99).

Waters chose four student activities most often: discussing answers and solution strategies (on 43% of the reported days), listening to the teacher or taking notes (39%), investigating problems (38%), and participating in whole-class discussions (34%; see Tables H53-H60 in this appendix). Each activity was given equal emphasis with other student activities about 75% of the time. Waters' students seldom took tests or quizzes (11% of the reported days), practiced computation (6%), began homework (4%), and reflected on or summarized lesson concepts (1%; Waters, Teacher Log, 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Waters reported in her teacher logs. The lesson observed on 11/10/98, for example, included: warm-up activity (12 minutes); large-group discussion of warm-up solutions (7 minutes); teacher presentation (15minutes); individual activity (14 minutes); large-group discussion of solutions and strategies (12 minutes); and large-group discussion (3 minutes; Waters, Observation 11/10/98).

Table H6
Subcategories of Lesson Planning: Waters, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual lessons from a conventional textbook, Wolfe gathered the materials, such as work sheets and overheads, worked through the story problems, and generally previewed the lesson. If students exhibited a lot of difficulties during a lesson or test, Wolfe modified her plans for the next day or two to include review and practice. Wolfe valued small-group work. She explained: "If a student doesn't understand the way I explain things, they may have a better chance of understanding it with another student explaining it. [Explaining the lesson] also teaches students communication skills" (Wolfe, Interview 4/29/99). However, Wolfe was not comfortable with small-group work because she felt that she lost control and could not be sure students were learning. On two occasions when Wolfe did plan for small-group work, she chose four or five students for each group and assigned each member a role such as facilitator, recorder, harmonizer, material manager, timekeeper, or reporter. She thought that small-group work was beneficial when students worked on word problems or reviewed for a chapter test. She commented: "I have yet to find a good way to teach word problems. I can get up there and explain to them how I did it, but they need to do them on their own. Maybe just sitting down and working with other kids on these problems would be good for them" (Wolfe, Interview 4/29/99). Wolfe's preferred form of instruction was whole-class direct teaching followed by students working on the problems individually. During whole-class instruction, any class discussion was between a student and the teacher. She said, "I think I like [students sitting] in rows on a regular basis and group work to give them a little variety. I do like the idea of pairs. [Students] just turn to the person near by and work on a problem. I've been experimenting with group work" (Wolfe, Interview 4/29/99).

Wolfe chose three instructional forms most often: review of previous material (on 74% of the reported days), teacher presentation (67%), and independent practice (55%; see Tables H47-H52 in this appendix). Each form was given a significant amount of class time. Review of previous material was given at least half of the class period on 44% of the days and equal emphasis with other instructional forms on 54% of the days. Teacher presentation was given at least half of the class time on 81% of the days and equal emphasis with other instructional forms on 18% of the days. Independent practice was given at least half of the class period on 45% of the days and equal emphasis with other instructional forms on 55% of the days. Wolfe began class with warm-up activities on 24% of the reported days and gave these activities at least half of the class period

on 34% of the days, equal emphasis with other instructional forms on 61% of the days, and less than 15% of class time on the remaining days. Wolfe rarely chose small-group work (7%) and did not report whole-class discussion (Wolfe, Teacher Log 1998-99).

Student activities that were important elements in Wolfe's instruction were: practicing computation (on 79% of the reported days); discussing answers and solution strategies (76%); investigating problems (76%); and participating in whole-class discussions (76%), listening to the teacher or taking notes (64%); and reflecting on or summarizing lesson concepts (58%; see Tables H53-H60 in this appendix). Each student activity was given at least half of the class period on 60% or more of the days and equal emphasis with other activities on about a third of the days. Students seldom took a test or quiz (11%) and began homework (9%; Wolfe, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Wolfe reported in her teacher logs. The lesson observed on 1/13/99, for example, included: housekeeping duties (11 minutes); large group, teacher gave answers to homework (6 minutes); teacher presentation with some students input (22 minutes); and large group, teacher assigned homework and announced quiz (4 minutes: Wolfe, Teacher Log 1998-99).

Table H7
Subcategories of Lesson Planning: Wolfe, Addams Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	1	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson

Data on lesson planning for Marin for 1998-1999 were unavailable. Kendall replaced Marin for the second quarter of the school year. Data on lesson planning for Kendall were unavailable.

DiMatteo replaced Kendall for the second semester of the school year. When planning to teach individual lessons from a conventional textbook, DiMatteo generally reviewed lesson objectives, worked through the example problems, and developed more example problems to work through with students. She thought the lessons were very straight forward except when there were mixed review problems. She explained, "I always go back through and make sure that I remind students of something they're going to need. I usually assign mixed review for class work so I can remind them of previous lessons" (DiMatteo, Interview 5/5/99).

Review of previous material was an important instructional form for DiMatteo, using it on 71% of the reported days and generally for at least half of the class period (see Tables H47-H52 in this appendix). DiMatteo chose four other instructional forms on approximately half of the reported days: warm-up activities (54%); independent practice (49%); whole-class discussion (46%); and teacher presentation (41%). Teacher presentation and whole-class discussion were always given at least half of the class time. Warm-up activities and independent practice were given at least half of the class period on about 85% of the days. DiMatteo chose small-group work on 24% of the reported days, and it was given at least half of the class period on 44% of the days, less than 15% of class time on 50% of the days, and equal emphasis with other instructional forms on the remaining days (DiMatteo, Teacher Log 1998-99).

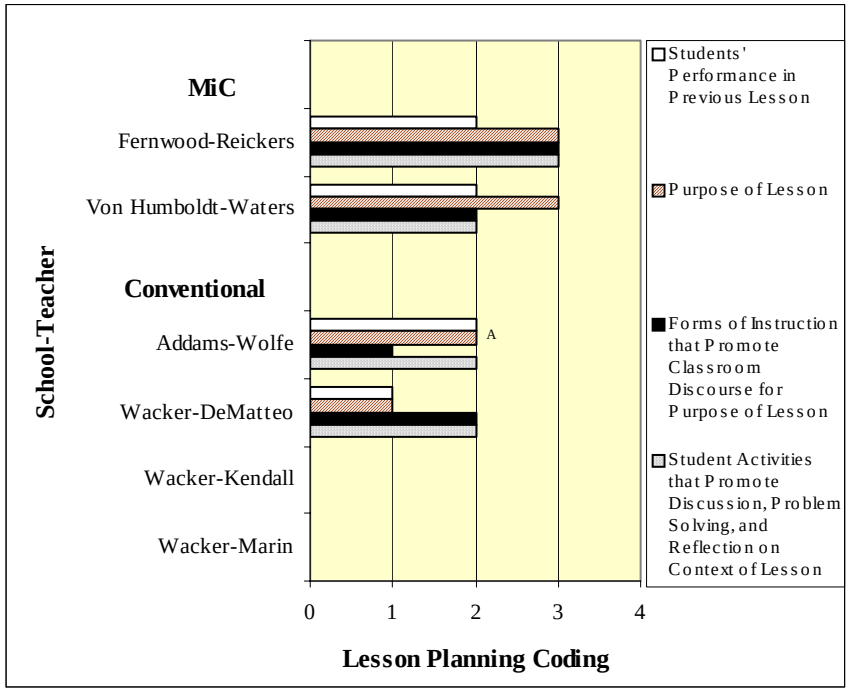
DiMatteo's students were engaged in five activities on approximately half of the reported days: listening to the teacher or taking notes (on 50% of the days); discussing answers and solution strategies (46%); investigating problems (46%); practicing computation (46%); and participating in whole-class discussions (45%; see Tables H53-H60 in this appendix). Listening to the teacher or taking notes and participating in whole-class discussion were always given at least half of the class period and other activities were given at least half of the class period on over 86% of the days. Students were engaged in three activities on approximately one fourth of the reported days: reflecting on or summarizing lesson concepts (25%), beginning homework (21%), and taking a test or quiz (21% DiMatteo, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information DiMatteo reported in her teacher logs. The lesson observed on 1/22/99, for example, included: individual warm-up activity (7 minutes); teacher presentation (6 minutes); large group, teacher and students worked 3 example problems (8 minutes); individual activity (5 minutes); teacher presentation (6 minutes); and individual activity (11 minutes; DiMatteo, Observation 1/22/99).

Table H8
Subcategories of Lesson Planning: DiMatteo, Wacker Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	1	Little or no planning to teach the specific lesson
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers ans solution strategies included in the lesson plan

In summary, the six eighth-grade teachers varied in all four subcategories of *lesson planning* (see Figure H2). Data on lesson planning for two teachers (Kendall and Marin) were unavailable. With respect to *students' performance in the previous lesson*, three teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The other teacher planned in the same manner daily without considering students' previous performance. Variation was also seen in teachers' attention to the purpose of the lesson. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. One teacher selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources. When the remaining teacher planned to teach a specific lesson, her purpose was to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction was to cover lessons in the textbook or curriculum; thus, no additional planning was deemed necessary.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F46 in this appendix.

Figure H2. Lesson planning, Grade 8, District 1.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. Two teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. The

remaining teacher seldom planned forms of instructions that encouraged classroom discourse. Teachers varied minimally in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher, investigation of problems discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan. The remaining three teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Reichers. The evidence gathered for lesson presentation and development for Reichers ranged from Level 2B to 5. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table H61 in this appendix). An overall rating of 3 was assigned for Reichers for the nature of inquiry that transpired during instruction (see Table H62). Reichers was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure H9). For Reichers, 36% of the decisions were coded as reflective of good standard pedagogy, 30% were most aligned with teaching for understanding, and 36% were least aligned with teaching for understanding.

Across all observations for Reichers, the mean rating for the index on students' explanations was 2.00, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H63). The mean rating for elicitation of multiple strategies was 1.88, indicating that multiple strategies were rarely elicited from students (see Table H64). A rating of 1 was assigned for Reichers on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H9
Overall Ratings for Mathematical Interaction for Reichers, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	2.00	Focus on procedures
Elicitation of Multiple Strategies	1.88	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Waters. The evidence gathered for lesson presentation and development for Waters ranged from Level 2B to 4. An overall rating of Level 3B was assigned indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table H61 in this appendix). An overall rating of 2 was assigned for Waters for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table H62). Waters was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure H10). For Waters, 53% of the decisions were coded as reflective of good standard pedagogy, 16% were most aligned with teaching for understanding, and 32% were least aligned with teaching for understanding.

Across all observations for Waters, the mean rating for the index on students' explanations was 1.33, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H63). The mean rating for elicitation of multiple strategies was 1.78, indicating that multiple strategies were rarely elicited from students (see Table H64). A rating of 1 was assigned for Waters on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H10
Overall Ratings for Mathematical Interaction for Waters, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.33	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.78	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Wolfe. The evidence gathered for lesson presentation and development for Wolfe ranged from Level 1 to 3B. An overall rating of Level 3B was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table H61 in this appendix). An overall rating of 1 was assigned for Wolfe for the nature of inquiry that transpired during instruction, indicating that inquiry was limited to lower-order thinking (see Table H62). Wolfe was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure H11). For Wolfe, 93% of the interactive decisions were coded as least aligned with teaching for understanding, and 7% were reflective of good standard pedagogy.

Across all observations for Wolfe, the mean rating for the index on students' explanations was 1.00, indicating that students only provided answers (see Table H63). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table H64). A rating of 1 was assigned for Wolfe on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H11
Overall Ratings for Mathematical Interaction for Wolfe, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Marin. The evidence gathered for lesson presentation and development for Marin were both 3B, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table H61 in this appendix). An overall rating of 1 was assigned for Marin for the nature of inquiry that transpired during instruction, indicating inquiry was limited to lower-order thinking (see Table H62). Marin was assigned was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure H12). For Marin, all interactive decisions were coded as least aligned with teaching for understanding.

Across all observations for Marin, the mean rating for the index on students' explanations was 1.00, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H63). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were generally not elicited from students (see Table H64). A rating of 1 was assigned for Marin on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H12
Overall Ratings for Mathematical Interaction for Marin, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Emphasis on review
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Kendall. Kendall replaced Marin during the first semester of the school year. The evidence gathered for lesson presentation and development for Kendall were at Level 2A and Level 3B. The overall rating assigned to Kendall was 3B, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table H61 in this appendix). An overall rating of 1 was assigned for Kendall for the nature of inquiry that transpired during instruction, indicating that the inquiry was limited to lower-order thinking (see Table H62). Kendall was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure H13). For Kendall, all decisions were coded as reflective of good standard pedagogy.

