Longitudinal/Cross-sectional Study of the Impact of *Mathematics in Context* on Student Performance

Instruction for 1999-2000
(Technical Report # 32)

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INTRODUCTION

The purposes of the longitudinal/cross-sectional study of the impact of *Mathematics in Context* (MiC; National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998) on student performance are (a) to determine the mathematical knowledge, understanding, attitudes, and levels of student performance as a consequence of studying MiC for over three years; and (b) to compare student knowledge, understanding, attitudes, and levels of performance of students using MiC with those using conventional mathematics curricula. The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). For this study, information is being gathered on 14 variables over a 3-year period for three groups of students (those in Grades 5, 6, and 7 in 1997). The variables have been organized in five categories (prior, independent, intervening, outcome, and consequent). (See Figure 1 for variables and hypothesized relationships.)

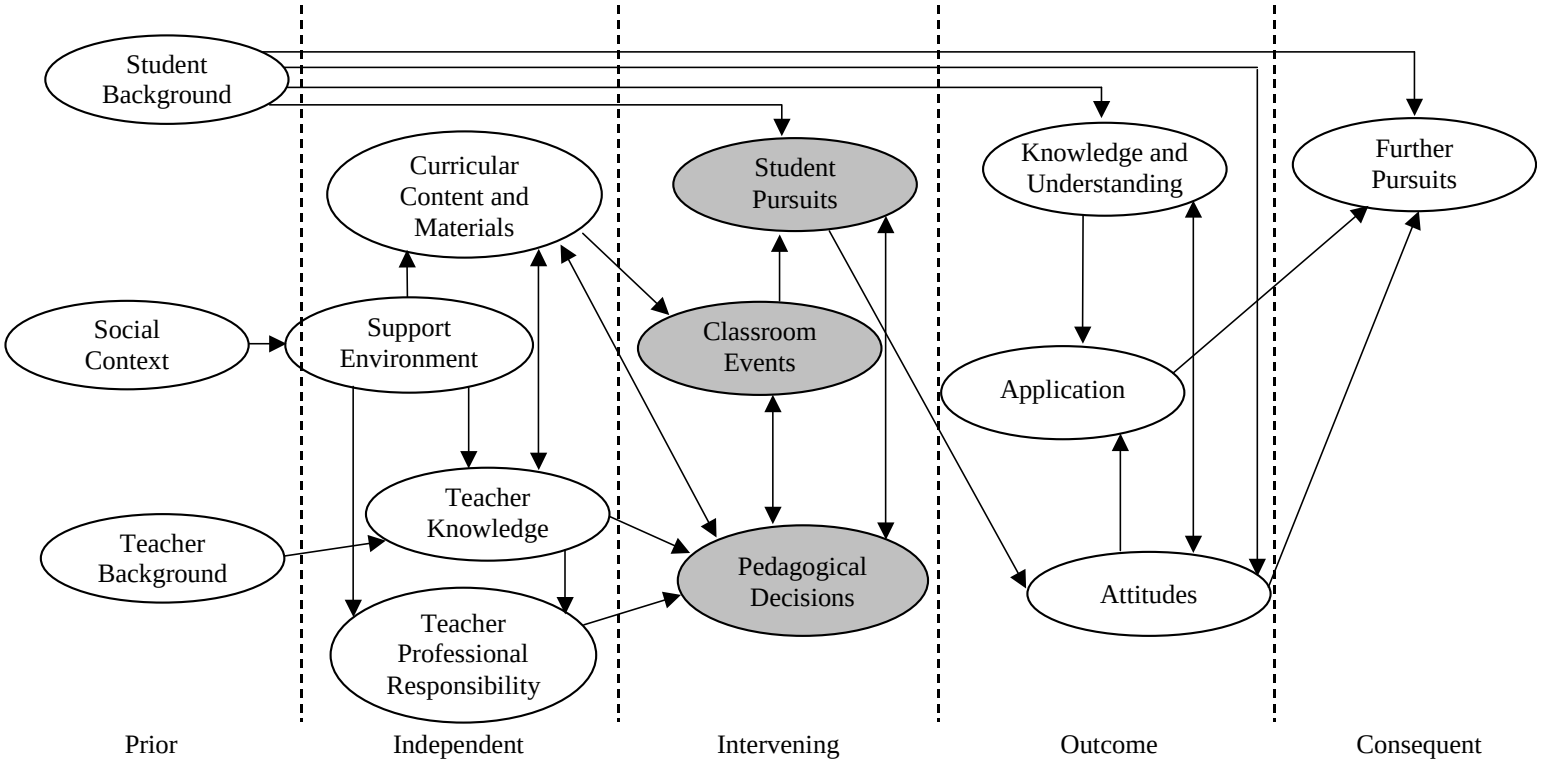

Figure 1. Revised model for the monitoring of school mathematics.

For analytical purposes, although the variation in each set of variables could be examined using structural equations, the number of classes at a given grade level is small, and collinearity across variables poses a serious interpretation problem. For this reason, a simplified research function will be used to make both the cross-sectional and longitudinal comparisons. Variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, can be attributed to variations in prior achievement (PA), method of instruction (I), opportunity to learn with understanding (OTL*u*), and school capacity (SC). This relationship can be expressed as—

$$CA = PA + I + OTLu + SC.$$

Each of these composite indices is being specified from the variables in the original model. This paper details the analysis of the *Instruction* variable.

Overview

The purpose of this working paper is to summarize information for the composite variable *Instruction* collected during the first year of the longitudinal/cross-section study, the 1999–2000 school year, for seventh- and eighth grade teachers and students. The purpose of gathering this information was to document the variation in instruction that study students experienced as they studied either MiC or conventional mathematics curricula. The composite index *Instruction* is specified from data gathered on the intervening variables (*pedagogical decisions, classroom events*, and *student pursuits*) in the structural research model. In the simplified research function, instruction includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice,*[1] and *student pursuits* during instruction (see Figure 2). Information on *unit planning, lesson planning*, and *classroom assessment practice* was gathered through the Teacher Interview: Instructional Planning and Classroom Interaction (see Appendix A; Shafer, Davis, & Wagner, 1998). Information on *lesson planning, mathematical interaction during instruction*, and *classroom assessment* was gathered through the Teacher Log (see Appendix B; Shafer, Wagner, & Davis, 1997). Information on *mathematical interaction during instruction, classroom assessment*, and *student pursuits* was gathered through the Classroom Observation Scale (see Appendix C; Davis, Wagner, & Shafer, 1998).

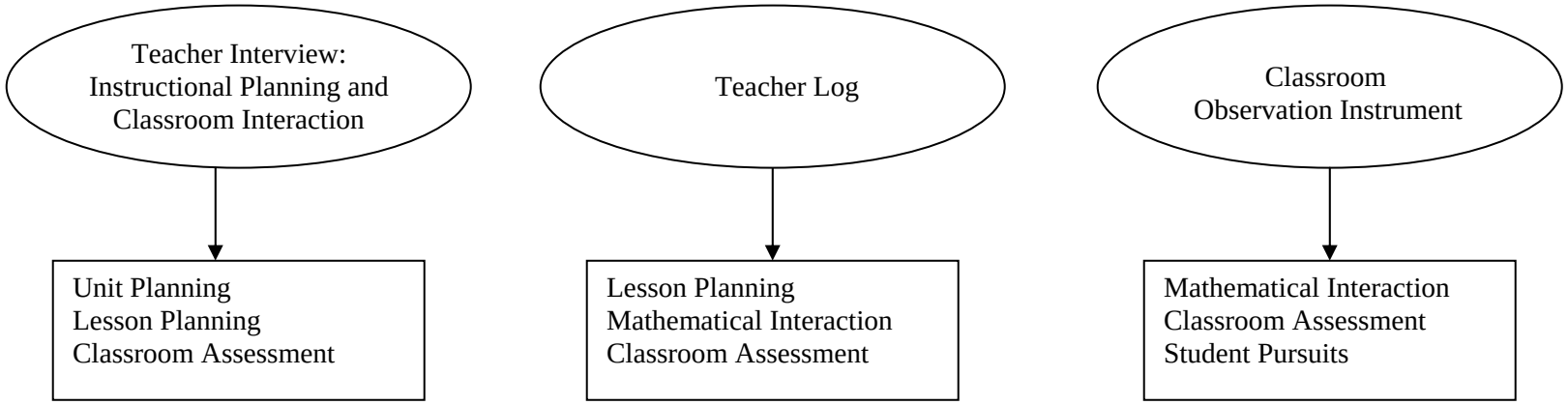

Figure 2. Categories of *Instruction* in the longitudinal/cross-sectional study of the impact of *Mathematics in Context* on student performance and their sources.

Fifty-three teachers and students in 94 classes from four school districts participated in the study. Districts are identified by number, and the classes by school and teacher (both pseudonyms). Also noted are the type of materials used (MiC materials or a conventional text).

[1] The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

The Sample

Districts 1 and 2

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. All MiC teachers used commercial versions of the units. Teachers using conventional curricula used whatever curriculum was already in place in the schools. The sample in Districts 1 and 2 consisted of 18 seventh- and eighth-grade teachers and their classes from diverse schools in Districts 1 and 2. These teachers taught a combined total of 45 classes involving 530 students Although 18 teachers participated in the study this year, complete data sets are available for 17 teachers because one teacher from District 1 did not complete the interview.

Table 1
Summary of Study Participants, Districts 1 and 2, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 1	
MiC	7	18	242
Conventional	3	4	71
		District 2	
MiC	7	21	191
Conventional	1	2	26
Total	18	45	530

Table 2
Characteristics of the Student Participants, Districts 1 and 2, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
District 1											
MiC	44	56	21	0	3	4	50	11	10	84	17
Conventional	48	52	7	0	1	6	76	10	0	96	4
District 2											
MiC	48	52	32	2	1	27	18	15	6	88	12
Conventional	46	54	4	0	0	23	38	27	8	81	19

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.
[2]Includes nonresponses.

District 1 was located in an urban region in the eastern part of the United States. In 13 elementary, 6 middle, and 4 high schools, 1052 teachers were responsible for teaching the district's 15,749 students. Four of the middle schools participated in the third year of the study. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. The teacher population was 17% minority with 14% African American teachers and 2% Hispanic teachers. Approximately 30–40% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. For preliminary teacher certification, 24 semester hours were recommended for fifth- and sixth-grade teachers; 24 semester hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements.

District 2, located in a large urban area in southeastern United States, had 14,994 teachers and 352,595 students housed in 200 elementary schools, 52 middle schools and numerous high schools. One elementary and two middle schools participated in the third year of the study. The district student population was predominantly minority with 33% African American students and 53% Hispanic students. The teacher population was more than half minority with 28% African American teachers and 34% Hispanic teachers. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students learned English as a second language. Teachers were provided 10 paid in-service days for professional development. Continuing certification required the completion of six semester hours in mathematics education every five years for middle and high school teachers and six semester hours in education, not specifically mathematics, for elementary school teachers. District requirements for preparation of mathematics teachers were the same as state requirements.

Districts 3 and 4

Districts 3 and 4 agreed to participate in a modified research design in which all MiC teachers used MiC. All teachers used commercial versions of MiC units. The sample in Districts 3 and 4 consisted of 9 seventh- and eighth-grade teachers who taught a combined total of 22 classes involving 285 students (see Tables 3 and 4). Although 9 teachers participated in the study this year, complete data sets are available for 8 teachers because one teacher (from District 3) did not return teacher questionnaires.

Table 3
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 3	
MiC	3	11	162
		District 4	
MiC	6	11	123
Total	9	22	285

Table 4
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	Female	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
					District 3						
MiC	49	51	1	1	1	2	86	9	1	98	2
					District 4						
MiC	46	54	28	2	2	24	3	16	25	89	11

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.

[2]Includes nonresponses.