Across all observations for Kendall, the mean rating for the index on students' explanations was 1.00, indicating that students only provided answers (see Table H63). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table H64). A rating of 1 was assigned for Kendall on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H13
Overall Ratings for Mathematical Interaction for Kendall, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

DiMatteo. DiMatteo replaced Kendall for the second semester of the school year. The evidence gathered for lesson presentation and development for Marin ranged from Level 2A to Level 3B. An overall rating of Level 2B was assigned, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table H61 in this appendix). An overall rating of 1 was assigned for Marin for the nature of inquiry that transpired during instruction, indicating inquiry was limited to lower-order thinking (see Table H62). DiMatteo was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure H14). For DiMatteo, all interactive decisions were coded as least aligned with teaching for understanding.

Across all observations for DiMatteo, the mean rating for the index on students' explanations was 1.00, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H63). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were generally not elicited from students (see Table H64). A rating of 1 was assigned for DiMatteo on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H14
Overall Ratings for Mathematical Interaction for DiMatteo, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the six Grade 8 teachers in District 1 varied from Level 2B to Level 5 *in lesson presentation and development*. One MiC teacher was assigned Level 5, indicating that the lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. Four teachers (one MiC) were assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. The remaining teacher using a conventional curriculum was assigned Level 2B, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident.

Table H15
Nature of Mathematical Interaction, Grade 8 Teachers in District 1, 1998-1999

School-Teacher	Lesson Presentation and Development	Nature of Inquiry	Teachers' Interactive Decisions	Nature of Student Explanations	Elicitation of Multiple Strategies	Lesson Closure, Reflection, or Summary	Total
			— MiC —				
Fernwood-Reichers	5	3	3	2.00	1.88	1	15.88
Von Humboldt-Waters	3B	2	3	1.33	1.78	1	12.11
			— Conventional —				
Addams-Wolfe	3B	1	1	1.00	1.00	1	8.00
Wacker-Kendall	3B	1	3	1.00	1.00	1	10.00
Wacker-Marin	3B	1	1	1.00	1.00	1	8.00
Wacker-DiMatteo	2B	1	1	1.00	1.00	1	7.00

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 3. One MiC teacher was assigned Level 3, indicating that attention was given to conceptual understanding during instruction, and one MiC teachers was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction. The remaining four teachers, all using conventional curricula, were assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to *teachers' interactive decisions*, teachers varied from Level 1 to Level 3. Three teachers (two MiC) were assigned Level 3, indicating that their interactive decisions provided greater attention to good standard pedagogy. The remaining three teachers using conventional curricula were assigned Level 1, indicating that their interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.00 to 2.20, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 1.88. In general, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, teachers offered few, if any, opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The eighth-grade teachers in District 1 varied in all three subcategories of *classroom assessment* (see Tables H16-H19). With respect to the *evidence sought* during classroom assessment, four teachers (two MiC) sought procedural competence of student learning. The remaining two teachers using conventional curricula sought little evidence of student learning. *Feedback* in the classes of five teachers (two MiC) was very teacher-directed, whereas one teacher using a conventional curriculum provided no feedback. Five teachers (two MiC) provided feedback that allowed students to know whether answers were correct or incorrect. The remaining teacher using a conventional curriculum withheld feedback or the feedback was misleading.

Table H16
Evidence Sought through Classroom Assessment Practice, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Reichers	2	Procedural competence
Von Humboldt-Waters	2	Procedural competence
Conventional		
Addams-Wolfe	2	Procedural competence
Wacker-Kendall	1	Limited evidence
Wacker-Marin	2	Procedural competence
Wacker-DiMatteo	1	Limited evidence

Table H17
Feedback Coherence and Purpose, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Reichers	2	Teacher-directed feedback
Von Humboldt-Waters	2	Teacher-directed feedback
Conventional		
Addams-Wolfe	2	Teacher-directed feedback
Wacker-Kendall	1	No feedback
Wacker-Marin	2	Teacher-directed feedback
Wacker-DiMatteo	2	Teacher-directed feedback

Table H18
Feedback Content, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Reichers	2	Answer-only feedback
Von Humboldt-Waters	2	Answer-only feedback
Conventional		
Addams-Wolfe	2	Answer-only feedback
Wacker-Kendall	2	Answer-only feedback
Wacker-Marin	1	Feedback withheld and/or misleading
Wacker-DiMatteo	2	Answer-only feedback

Table H19
Classroom Assessment, Grade 8, District 1

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Fernwood-Reichers	2	2	2	6
Von Humboldt-Waters	2	2	2	6
		— Conventional —		
Addams-Wolfe	2	2	2	6
Wacker-Kendall	1	1	1	3
Wacker-Marin	2	2	2	6
Wacker-DiMatteo	1	2	2	5

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction: nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Reichers ranged from Level 1 to Level 4 (see Table H65 in this appendix). While the mean rating across observations was 1.75, on five of the eight observations student–student conversation was rated at Level 1, indicating that student–student conversation usually was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Reichers are described. The observer noted, "There was a lot of social conversation. Even though students were seated in pairs, most of them worked alone on the mathematics. There was little substantive exchange between partners. They waited for the teacher's input if they couldn't do the problem right away" (Reichers, Observation 5/10/99). In this example, students conversed with the teacher; student–student conversation was social in nature.

The index ratings about the nature of student–student conversation for Waters ranged from Level 1 to Level 2 (see Table H65 in this appendix). The mean rating across observations was 1.33, indicating that student–student conversation on most occasions was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Waters are described. The observer noted, "When the students were in the large group or with a partner, they did not discuss mathematics or have any conversations about the lesson" (Waters, Observation 6/1/99). In this example, student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Wolfe were all at Level 1 (see Table H65 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Wolfe are described. The observer noted "There was no exchange of mathematical ideas in small or large groups" (Wolfe, Observation 3/17/99). In this example, student–student conversation did not exist.

The index ratings about the nature of student–student conversation for Marin were all at Level 1 (see Table H65 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Marin are described. The observer noted, "There was no exchange of mathematical ideas with peers individually or in groups. The only interaction was between the teacher and individual students and this was rare" (Marin, Observation 11/5/98). In this example, students occasionally conversed with the teacher; student–student conversation did not take place.

Kendall replaced Marin during the second quarter of the school year. The index ratings about the nature of student–student conversation for Kendall were all at Level 1 (see Table H65 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Kendall are described. The observer noted "Students

did not discuss mathematics with each other during this class" (Kendall, Observation 11/19/98). In this example, student–student conversation did not exist.

DiMatteo replaced Kendall at the beginning of the second semester. The index ratings about the nature of student–student conversation for DiMatteo were all at Level 1 (see Table H65 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by DiMatteo are described. The observer noted, "Students did not exchange any mathematical ideas with their peers. Some students answered questions posed by the teacher" (DiMatteo, Observation 1/22/99). In this example, some students conversed with the teacher; student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the six Grade 8 teachers in District 1 (one class using conventional curricula had three teachers during the school year) varied from 1.00 to 1.75. Two MiC teachers had mean ratings greater than 1.00, but on the majority of observations, they received a rating of Level 1, indicating that usually student–student conversation was not encouraged and did not exist. Four teachers who used conventional curricula each received a rating of 1.00, indicating that student–student conversation was not encouraged and did not exist.

Collaborative Working Relationships Among Students

The index ratings about the nature of students' collaboration in the classroom for Reichers ranged from Level 1 to Level 4 (see Table H66 in this appendix). The mean rating across observations was 2.14, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Reichers is described. The observer noted, "When students worked in pairs, they did not work together to solve a problem. Usually they were working on different problems and did not share solutions or strategies" (Reichers, Observation 11/4/98). In this example, students did not share ideas or discuss how a problem should be solved. Many of the students in a group worked on different problems at different paces.

The index ratings about the nature of students' collaboration in the classroom for Waters ranged from Level 1 to Level 2 (see Table H66 in this appendix). The mean rating across observations was 1.22, indicating that on most occasions none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Waters is described. The observer noted, "Students were sitting in groups but did not share ideas. In most groups, one student solved the problem and another one wrote it on the graph. The other two members were idle" (Waters, Observation 9/24/98). In this example, students worked in small groups, but not collaboratively to solve the problems. Two students in a group did the work and gave the answers to other members of the group without explanation.

The index ratings about the nature of students' collaboration in the classroom for Wolfe were all Level 1 (see Table H66 in this appendix), indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Wolfe is described. The observer noted, "When the students were working in groups they were mostly working on their own and not sharing with each other. Sometimes one student would give another one an answer with no explanation" (Wolfe, Observation 11/5/998). In this example, students worked in small groups, but not collaboratively. At times, one student gave answers to other members of the group without explanation.

No index rating about the nature of students' collaboration in the classroom was given for Marin. During each of the two observations, no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work (see Table H66 in this appendix). The observer noted, "The main purpose of the lesson was to lecture on procedure and then have the students practice the procedure for about half of the class period" (Marin, Observation 11/5/98).

Kendall replaced Marin during the second quarter. No index rating about the nature of students' collaboration in the classroom was given for Kendall. During each of the two observations no rating for student collaboration was given because the main purpose of the lessons were to give students needed individual practice or independent work (see Table H66 in this appendix). The observer noted, "All work assigned was to be completed individually. Students were not supposed to work with other students" (Kendall, Observation 11/19/98).

DiMatteo replaced Kendall at the beginning of the second semester. The index rating about the nature of students' collaboration in the classroom for DiMatteo during one observation was Level 1 (see Table H66 in this appendix), indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. On four of the five observations, no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by DiMatteo is described. The observer noted, "The purpose of the lesson was not individual practice. No one worked together" (DiMatteo, Observation 2/4/99). In this example, none of the students worked collaboratively.

In summary, the mean ratings for *student collaboration* for the six Grade 8 teachers in District 1 varied from 1.00 to 2.14. One MiC teacher had a mean rating greater than 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. Three teachers (one MiC) had mean ratings at or near 1.00, indicating that none of the students were working collaboratively. Two teachers using conventional curricula had no ratings because during each observation the main purpose of the lesson was to give students needed individual practice or independent work.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Reichers ranged from Level 1 to Level 4 (see Table H67 in this appendix). The mean rating across observations was 2.88, indicating that usually student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Reichers is described. The observer noted, "The teacher had to keep reminding the students of their jobs and what they were supposed to be doing. It was difficult to keep them on task" (Reichers, Observation 11/4/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and dependent on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Waters ranged from Level 1 to Level 4 (see Table H67 in this appendix). The mean rating across observations was 2.22, indicating that on most occasions students appeared lethargic, were only occasionally on task, and did not seem to put forth much effort. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by Waters is described. The observer noted, "Students wasted most of the period. They seemed to already know how to plot points on a plane. They were not serious about the lesson. They did not pay attention. Most students did not listen or participate" (Waters, Observation 1/21/99). In this example, students were passively disengaged.

The index ratings about the extent to which students were engaged during the lesson for Wolfe ranged from Level 1 to Level 2 (see Table H67 in this appendix). The mean rating across observations was 1.71, indicating that often students appeared lethargic, were only occasionally on task, and did not seem to put forth much effort. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by Wolfe is described. The observer noted, "Many students were off task and needed constant prodding to do the work. They were not interested in the project at all" (Wolfe, Observation 12/16/98). In this example, students were only occasionally on task and did not seem to put forth much effort.

The index ratings about the extent to which students were engaged during the lesson for Marin were Level 1 (see Table H67 in this appendix), indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. To illustrate a rating for student engagement at Level 1, students' on-task behavior that occurred during a lesson by Marin is described. The observer noted, "Students were making noise, talking, getting out of their seats, and generally not paying attention to the lesson except when they were called on" (Marin, Observation 10/1/98). In this example, students were frequently off task as evidenced by their gross inattention.

Kendall replaced Marin during the second quarter. The index ratings about the extent to which students were engaged during the lesson for Kendall were Level 1 (see Table H67 in this appendix), indicating that students were frequently off task, as evidenced by gross inattention or serious disruptions. To illustrate a rating for student engagement at Level 1, students' on-task behavior that occurred during a lesson by Kendall is described. The observer noted, "About 15 minutes into the class period students started to work pretty well. Before that they were unruly and not doing any work at all" (Kendall, Observation 11/19/98). In this example, students were frequently off task.

DiMatteo replaced Kendall at the beginning of the second semester. The index ratings about the extent to which students were engaged during the lesson for DiMatteo ranged from Level 2 to Level 3 (see Table H67 in this appendix). The mean rating across observations was 2.40, indicating that on most occasions students appeared lethargic, were only occasionally on task, and did not seem to put forth much effort. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by DiMatteo is described. The observer noted, "There were very few disruptions. Students were quiet, but some were doing no work at all. The others were doing the assigned problems and trying to answer [DiMatteo's] questions" (DiMatteo, Observation 1/22/99). In this example, some students were clearly off task for substantial portions of time.

In summary, the mean ratings for *student engagement during lessons* for the six Grade 8 teachers in District 1 varied from 1.00 to 2.88. One MiC teacher had a mean rating near 3.00, indicating that student engagement was sporadic or episodic with most students engaged in class activities some of the time. Three teachers (two MiC) had mean ratings near 2.00, indicating that many students were passively disengaged as evidenced by their being either clearly off task or nominally on task. Two teachers, using conventional curriculum, had mean ratings of 1.00, indicating that students were frequently off task, as evidenced by their gross inattention or serious disruptions.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table H20
Teacher level of Instruction, Grade8, District 1

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Fernwood-Reichers	3.75	3.75	5	3	3	3.33	2.35	1.67	2	2	2	2.19	2.68	3.60	40.32	**4**
Von Humboldt-Waters	2.5	2.5	3	2	3	2.22	2.23	1.67	2	2	2	1.66	1.53	2.78	31.09	**2**
— Conventional —																
Addams-Wolfe	1.25	2.5	3	1	1	1.67	1.25	1.67	2	2	2	1.25	1.25	2.14	23.98	1
Wacker-Kendall	2.5	2.5	3	1	3	1.67	1.25	1.67	1	1	1	1.25	1.25	1.25	23.34	1
Wacker-Marin	2.5	2.5	3	1	1	1.67	1.25	1.67	2	2	2	1.25	1.25	1.25	24.34	1
Wacker-DiMatteo	2.5	2.5	2	1	1	1.67	1.25	1.67	1	2	2	1.25	1.25	3.00	24.09	**1**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

District 2

In District 2, four Grade 8 teachers participated in the study. Two teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Unit Planning

In this study, three subcategories characterized *unit planning: consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Keeton generally followed the recommended sequence of MiC units. In planning to teach a MiC unit, she studied the unit and worked the problems, noting ideas for accommodating groups that might finish before the rest of the class and listing the materials she needed to organize. She used the time it took her to work through the problems as a basis for determining the pace for instruction, recognizing that students usually needed less time to work through the problems than she did. She explained:

> When I'm going to decide for the week which pages [students] are going to do on two or three days, I go through the unit again and I try to judge how much time it takes me to go through it. Then I decide how much time it's going to take the [students] to do it. I usually divide my time in half or even more than that because they're pretty good. In other words, if it takes me about an hour, it will take the kids maybe 30 minutes or less. Judging the pacing has been part of the problem for me because I may think they can do four pages in a 30-minute time [period], and they do more or they do less. (Keeton, Interview 4/12/99)

Keeton then divided the unit into portions reasonable for one class period and organized the needed materials. She considered students' prior knowledge in her planning process because she taught the same core group of students from Grade 6–8: "I would say 85% of the [students] came up with me from Grade 6 and Grade 7. So I'm pretty much aware without doing a lot of formalized testing of what their prior knowledge is. I do use their prior knowledge" (Keeton, Interview 4/12/99). Keeton used supplementary materials from conventional mathematics textbooks to reinforce skills. She thought that district standardized tests reflected MiC content (Keeton, Interview 4/12/99).

Table H21
Subcategories of Unit Planning: Keeton, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	3	Consideration of the needs of current students

Teague generally followed the recommended sequence of MiC units. When planning to teach a unit, she worked through it and studied the introduction to the unit in the teacher guide. If she had difficulties with any part of the unit she referred to the unit notebook prepared by the department chairperson and/or discussed her concerns at the mathematics department meetings. She explained:

> If I get stumped on any problems, I bring those to my math department and we discuss them at the meeting or I use the notebook that the math department chairperson made up and we copied for our own use. Plus, I always copy the front of the MiC teacher guide and put that in my lesson plans so that I have the goals and the objectives right in there. I have to go back and refresh myself almost daily if I'm doing a different lesson in each class so that I know where I stand. (Teague, Interview 4/13/99)

Teague used the amount of time she needed to work through the unit to estimate the pace of instruction. She also considered that many of her students had below average skills in mathematics and reading. She commented:

> If it takes me an hour to do an assignment, I know that it's going to take my students probably an hour and 40 minutes or two hours. Sometimes it's just too long to do this for the whole two-hour block. You look at [the students]. If they're getting restless, you know it's time to change the activity. But if they're into it and they're working along, I'm certainly not going to stop. (Teague, Interview 4/13/99)

Teague supplemented units with a variety of resource materials, such as a conventional textbook, the SAT Practice Materials, and commercial worksheets. She thought MiC was well aligned with the state standards and that the new state standardized tests reflected MiC content. However, she reviewed basic skills before administering the SAT (Teague, Interview 4/13/99).

Table H22
Subcategories of Unit Planning: Teague, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	3	Consideration of the needs of current students

For description of Cunningham's unit planning, see Appendix G for Grade 7.

Stark sequenced the chapters in the conventional textbook that she used in order to meet the demands of the district standardized tests. She explained, "We have to answer for our test scores. In the very near future in [our state] with our new governor, we're going to be graded on how well our students perform on statewide tests and that will determine how much money each school gets as well as teacher placement. We're under a lot of pressure" (Stark, Interview 4/14/99). When planning the pace of instruction, Stark used her experience

teaching the topic, her understanding of the students' abilities gained by working with them, and. the topics to be covered before standardized testing. "Sometimes I have to go more quickly than I want to, especially before the SAT. I need to review a whole bunch of stuff so that they get as many right as they can" (Stark, Interview 4/14/99). Stark used supplementary materials with the pre-algebra class so students would be prepared for high school algebra (Stark, Interview 4/14/99).

Table H23
Subcategories of Unit Planning: Stark, Newberry Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of students abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the four eighth-grade teachers varied minimally in all three subcategories of *unit planning* (see Figure H3). One teacher based unit plans on her perceptions of students' needs related to the development of concepts and procedures. Two teachers based unit plans on their perceptions of students' mathematics skills or reading ability without informal or formal assessment.

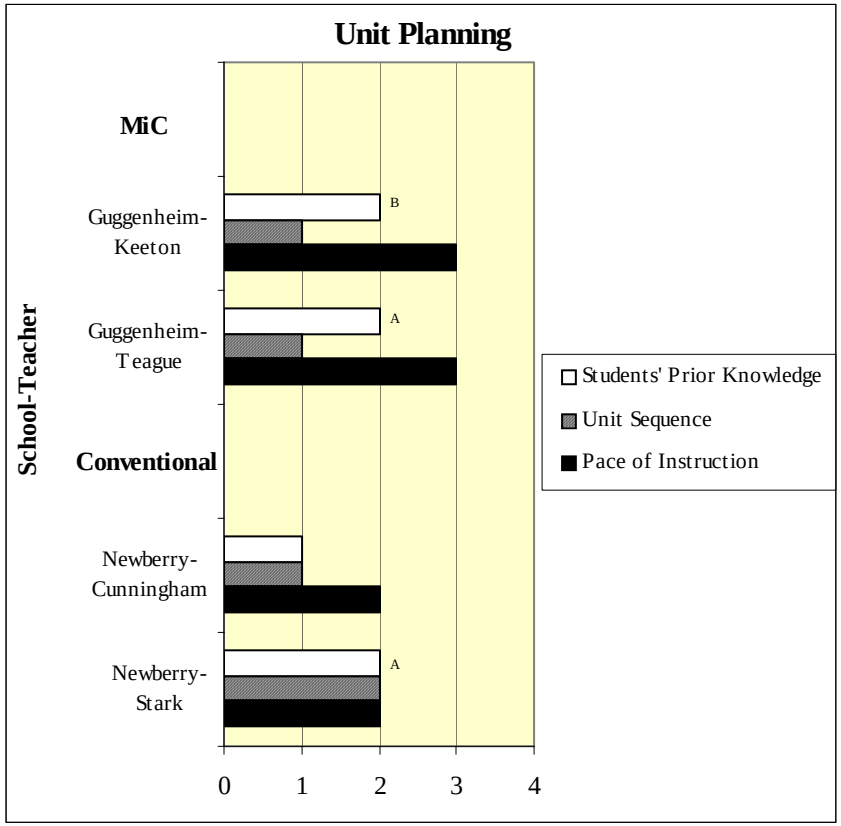


Figure H3. Unit planning, Grade 8, District 2.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table H68 in this appendix.

In determining the *sequence of instructional units*, one teacher considered external factors such as the content of the statewide testing program. Two teachers followed the unit sequence recommended in teacher support materials. In determining the pace for instruction, two teachers considered the learning styles and reasoning skills of their current students. One teacher anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended *pace for instruction*. See Appendix X for Grade 7 for the unit-planning summary that includes Cunningham.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Keeton worked through all of the problems in the lesson to help her anticipate any difficulties students might encounter and prepare ways to help students overcome the difficulties. She explained, "I can tell [students] that they should see a correlation between this lesson and a [specific previous] one. Or that when I did this lesson at home I used millimeter graph paper and counted off my numbers" (Keeton, Interview 4/12/99). Keeton considered students' performance on previous lessons when setting the pace for instruction, deciding the number of pages assigned for each day, and determining if any difficulties needed to be addressed during whole-class introduction. Keeton valued small-group work. Students worked in small groups every day after she presented the lesson to the whole class. Keeton had activities at four stations, labeled A, B, C, and D. One activity was an assignment on the computers and another was review and practice that students completed independently in their group. Two groups worked on the MiC lesson. She commented:

> When students break off into the groups, I get to see who really understood [the introduction]. I could stand up in the front of [the class] and ask them questions and lead them through a lesson. Some [students] will just sit there nicely and quietly, but they won't have understood one thing. When [students] are in groups, I go over to the two groups that are working with MiC, I watch, listen, and see what they're doing. I can interject questions when I see someone is not on task. My biggest clue that somebody needs help is when that student is looking at somebody else's paper. I think [small-group work] is good when it's a new skill, a new set of problems. After we've done the first few pages, students need to work together because they're exploring it, discovering it, hearing it for the first time, and they're thinking of something new. (Keeton, Interview 4/12/99)

On a monthly basis, Keeton randomly assigned students to groups, adjusting for racial integration. When behavior problems occurred, group membership was altered. Although students were in small groups every day, they did not always work cooperatively as a group. During whole-class discussions, students explained solution strategies, demonstrated at the board, defended their reasoning, made conjectures, and drew conclusions (Keeton, Interview 4/12/99).

Keeton chose two forms of instruction on approximately three-fourths of the reported days: independent practice (on 73% of the days) and warm-up activities (70%; see Tables H70-H75 in this appendix). Independent practice was given more class time than any other instructional form, at least half of the class period on 71% of the days, equal emphasis with other instructional forms on 26% of the days, and less than 15% of class time on the remaining days. Warm-up activities were given less than 15% of class time on 50% of the days and equal emphasis with other instructional forms on 48% of the days. Keeton chose four other instructional forms regularly: review of previous material (54%), whole-class discussion (53%), teacher presentation (43%), and small-group work (37%). Whole-class discussion, review of previous material, and teacher presentation were given similar amounts of class time: at least half the class period on about a third of the days and equal emphasis with other

forms of instruction on about half of the days. Small-group work was given at least half of the class period on 22% of the days, equal emphasis with other instructional forms on 68% of the days, and less than 15% of class time on the remaining days (Keeton, Teacher Log 1998-99). Keeton's students engaged in three activities most frequently: investigating problems (58% of the reported days), participating in whole-class discussions (49%), and discussing answers and solution strategies (48%; see Tables H76-H83 in this appendix). Each activity was given significant amounts of class time. Investigating problems and participating in whole-class discussions were given at least half of the class period on 45% of the days, equal emphasis with other activities on about 50% of the days, and less than 15% of class time on the remaining days. Discussing answers and solution strategies was given at least half of the class period on 33% of the days, equal emphasis with other activities on 60% of the days, and less than 15% of class time on the remaining days. On approximately one third of the reported days, students practiced computation (36%), listened to the teacher or took notes (35%), and reflected on or summarized lesson concepts (30%). Of these three activities, practicing computation was given the most class time: at least half of the class period on 41% of the days, equal emphasis with other activities on 50% of the days, and less than 15% of class time on the remaining days. Listening to the teacher or taking notes and reflecting on or summarizing lesson concepts were given at least half of the class period on about 30% of the days, equal emphasis with other activities on about 60% of the days, and less than 15% of class time on the remaining days. Keeton's students less frequently took a test or quiz (20%) or began homework during class time (10%; Keeton, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Keeton reported in her teacher logs. The lesson observed on 2/11/99, for example, included: large-group discussion (32 minutes); individual seatwork, SAT practice (9 minutes); and small-group activities, 4 groups rotated through various activities (70 minutes; Keeton, Observation 21199).