District 3 was located in a suburban area of a large western state. In the district's two elementary and one middle school, 63 teachers taught 1427 students. Study participants are from one middle school in this district. Two of the study classes were in self-contained classrooms. The rest of the study students had several subject-matter teachers. The district's student population was 98% White and the teacher population was 100% White. Approximately 10–20% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. The district provided teachers with three paid in-service days for professional development. For preliminary teacher certification, the state mandated single-subject credentials for Grades 7–8. Although the district provided mathematics courses and staff development opportunities, it did not require additional certification and courses for experienced teachers.

District 4 was one of many districts located in a large urban area in the eastern part of the United States. The district's 2500 teachers were responsible for teaching the 29,000 students in 23 elementary schools, seven middle schools, and several high schools. In District 4, Grades 6–8 are contained in middle schools in which students have several subject-matter teachers. Study participants are from one middle school in this district. The student population was predominately minority with 51% African American students and 34% Hispanic students. Teacher ethnicity data were unavailable. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. To be provisionally certified, the state required teachers to earn 36 semester credits in mathematics. A master's degree was required for permanent certification, but not necessarily in mathematics. District requirements were the same as the state requirements. Professional development opportunities related to mathematics teaching varied from school to school. All teachers were provided four paid in-service days for general professional development.

The *Instruction* Composite Variable

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. The five categories of *Instruction* were further subdivided into 19 subcategories, and an index was created for each (see Figure 3). In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence*; and *pace of instruction*. Four subcategories characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies; and lesson reflection, summary, or closure*. Three subcategories characterized *classroom assessment practice: evidence sought during classroom assessment; purpose and coherence of feedback given in response to classroom assessment*; and *content of feedback* provided in response to classroom assessment. Finally, three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students*; and *level of student engagement*.

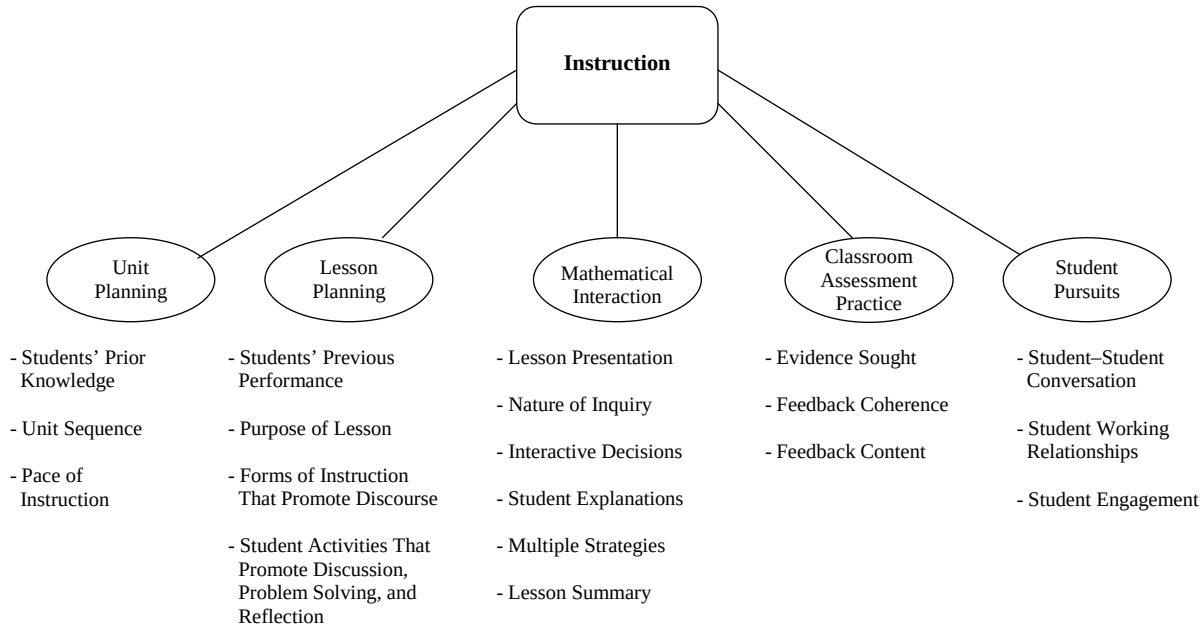

Figure 3. Major categories and subcategories of *Instruction*.

A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the simplified research function. Description of the theoretical framework that guided the analysis of *Instruction* is contained in Appendix D, and description of the methodology used in analyzing the data is contained in Appendix E. For complete sets of data, see Appendices F, G, and H for fifth-, sixth-, and seventh-grade teachers, respectively.

Categories of Instruction

Unit Planning

In this study, three subcategories of *Instruction* characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*. Forty-eight percent of the teachers in Districts 1-4 based unit planning on their perceptions of students' prior knowledge with respect to mathematics skills and/or reading ability and vocabulary. Thirty percent planned units on the basis of information gathered through informal or formal assessment and included remedial skill-based activities to address weaknesses or extension activities for students who might be ready for such challenges. Fifteen percent of the teachers planned units with little or no consideration of the prior knowledge of students in the current class. Seven percent of the teachers planned conceptually-based activities designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit. With respect to the sequence of units, 44% of the teachers did not vary from the unit/chapter sequence recommended in teacher support materials. Thirty-three percent of the teachers based decisions about unit sequence on external factors such as the content and dates of district or state standardized testing or on various seasonal events. Fifteen percent of the teachers sequenced units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests. Seven percent of the teachers sequenced units to support the development of mathematics concepts. Finally, 79% of the teachers considered the recommendations for pacing in teacher support materials but planned to adjust the pace as the unit developed or as a result of collaboration with other teachers.

Lesson Planning

Four subcategories of instruction characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Sixty-three percent of the teachers in Districts 1-4 made changes in the next day's plans because of students' performance in the previous lesson. The teachers extended the previous lesson to complete a task, disregarded time constraints, or added a review. Twenty-two percent of the teachers made changes that focused on students' understanding of the mathematical content of the lesson. These teachers allowed for a more in-depth exploration of the mathematical content or introduced another approach to encourage students' understanding. The remaining teachers planned lessons with little or no regard for students' performance on the previous lesson. The teachers might have noted students' performance but lessons were planned in the same way as previous lessons. None of the teachers indicated that they made changes focused on encouraging thinking at higher levels by varying problem structure or setting or by emphasizing connections with related concepts. With respect to the purpose of the lesson, 7% of the teachers selected lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials included problem solving, applications of mathematics, or

practice. Nineteen percent of the teachers planned lessons from unit/chapter materials to become familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched. Sixty-seven percent of the teachers went beyond becoming familiar with the content, presentation, and context to make decisions for student learning, for example, potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit. None of the teachers planned lessons that emphasized higher order thinking, depth of knowledge, and/or understanding.

Planning *forms of instruction that promote classroom discourse for the purpose of the lesson* varied among all teachers in Districts 1-4. Forty-eight percent of the teachers anticipated using whole-class discussion and small-group or pair work. Although these teachers planned for such lesson formats, the focus was primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. Thirty-seven percent planned for students' participation and collaboration for during instruction, but these still were not the primary focus of the lesson plan. Fifteen percent of the teachers planned forms of instruction that promoted substantive conversation. The teacher planned classroom activities that encouraged students to participate in discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. Similarly, planned *student activities that promoted discussion, problem solving, and reflection on the content of the lesson* varied among the teachers. Forty-four percent of the teachers included investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) in lesson plans. Thirty-three percent of the teachers included investigation of problems and discussion of answers and solution strategies as important elements in lesson plans, although questions or activities that encouraged students to reflect on or summarize lessons were rarely included. For 15% of the teachers, investigation of problems and discussion of answers and solution strategies were dominant in lesson plans. Seven percent of the teachers rarely planned investigation of problems and discussion of mathematical ideas for the lessons. Emphasis, instead, was placed on practicing routine calculations, and little discussion among students was anticipated.

Mathematical Interaction during Instruction

Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*. With respect to lesson presentation and development, the results for the 18 teachers in Districts 1 and 2 revealed differences by grade level, type of curriculum taught, and by district. By grade level, more eighth -grade teachers taught mathematics for conceptual understanding than seventh-grade teachers (3 of 9 eighth-grade teachers compared to 1 of 9 seventh-grade teachers). These teachers emphasized conceptual understanding with active participation by students and teacher. Lesson presentations featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher. The greatest number of teachers, however, demonstrated procedures and strategies for students to use. Three of the 7 seventh-grade teachers and 4 of 7 eighth-grade teachers presented and emphasized particular procedures or strategies.

Because the number of teachers using conventional curriculum in the third study year is small, it is difficult to make comparisons among the treatments. Five of the 14 teachers using MiC either attempted to teach for or clearly emphasized conceptual understanding in contrast to only 1 of the 4 teachers using conventional curricula. On the other hand, 5 of the 14 MiC teachers presented particular procedures or strategies in class in

comparison to 2 of the 4 teachers using conventional curricula, and 4 of the MiC teachers emphasized review in comparison to 1 of the teachers using a conventional curriculum. When these results were reviewed by district, however, differences became apparent for teachers using MiC. Although teachers in both districts had comparable numbers of teachers at Levels 4 and 5, differences are evident in Levels 2 and 3. Four of the 7 MiC teachers in District 1 demonstrated procedures or strategies compared to 1 of the 7 MiC teachers in District 2. On the other hand, 3 of the 7 MiC teachers in District 2 emphasized review in comparison to 1 of the 7 MiC teachers in District 1.

Less variation was noted with respect to the *nature of inquiry during instruction*. An equal number of teachers at each grade level (4 out of 7) emphasized conceptual understanding during instruction. Also, the number of teachers who provided limited attention to conceptual understanding or promoted lower level thinking was about the same.

Teachers' interactive decisions also varied among the teachers in Districts 1 and 2. Differences were revealed by grade level and by curriculum taught. By grade level, Although the same number of teachers made interactive decisions that were aligned with teaching mathematics for understanding, 5 of the 9 seventh-grade teachers exhibited strong standard pedagogy in comparison to 2 of the 9 eighth-grade teachers. An additional pattern of variation was found when reviewed by curriculum taught. Five of the 14 MiC teachers made interactive decisions that supported teaching mathematics for understanding in contrast to 1 of the 4 teachers using conventional curricula. On the other hand, 5 of the 14 MiC teachers exhibited strong standard pedagogy in comparison to 2 of the 4 teachers using conventional curricula.

Student explanations in most classes were focused on procedures or students' responses were limited to answers only. Multiple strategies were generally not elicited from students during instruction. Few opportunities, if any, were provided for reflection on or summary of lesson content. Three teachers provided some opportunities each week for students to reflect on the mathematics or summarize what they had learned in a lesson or in a series of lessons.

Classroom Assessment Practice

Three subcategories of instruction characterized *classroom assessment practice*: *evidence sought during classroom assessment; purpose and coherence of feedback given in response to classroom assessment*; and *content of feedback* provided in response to classroom assessment. In Districts 1 and 2, half of the teachers (7 of the 14 MiC teachers and 2 of the 4 teachers using conventional curricula) gathered limited evidence of their students' understanding. Seventeen percent of the teachers (2 of the 14 MiC teachers and 1 of the 4 teachers using conventional curricula) sought evidence of procedural competence based on student homework and classwork during their assessment practice. Five teachers (all MiC) sought student explanations in addition to procedural competence and answers. However, these explanations were often void of mathematical substance and were used to generate some form of communication rather than assess student understanding. One MiC teacher was somewhat effective at eliciting student responses and orchestrating substantive whole-class discussions. However, the overriding focus of their classroom practice was to use correct answers and procedures as evidence of student learning.

With respect to the *purpose and coherence of feedback*, 78% of the teachers (11 MiC and 3 using conventional curricula) provided feedback that was indirectly responsive to student needs. That is, teacher-directed feedback involved reteaching using more of the same type of instruction and

practice sets that were used in the presentation of the lesson. Feedback may have also occurred in response to specific student questions or procedural errors when the teacher moved around the classroom during seatwork. Feedback provided by other students was minimal or non-existent. The *content of feedback* varied widely among the teachers. Nine teachers (7 MiC) provided limited feedback (praise or criticism) that involved checking the correctness of answers but seldom addressed student misconceptions. Five teachers (4 MiC) provided clear and mathematically sound feedback that addressed skills, procedures, and concepts. Five teachers (4 MiC) directed feedback toward skills and procedures and the format of the answer (such as simplified form) rather than clarifying explanations or developing student understanding. Four teachers (3 MiC) provided feedback that addressed skills, procedures, and concepts and was mathematically sound.

Students' Pursuits

Three subcategories of instruction characterized *students' pursuits during instruction*: *student–student conversation, collaborative working relationships among students*, and *level of student engagement*. During most class periods, *student–student conversation* was limited or was not encouraged. On only a few occasions did student–student conversation reach a substantive level characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level. Mean ratings for student–student conversation were generally higher in District 2 than in District 1.

Study students rarely collaborated with one another during lessons. Students shared ideas or discussed how problems could be solved. Even though they physically sat together, students worked on different problems at different paces. Substantive collaboration among students was rarely noted. Mean ratings for student collaboration were generally higher in District 2 than in District 1.

Student engagement during instruction also varied among the teachers. On many occasions, student engagement was widespread. Most students were on task pursuing the substance of the lesson most of the time. They seemed to take the work seriously and put forth much effort. During other lessons, most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

Composite Index for *Instruction*

Although teachers in all four research sites completed interviews, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index *Instruction*, therefore, was created only for teachers in Districts 1 and 2 for whom there was a complete set of ratings on all 19 indices. Seventeen teachers were involved in the analysis. One teacher was not included because she did not teach a full semester during the study.

The composite index *Instruction* was created in a multiple-step process. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the Instruction Total.[2] Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subsubcategories and the Instruction Total (see Table 5).

[2] The sum of the weighted results was taken as a measure of the quality of instruction. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of instruction.

Table 5.

Correlation between the Instruction Total and the Subcategories of Instruction

Subcategory	Unit Planning			Lesson Planning				Mathematical Interaction						Classroom Assessment			Student Pursuits		
	SPK	US	PI	SPPL	PL	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE
SPK																			
US	0.092																		
PI	0.321**	0.017																	
SPPL	0.164	0.295**	0.051																
PL	0.136	-0.026	0.147	0.409***															
FIFD	0.038	-0.077	0.229*	0.177	0.326**														
SAPD	0.016	-0.181	0.134	0.118	0.474***	0.640***													
LPD	-0.023	0.104	0.176	0.157	0.312**	0.537***	0.546***												
NI	-0.026	-0.116	0.208	0.217*	0.506***	0.665***	0.549***	0.647***											
ID	-0.033	-0.015	0.285**	0.099	0.309**	0.568***	0.527***	0.676***	0.702***										
SE	-0.008	-0.072	0.141	0.175	0.260*	0.497***	0.468***	0.559***	0.692***	0.660***									
MS	-0.088	-0.060	0.048	0.025	0.227*	0.523***	0.426***	0.552***	0.635***	0.619***	0.788***								
LCS	-0.032	0.000	0.084	0.172	0.291**	0.476***	0.534***	0.556***	0.476***	0.495***	0.445***	0.471***							
ES	0.005	0.000	0.015	0.152	0.257*	0.408***	0.457***	0.650***	0.573***	0.602***	0.635***	0.599***	0.537***						
FCP	-0.036	0.055	0.108	0.079	0.266*	0.442***	0.423***	0.570***	0.520***	0.532***	0.561***	0.523***	0.480***	0.791***					
FC	-0.136	0.030	0.142	0.159	0.228*	0.398***	0.401***	0.624***	0.547***	0.613***	0.551***	0.469***	0.464***	0.695***	0.753***				
SC	0.015	-0.002	0.101	0.189	0.375**	0.493***	0.425***	0.436***	0.624***	0.532***	0.506***	0.475***	0.409***	0.483***	0.503***	0.578***			
SWR	0.073	0.008	0.063	0.109	0.336**	0.437***	0.422***	0.369***	0.532***	0.487***	0.521***	0.480***	0.385***	0.450***	0.486***	0.492***	0.845***		
OSE	-0.049	0.015	0.082	0.220	0.272*	0.322**	0.336**	0.525***	0.562***	0.512***	0.623***	0.572***	0.364***	0.548***	0.489***	0.518***	0.592***	0.593***	
Instr. Total	0.089	0.081	0.272*	0.309**	0.501**	0.697***	0.665***	0.801***	0.833***	0.803***	0.781***	0.721***	0.654***	0.787***	0.748***	0.751***	0.723***	0.667***	0.688***

*p<.05
**p<.01
***p<.001

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction
Instr. Total--Instruction Total

Five subcategories were not well correlated with the Instruction Total and other subcategories: consideration of students' prior knowledge; unit sequence; pace of instruction; students' performance in the previous lesson; and the purpose of the lesson. To verify these results, a principle component factor analysis was completed using SAS. Factors 1 and 2 accounted for a significant amount of the variance among the subcategories. Fourteen subcategories were included in Factors 1 and 2 (see Table 6). The five subcategories that had weak correlations to the Instruction Total were not influential in Factors 1 and 2 and were important only in the composition of other factors. Consequently, these subcategories were excluded from the analysis. The Instruction Total for each teacher was then recalculated.

Table 6.
Contribution of Subcategories to Principle Component Factors

Subcategory	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
SPK	-18	11	16	72*	9
US	4	-5	7	18	83*
PI	21	-5	-4	85*	7
SPPL	5	9	75*	0	46*
PL	19	21	78*	11	-12
FIFD	60*	15	33	23	-32
SAPD	59*	9	44*	9	-41*
LPD	83*	10	17	7	6
NI	65*	39	33	12	-23
ID	76*	28	8	19	-11
SE	68*	45*	5	5	-10
MS	68*	39	-2	-3	-17
LCS	68*	6	29	-4	-5
ES	79*	30	4	-10	12
FCP	75*	332	-1	-4	15
FC	74*	35	0	-8	17
SC	36	80*	22	6	-6
SWR	29	85*	15	8	-8
OSE	46*	65*	8	-5	9

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

Using the revised Instruction Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into six groups. For each group of teachers, common characteristics from the subcategories of instruction were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each subcategory of *Instruction*. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. Similar to the index for each subcategory, the underlying single dimension of the composite index *Instruction* was teaching mathematics for understanding. The levels of the composite index were on a continuum from least to most reflective of teaching mathematics for understanding. The six levels are summarized in Table 7.

Level 6: Most reflective of teaching for understanding
Level 5: Reflective of teaching for understanding
Level 4: Attempted to teach mathematics for understanding
Level 3: Limited attention to conceptual understanding
Level 2: Focus on procedures
Level 1: Underdeveloped lessons.

Table 7.
Summary of the Levels of the Composite Index for Instruction

Level 6: Most Reflective of Teaching for Understanding	**Level 5: Reflective of Teaching for Understanding**	**Level 4: Attempt to Teach for Conceptual Understanding**
Mathematical Interaction Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students with teacher support • Discussion of solutions, generalizations, connections Interactive decisions • Predominantly aligned with understanding • Frequent questions on articulation of thinking, understanding mathematics, or reasonable solutions *Classroom Assessment Practice* • Attention to mathematical processes • Ongoing, purposeful feedback from teacher, students • Feedback: making sense of mathematics, solutions • Student assessment of own work and others' work *Student Pursuits* • Occasional substantive conversation • Student-student conversation about procedures *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students and teacher • Discussion of solutions Interactive decisions • Attentive to teaching for understanding • Teacher explanations promote connections *Classroom Assessment Practice* • Student explanations as evidence of mathematical processes *or* procedural understanding • Feedback consistent with Level 6 *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Attempt for conceptual understanding, but focus on procedural understanding • General acceptance of teacher's procedures Interactive decisions • More attentive to good standard pedagogy • Additional exercises, mini-lessons, contexts, review *Classroom Assessment Practice* • Evidence from student explanations • Focus on procedural understanding • Teacher feedback related to concepts, contexts • Student-student feedback: answers, procedures *Student Pursuits* • Engagement mildly enthusiastic, teacher encouraged *Lesson Planning* • Student discussion, problem solving, planned

Table 7 (continued).
Summary of the Levels of the Composite Index for Instruction

Level 3: Limited Attention to Conceptual Understanding	**Level 2: Focus on Procedures**	**Level 1: Underdeveloped Lessons**
Mathematical Interaction Inquiry and lesson presentation • Students use invented or demonstrated strategies • Student explanations focused on procedures Interactive decisions • More reflective of good standard pedagogy • Some attention to articulation of thinking, reasonable solutions • Occasional addition of different context or review *Classroom Assessment Practice* • Evidence from homework, classwork, occasionally student explanations • Teacher feedback: concepts, contexts, *or* procedures, answer format • Student-student feedback: answers *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion of vocabulary, steps in procedures planned, not elaboration of thinking	*Mathematical Interaction* Inquiry and lesson presentation • Predominantly lower order thinking • Students expected to use demonstrated procedures Interactive decisions • Predominantly least aligned with understanding • Limited changes in response to student difficulties, misunderstanding *Classroom Assessment Practice* • Evidence from homework, classwork • Emphasis on procedures, format of answers • Teacher feedback indirectly responsive to students, inattentive to student misconceptions • Student-student feedback: minimal *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion anticipated but not planned	*Mathematical Interaction* Inquiry and lesson presentation • No formal lesson presentation • Procedures demonstrated to individual students • Student dependence on teacher for mathematical work • Frequent confusion or misunderstanding Interactive decisions • Least likely to support teaching for understanding • Teacher explanations preferred, no changes to address student needs *Classroom Assessment Practice* • Teacher feedback inattentive to student misconceptions, misleading, lacked mathematical substance • Student-student feedback: nonexistent *Student Pursuits*: Conversation not encouraged *Lesson Planning* • Student discussion, problem solving not considered

Results

The results for 17 teachers in the third year of data collection were similar to previous years. By grade level, 5 of the teachers (4 eighth-grade) were at Levels 4and 5, no one was at Level 6, and 7 of the 17 (3 seventh-grade) were at Levels 1 and 2 (see Figure 4a). By curriculum taught, 4 of the 13 teachers using MiC were at Levels 4 and 5, and 4 were at Levels 1 and 2 (see Figure 4b). Comparisons by curriculum taught are difficult to make in the third year of data collection due to a small sample size for teachers using conventional curricula (only 4 teachers), and in District 2, no teachers using a conventional curriculum were available in eighth grade. When analyzed by district, differences became apparent for MiC teachers. As in the other years, MiC teachers in District 2 were more likely to teach mathematics for understanding than MiC teachers in District 1 (see Figures 1-34c and 1-34d).