Table H24
Subcategories of Lesson Planning: Keeton, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Teague checked the school calendar. If many school-wide interruptions had been planned, she did not plan MiC lessons because she felt they needed continuity.When planning MiC lessons, Teague worked through all of the problems to make sure she understood them. She planned the pace of instruction to be guided by students' performance and their attitude toward the activity. Since

classes met every other day for two-hour blocks, Teague planned for a variety of activities because the quality of whole-class discussions varied from students talking with each other about solution strategies and trying to prove a point, to minimally answering teacher's questions. Teague thought that having students work in small groups was valuable, when students had the necessary work skills. She tried several organization strategies, including assigning roles to members of the groups, assigning different activities to each group, and dividing students into groups of two, three, or four. However, Teague did not think any small group organization worked with her class. She explained:

> I tell students that I'm going to start [the activity] and they are to finish it in their groups. But they have no clue [of where to begin]. They just sit there and end up talking and having a grand old time. At the end of ten minutes, I might have one student who might have an answer. I don't feel very successful that way. Or sometimes somebody from every group comes up to me to ask, "How do you do this? We need some help here." I can't be at eight different places at the same time. My kids have a very hard time interpreting what MiC is saying. Even when I try whole-group instruction, they ask about what the lesson means. They're not comfortable or maybe their skills aren't high enough yet to do MiC individually or in small groups. (Teague, Interview 4/13/99)

Teague usually planned to use whole-class direct teaching followed by students working individually (Teague, Interview 4/13/99).

Teague regularly used four instructional forms: teacher presentation (on 40% of the days), warm-up activities (40%), whole-class discussion (33%), and small-group work (32%; see Tables H70-H75 in this appendix). Small-group work was given the most class time, at least half of the class period on 19% of the days and equal emphasis with other instructional forms on the remaining days. Whole-class discussion was frequently given equal emphasis with other instructional forms (89%). Warm-up activities were given equal emphasis with other instructional forms on 65% of the days and less than 15% of class time on 29% of the days. Teacher presentation was given equal emphasis with other instructional forms on 38% of the days and less than 15% of class time on 62% of the days. Teague reviewed previous material on 27% of the reported days generally for less than 15% of class time. Teague seldom chose independent practice (8%; Teague, Teacher Log 1998-99).

Teague's students engaged in four activities on approximately one third of the reported days: discussing answers and solution strategies (on 39% of the days), investigating problems (38%), listening to the teacher or taking notes (35%), and participating in whole-class discussions (32%; see Tables H76-H83 in this appendix). Investigating problems was given the most class time: at least half of the class period on 19% of the days and equal emphasis with other activities on 78% of the days. Listening to the teacher or taking notes was generally given less than 15% of class time. Participating in whole-class discussion and discussing answers and solution strategies were given equal emphasis with other activities on most days. Practicing computation was used on 20% of the reported days and was given equal emphasis with other activities on 53% of the days and less than 15% of class time on 47% of the days. Taking a test or quiz (5% of the reported days), reflecting on or summarizing lesson concepts (1%), and beginning homework (1%) were rarely used (Teague, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Teague reported in her teacher logs. The lesson observed on 2/11/99, for example, included: housekeeping duties (11 minutes); large-group discussion alternating with student work time, individually or with a partner (91 minutes); and individual skill practice for the SAT (6 minutes; Teague, Observation 2/11/99).

Table H25
Subcategories of Lesson Planning: Teague, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

For description of Cunningham's lesson planning, see Appendix G for Grade 7.

When planning to teach individual lessons from a conventional textbook, Stark read through the lesson to identify the concepts and skills to be presented and worked through some of the problems. She added review when she felt it was necessary. Stark planned for small-group work except for when students were taking quizzes or tests. She explained:

> It's much more interesting if students can share their ideas. They're so afraid of doing math. They think that they're going to get it wrong. So if they're working with other kids, one might say, "Well, what do you think about 6?" And then another kid might say, "Well, I thought it was 7." And they get into a discussion. It gives the kids with different abilities a chance to work with different kids, and it's more interesting. It's more fun. I mean, who wants to sit at a desk and work individually all the time. I don't. (Stark, Interview 4/14/99)

Stark used whole-group direct instruction to introduce the lesson, a whole-class question-and-answer period, and small-group work. She thought that ideally students worked best in heterogeneous small groups of three to five students. She commented, "It's easier for me if they're working individually. It's quiet and I can think. They get it much better, I think, if they work in small groups" (Stark, Interview 4/14/99).

Stark used five forms of instruction regularly: review of previous material (on 35% of the days), teacher presentation (30%), independent practice (30%), small-group work (28%), and whole-class discussion (26%; see Tables H70-H75 in this appendix). Small-group work was given the most class time: at least half of the class period on 55% of the days, equal emphasis with other instructional forms on 42% of the days, and less than 15% of class time on the remaining days. Independent practice and whole-class discussion were given significant amounts of class time.

Independent practice was given at least half of the class period on 42% of the days, equal emphasis with other instructional forms on 54% of the days, and less than 15% of class time on the remaining days. Whole-class discussion was given at least half of the class period on 23% of the days and equal emphasis with other instructional forms on the remaining days. Less class time was given to teacher presentation and review of previous material. Teacher presentation was given at least half of the class period on 23% of the days, equal emphasis with other instructional forms on 50% of the days, and less than 15% of class time on 27% of the days. Review of previous material was given at least half of the class period on 13% of the days, equal emphasis with other instructional forms on 47% of the days, and less than 15% of class time on 40% of the days. Stark rarely began the period with warm-up activities (2%; Stark, Teacher Log 1998-99).

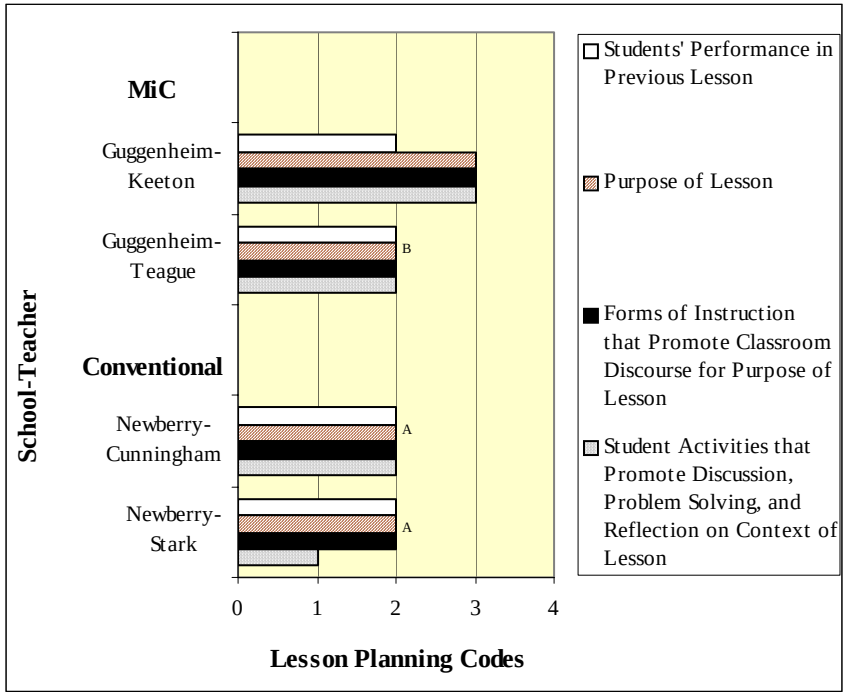
Practicing computation was an important element in Stark's instruction. It was used on 50% of the reported days and was given a considerable amount of class time: at least half of the class period on 45% of the days and equal emphasis with other student activities on 49% of the days (see Tables H76-H83 in this appendix). Stark's students listened to their teacher or took notes (41%), discussed answers and solution strategies (36%), and participated in whole-class discussion (21%). These activities were given similar amounts of class time. Listening to the teacher or taking notes was given equal emphasis with other activities on 60% of the days and less than 15% of class time on 31% of the days. Discussing answers and solution strategies was given equal emphasis with other activities on 71% of the days, and less than 15% of class time on 16% of the days. Participating in whole-class discussions was given at least half of the class period on 22% of the days, equal emphasis with other activities on 50% of the days and less than 15% of class time on 28% of the days. Students rarely took a test or quiz (9%), investigated problems (7%), reflected on or summarized lesson concepts (1%), and began homework (1%; Stark, Teacher Log 1998-99).

In general, observation reports completed during 1998-99 classes supported the information Stark reported in her teacher logs. The lesson observed on 3/10/99, for example, included: teacher presentation with some discussion (22 minutes); and individual seatwork (58 minutes; Stark, Observation 3/10/99).

Table H26
Subcategories of Lesson Planning: Stark, Newberry Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	1	Investigation of problems and discussion of mathematical ideas seldom planned for the lesson

In summary, the four eighth-grade teachers in District 2 varied in three of the four subcategories of *lesson planning, teachers' attention to the purpose of the lesson, forms of instruction that promote classroom discourse for the purpose of the lesson*, and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* (see Figure H4). (Discussion of the four subcategories of lesson planning with respect to Cunningham is included with Grade 7 teachers in Appendix G.) With respect to *students' performance in the previous lesson*, all three teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Variation was seen in teachers' attention to the *purpose of the lesson*. One teacher went beyond checking her own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. One teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context. The remaining teacher selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F69 in this appendix.

Figure H4. Lesson planning, Grade 8, District 2.

Teachers varied minimally in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. Two teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. The teachers varied more in the fourth subcategory of *lesson planning, student activities that promote discussion, problems solving, and reflection on the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. One teacher included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered. The remaining teacher seldom planned for investigation of problems and discussion of mathematical ideas.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Keeton. The evidence gathered for lesson presentation and development for Keeton ranged from Level 2A to 5. An overall rating of Level 5 was assigned, indicating that the lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and their teacher (see Table H84 in this appendix). An overall rating of 4 was assigned for Reichers for the nature of inquiry, indicating that in-depth exploration of mathematics transpired during instruction (see Table H85). Keeton was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure H15). For Keeton, 63% of the interactive decisions were coded as most aligned with teaching for understanding, 34% were reflective of good standard pedagogy, and 6% were least aligned with teaching for understanding.

Across all observations for Keeton, the mean rating for the index on students' explanations was 2.14, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H86). The mean rating for elicitation of multiple strategies was 1.86, indicating that multiple strategies were rarely elicited from students (see Table H87). A rating of 2 was assigned for Keeton on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H27
Overall Ratings for Mathematical Interaction for Keeton, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.14	Focus on procedures
Elicitation of Multiple Strategies	1.86	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Teague. The evidence gathered for lesson presentation and development for Teague ranged from Level 3B to 5. An overall rating of Level 4 was assigned indicating that attempts were made to develop conceptual understanding (see Table H84 in this appendix). An overall rating of 4 was assigned for Teague for the nature of inquiry, indicating that in-depth exploration of mathematics transpired during instruction (see Table H85). Teague was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure H16). For Teague, 4% of the interactive decisions were coded as reflective of good standard pedagogy, 35% were most aligned with teaching for understanding, and 14% were least aligned for teaching for understanding.

Across all observations for Teague, the mean rating for the index on students' explanations was 2.20, indicating student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H86). The mean rating for elicitation of multiple strategies was 1.80, indicating that multiple strategies were rarely elicited from students (see Table H87). A rating of 1 was assigned for Teague on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H28
Overall Ratings for Mathematical Interaction for Teague, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	4	Demonstration of strategy or procedure
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	2.20	Focus on procedures
Elicitation of Multiple Strategies	1.80	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Cunningham. The evidence gathered for lesson presentation and development for Cunningham ranged from Level 2A to 5. An overall rating of at Level 3B, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table H84 in this appendix). An overall rating of 3 was assigned for Cunningham for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table H85). Cunningham was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure H17). For Cunningham, 22% of the interactive decisions were coded as reflective of good standard pedagogy, 55% were most aligned with teaching for understanding, and 22% were least aligned for teaching for understanding.