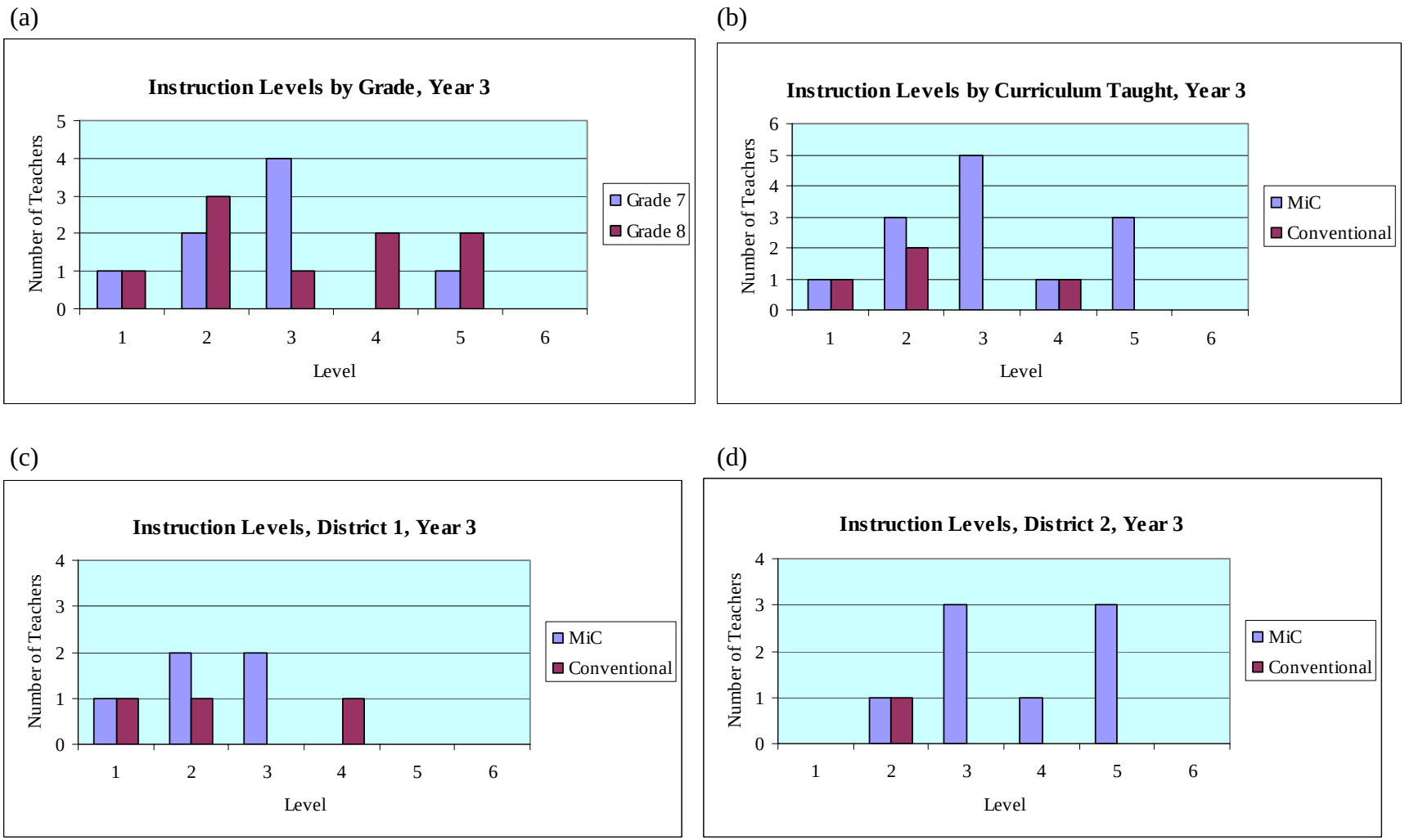

Figure 4. Teacher level on the *Instruction* composite index, Year 3: (a) by grade; (b) by curriculum taught; (c) by curriculum taught, District 1; (d) by curriculum taught, District 2

Analysis of Variance

Using data for study teachers in all three years, analysis of the variance was completed to check for significant differences among teachers by grade level, curriculum taught, and district. The Instruction Total was used as the dependent variable. Results suggest that 71% of the variance in the Instruction Total was accounted for by differences in grade level, curriculum taught, and district (see Table 8). The results show that there is an effect for curriculum, grade, and district.

Table 8.
ANOVA with Instruction Total as the Dependent Variable

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	6425.05	1070.84	33.76	<.0001
Error	80	2537.28	31.71		
Corrected Total	86	8962.33			

R-Square	Coeff Var	Root MSE	Instruction Total Mean
0.71	15.47	5.63	36.38

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Curriculum Taught	2	4317.18	2158.59	68.06	<.0001
Grade Level	3	424.17	141.39	4.46	0.006
District	1	306.15	306.15	9.65	0.002

When the contrasts were examined (see Tables 9 and 10), there were significant differences between the means of teachers using MiC and teachers using conventional curricula, between Grade 5 and the other grade levels, and between districts. The results suggest that students experienced significantly different instruction when studying MiC than when studying conventional curricula, in fifth-grade classrooms than in middle-school classrooms, and in District 2 than in District 1.

Table 9.
Least Squares Means for Curriculum Taught, Grade Level, and District

Least Squares Means	
Curriculum Taught	
MiC	40.61
Conventional	31.20
Grade Level	
Grade 5	41.86
Grade 6	35.07
Grade 7	36.46
Grade 8	35.44
District	
District 1	35.27
District 2	39.15

Table 10.
Contrasts in Means for the Instruction Total by Curriculum Taught, Grade, and District

Parameter	Standard Estimate	Error	t Value	Pr > |t|
		Curriculum Taught		
MiC vs Conventional	9.41	1.86	5.05	<.0001
		Grade Level		
Grade 5 vs Grade 6	6.79	1.98	3.43	0.001
Grade 5 vs Grade 7	5.40	1.90	2.83	0.005
Grade 5 vs Grade 8	6.42	2.05	3.13	0.002
Grade 6 vs Grade 7	-1.39	1.53	-0.91	0.364
Grade 6 vs Grade 8	-0.37	1.74	-0.22	0.830
Grade 7 vs Grade 8	1.02	1.65	0.62	0.538
		District		
District 1 vs District 2	-3.87	1.24	-3.11	0.002

Conclusion

In summary, the development of the composite index *Instruction* was described. Based on 19 subcategories of *Instruction*, the composite index served as a useful tool in developing profiles of the instruction for each teacher. An extensive, diverse set of data from classroom observations, interviews, teaching logs, and journal entries was used to identify and scale variation in the instruction study students experienced. The results suggest significant differences between the instruction experienced by students in MiC classrooms and students who studied conventional curricula, between fifth-grade students and students in other grade levels, and between students in District 2 and students in District 1.

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Appendix A

Teacher Interview: Instructional Planning and Classroom Interaction

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A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context*
on Student Mathematical Performance

Teacher Interview Protocol: Instructional Planning and Classroom Interaction

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Shafer, M. C., Davis, J., & Wagner, L. R. (1998) *Teacher interview: Instructional planning and classroom interaction*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 3). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

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Description of Teacher Interview on Instructional Planning and Classroom Interaction

The teacher interview on instructional planning and classroom interaction was the primary instrument used to gather information about the considerations teachers made when planning for teaching instructional units and individual lessons prior to instruction. The interview also gathered data on particular aspects of classroom interaction and differences between teaching MiC and teaching conventional mathematics curricula.

The interview was composed of five questions, one of which was reserved for teachers who used MiC. A list of probing questions accompanied each interview question. The probes were designed to enhance both the breadth and depth of teacher responses and to ensure consistency of the data gathered from all study teachers. Two interview questions included additional probes about the comparison of instructional planning and the nature of student conversations with MiC and conventional curricula. A brief introductory statement to the teacher provided a description of the interview, assurance that there were no right or wrong answers to interview questions, and notification that the interview would be audiotaped.

The first interview question asked teachers about their planning for each mathematics unit/chapter. The probes provided specific attention to (a) whether the teacher planned with others; (b) the considerations given in planning at the unit level, specifically, students' prior knowledge, textbook scope and sequence, district curriculum guidelines, state standards, district or state standardized tests, and other resources; and (c) setting the pace for instruction. An additional probe was reserved for teachers who used MiC. The focus of this probe was comparison of planning to teach MiC with planning to teach other mathematics textbooks used in the past.

The second interview question was dedicated to planning individual lessons. Probing questions provided attention to (a) considerations of students' performance in the previous lesson, and (b) whether the teacher solved unit/chapter problems or exercises before teaching, and, if so, the effect this preparation had on teaching the lesson.

The third interview question was designed to collect information on classroom interaction. In particular, the question addressed the issue of what counts as an acceptable answer. Related probes were (a) how students determine if their answers are acceptable; and (b) how the teacher determines if a student's answer is acceptable. The third probe elicited information about the ways in which students contribute to classroom discussions. A fourth probe was reserved for teachers who used MiC. The focus of this probe was differences in student participation when using MiC in comparison to student participation when using conventional mathematics curricula in the past. Additional probes for this question provided attention to differences in the types of conversation generated with each type of curriculum such as student conjectures, answers, and explanations.

The fourth interview question focused on the value of students working in small groups. Probing questions centered on (a) times when working in small groups was useful; (b) planning for small-group instruction; and (c) the ways grouping varied for different instructional purposes.

An additional interview question was reserved for teachers using MiC. This question focused on the ways that teaching MiC was different from teaching conventional mathematics curricula in the past for the students and the teacher. Probes provided attention to advantages and disadvantages of teaching MiC and difficulties in implementing MiC.

The interview protocol for instructional planning and classroom interaction was used during the spring semester of each study year. The interviews with teachers in Districts 1 and 2 were conducted by the on-

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site observer in each district. Audiotaped interviews were promptly sent to the research center. Interviews with teachers in Districts 3 and 4 were conducted via telephone by the project director or a project assistant at the research center. The interviews were conducted at times that were convenient for the teacher and that did not interfere with classroom instruction such as during the teachers' planning time or before or after school. The interviews varied from 20-30 minutes, depending on teacher responses and the need to use probing questions. Interviewers were instructed to follow particular procedures, including asking probing questions and interjecting nonjudgmental comments when appropriate (see Instructions for Interviews in this appendix). Teachers received an honorarium of $25 per interview. Interviews were transcribed for analysis. All teachers in Districts 1 and 2 completed the interview on instructional planning and classroom interaction. One teacher in each of Districts 3 and 4 did not complete the interview. The combined completion rate for teachers in all four districts in 1997-1998 was 98%.

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Teacher Interview Protocol: Instructional Planning and Classroom Interaction

Instructions for Interviewer

1. Be sure that you and the principal/teacher are alone and in a quiet room where responses can be recorded. Be sure that the tape is labeled with the principal/teacher's full name and the date of the interview.

2. Remember to follow the written protocol faithfully. You should always probe once if you think that the principal/teacher has not answered the question asked. In most cases, probes are given. In other cases, you may use one of the following.
   Anything else?
   Can you tell me more about ( )?
   Rephrase the question.

3. In responding to some questions, the principal/teacher may describe one aspect of the question in depth while not addressing the breadth of the question. Probes are provided to assist you in eliciting a broad response to each question. As you listen to the person's responses, glance at the list of probes and use the probe(s) that will give a more complete answer to the question. You might say, for example, "What about (probe)?" or "How important is (probe)?"

4. 3. If the principal/teacher has already answered a question you are about to ask, you should say: "The next question is ( ). I think you have already answered it. Do you think you have answered it? Is there anything else you want to add?

5. Your responses to the principal/teacher's statements should be non-committal and non-judgmental. Use responses such as "Thanks," "That's fine," "Alright," and "Okay."

6. If you forget to ask a question, make sure that you go back and ask it even if it is out of order.

7. Thank the principal/teacher for his/her time.

Instructions adapted from:
Fennema, E., Carpenter, T., & Loef, M. (1990). Belief Interview: CGI-2. Madison, WI: Wisconsin Center for Education Research, University of Wisconsin–Madison.

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Teacher Interview Protocol: Instructional Planning and Classroom Interaction

Turn on the tape recorder and record the following:

This is (your first and last names). I am interviewing (teacher's first and last names) who teaches ( ) grade at (school name). Today is (month, date, year).

Say to teacher:

I will be asking you questions about how you decide what mathematics students should know and how you plan for your teaching. I am also interested in how you monitor student learning and how students contribute to classroom discussions. Please answer the following questions as truthfully as possible. There are no right or wrong answers to these questions. I am only interested in your opinions and ideas. Your responses will be audiotaped.

1. In general, how do you plan for each mathematics unit/chapter that you teach?
   - A. With whom do you plan?
   - B. Which of these do you take into consideration in your planning? Explain how.
     1. Students' prior knowledge
     2. Textbook scope and sequence
     3. District curriculum scope and sequence
     4. State standards
     5. District tests or other large-scale testing
     6. Other resources
   - C. How do you set the pace for instruction?
   - **D. For MiC teachers:**
     How does your *planning* for teaching MiC compare with your planning for other mathematics textbooks you have used?

2. How do you plan for *individual* lessons?
   - A. What considerations do you give in your planning to how students performed in previous lessons?
   - B. Do you work through the problems in the unit/chapter before teaching? How does this affect how you teach the lessons?

3. What counts as an acceptable answer?
   - A. How do students determine if their answers are acceptable?
   - B. How do you determine if a student's answer is acceptable?
   - C. In what ways do students contribute to whole class discussions?
   - **D. For MiC teachers:**
     In comparison to other mathematics curricula you have used, how does student participation in discussions differ when using MiC?
     1. **Type of answers and explanations**
     2. **Type of conversation (e.g., conjectures, support for their reasoning)**

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4. Do you think it is valuable for students to work in small groups? Why?
   A. When is working in small groups useful?
   B. How do you plan for small group instruction?
   C. What type of grouping have you found to be the best for you and your students in terms of instruction (e.g., individual, occasional small groups, small groups that change over time, large group, etc.)?

5. For MiC teachers:
How is *teaching* MiC different than other mathematics curricula you have used?
   A. For the students?
   B. For you as the teacher?
   C. What advantages do you see in teaching the MiC curriculum?
   D. Do you see any disadvantages in using this curriculum?
   E. Have you encountered any difficulties implementing this curriculum?

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Appendix B

Teaching Log

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A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teaching Log

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Shafer, M. C., Davis, J., & Wagner, L. R. (1997) *Teaching log*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 5). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

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Description of the Teaching Log

The teaching log compiled by study teachers daily was one of the instruments designed to record information about daily instruction in study classes for the longitudinal/cross-sectional study. Information from the teaching logs was used in the analysis of the content of the actual curriculum, the use and modification of curricular materials, lesson planning, mathematical interaction during instruction, and classroom assessment. The teaching log was pilot-tested with nonstudy teachers during the spring semester prior to the study. Based on feedback from pilot teachers and input from district personnel involved in the study, the log was refined to clarify items and make the format easier for teachers to complete in five to ten minutes daily.

The teaching log consisted of Introductory Information, Daily Logs, and Journal Questions. The purpose of the Introductory Information was to document the unit/chapter taught, changes in class rosters, information about grouping for instruction, and the physical arrangement of the classroom. After indicating their name, the school, city, and date, teachers identified the text and the unit/chapter currently taught. Because the study was longitudinal, teachers noted names of students who were added (Item 1) or dropped from the class (Item 2). Over time, tables were used for teachers to record specific information. For students who were added to the class, teachers noted the approximate date students joined the class and the class periods to which the students were assigned. For students who dropped the class, teachers noted the approximate date students dropped and reasons they left the class. For Item 3, teachers indicated whether they intended for students to work in small groups or pairs during the teaching of the particular unit/chapter. They also described the reason for grouping students in that way and criteria for grouping the students. Finally, teachers sketched the physical arrangement of the classroom. The Introductory Information was completed once a month.

The daily log was printed on both sides of a single sheet of paper. The first side of the log was designed to document content taught, forms of instruction, and student activities. After noting the date, unit/chapter and pages taught on a particular day, teachers indicated if the lesson was a continuation of the previous lesson. If the lesson was continued, teachers were asked to indicate activities that were new to the current lesson. For Item 2, teachers checked whether all students in the class covered the same content. If they did not, they described the ways the content differed and the reasons for these differences. Item 3 was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). In Item 4, teachers noted whether the instructional activities precipitated changes in the physical arrangement of the classroom, and they described the reason for such changes. The last item on the first page was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. Items 2, 3, and 5 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

The second page of the log was designed to document supplemental materials, classroom assessment, homework, and formal assessment. For Item 6, teachers checked the additional materials used during the lesson: teacher-designed materials, work from text resource materials, work from other resources, quiz, calculators, or another resource specified by the teacher. Teachers were asked to date and attach teacher-designed materials, worksheets from other resources, and quizzes to the daily log page. Item 7 was designed to gather information about teachers' use of classroom assessment. If they assessed students

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informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered. After the first semester of the study, parts (a) and (b) were changed to a checklist format for easier use by the teachers. In Item 8, teachers checked the type of homework assignment, if given: exercises form the text, completion of work begun in class, teacher-designed work, work from text resource materials, exercises from another text, supplementary practice, investigation or project, or other assignment specified by the teacher. After the first semester of the study, Item 8 was revised to be less time-intensive for teachers. In the original log, teachers were asked to list the pages and exercise numbers for text assignments and to attach exercises from supplemental resources and investigations or projects. For the revised item, a more inclusive checklist was used (adding teacher-designed materials, work from text resources materials, and supplemental practice). Teachers were asked to briefly describe the content of teacher-designed and supplemental practice in lieu of attaching copies of such materials, and listing exercise numbers was eliminated. In the final item on the second page, for lessons during which a formal assessment was given, teachers checked the type of formal assessment used: end-of-unit or chapter test, district or state test, student presentation, or student projects. Teachers were asked to date and attach copies of end-of-unit or chapter tests that were representative of below average, average, and above average performance and copies instructions given for student presentations or projects. Items 6 and 8 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

Journal questions were printed on one side of a paper, and one sheet was inserted after each daily log sheet. The journal questions were designed to document lesson content that was emphasized or modified and notable classroom events. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them. In preparation for analysis, journal entries for each teacher were typed and collated by research staff.

Teachers were instructed to complete to complete a daily log sheet for each day of instruction as soon as possible after the lesson and complete at least one set of journal entries per week for the entire school year. If teachers taught multiple mathematics classes, they were asked to complete the log for the class that was observed monthly by the on-site observer. In this way, the information gathered through the log would add the teacher's perspective on the particular lessons for which observation reports were completed, thereby adding a means of triangulating data from observations and teaching logs. Each month the teaching log was a different color for ease in documenting the receipt of teacher logs. A binder was given to each teacher at the beginning of the school year. This binder contained the one-page Introductory Information, daily log sheets and journal questions for each instructional day for one month, a pocket folder for holding supplementary resources, quizzes, and formal assessments used by the teacher during instruction, and a postage-paid envelope for sending the log to the research team. Instructions for completing the teaching log and models of completed logs were reviewed with the teachers each August during the Summer Institute sponsored by the project for study teachers. District contact persons reviewed

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the instructions with teachers who were unable to attend the Summer Institutes. Subsequent teaching logs with postage-paid envelopes were sent to each teacher monthly. Logs were sent to a contact teacher at each school for distribution. The contact teacher was given an honorarium of $50 per semester for distributing all study instruments to teachers on a timely basis. Teachers received an honorarium of $50 per teaching log upon receipt of the log at the research center. (As a result of negotiation with the teachers, the honorarium was increased to $125 per log during the second and third years of the study.)

Graduate project assistants were liaisons between the research staff and study teachers. Each project assistant read and commented on teacher logs received from one of two research sites (Districts 1 and 2). The numbers of teachers in Districts 1 and 2 who sent logs and journal entries to the research center monthly varied greatly (see Table B1), despite our extensive efforts to collect a full set of teaching logs from each teacher. During the first and second years of data collection, reminders were sent to teachers from the research staff, and graduate research assistants encouraged teachers to continue completing this important source of data through personalized letters of interest in the teachers' work.

Table B1
Number of Teaching Logs Received, by Grade and Year

Grade *(No. of Teachers*)*	Number of Teaching Logs Per Teacher	Percent of Teachers Submitting Teaching Logs			
		0-2 Logs	3-6 Logs	7-8 Logs	9 Logs
1997-1998					
5 *(13)*	0-9	23	15	23	38
6 *(12)*	0-9	33	42	8	17
7 *(10)*	0-9	10	20	30	40
1998-1999					
6 *(12)*	0-9	33	25	0	42
7 *(12)*	0-9	50	17	8	25
8 *(10)*	0-9	30	20	0	50
1999-2000					
7 *(9)*	0-9	44	22	0	33
8 *(9)*	0-9	22	11	11	56

*Includes teachers who taught portions of the school year

Reference:

Porter, A. C., Kirst, M. W., Osthoff, E. J., Smithson, J. L., & Schneider, S. A. (1993). *Reform up close: A classroom analysis*. Madison, WI: University of Wisconsin–Madison.

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DAILY LOG PROCEDURES

The daily logs you complete are crucial components of the longitudinal study. These logs are designed to record daily practices in your mathematics classroom. No single instrument can characterize the complexities of classroom life, but the logs are intended to facilitate the general description of your teaching practices and your students' activities in the classroom. Your thoroughness in completing the daily logs is a most vital and appreciated aspect of this study.

INSTRUCTIONS:

At the beginning of month, please complete the introductory information.

The daily log is intended to reflect the character of your mathematics classroom. As its name implies, this log should be completed on a daily basis, as soon after the math class meets as possible. Most questions on the daily log can be completed with a check mark or brief descriptions.

The most crucial components of the daily log, the journal questions, require as thorough description as you can give to accurately reflect the lesson flow and classroom events. The importance of your thoroughness in answering these two questions cannot be underestimated. We would like you to respond to these questions as often as you notice events reflective of the suggested topics in your classroom, but we expect them to be completed at least once a week as these events present themselves.

If you do not have enough room to complete these, or any other questions, please complete them on an additional sheet of paper and attach the paper behind its corresponding entry.

In addition to filling in the daily log, where requested, we would like you to place hand-outs and/or student work in the folder provided and send these items along each month when you send your daily log entries to us. As you make copies of materials for students, please and date an additional copy to include with its corresponding daily log entry.

We estimate that it should take no more than 10 minutes each day to complete the daily log. The journal questions will require additional time to answer each week.

At the end of month, please staple or clip the log together with the introductory information in front, and return the completed log, the requested materials, and student work in the provided envelope.

If at any time you have questions about these procedures or the completion of the logs, please contact Lesley Wagner at 1-800-862-1055 or via e-mail at lrwagner@students.wisc.edu.

Thank you for your invaluable time in completing the daily log.

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Please complete this information at the beginning of each month.

INTRODUCTORY INFORMATION

Name _____ Date ____/____/_____
School _____ Text _____
City _____ Unit/Chapter _____

1. Please list the names of students added to the class, the date they were added, and the period to which they were added:

Name of Student	Date Added to Class	Class Period

2. Please list the names of students dropped from the class, the date they were dropped, and the reason they were dropped (e.g., transferred to new class, transferred to new school):

Name of Student	Date Dropped from Class	Reason Dropped

3. If students will work in groups or pairs during the teaching of this unit or chapter, please describe how these groups or pairs are chosen and the reason for grouping the students this way.

4. Please sketch the physical arrangement of the classroom.

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DAILY LOG

Date ____/___/_____ Pages _____ Unit/Chapter _____

1. __ Check here if today's lesson is a continuation of yesterday's. If so, only check activities which are new to today's lesson.

2. __ All students covered the same content.
   __ Content covered varied by student/group. Please describe how the instruction differed and the reasons for these differences.

3. Please check the instructional formats utilized during the lesson and circle the emphasis each received.

Instructional Formats	Emphasis Received			
__ Warm-up	1	2	3	4
__ Review of previous material	1	2	3	4
__ Teacher presentation of material	1	2	3	4
__ Whole class discussion during lesson	1	2	3	4
__ Small group or pair work	1	2	3	4
__ Independent practice	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary instructional format, used for more than 75% of the lesson.
3-Significant instructional format, used for 50-75% of the class period.
2-One of several instructional formats used with approximately equal emphasis.
1-An instructional activity used for a short period of time, approximately 15% or less of the class period.