Across all observations for Cunningham, the mean rating for the index on students' explanations was 1.50, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table H86). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were generally not elicited from students (see Table H87). A rating of 1 was assigned for Cunningham on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H29
Overall Ratings for Mathematical Interaction for Cunningham, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.50	Answers only
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Stark. The evidence gathered for lesson presentation and development for Stark ranged from Level 2A to 3B. An overall rating of Level 3B was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table H84 in this appendix). An overall rating of 2 was assigned for Stark for the nature of inquiry that transpired during instruction, indicating that inquiry was limited to lower-order thinking (see Table G85). Stark was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure H18). For Stark, 62% of the interactive decisions were coded as least aligned with teaching for understanding, 31% were reflective of good standard pedagogy, and 8% were most aligned with teaching for understanding.

Across all observations for Stark, the mean rating for the index on students' explanations was 1.17, indicating that students generally only provided answers (see Table H86). The mean rating for elicitation of multiple strategies was 1.17, indicating that multiple strategies were rarely elicited from students (see Table H87). A rating of 1 was assigned for Stark on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table H30
Overall Ratings for Mathematical Interaction for Stark, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.17	Answers only
Elicitation of Multiple Strategies	1.17	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the four Grade 8 teachers in District 2 varied from Level 3B to Level 5 in lesson presentation and development (see Table H31). One MiC teacher was assigned Level 5, indicating that the lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. One MiC teacher was assigned Level 4, indicating that she attempted to teach for conceptual understanding. Both teachers using conventional curricula were assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion.

Table H31
Nature of Mathematical Interaction, Grade 8 Teachers in District 2, 1998-1999

School-Teacher	**Lesson Presentation and Development**	**Nature of Inquiry**	**Teachers' Interactive Decisions**	**Nature of Student Explanations**	**Elicitation of Multiple Strategies**	**Lesson Closure, Reflection, or Summary**	**Total**
			— MiC —				
Guggenheim-Keeton	5	4	5	2.14	1.86	2	20.00
Guggenheim-Teague	4	4	4	2.20	1.80	1	17.00
			— Conventional —				
Newberry-Cunningham	3B	3	4	1.50	1.50	1	14.00
Newberry-Stark	3B	2	1	1.17	1.17	1	9.34

With respect to the nature of inquiry during instruction, teachers varied from Level 2 to Level 4. Both MiC teachers were assigned Level4, indicating in-depth exploration of mathematics occurred during instruction. One teacher using a conventional curriculum was assigned Level 3, indicating that attention was given to conceptual understanding during instruction, and the remaining teacher using a conventional curriculum was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction.

With respect to teachers' interactive decisions, teachers varied from Level 1 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding. Two teachers (one MiC) were assigned Level 4, indicating that their interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding. The remaining teacher using conventional curricula was assigned Level 1, indicating that her interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.17 to 2.20, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the elicitation of multiple strategies ranged from 1.17 to 1.86. In genera, teachers rarely elicited multiple strategies. With respect to lesson reflection, summary, or closure, teachers offered few, if any, opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought, purpose and coherence of feedback*, and *content of feedback*. The eighth-grade teachers in District 2 varied in all three subcategories of classroom assessment (see Tables H32-H35). With respect to the *evidence sought* during classroom assessment, one MiC teacher maintained conservative process orientation. Three teachers (two MiC) sought procedural competence of student learning. Students in classes of one MiC teacher experienced emerging shared responsibilities for providing *feedback*. In the classes of the remaining teachers (one MiC), students experienced teacher directed feedback. Both MiC teachers mixed, superficial feedback, whereas both teachers using conventional curricular provided low-level closed feedback.

Table H32
Evidence Sought through Classroom Assessment Practice, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Keeton	4	Conservative process orientation
Guggenheim-Teague	2	Procedural competence
Conventional		
Newberry-Cunningham	2	Procedural competence
Newberry-Stark	2	Procedural competence

Table H33
Feedback Coherence and Purpose, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Keeton	3	Emerging shared responsibility
Guggenheim-Teague	2	Teacher-directed feedback
Conventional		
Newberry-Cunningham	2	Teacher-directed feedback
Newberry-Stark	2	Teacher-directed feedback

Table H34
Feedback Content, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Keeton	4	Mixed, superficial feedback
Guggenheim-Teague	4	Mixed, superficial feedback
Conventional		
Newberry-Cunningham	3	Low-level closed feedback
Newberry-Stark	3	Low-level closed feedback

Table H35
Classroom Assessment, Grade 8, District 2

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Guggenheim-Keeton	4	3	4	11
Guggenheim-Teague	2	2	4	8
		— Conventional —		
Newberry-Cunningham	2	2	3	7
Newberry-Stark	2	2	3	7

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation*, *collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Keeton ranged from Level 1 to Level 4 (see Table H88 in this appendix). Although the mean rating across observations was 2.29, on four of the seven observations student–student conversation was rated at Level 3 or Level 4, indicating that student–student conversation occurred frequently, but was not always substantive in nature. To illustrate a rating at Level 3, student–student conversations that occurred during a lesson by Keeton are described. The observer noted, "Students were seated in groups and did share information, but not in detail" (Keeton, Observation 1/19/99). In this example, student–student conversation was frequent, but it was not substantive in nature.

The index ratings about the nature of student–student conversation for Teague ranged from Level 2 to Level 3 (see Table H88 in this appendix). The mean rating across observations was 2.40, indicating that student–student conversation often was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Teague are described. The observer noted, "When given time to do problem 4, two ways to factor 1092, two different set of students were conferring with each other" (Teague, Observation 2/11/99). In this example, student–student conversation was limited and consisted of sharing answers.

The index ratings about the nature of student–student conversation for Cunningham ranged from Level 1 to Level 3 (see Table H88 in this appendix). Although the mean rating across observations was 1.50, on four of the six observations student–student conversation was rated at Level 1, indicating that student–student conversation often was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Cunningham are described. The observer noted, "Students didn't discuss any of the mathematics with the teacher or with their teammates" (Cunningham, Observation 2/18/99). In this example, student–student conversation did not exist.

The index ratings about the nature of student–student conversation for Stark ranged from Level 1 to Level 3 (see Table H88 in this appendix). Although the mean rating across observations was 1.50, on four of the six observations student–student conversation was rated at Level 1, indicating that student–student conversation often was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Stark are described. The observer noted, "Students did not exchange ideas with each other during the class" (Stark, Observation 2/16/99). In this example, student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the four Grade 8 teachers in District 2 varied from 1.50 to 2.40. One MiC teacher had a mean rating close to 2.00, but received a rating of 3.00 or above on the majority of observations, indicating that student–student conversation occurred frequently but was not substantive in nature. One MiC teacher had a mean rating near 2.00, indicating that student–student

conversation was limited, consisted of sharing answers, or focused on procedures. Two teachers using conventional curricula had mean ratings between 1.00 and 2.00, but the majority of their ratings were 1.00, indicating that student–student conversation was not encouraged and did not take place.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Keeton ranged from Level 1 to Level 2 (see Table H89 in this appendix). The mean rating across observations was 1.67, indicating that usually few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Keeton is described. The observer noted, "Students did not collaborate in a way to insure that all students in a group were keeping up with the group. Most groups were not working on the same problems at the same time" (Keeton, Observation 1/19/99). In this example, many of the students in a group worked on different problems at different paces.

The index ratings about the nature of students' collaboration in the classroom for Teague ranged from Level 1 to Level 3 (see Table H89 in this appendix). The mean rating across observations was 2.00, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Teague is described. The observer noted, "During the seatwork, very few students shared in the problem-solving processes" (Teague, Observation 1/19/99). In this example, few students shared ideas or discussed how a problem should be solved.

The index ratings about the nature of students' collaboration in the classroom for Cunningham ranged from Level 1 to Level 3 (see Table H89 in this appendix). The mean rating across observations was 1.33, indicating that usually none of the students were working together in small-group or large-group settings, or if they worked in small groups, one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Cunningham is described. The observer noted, "Students were not working in groups today" (Cunningham, Observation 4/14/99). In this example, students did not work together in small or large-group settings.

The index ratings about the nature of students' collaboration in the classroom for Stark ranged from Level 1 to Level 2 (see Table H89 in this appendix). The mean rating across observations was 1.25, indicating that on most occasions none of the students worked together in small or large-group settings. However, on two of the six observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Stark is described. The observer noted, "Students were not seated in groups" (Stark, Observation 2/16/99). In this example, none of the students worked in small or large-group settings.

In summary, the mean ratings for *student collaboration* for the four Grade 8 teachers in District 2 varied from 1.25 to 2.00. Two MiC teachers had mean ratings at or close to 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. Two teachers using conventional curricula had mean ratings near 1.00, indicating that on most occasions none of the students worked collaboratively.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Keeton ranged from Level 3 to Level 4 (see Table H90 in this appendix). The mean rating across observations was 3.71, indicating that often student engagement was widespread with students on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Keeton is described. The observer noted, "Students were attentive and engaged in the large-group discussions" (Keeton, Observation 3/24/99). In this example, most students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Teague ranged from Level 3 to Level 4 (see Table H90 in this appendix). The mean rating across observations was 3.80, indicating that usually students were on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Teague is described. The observer noted, "Most students were on task most of the time" (Teague, Observation 1/19/99). In this example, student engagement was widespread with students on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Cunningham ranged from Level 2 to Level 4 (see Table H90 in this appendix). The mean rating across observations was 3.00, indicating that student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Cunningham is described. The observer noted, "Students were well behaved and usually on task. When the class moved on to the next activity, it was observed that several students had not begun the first activity" (Cunningham, Observation 11/4/98). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during another lesson by Cunningham is described. The observed noted, "Students were well behaved and attentive but so passive that engagement was hard to observe" (Cunningham, Observation 3/10/99). In this example, students were passively engaged.

The index ratings about the extent to which students were engaged during the lesson for Stark ranged from Level 2 to Level 4 (see Table H90 in this appendix). The mean rating across observations was 3.00, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Stark is described. The observer noted, "During the first 14-minute period of class time, students were quiet and attentive. They seemed to be interested and involved in what the teacher was saying about the homework. Students were slow to get started on the seatwork assignment; one student had completed 13 of the 14 assigned problems while other students had not begun. During the last part of the period, students were noisy and exhibited off-task behavior" (Stark, Observation 3/18/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

In summary, the mean ratings for *student engagement during the lesson* for the four Grade 8 teachers in District 2 varied from 2.67 to 3.80. Two MiC teachers had mean ratings near 4.00, indicating that on most occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time. Two teachers using conventional curriculum had mean ratings at or near 3.00, indicating that student engagement was sporadic or episodic and often dependent on frequent prodding from the teacher.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table H36
Teacher level of instruction, Grade 8, District 2

School-Teacher	**Lesson Planning**		**Mathematical Interaction**						**Classroom Assessment**			**Student Pursuits**			**Weighted Sum**	**Composite Level**
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Guggenheim-Keeton	3.75	3.75	5	5	5	3.57	2.33	3.33	4	3	4	2.86	2.09	4.64	52.32	**5**
Guggenheim-Teague	2.5	2.5	4	4	4	3.67	2.25	1.67	2	2	4	3.00	2.50	4.75	42.84	**4**
— Conventional —																
Newberry-Cunningham	2.5	2.5	3	3	4	2.5	1.88	1.67	2	2	3	1.88	1.66	3.75	45.30	**3**
Newberry-Stark	2.5	1.25	3	2	1	1.95	1.46	1.67	2	2	3	1.88	1.56	3.75	29.02	**2**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

The ratings assigned to the teacher in this district on each of the indices related to *unit and lesson planning* are described and illustrated with evidence from the teacher interview. In the modified research design, data from classroom observations and teaching logs were not gathered from the teacher in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for this teacher.