4. If the activities in the lesson necessitated any physical change in the classroom or significant student movement, please describe.

5. Please check the student activities completed during the lesson and circle the emphasis each received.

Student Activities	Emphasis Received			
__ Listened to teacher and/or took notes	1	2	3	4
__ Investigated problem	1	2	3	4
__ Discussed answers and solution strategies	1	2	3	4
__ Participated in whole class discussion	1	2	3	4
__ Practiced computation	1	2	3	4
__ Took a test or quiz	1	2	3	4
__ Reflected on/summarized lesson concepts	1	2	3	4
__ Began homework	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary student activity, engaged in for more than 75% of the class period.
3-Significant student activity, engaged in for 50-75% of the class period.
2-One of several activity types, engaged in with approximately equal emphasis.
1-An activity engaged in for a short period of time, approximately 15% or less of the class period.

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6. Please check the additional materials used during the lesson:

__ Teacher designed materials (Please date and attach)
__ Work from publisher resource materials
__ Worksheets or activities from sources other than the text or unit (Please date and attach)
__ Quiz (Please date and attach)
__ Calculators
__ Other (please specify) _____

7. If you informally assessed students during the class period, please answer the following questions:

a) Please check what you were assessing
__ Students' understanding of _____
__ Students' efforts in working as a group
__ Students' attitudes toward math (e.g., confidence, perseverance)
__ Other, please describe_____

b) Please check way(s) in which you informally assessed students
__ Observation
__ Listening during group work
__ Questioning
__ Checklists
__ Checking their work

c) Did the information you gained affect your instruction? __Yes __No
If yes, please describe.

8. Please check all student homework assignments that apply.
__ Exercises from text/unit
__ Completion of work begun in class
__ Teacher designed, please indicate content_____
__ Work from publisher resource materials
__ Exercises from source other than text
__ Supplementary practice, please indicate content _____
__ Investigation/Project
__ related to the unit, please describe_____
__ supplementary to the unit, please describe_____
__ Other, _____

9. If a formal assessment was part of the lesson, please indicate the type of assessment. Please attach copies of student assessments that are representative of below average, average, and above average performance as well as copies of student papers that show any interesting or unusual work.
__ Test
__ District or state developed test, please specify_____
__ Student presentations (Please date and attach the instructions or options given to students)
__ Student projects (Please date and attach the instructions or options given to students)

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Please remember to reflect on the following aspects of classroom instruction at least once a week.

Date___/___/___

Journal Questions

1. Please describe the parts of the lesson you emphasized and any modifications you made in the lesson as compared to its presentation in the unit or chapter of the text. Please check and reflect on one or more of the following occurrences:

   ___ particular problems or aspects of the lesson that were emphasized and explain why they were emphasized
   ___ particular problems or aspects of the lesson that were deleted and explain why they were deleted
   ___ additional activities, problems, or procedures that were included in the lesson and explain why they were added
   ___ the order of presentation of lesson activities and/or content as compared to its presentation in the unit or chapter; if you changed the order of presentation, please describe how it was changed and explain why
   ___ other changes, please describe
   _____

2. Please describe any notable classroom event(s) related to the lesson.
   Please check and reflect on one or more of the following events:

   ___ the lesson or part of the lesson went exceptionally well
   ___ something surprising occurred
   ___ an idea was particularly difficult for the students
   ___ students seemed to comprehend an idea that had previously been troublesome
   ___ student misconceptions emerged
   ___ a student offered an unusual or unexpectedly sophisticated strategy
   ___ a student's question caused a modification in the lesson
   ___ other(s), please describe_____

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Appendix B

Teaching Log

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A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teaching Log

Mary C. Shafer, Lesley R. Wagner, and Jon Davis

Wisconsin Center for Education Research
University of Wisconsin–Madison

Shafer, M. C., Davis, J., & Wagner, L. R. (1997) *Teaching log*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 5). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

The development of this instrument was supported in part by the National Science Foundation #REC-9553889.

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Description of the Teaching Log

The teaching log compiled by study teachers daily was one of the instruments designed to record information about daily instruction in study classes for the longitudinal/cross-sectional study. Information from the teaching logs was used in the analysis of the content of the actual curriculum, the use and modification of curricular materials, lesson planning, mathematical interaction during instruction, and classroom assessment. The teaching log was pilot-tested with nonstudy teachers during the spring semester prior to the study. Based on feedback from pilot teachers and input from district personnel involved in the study, the log was refined to clarify items and make the format easier for teachers to complete in five to ten minutes daily.

The teaching log consisted of Introductory Information, Daily Logs, and Journal Questions. The purpose of the Introductory Information was to document the unit/chapter taught, changes in class rosters, information about grouping for instruction, and the physical arrangement of the classroom. After indicating their name, the school, city, and date, teachers identified the text and the unit/chapter currently taught. Because the study was longitudinal, teachers noted names of students who were added (Item 1) or dropped from the class (Item 2). Over time, tables were used for teachers to record specific information. For students who were added to the class, teachers noted the approximate date students joined the class and the class periods to which the students were assigned. For students who dropped the class, teachers noted the approximate date students dropped and reasons they left the class. For Item 3, teachers indicated whether they intended for students to work in small groups or pairs during the teaching of the particular unit/chapter. They also described the reason for grouping students in that way and criteria for grouping the students. Finally, teachers sketched the physical arrangement of the classroom. The Introductory Information was completed once a month.

The daily log was printed on both sides of a single sheet of paper. The first side of the log was designed to document content taught, forms of instruction, and student activities. After noting the date, unit/chapter and pages taught on a particular day, teachers indicated if the lesson was a continuation of the previous lesson. If the lesson was continued, teachers were asked to indicate activities that were new to the current lesson. For Item 2, teachers checked whether all students in the class covered the same content. If they did not, they described the ways the content differed and the reasons for these differences. Item 3 was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). In Item 4, teachers noted whether the instructional activities precipitated changes in the physical arrangement of the classroom, and they described the reason for such changes. The last item on the first page was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. Items 2, 3, and 5 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

The second page of the log was designed to document supplemental materials, classroom assessment, homework, and formal assessment. For Item 6, teachers checked the additional materials used during the lesson: teacher-designed materials, work from text resource materials, work from other resources, quiz, calculators, or another resource specified by the teacher. Teachers were asked to date and attach teacher-designed materials, worksheets from other resources, and quizzes to the daily log page. Item 7 was designed to gather information about teachers' use of classroom assessment. If they assessed students

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informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered. After the first semester of the study, parts (a) and (b) were changed to a checklist format for easier use by the teachers. In Item 8, teachers checked the type of homework assignment, if given: exercises form the text, completion of work begun in class, teacher-designed work, work from text resource materials, exercises from another text, supplementary practice, investigation or project, or other assignment specified by the teacher. After the first semester of the study, Item 8 was revised to be less time-intensive for teachers. In the original log, teachers were asked to list the pages and exercise numbers for text assignments and to attach exercises from supplemental resources and investigations or projects. For the revised item, a more inclusive checklist was used (adding teacher-designed materials, work from text resources materials, and supplemental practice). Teachers were asked to briefly describe the content of teacher-designed and supplemental practice in lieu of attaching copies of such materials, and listing exercise numbers was eliminated. In the final item on the second page, for lessons during which a formal assessment was given, teachers checked the type of formal assessment used: end-of-unit or chapter test, district or state test, student presentation, or student projects. Teachers were asked to date and attach copies of end-of-unit or chapter tests that were representative of below average, average, and above average performance and copies instructions given for student presentations or projects. Items 6 and 8 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

Journal questions were printed on one side of a paper, and one sheet was inserted after each daily log sheet. The journal questions were designed to document lesson content that was emphasized or modified and notable classroom events. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them. In preparation for analysis, journal entries for each teacher were typed and collated by research staff.

Teachers were instructed to complete to complete a daily log sheet for each day of instruction as soon as possible after the lesson and complete at least one set of journal entries per week for the entire school year. If teachers taught multiple mathematics classes, they were asked to complete the log for the class that was observed monthly by the on-site observer. In this way, the information gathered through the log would add the teacher's perspective on the particular lessons for which observation reports were completed, thereby adding a means of triangulating data from observations and teaching logs. Each month the teaching log was a different color for ease in documenting the receipt of teacher logs. A binder was given to each teacher at the beginning of the school year. This binder contained the one-page Introductory Information, daily log sheets and journal questions for each instructional day for one month, a pocket folder for holding supplementary resources, quizzes, and formal assessments used by the teacher during instruction, and a postage-paid envelope for sending the log to the research team. Instructions for completing the teaching log and models of completed logs were reviewed with the teachers each August during the Summer Institute sponsored by the project for study teachers. District contact persons reviewed

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the instructions with teachers who were unable to attend the Summer Institutes. Subsequent teaching logs with postage-paid envelopes were sent to each teacher monthly. Logs were sent to a contact teacher at each school for distribution. The contact teacher was given an honorarium of $50 per semester for distributing all study instruments to teachers on a timely basis. Teachers received an honorarium of $50 per teaching log upon receipt of the log at the research center. (As a result of negotiation with the teachers, the honorarium was increased to $125 per log during the second and third years of the study.)

Graduate project assistants were liaisons between the research staff and study teachers. Each project assistant read and commented on teacher logs received from one of two research sites (Districts 1 and 2). The numbers of teachers in Districts 1 and 2 who sent logs and journal entries to the research center monthly varied greatly (see Table B1), despite our extensive efforts to collect a full set of teaching logs from each teacher. During the first and second years of data collection, reminders were sent to teachers from the research staff, and graduate research assistants encouraged teachers to continue completing this important source of data through personalized letters of interest in the teachers' work.

Table B1
Number of Teaching Logs Received, by Grade and Year

Grade *(No. of Teachers*)*	Number of Teaching Logs Per Teacher	Percent of Teachers Submitting Teaching Logs			
		0-2 Logs	3-6 Logs	7-8 Logs	9 Logs
1997-1998					
5 *(13)*	0-9	23	15	23	38
6 *(12)*	0-9	33	42	8	17
7 *(10)*	0-9	10	20	30	40
1998-1999					
6 *(12)*	0-9	33	25	0	42
7 *(12)*	0-9	50	17	8	25
8 *(10)*	0-9	30	20	0	50
1999-2000					
7 *(9)*	0-9	44	22	0	33
8 *(9)*	0-9	22	11	11	56

*Includes teachers who taught portions of the school year

Reference:

Porter, A. C., Kirst, M. W., Osthoff, E. J., Smithson, J. L., & Schneider, S. A. (1993). *Reform up close: A classroom analysis*. Madison, WI: University of Wisconsin–Madison.

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DAILY LOG PROCEDURES

The daily logs you complete are crucial components of the longitudinal study. These logs are designed to record daily practices in your mathematics classroom. No single instrument can characterize the complexities of classroom life, but the logs are intended to facilitate the general description of your teaching practices and your students' activities in the classroom. Your thoroughness in completing the daily logs is a most vital and appreciated aspect of this study.

INSTRUCTIONS:

At the beginning of month, please complete the introductory information.

The daily log is intended to reflect the character of your mathematics classroom. As its name implies, this log should be completed on a daily basis, as soon after the math class meets as possible. Most questions on the daily log can be completed with a check mark or brief descriptions.

The most crucial components of the daily log, the journal questions, require as thorough description as you can give to accurately reflect the lesson flow and classroom events. The importance of your thoroughness in answering these two questions cannot be underestimated. We would like you to respond to these questions as often as you notice events reflective of the suggested topics in your classroom, but we expect them to be completed at least once a week as these events present themselves.

If you do not have enough room to complete these, or any other questions, please complete them on an additional sheet of paper and attach the paper behind its corresponding entry.