District 3

In District 3, one Grade 8 teacher participated in the study. She used MiC. Classroom observation and teacher log data were not collected from this teacher.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Wells generally followed the recommended sequence of MiC units. She felt that a problem inherent with most of the eighth-grade units was the lack of program continuity. Because MiC was designed with a spiral format (mastery over time), the development of the concepts studied in eighth-grade units had begun years earlier. Students whose mathematics instruction was not consistently MiC or had not included all of the units for each year from Grades 5–7 might not have prerequisite conceptual understanding or skill in procedures unique to MiC such as using a ratio table for the division of fractions or decimals. She explained:

> For example, I usually start with *Graphing Equations*, but I felt that students would have a really difficult time with *Graphing Equations* without being familiar with *Building Formulas*. *Building Formulas*, a seventh-grade unit, is also a cornerstone unit for algebra and the algebra strand. Since our seventh-grade teachers didn't get to it or to *Ups and Downs* last year, that's the unit I started with this year. (Wells, Interview 9/28/99)

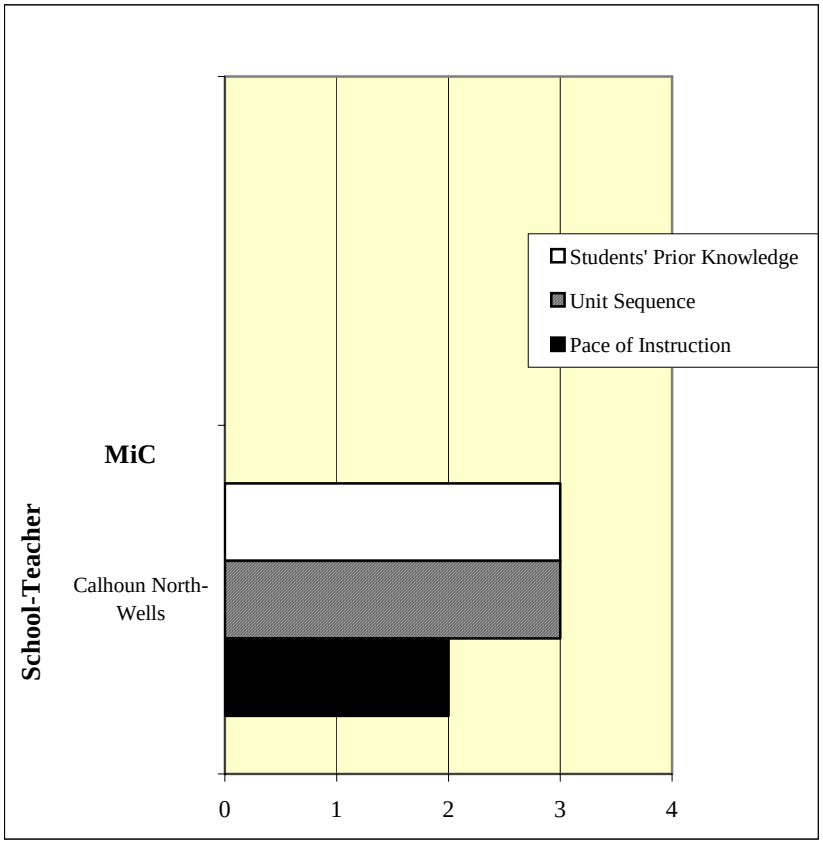
When planning to teach a MiC unit, she read through the unit and assessed students' prior knowledge of the concepts and skills. Wells used that information to determine the questions to discuss with the whole class and those they could work through with a partner. Wells did not determine the pace of instruction prior to teaching the unit. She anticipated the pace of instruction would vary from class to class, based on students' learning.

Wells thought that MiC was well aligned with the district mathematics guidelines because they were based on MiC and that MiC exceeded the state standards (Wells, Interview 9/28/99).

Table H37
Subcategories of Unit Planning: Wells, Calhoun North Middle School, District 3, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

In summary, the one eighth-grade teacher in District 3 with respect to *students' prior knowledge* when planning to teach a unit, assessed students' understanding of skills or concepts needed in a particular unit; considered the mathematical topics students had previously studied when determining the *sequence of instructional units*; and anticipated that various factors such as needed prerequisite skills or the difficulty of content would necessitate adjusting the recommended *pace for instruction* (see Figure H5).



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure H5. Unit planning, Grade 8, District 3.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

When planning to teach individual MiC lessons, Wells worked through all of the problems in the lesson. She noted when the problems changed focus so that she did not assign problems for homework that needed to be preceded by discussion or explanation. Wells marked in her teacher's guide when it was appropriate for students to have a whole-class discussion, work as a whole class, and work in small groups. She always planned for an in-depth discussion of the summary box. Wells explained:

> The kids know that the summary box is really the essence of the section. We talk about if you really understand what the summary box is telling you, you've learned something from the section. If, in fact, something is not clear, then we need to go back over it. They're really taking that to heart. (Wells, Interview 9/28/99)

Wells valued students working collaboratively. She facilitated group work by having the students sit with their partners. They could discuss issues with partners during whole-class discussions as well as during small-group work time. Wells found there was more individual accountability when students worked just with one partner. For big problems that needed more input, several partners formed pre-determined study teams. Wells explained that there is one situation when she is not particularly happy with group work:

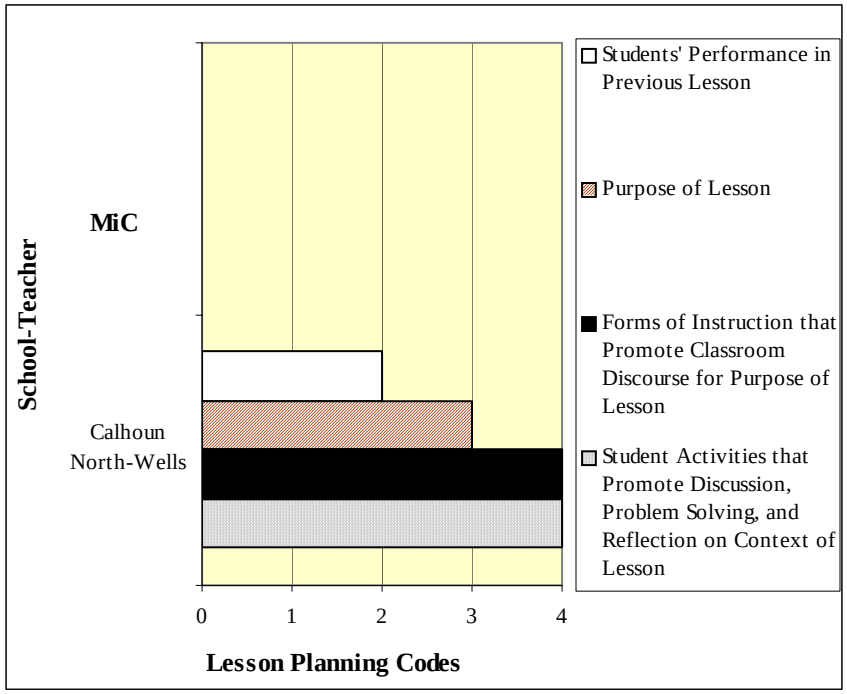
> It's when I have to spend an inordinate amount of time with one study team. When I'm not walking around the room and monitoring the teams, asking them questions, redirecting their discussion, the study teams get off task. However, there are times when I have to spend some time with one study team because they're just really off track. The magic number for me was three. If I saw that three teams were struggling or going in the wrong direction, I'd pull the whole class together. We would brainstorm together how to get through this. I don't want to stop the other teams from working on the problem if there is only one study team that's struggling. (Wells, Interview 9/28/99)

Wells encouraged students to participate in whole-class discussions. Students were free to openly and spontaneously share ideas, strategies, challenges, and questions. She explained: “It’s exciting actually, but maybe for someone who likes a quiet class, they might think it’s a little chaotic” (Wells, Interview 9/18/99).

Table H38
Subcategories of Lesson Planning: Wells, Calhoun North Middle School, District 3, Grade 8

Subcategory	Rating	Description of Rating
Students’ Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	4	Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan

In summary, the one eighth-grade teacher in District 3 with respect to *students' performance in the previous lesson*, made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences (see Figure H6). With respect to teachers' attention to the *purpose of the lesson*, Wells went beyond checking her own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure H6. Lesson planning, Grade 8, District 3.

In the third subcategory of *lesson planning, forms of instruction that promote classroom discourse for the purpose of the lesson*, Wells planned formats that promoted substantive conversation by encouraging students to contribute to discussions, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. In the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection of the content of the lesson*, investigation of problems and discussion of answers and solution strategies were dominant in Wells' lesson plan and opportunity to reflect on or summarize lesson concepts were important elements in her instruction.

The ratings assigned to each teacher in this district on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 4

In District 4, three Grade 8 teachers participated in the study. All three teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Catalano generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she looked through the entire unit to get a general idea of its content. She then studied each section and identified the concepts or ideas she wanted students to learn during each class period. Cognizant of the fact that many of her students had weak mathematics skills and had scored very poorly on standardized tests, Catalano planned to provide more guidance for them. She was aware of the suggested pace for instruction, but anticipated that many students would need more time to develop the concepts. She commented, "I can calculate how far I think I'm going to get with a lesson based on the information that's in the teacher guide. Then, based on experience with the students, I determine the pacing" (Catalano, Interview 7/8/99). Catalano thought planning to teach a MiC unit was very different from planning to teach a unit from a conventional textbook because MiC was a new program. She commented:

> It is not clear what you are to do on each page, what the objective is. You can't say I'll check tomorrow and make sure all the kids have learned it, because it's an ongoing learning process as you work through the entire unit. It also took a lot of time for the preparation, making sure that all the materials were in the classroom. There was just a lot more planning involved with MfiC units than with the old-fashioned type lessons. (Catalano, Interview 7/8/99)

Catalano and other mathematics teachers at Kelvyn Park Middle School found it difficult to find appropriate daily homework, which was a requirement at their school, because many MiC problems needed teacher guidance. Some of the eighth-grade teachers planned to develop a group of problems related to different MiC units that could be used for homework (Catalano, Interview 7/8/99).

Table H39
Subcategories of Unit Planning: Catalano, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Novak noted the recommended sequence of MiC units, but chose to sequence units by considering connections among them. He explained, "I try to combine the units that are cohesive. With MiC, most of the units tend to be cohesive. They wrap around each other. I do vary from the [recommended] sequence" (Novak, Interview 7/15/99). When planning to teach a MiC unit, Novak reviewed the unit, obtained the needed supplies together, and tried the unit with his two sons: "I usually go through a practice run with my two boys at home. They usually tell me what I'm doing right and what I'm doing wrong. I go through it at least once before I present it to the class. When I feel satisfied, then I'll give it out to my students" (Novak, Interview 7/15/99). Factors that influenced the pace of instruction were students' performance on the assessment he gave at the beginning of the year, events scheduled on various school days, the amount of group discussion, the number of student questions, and daily student performance. Novak commented, "Normally I don't go to another unit until I finish one. So it might be that a section might take me two days or three days. I don't let time limit me. I try to get the point across whether it takes one, two, three, or four days. With certain classes it might take more days than others" (Novak, Interview 7/15/99). Novak did not consider district mathematics guidelines when he planned (Novak, Interview 7/15/99).

Table H40
Subcategories of Unit Planning: Novak, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of needs of current students
Pace of Instruction	2	Adjustment anticipated

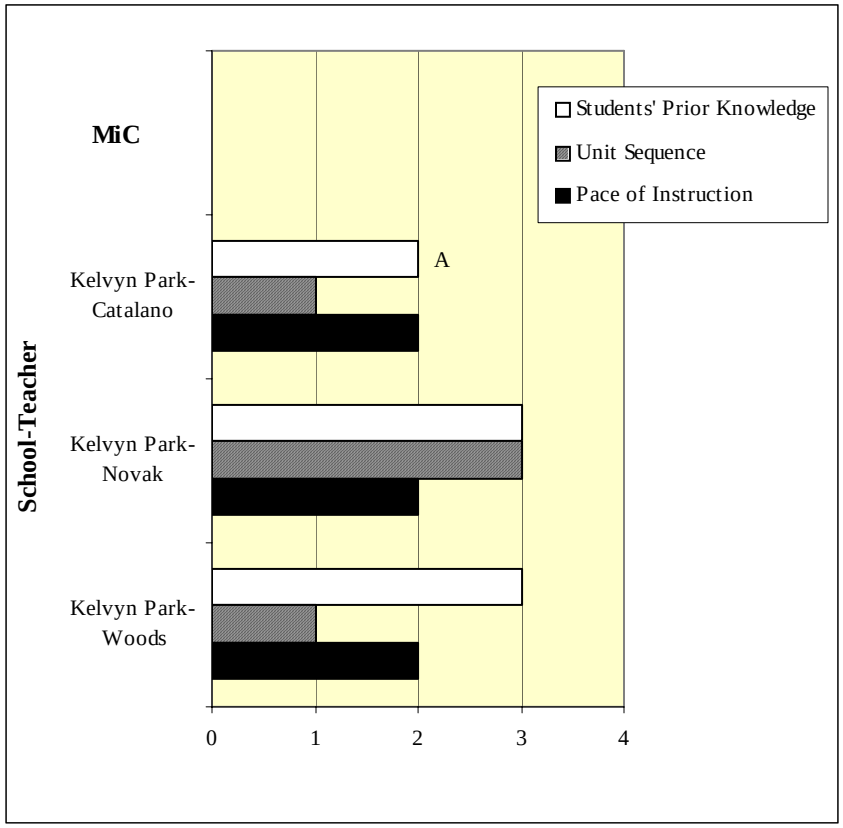
Woods followed the recommended sequence of MiC units. When planning to teach a MiC unit, he worked through each activity, decided which ones were appropriate for his students, identified any problems he thought students would have based on information from an assessment given at the beginning of the year, and determined supplementary materials. Woods did not determine the pace for instruction prior to beginning the unit.

He explained, “Pacing is intuitive. I mean I try to go as quickly as I can and make sure there’s understanding. If I find that I’m going too fast, I’ll slow down. If I’m going too slowly and the kids get a little bored, I’ll pick up the pace” (Woods, Interview 7/1/99).

Table H41
Subcategories of Unit Planning: Woods, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students’ Prior Knowledge	3	Informal or formal assessment of students’ understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the three eighth-grade teachers in District 4 varied in two of the subcategories of *unit planning*, *prior knowledge* and *unit sequence* (see Figure H7). With respect to *students' prior knowledge* when planning to teach a unit, two teachers assessed students' understanding of skills or concepts needed in a particular unit. One teacher based unit planning on her perceptions of students' mathematics skills or reading ability without formally or informally assessing.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure H7. Unit planning, Grade 8, District 4.

In determining the *sequence of instructional units*, one teacher considered linkages across units of the same content strand. Two teachers followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, all of the teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Catalano worked through all of the problems in the lesson to become familiar with its context, concepts, and skills and to estimate the amount of time students needed to complete the lesson. She also identified where natural breaks occurred within the lesson, so she would not start a problem needing twenty minutes to work through if there were only five minutes left in the class period. In Kelvyn Park Middle School, each class was to begin with a warm-up activity and discussion of the completed homework. The remaining 40-45 minutes was lesson time. Catalano thought the kind of lesson activity determined whether students worked individually, in groups, or as a whole class. She explained:

> If there was a task or a problem that had to be done where information had to be gathered, or charts had to be formulated, or even if results had to be explained, putting students in groups was helpful. Everybody had a certain job in getting the problem done. Other questions were very straightforward, very simple. One person could do it alone. Some problems were enriched by group discussions. Small-group discussions gave more students an opportunity to share ideas than whole class discussions did. So it depended on the type of the problem. (Catalano, Interview 7/8/99)

Students felt comfortable contributing to whole-class discussions. However, some discussions were more context-related than mathematics-related (Catalano, Interview 7/8/99).

Table H42
Subcategories of Lesson Planning: Catalano, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or not regard for students' performance on the previous lesson
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Novak considered each student's profile as evidenced in his/her portfolio. He also considered the Individual Education Program written for some of his students. He explained that large class sizes, approximately 30 students per class, and little planning time made it difficult to individualize instruction. To meet some of the students' needs, Novak tutored after school hours and between class periods. The type of lesson influenced Novak's choice of instruction. He explained:

> Some lessons would be a small group. Some lessons I would divide the classroom in half. I might offer a competition type situation where one-half of the class competes against the other half in finding a logical solution. So, depending on the lesson, [I might choose] groups of half the class each, groups of four students, or part whole-class lecture and part small-group work. I try to do more cooperative work. I try to make a heterogeneous blend of students so one or two students will motivate the group. By blending students heterogeneously, all students generally do communicate among themselves. I usually rotate around the room to be available to answer any questions. I always hear open discussions (Novak, Interview 71599).

Table H43
Subcategories of Lesson Planning: Novak, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

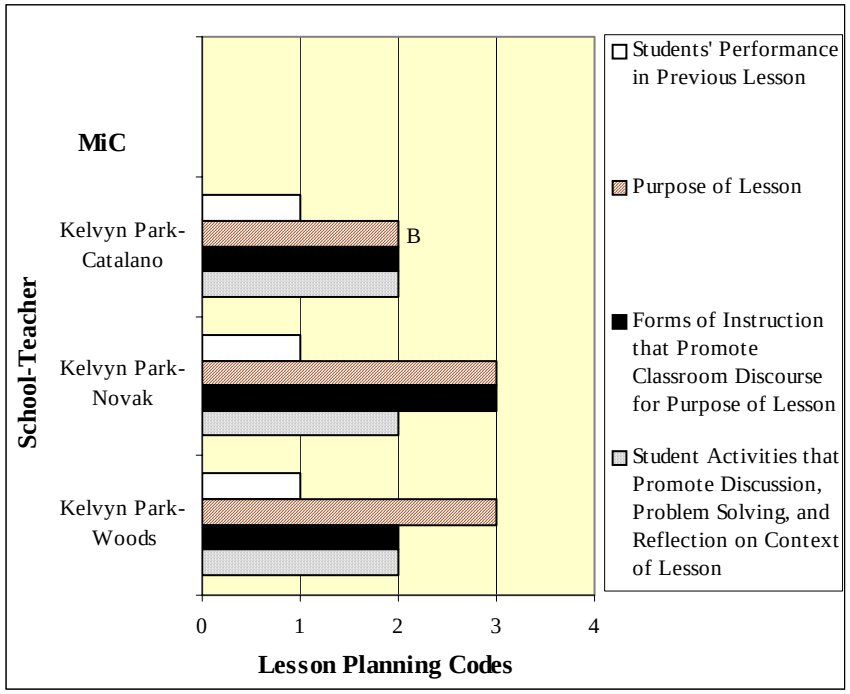
When planning to teach individual MiC lessons, Woods planned in a manner similar to how he planned to teach a MiC unit. He worked through all of the problems in the lesson so he could anticipate some of the difficulties students might have. He identified the goal of the lesson and his plan for the students' attainment of that goal, whether it was the completion of a task or the development of a problem solving skill. Woods preferred whole-class direct teaching because he thought that was most effective. He stated: "If you can stand in front of the class and expound very well, it's very effective" (Woods, Interview 7/1/99). Woods chose small-group work when it was task appropriate. He explained: "I think it has to be appropriate to the task. So when there's a project involved in which you want a heterogeneous group so that kids can help each other and contribute their own individual skills, then I think [cooperative groups] are great" (Woods, Interview 7/1/99). The quantity and quality of whole-class discussions varied in Woods' classes. Some discussions were limited because few students participated. Some students' contributions were

short answers to the teacher's questions. Other discussions were more complex with several students participating, eager to share their solution strategies, to support their reasoning, and to challenge other students' thinking (Woods, Interview 7/1/99).

Table H44
Subcategories of Lesson Planning: Woods, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the three eighth-grade teachers in District 4 varied in two of the four subcategories of *lesson planning*, teachers' attention to the *purpose of the lesson* and *forms of instruction that promote classroom discourse for the purpose of the lesson* (see Figure H8). With respect to *students' performance in the previous lesson*, all three teachers planned in the same manner daily without considering students' previous performance. Little variation was seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. The other teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.



Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure H8. Lesson planning, Grade 8, District 4.

Teachers varied little in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The other two teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. No variance was seen in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson.* All of the teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Table H45
Unit Planning, District 1, Grade 8, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Fernwood-Reichers	2B	2	2	6
Von Humboldt-Waters	2B	1	1B	4
		— Conventional —		
Addams-Wolfe	1	1	1A	3
Wacker-Kendall	2A	1	2	5
Wacker-Marin	2A	1	2	5
Wacker-DiMatteo	2A	1	2	5

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.

2. Consideration of student abilities.
   A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
   B. The teacher planned the unit based on perceptions of students' mathematics skills.

3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.

4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.

1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.

2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.

3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.

4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. *Little or no consideration of pacing*.
   A. The teacher follows the recommendations for pacing in teacher support materials.
   B. The teacher does not plan unit pacing because the curriculum is unfamiliar.

2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.

3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.

4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table H46
Lesson Planning, District 1, Grade 8, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Context of the Lesson	Total
			— MiC —		
Fernwood-Reickers	2	3	3	3	11
Von Humboldt-Waters	2	3	2	2	9
			— Conventional —		
Addams-Wolfe	2	2	1	2	7
Wacker-DeMatteo	1	1	2	2	6
Wacker-Kendall	NA	NA	NA	NA	
Wacker-Marin	NA	NA	NA	NA	

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.

2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.

3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.

4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.

Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.

2. *Overall curriculum plan in mind.*

A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.

B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).

3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.

2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)

3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.

4. *Forms of instruction that promote substantive conversation planned*. The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.

2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.

3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.

4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table H47

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	35	11	74	15	0
Von Humboldt-Waters	164	26	91	5	2	2
Conventional						
Addams-Wolfe	76	24	6	61	28	6
Wacker-DiMatteo	76	54	12	2	2	83

Table H48

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	49	12	76	9	3
Von Humboldt-Waters	164	34	18	38	24	20
Conventional						
Addams-Wolfe	76	74	2	54	21	23
Wacker-DiMatteo	76	71	6	6	19	70

Table H49

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	47	10	67	22	1
Von Humboldt-Waters	164	42	12	57	30	1
Conventional						
Addams-Wolfe	76	67	2	18	61	20
Wacker-DiMatteo	76	41	0	0	35	68

Table H50

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	30	4	30	52	13
Von Humboldt-Waters	164	15	4	44	52	0
Conventional						
Addams-Wolfe	76	0				
Wacker-DiMatteo	76	46	0	0	29	71

Table H51

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	54	1	14	61	23
Von Humboldt-Waters	164	35	5	58	35	2
Conventional						
Addams-Wolfe	76	7	0	0	20	80
Wacker-DiMatteo	76	24	50	6	11	33

Table H52

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	27	5	36	33	26
Von Humboldt-Waters	164	8	0	54	23	23
Conventional						
Addams-Wolfe	76	55	0	55	19	26
Wacker-DiMatteo	76	49	8	8	32	51

Table H53

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	55	14	72	11	4
Von Humboldt-Waters	164	39	9	66	22	3
Conventional						
Addams-Wolfe	76	64	2	27	49	22
Wacker-DiMatteo	76	50	0	0	5	95

Table H54

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	34	0	34	55	11
Von Humboldt-Waters	164	38	0	84	14	2
Conventional						
Addams-Wolfe	76	76	0	34	40	26
Wacker-DiMatteo	76	46	0	9	3	89

Table H55

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	63	1	24	66	8
Von Humboldt-Waters	164	43	0	76	16	9
Conventional						
Addams-Wolfe	76	76	0	29	53	17
Wacker-DiMatteo	76	46	3	3	11	83

Table H56

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	49	3	36	55	7
Von Humboldt-Waters	164	34	0	79	16	5
Conventional						
Addams-Wolfe	76	76	0	36	48	16
Wacker-DiMatteo	76	45	0	0	32	68

Table H57

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	23	3	33	36	28
Von Humboldt-Waters	164	6	0	40	50	10
Conventional						
Addams-Wolfe	76	79	0	40	38	22
Wacker-DiMatteo	76	46	11	3	37	49

Table H58

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	12	5	47	16	32
Von Humboldt-Waters	164	11	0	0	28	72
Conventional						
Addams-Wolfe	76	11	0	0	13	88
Wacker-DiMatteo	76	21	0	0	63	38

Table H59

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	37	0	79	19	2
Von Humboldt-Waters	164	1	0	100	0	0
Conventional						
Addams-Wolfe	76	58	0	59	34	7
Wacker-DiMatteo	76	25	5	5	53	37

Table H60

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 1, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Fernwood-Reichers	155	7	27	73	0	0
Von Humboldt-Waters	164	4	17	67	0	17
Conventional						
Addams-Wolfe	76	9	0	29	43	29
Wacker-DiMatteo	76	21	6	6	56	31

Table G61

Teacher Level of Lesson Presentation and Development, Grade 8, District 1

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Fernwood-Reichers	8	0	0	2	1	0	1	4	0	5
Von Humboldt-Waters	9	0	0	1	3	3	2	0	0	3B
Conventional										
Addams-Wolfe	7	1	3	0	0	3	0	0	0	3B
Wacker-Kendall	2	0	1	0	0	1	0	0	0	3B
Wacker-Marin	2	0	0	0	0	2	0	0	0	3B
Wacker-DiMatteo	4	0	2	1	0	1	0	0	0	2B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table H62