In addition to filling in the daily log, where requested, we would like you to place hand-outs and/or student work in the folder provided and send these items along each month when you send your daily log entries to us. As you make copies of materials for students, please and date an additional copy to include with its corresponding daily log entry.

We estimate that it should take no more than 10 minutes each day to complete the daily log. The journal questions will require additional time to answer each week.

At the end of month, please staple or clip the log together with the introductory information in front, and return the completed log, the requested materials, and student work in the provided envelope.

If at any time you have questions about these procedures or the completion of the logs, please contact Lesley Wagner at 1-800-862-1055 or via e-mail at lrwagner@students.wisc.edu.

Thank you for your invaluable time in completing the daily log.

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Please complete this information at the beginning of each month.

INTRODUCTORY INFORMATION

Name _____ Date ____/____/_____
School _____ Text _____
City _____ Unit/Chapter _____

2. Please list the names of students added to the class, the date they were added, and the period to which they were added:

Name of Student	Date Added to Class	Class Period

3. Please list the names of students dropped from the class, the date they were dropped, and the reason they were dropped (e.g., transferred to new class, transferred to new school):

Name of Student	Date Dropped from Class	Reason Dropped

3. If students will work in groups or pairs during the teaching of this unit or chapter, please describe how these groups or pairs are chosen and the reason for grouping the students this way.

4. Please sketch the physical arrangement of the classroom.

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DAILY LOG

Date ____/___/_____ Pages _____ Unit/Chapter _____

1. __ Check here if today's lesson is a continuation of yesterday's. If so, only check activities which are new to today's lesson.

2. __ All students covered the same content.
   __ Content covered varied by student/group. Please describe how the instruction differed and the reasons for these differences.

3. Please check the instructional formats utilized during the lesson and circle the emphasis each received.

Instructional Formats	Emphasis Received			
__ Warm-up	1	2	3	4
__ Review of previous material	1	2	3	4
__ Teacher presentation of material	1	2	3	4
__ Whole class discussion during lesson	1	2	3	4
__ Small group or pair work	1	2	3	4
__ Independent practice	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary instructional format, used for more than 75% of the lesson.
3-Significant instructional format, used for 50-75% of the class period.
2-One of several instructional formats used with approximately equal emphasis.
1-An instructional activity used for a short period of time, approximately 15% or less of the class period.

4. If the activities in the lesson necessitated any physical change in the classroom or significant student movement, please describe.

5. Please check the student activities completed during the lesson and circle the emphasis each received.

Student Activities	Emphasis Received			
__ Listened to teacher and/or took notes	1	2	3	4
__ Investigated problem	1	2	3	4
__ Discussed answers and solution strategies	1	2	3	4
__ Participated in whole class discussion	1	2	3	4
__ Practiced computation	1	2	3	4
__ Took a test or quiz	1	2	3	4
__ Reflected on/summarized lesson concepts	1	2	3	4
__ Began homework	1	2	3	4
__ Other (please specify) _____	1	2	3	4

Emphasis Scale
4-Primary student activity, engaged in for more than 75% of the class period.
3-Significant student activity, engaged in for 50-75% of the class period.
2-One of several activity types, engaged in with approximately equal emphasis.
1-An activity engaged in for a short period of time, approximately 15% or less of the class period.

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6. Please check the additional materials used during the lesson:

__ Teacher designed materials (Please date and attach)
__ Work from publisher resource materials
__ Worksheets or activities from sources other than the text or unit (Please date and attach)
__ Quiz (Please date and attach)
__ Calculators
__ Other (please specify) _____

9. If you informally assessed students during the class period, please answer the following questions:

a) Please check what you were assessing
__ Students' understanding of _____
__ Students' efforts in working as a group
__ Students' attitudes toward math (e.g., confidence, perseverance)
__ Other, please describe_____

b) Please check way(s) in which you informally assessed students
__ Observation
__ Listening during group work
__ Questioning
__ Checklists
__ Checking their work

c) Did the information you gained affect your instruction? __Yes __No
If yes, please describe.

10. Please check all student homework assignments that apply.
__ Exercises from text/unit
__ Completion of work begun in class
__ Teacher designed, please indicate content_____
__ Work from publisher resource materials
__ Exercises from source other than text
__ Supplementary practice, please indicate content _____
__ Investigation/Project
__ related to the unit, please describe_____
__ supplementary to the unit, please describe_____
__ Other, _____

10. If a formal assessment was part of the lesson, please indicate the type of assessment. Please attach copies of student assessments that are representative of below average, average, and above average performance as well as copies of student papers that show any interesting or unusual work.
__ Test
__ District or state developed test, please specify_____
__ Student presentations (Please date and attach the instructions or options given to students)
__ Student projects (Please date and attach the instructions or options given to students)

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Please remember to reflect on the following aspects of classroom instruction at least once a week.

Date___/___/___

Journal Questions

1. Please describe the parts of the lesson you emphasized and any modifications you made in the lesson as compared to its presentation in the unit or chapter of the text. Please check and reflect on one or more of the following occurrences:

   ___ particular problems or aspects of the lesson that were emphasized and explain why they were emphasized
   ___ particular problems or aspects of the lesson that were deleted and explain why they were deleted
   ___ additional activities, problems, or procedures that were included in the lesson and explain why they were added
   ___ the order of presentation of lesson activities and/or content as compared to its presentation in the unit or chapter; if you changed the order of presentation, please describe how it was changed and explain why
   ___ other changes, please describe
   _____

2. Please describe any notable classroom event(s) related to the lesson.
   Please check and reflect on one or more of the following events:

   ___ the lesson or part of the lesson went exceptionally well
   ___ something surprising occurred
   ___ an idea was particularly difficult for the students
   ___ students seemed to comprehend an idea that had previously been troublesome
   ___ student misconceptions emerged
   ___ a student offered an unusual or unexpectedly sophisticated strategy
   ___ a student's question caused a modification in the lesson
   ___ other(s), please describe_____

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Appendix C

Classroom Observation Instrument

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Observation Scale

Mary C. Shafer, Lesley R. Wagner, and Jon Davis

Wisconsin Center for Education Research
University of Wisconsin–Madison

Davis, J., Wagner, L. R., & Shafer, M. C. (1997). *Classroom observation scale*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 6). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

The research reported in this paper was supported in part by the National Science Foundation #REC-9553889 and by the Wisconsin Center for Education Research, School of Education, University of Wisconsin–Madison.

Description of the Observation Scale

The observation instrument for the longitudinal/cross-sectional study was designed to measure one independent variable (curricular content and materials—the actual curriculum) and the three intervening variables: pedagogical decisions, classroom events, and student pursuits. The observation instrument is composed of seven sections. In the first section, the observer recorded pertinent information related to the teacher and students: the name of the teacher, the school, and the grade level of the students in the class. The observer also recorded information pertinent to the particular lesson: the date of the observation, times the lesson began and ended, text used, unit/chapter taught, and the page numbers taught during the lesson. In the second section of the observation instrument, the observer conducted and recorded notes from a brief preobservation interview of the teacher during which the teacher was asked to identify the mathematical content to be explored or conveyed in the lesson and the location of the lesson with respect to the development of concepts in the instructional unit/chapter. In the third section, the observer recorded the flow of the lesson, which was a list of lesson activities along with the time allotted to each.

The next two sections of the observation instrument were collectively composed of 12 indices for various dimensions of instruction, which addressed the three intervening variables in the research model for the study. Nine of these indices focused on classroom events; the remaining three indices focused on student pursuits. Pedagogical decisions, although not presented in a separate section of the observation instrument, were central to both classroom events and student pursuits.

The indices used to characterize each dimension were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995), Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993), and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through a review of literature that was specific to each dimension. The indices were refined as a result of classroom observations of nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Three to four levels were identified for each dimension in order to identify differences in these variables between conventional and reform-based approaches to teaching and learning mathematics. The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the dimension being scaled. For example, levels of lessons that fostered conceptual understanding range from no attention to conceptual understanding during instruction to lessons in which the continual focus was on building connections among mathematical ideas.

The observation instrument was pilot-tested by project staff numerous times in both MiC and conventional classrooms in order to define and clarify descriptors for each item and to determine ways to achieve interrater reliability. Before the observation instrument was used in study classrooms, administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each dimension. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

In the sixth section of the observation instruction, the observer conducted and recorded notes from a brief postobservation interview of the teacher during which the teacher was asked to rate and comment on the degree to which the teacher felt the lesson achieved the purpose noted in the preobservation interview. The teacher was also asked whether any incidents occurred during the lesson which revealed student misunderstanding or provided opportunities to facilitated student understanding in any way. In this way, teachers had an opportunity to describe and explain modifications made during the lesson. In the final section of the observation instrument, the observer recorded any additional comments about the lesson.

Indices

Classroom Events

The lesson provided opportunities for students to make conjectures about mathematical ideas. In the conceptualization of conjectures in the longitudinal study, three types of student conjectures are described and sought in classroom interaction. First, students can make conjectures that are realizations of the connections between existing knowledge and the application of these concepts in new contexts. That is, students might see a connection between a new problems and problems they have already solved. Second, students may investigate the truthfulness of particular statements. Third, conjecturing may permeate a lesson. Given a pattern, for example, students are asked to devise a formula that captures the essence of the pattern in a concise form, which in turn leads to generalizations. Each type of conjecture is given a specific rating in the index, with an observation of the third type given the highest rating.

The lesson fostered the development of conceptual understanding. Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. Lower ratings in this category describe classrooms in which teaching for conceptual understanding occurs, but is often overshadowed by an emphasis on procedural knowledge. The highest rating describes a lesson in which links between conceptual and procedural understanding are the main emphasis of the instruction.

Connections within mathematics were explored. In this index, mathematical topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios which connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating is meant to reflect both the problems and instruction.

Connections between mathematics and students' daily lives were apparent in the lesson. This index measures whether connections between mathematics and students' daily lives were apparent in text problems or problems presented in class or were discussed by the teacher or students.

Students explained their responses or solution strategies. This index is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

Multiple strategies were encouraged and valued. This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. Higher ratings on this index refer to lessons in which discussion of alternative strategies is a frequent, important element of classroom instruction.

The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class. This index is intended to measure the ways in which the

teacher uses student responses during instruction. The highest rating is reserved for lessons in which the teacher not only probed individual students' thinking but also encouraged other students to comment on the solution strategies or used students' thinking processes to open discussions that encourage deeper understanding of mathematics.

The teacher used student inquiries as a guide for instructional mathematics investigation or as a guide to shape the mathematical content of the lesson. Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. This index measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

The teacher encouraged students to reflect on the reasonableness of their responses. This index is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

Student Pursuits

Student exchanges with peers reflected substantive conversation of mathematical ideas. Substantive conversation by students is characterized by interaction that is reciprocal, which involves listening carefully to others' ideas in order to understand them, building conversation on others' ideas, or extending an idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions. (Newmann, Secada, and Wehlage, 1995). While other items in this observation scale refer to the role of the teacher in mediating discourse, this item measures student discourse between peers in either large-group or small-group settings.

Interactions among students reflected collaborative working relationships. A low rating is given when students are physically sitting in groups but rarely working together. In contrast, the highest rating denotes a lesson in which students are actively involved in solving problems with their classmates and in which students made sure that all students in the group understood one problem before moving on to the next. N/A is reserved for lessons in which the goal is for students to work on problems independently.

The overall level of student engagement throughout the lesson was serious. This index measures the extent to which students remained on task during the lesson. Engagement is exemplified by behaviors in which students are attentive, complete assigned work, participate by raising questions, contribute to both large-group and small-group discussions, and help their peers (Secada and Byrd, 1993).