Nature of Mathematical Inquiry, Grade 8, District 1

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																							
Fernwood-Reichers	3	3	4	3	1	4	2	2		3	2	2	2	1	2	1	1		2	3	1	1	1	3	2	2		3	2	3	1	1	3	2	3		*2.16*	***3***
Von Humboldt-Waters	2	4	2	2	2	1	1	1	1	3	2	2	2	1	1	1	1	1	2	3	2	2	2	1	1	1	1	1	2	2	1	2	1	1	1	1	*1.58*	***2***
Conventional																																							
Addams-Wolfe	1	1	1	1	1	1	1			1	1	1	1	1	1	1			2	1	1	1	1	1	1			1	1	1	1	1	1	1			*1.04*	***1***
Wacker-Kendall	1	1								1	1								1	1								1	1								*1.00*	***1***
Wacker-Marin	1	1								1	1								1	1								1	1								*1.00*	***1***
Wacker-DiMatteo	1	1	1	1	1					1	1	1	1	1					1	1	1	1	1					1	1	1	1	1					*1.00*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table H63

Nature of Students' Explanations, Grade 8, District 1

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Reichers	3	2	2	2	1	2	2	2		*2.00*
Von Humboldt-Waters	1	1	2	2	2	1	1	1	1	*1.33*
Conventional										
Addams-Wolfe	1	1	1	1	1	1	1			*1.00*
Wacker-Kendall	1	1								*1.00*
Wacker-Marin	1	1								*1.00*
Wacker-DiMatteo	1	1	1	1	1					*1.00*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table H64

Elicitation of Multiple Strategies, Grade 8, District 1

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Reichers	2	2	4	2	1	2	1	1		*1.88*
Von Humboldt-Waters	2	2	3	2	3	1	1	1	1	*1.78*
Conventional										
Addams-Wolfe	1	1	1	1	1	1	1			*1.00*
Wacker-Kendall	1	1								*1.00*
Wacker-Marin	1	1								*1.00*
Wacker-DiMatteo	1	1	1	1	1					*1.00*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table H65

Nature of Student–Student Conversation, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Reichers	4	2	1	1	1	3	1	1		*1.75*
Von Humboldt-Waters	2	2	1	1	1	2	1	1	1	*1.00*
Conventional										
Addams-Wolfe	1	1	1	1	1	1	1			*1.00*
Wacker-Kendall	1	1								*1.00*
Wacker-Marin	1	1								*1.00*
Wacker-DiMatteo	1	1	1	1	1					*1.00*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table H66

Students' Collaborative Working Relationships, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Reichers	3	2	N/A	1	1	4	1	3		*2.14*
Von Humboldt-Waters	1	2	1	1	1	2	1	1	1	*1.22*
Conventional										
Addams-Wolfe	N/A	1	1	1	1	1	N/A			*1.00*
Wacker-Kendall	N/A	N/A								N/A
Wacker-Marin	N/A	N/A								N/A
Wacker-DiMatteo	N/A	1	N/A	N/A	N/A					*1.00*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table H67

Student Engagement during Instruction, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Reichers	4	3	3	3	1	4	2	3		*2.88*
Von Humboldt-Waters	1	3	3	2	2	4	2	2	1	*2.22*
Conventional										
Addams-Wolfe	2	2	2	2	1	1	2			*1.71*
Wacker-Kendall	1	1								*1.00*
Wacker-Marin	1	1								*1.00*
Wacker-DiMatteo	2	2	3	3	2					*2.40*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table H68

Unit Planning, District 2, Grade 8, 1998-1999

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Guggenheim-Keeton	2B	1	3	6
Guggenheim-Teague	2A	1	3	6
		— Conventional —		
Newberry-Cunningham	1	1	2	4
Newberry-Stark	2A	2	2	6

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.

2. Consideration of student abilities.

A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.

B. The teacher planned the unit based on perceptions of students' mathematics skills.

3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.

4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.

1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.

2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.

3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.

4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. *Little or no consideration of pacing*.

A. The teacher follows the recommendations for pacing in teacher support materials.

B. The teacher does not plan unit pacing because the curriculum is unfamiliar.

2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.

3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.

4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table H69

Lesson Planning, District 2, Grade 8, 1998-1999

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Guggenheim-Keeton	2	3	3	3	11
Guggenheim-Teague	2	2B	2	2	8
			— Conventional —		
Newberry-Cunningham	2	2A	2	2	8
Newberry-Stark	2	2A	2	1	7

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.

2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.

3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.

4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.

Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.

2. *Overall curriculum plan in mind.*

A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.

B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).

3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.

2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)

3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.

4. *Forms of instruction that promote substantive conversation planned*. The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.* Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.

2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.

3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.

4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table H70

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Warm-Up Activities, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	70	50	48	2	
Guggenheim-Teague	85	40	29	65	6	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	2	50	50	0	0

Table H71

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Review of Previous Material, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	54	20	47	27	7
Guggenheim-Teague	85	27	87	13	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	35	40	47	3	10

Table H72

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Teacher Presentation, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	43	19	47	31	3
Guggenheim-Teague	85	40	62	38	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	30	27	50	23	0

Table H73

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Whole-Class Discussion, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	53	7	52	30	11
Guggenheim-Teague	85	33	4	89	7	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	26	0	77	18	5

Table H74

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Small-Group Work, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	37	10	68	19	3
Guggenheim-Teague	85	32	0	81	19	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	28	4	42	38	17

Table H75

Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson: Independent Practice, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	73	3	26	43	28
Guggenheim-Teague	85	8	0	43	43	14
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	30	4	54	27	15

Table H76

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Listened to Teacher or Took Notes, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	35	14	55	28	3
Guggenheim-Teague	85	35	87	13	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	41	31	60	9	0

Table H77

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Investigated Problems, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	58	8	48	38	6
Guggenheim-Teague	85	38	3	78	19	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	7	33	33	17	17

Table H78

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Discussed Answers and Solution Strategies, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	48	8	60	30	3
Guggenheim-Teague	85	39	6	91	3	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	36	16	71	6	6

Table H79

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Participated in Whole-Class Discussions, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	49	2	51	44	2
Guggenheim-Teague	85	32	4	96	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	21	28	50	22	0

Table H80

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Practiced Computation, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	36	7	50	37	7
Guggenheim-Teague	85	20	47	53	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	50	7	49	33	12

Table H81

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Took Test or Quiz, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	20	6	29	35	29
Guggenheim-Teague	85	5	50	25	0	25
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	9	0	0	0	100

Table H82

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Reflected on or Summarized Lesson Concepts, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	30	4	68	28	0
Guggenheim-Teague	85	1	100	0	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	1	100	0	0	0

Table H83

Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: Began Homework, District 2, Grade 8, 1998-1999

School-Teacher	Number of Days Reported	(%) of Class Periods Reported	<15% of Class (%)	Equal Emphasis (%)	50-75% of Class (%)	>75% of Class (%)
MiC						
Guggenheim-Keeton	83	10	63	38	0	0
Guggenheim-Teague	85	1	100	0	0	0
Conventional						
Newberry-Cunningham	NA	NA				
Newberry-Stark	86	1	100	0	0	0

Table H84

Teacher Level of Lesson Presentation and Development, Grade 8, District 2

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Keeton	7		2				3	2		5
Guggenheim-Teague	5					2	2	1		4
Conventional										
Newberry-Cunningham	6		1			3	1	1		3B
Newberry-Stark	6		2		1	3				3B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table H85

Nature of Mathematical Inquiry, Grade 8, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation										
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	*Mean*	***Rating***
MiC																																						
Guggenheim-Keeton	4	4	4	4	4	4	3			3	4	3	4	2	2	1			3	3	3	3	3	2	1			2	1	3	2	3	2	3			*2.86*	***4***
Guggenheim-Teague	3	4	3	4	2					2	2	4	4	1					3	3	3	2	1					3	2	2	3	2					*2.65*	***4***
Conventional																																						
Newberry-Cunningham	4	2	3	1	3	2				3	1	3	1	1	1				1	2	2	1	1	1				3	3	3	1	1	1				*1.88*	***3***
Newberry-Stark	1	2	2	2	3	1				1	1	1	1	2	1				1	2	1	2	2	1				1	1	1	2	2	1				*1.46*	***2***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table H86

Nature of Students' Explanations, Grade 8, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Keeton	2	3	2	3	2	2	1			*2.14*
Guggenheim-Teague	1	2	3	3	2					*2.20*
Conventional										
Newberry-Cunningham	2	1	2	1	1	2				*1.50*
Newberry-Stark	1	2	1	1	1	1				*1.17*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table H87

Elicitation of Multiple Strategies, Grade 8, District 2

	Observation									
School-Teacher (No. of Observations)	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Keeton	3	2	3	2	1	1	1			*1.86*
Guggenheim-Teague	1	3	2	2	1					*1.80*
Conventional										
Newberry-Cunningham	3	1	2	1	1	1				*1.50*
Newberry-Stark	1	1	1	2	1	1				*1.17*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table H88

Nature of Student–Student Conversation, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Keeton	1	4	3	3	1	3	1			*2.29*
Guggenheim-Teague	2	2	2	3	3					*2.40*
Conventional										
Newberry-Cunningham	3	1	1	1	2	1				*1.50*
Newberry-Stark	1	1	1	2	3	1				*1.50*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table H89

Students' Collaborative Working Relationships, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Keeton	1	2	2	2	N/A	2	1			*1.67*
Guggenheim-Teague	2	2	1	3	2					*2.00*
Conventional										
Newberry-Cunningham	3	1	1	1	1	1				*1.33*
Newberry-Stark	N/A	1	1	1	2	N/A				*1.25*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table H90

Student Engagement during Instruction, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Keeton	3	4	4	4	4	3	4			*3.71*
Guggenheim-Teague	3	4	4	4	4					*3.80*
Conventional										
Newberry-Cunningham	3	2	4	4	2	3				*3.00*
Newberry-Stark	3	4	2	3	3	3				*3.00*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		12%	*6%*	*12%*	*12%*		*6%*		*6%*	
Task-Oriented					1	2	3	4		
						6%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	6%					*18%*				*18%*
(17 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H9. Interactive Decisions, Reichers, Fernwood Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
			5%	*21%*	*11%*					
Task-Oriented					1	2	3	4		
						5%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	5%				*21%*	*16%*	*11%*			*5%*
(19 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H10. Interactive Decisions, Waters, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	21%	*7%*		*36%*						
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
		29%			*7%*					
(14 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H11. Interactive Decisions, Wolfe, Addams Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
	22%		*11%*	*22%*						
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
	33%	*11%*								
(9 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H12. Interactive Decisions, Marin, Wacker Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4 *100%*	5	6	7	8
(1 item coded)	Least			Teaching Mathematics for Understanding						Most

Figure H13. Interactive Decisions, Kendall, Wacker Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2 *100%*			3	4	5	6	7	8
(1 item coded)	Least			Teaching Mathematics for Understanding						Most

Figure H14. Interactive Decisions, DiMatteo, Wacker Middle School.

Explanation-Oriented	1	2	3	4	5 *22%*		6 *17%*	7 *28%*	8 *6%*	
Task-Oriented					1	2 *6%*	3	4 *6%*		
Shift in Pedagogical Approach	1 *6%*	2			3	4 *6%*	5 *6%*	6	7	8
(18 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H15 Interactive Decisions, Keeton, Guggenheim Middle School.

Explanation-Oriented	1 *7%*	2	3	4	5 *7%*		6 *7%*	7 *7%*	8 *7%*	
Task-Oriented					1 *21%*	2 *14%*	3	4		
Shift in Pedagogical Approach	1 *7%*	2			3	4 *7%*	5	6	7 *7%*	8 *7%*
(14 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H16. Interactive Decisions, Teague, Guggenheim Middle School.

Explanation-Oriented	1 *11%*	2	3 *11%*	4	5 *11%*		6	7 *44%*	8	
Task-Oriented					1 *11%*	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7 *11%*	8
(9 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H17. Interactive Decisions, Cunningham, Newberry Middle School.

Explanation-Oriented	1 *8%*	2	3	4 *39%*	5		6	7 *8%*	8	
Task-Oriented					1 *8%*	2	3	4		
Shift in Pedagogical Approach	1 *15%*	2			3 *15%*	4 *8%*	5	6	7	8
(13 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure H18. Interactive Decisions, Stark, Newberry Middle School.