Observations

The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. Throughout the class period, the observer continually judged the levels of each dimension of classroom events and student pursuits. During each observation the observer took field notes that pertained to the 12 indices. Immediately after observing a lesson, the observer rated each item and recorded evidence from the lesson (consisting of dialogue or an artifact) to support the given rating. In general, a rating of 1 on a particular item indicated that the dimension was rarely or never seen in the lesson; the highest rating indicated that the dimension received major emphasis in the classroom. In practice, high ratings were rarely attained on every item during one observation. Ratings also varied in different observations of the same teacher.

The number of observations per teacher varied in each district (see Table C1). Most teachers in District 1 were observed once a month for a total of nine observations per teacher. During the first year of data collection, one teacher in District 1 accepted an administrative position in December; consequently, she was observed three times, and the newly assigned teacher was observed five times. During the second and third years of data collection, one eighth-grade control class had three teachers, and two seventh-grade experimental classes had two teachers over the course of the school year. As a result, each teacher was observed only a few times. Teachers in District 2 were observed a total of two to nine times each. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. In addition, four teachers from one school in District 2 withdrew from participation in the study during the spring semester of the first year of data collection; consequently, they were observed only three times. During the third year of data collection, two seventh-grade experimental classes were observed twice because the teacher had been on parental leave.

Table C1
Number of Observations Conducted, by Grade and Year

Grade *(No. of Teachers*)*	Number of Observations Per Teacher	Percent of Teachers Observed			
		1-3 Times	4-6 Times	7-8 Times	9 Times
1997-1998					
5 *(13)*	5-9	0	38	0	62
6 *(12)*	3-9	25	33	8	33
7 *(10)*	3-9	20	40	0	40
1998-1999					
6 *(12)*	5-9	0	33	25	42
7 *(12)*	5-9	0	42	25	33
8 *(10)*	2-9	20	40	30	10
1999-2000					
7 *(9)*	2-9	22	11	11	56
8 *(9)*	8-9	0	0	44	56

*Includes teachers who taught portions of the school year

Interrater Reliability

In the August prior to the study, each observer viewed two videotaped lessons with a graduate project assistant who developed the observation instrument and rated the lessons using the instrument. During these meetings discussions centered on consistency of ratings and descriptions of the types of conjectures observed, the nature of student–student conversation, and instances in which teachers used student inquiries to shape the lesson.

In the fall of 1997, each observer and a project assistant visited five classes in District 1 and nine classes in District 2. During the first few observations at each site, the project assistant's and observer's ratings of several items differed by one point. By the last observation, however, this disagreement had subsided considerably. The first dimension, student conjectures, initially caused difficulty for both observers. For example, observers initially categorized the repetitious practice of problems using a single prescribed algorithm as a first level conjecture, when the first level of conjecture is meant to describe the preponderance of students making conjectures that link concepts they have studied in the past with the same concept set within a new context. Another dimension, students' level of collaboration in the classroom, one observer tended to give the highest rating if students were physically sitting in groups. The project assistant emphasized the importance of circulating around the classroom to determine if

students actually worked in groups to support each other's learning. After this training, the observers began observing each study teacher once a month and completed a report for each observation. Completed reports were sent electronically to the research center for analysis. Each observer was compensated an amount per observation as part of a subcontract between the observer and the University of Wisconsin. The amount of payment varied according to the length of the class period observed. In September 1998, both observers worked on interrater reliability with a project assistant during on-site classroom observations in District 1. Because of the lack of funds, on-site work for interrater reliability between observers and a project assistant were not conducted in the fall of 1999.

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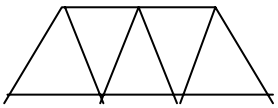
Observation Scale Descriptors

C.1. The lesson provided opportunities for students to make conjectures about mathematical ideas.

This scale measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas. There are three types of conjectures that students might make. One type of conjecture involves the student in making a guess about how to solve a particular problem based on experience solving problems with similar solution strategies. For example, students were solving problems in which they used properties of similar triangles. When asked to determine the height of a tree, students conjectured that an appropriate solution strategy would involve similar triangles. The students made a connection between the new problem and problems that they had previously solved. A second type of conjecture occurs when a student makes a guess about the truthfulness of a particular statement and subsequently plans and conducts an investigation to determine whether the statement is true or false. For example, a 12-year-old student disagreed with a statement that she was half as tall as she is now when she was 6-years old, and proceeded to support her argument by comparing her present height with heights of 6-year-old children. A third type of conjecture is a generalization. A generalization is created by reasoning from specific cases of a particular event, is tested in specific cases, and is logically reasoned to be acceptable for all cases of the event. For example, given that a beam is constructed of rods in the following configuration,



students are asked to describe the relation between the number of rods and the length of the beam[3] (Wijers, Roodhardt, van Reeuwijk, Burrill, Cole, & Pligge, 1998). Using a table to organize their reasoning, students described the pattern that emerged, explained how the pattern fit the given diagram, and generated formulas for the relationship. In this situation, students reasoned from specific cases, tested and supported their ideas with evidence from drawings and the table, and described the relation in a formula.

1. No conjectures of any type were observed in the lesson. Students were not encouraged to make connections between a new problem and problems previously seen, investigate the validity of their own guesses, look for patterns, or make generalizations.

2. Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

3. Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

4. Students made generalizations about mathematical ideas.

[3] The length of the beam is the number of rods on the bottom of the beam.

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C.2. The lesson fostered the development of conceptual understanding.

Conceptual knowledge is described as the "facts and properties of mathematics that are recognized as being related in some way" (Hiebert & Wearne, 1986, p. 200), or as a network of relationships that link pieces of knowledge (Hiebert & Lefevre, 1986). In the primary grades, for example, students learn the labels for whole-number place-value positions. If this information is stored as isolated pieces of information, the knowledge is not conceptual. If this knowledge, however, is linked with other information about numbers, such as grouping objects into sets of ten or counting by tens or hundreds, then the information becomes conceptual knowledge. The network of relationships about place value grows as other pieces of knowledge related to place value, such as regrouping in subtraction, are recognized. Procedural knowledge, in contrast, is described as having two parts. One category comprises the written mathematical symbols, which are devoid of meaning and are acted upon through knowledge of the syntax of the system. A second category is composed of rules and algorithms for solving mathematics problems, step-by-step procedures that progress from problem statement to solution in a predetermined order. Procedural knowledge is rich in rules and strategies for solving problems, but it is not rich in relationships (Hiebert & Wearne, 1986).

Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. In addition and subtraction of decimals, for example, lining up the decimal points should be linked with combining like quantities. Instruction might explicitly bring out the relationships between lining up the decimal point in addition and subtraction and lining up whole numbers on the right side for the same operations (Hiebert & Wearne, 1986).

1. The lesson as presented did not promote conceptual understanding.

2. The lesson asked few questions that fostered students' conceptual development of mathematical ideas, or conceptual understanding was a small part of lesson design.

3. Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

4. The continual focus of the lesson was on building connections between disparate pieces of information or linking procedural knowledge with conceptual knowledge.

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C.3. Connections within mathematics were explored in the lesson.

This scale measures the extent to which instruction addressed mathematical topics thoroughly enough to explore relationships and connections among them.[4] A low rating is given when the mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning, and instruction treated this topic in isolation of other mathematical topics. A high rating is given when the mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. Rather than examining fragmented pieces of information, students looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical topics, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

Topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios, as in the following problem. Students are asked to determine the probability of a frog jumping from a cage and landing on white or black floor tiles and to express this probability as a fraction or percent (Jonker, van Galen, Boswinkel, Wijers, Simon, Burrill, & Middleton, 1998). In solving this problem, students use area, number, and probability concepts. Second, connections can be made among more narrowly defined areas such as a lesson involving the solution of quadratic equations. In this lesson, connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating should reflect both the problems and instruction.

1. The mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning. The mathematical topic was presented in isolation of other topics, and the teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

2. Connections among mathematical topics were present in the lesson. The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail by the teacher or the students.

3. Connections among mathematical topics were discussed by teacher and students during the lesson, or connections were clearly explained by the teacher.

4. The mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. During instruction, many students did at least one of the following: looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical relationships, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

[4] Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Deep Knowledge (pp. 31-35).

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C.4. Connections between mathematics and students' daily lives were apparent in the lesson.

This scale measures whether connections between mathematics and students' daily lives were apparent in text problems or discussed by the teacher or students. Examples of problems that foster such connections are estimating the sale price of an item or determining the amount of ingredients required to serve four people when a recipe serves seven. In contrast, word problems such as "Bart is two years older than Lisa. In five years Bart will be twice as old as Lisa. How old are they now?" are devoid of connections between mathematics and students' lives.

1. Connections between mathematics and students' daily lives were not apparent in the lesson.

2. Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

3. Connections between mathematics and students' daily lives were clearly apparent in the lesson.

C.5. Students explained their responses or solution strategies.

This scale is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. Students simply stated answers to problems. They did not explain their responses or solution strategies orally or in written form.

2. Students explained how they arrived at an answer, but these explanations focused on the execution of procedures for solving problems rather than an elaboration on their thinking and solution path.

3. Students explained their responses or solution strategies. They elaborated on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results.

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C.6. Multiple strategies were encouraged and valued.

This scale measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. In a classroom where multiple strategies are encouraged and valued, students spend much of their time discussing different strategies in a substantive manner, and this discourse is an important element within the classroom. Multiple strategies might be elicited by the teacher during whole-class or small-group discussion in which students explicitly share their strategies. The task itself might clearly involve students in solving the problem in different ways (e.g., find the discount in another way), or the task may require students to consider alternative approaches for successful completion (e.g., list as many ways as you can to calculate 15 x $1.98).

1. Multiple strategies were not elicited from students.

2. Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

3. Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

4. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

C.7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

This scale is intended to measure the ways in which the teacher uses student responses during instruction. Teachers can give credence to students' responses by inviting students to listen carefully to other students, to ask each other questions that clarify meaning, and to compare other students' strategies with their own. Teachers can also use student responses to pose questions that stimulate further discussion, to illustrate a point, or to relate them to other aspects of the lesson.

1. The teacher was interested only in correct answers. The majority of the teacher's remarks about student responses were neutral short comments such as "Okay," "All right," or "Fine." No attempt was made to use students' responses to further discussion.

2. The teacher established a dialogue with the student by asking probing questions in an attempt to elicit a student's thinking processes or solution strategies.

3. The teacher valued students' statements about mathematics by using them to foment discussion or to relate them to the lesson in some way. The teacher opened up discussion about the student response by asking other students questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

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C.8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. A student's question about whether the sum of the angles of every triangle is always 180°, for example, might lead to a discussion of non-Euclidean geometry. This scale measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

Circle Yes, if the teacher used students' inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Circle No, if a student's comment or question potentially could have led to such a discussion, but the teacher did not pursue it.

Circle N/A, if no such opportunities came about during the lesson.

C.9. The teacher encouraged students to reflect on the reasonableness of their responses.

An unreasonable response refers to a response that is mathematically distant from the correct answer and might even be distant from an answer that students recognize as reasonable in contexts outside the classroom. One explanation for unreasonable responses is that students do not check the reasonableness of their answers. Although this may be true in some cases, unreasonable responses may also be the result of the lack of connections between symbols and their meaning. Evaluating the reasonableness of a solution involves connections between conceptual and procedural knowledge. These connections are especially significant at the end of the problem-solving process. Lining up decimal points when adding or subtracting decimals, for example, without connecting the process to place value concepts, may lead to unreasonable responses. Students might rely on rules or procedures to obtain correct answers and not have the conceptual knowledge to help them evaluate reasonableness of the answer (Hiebert & Wearne, 1986). This scale is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

1. The teacher rarely asked students whether their answers were reasonable. If a student gave an incorrect response, another student provided or was asked to provide a correct answer.

2. The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding.

3. The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

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D.1. Student exchanges with peers reflected substantive conversation of mathematical ideas.

With this scale we are attempting to capture the quality of student communication. Substantive conversation by students is characterized by interaction that is reciprocal, involving listening carefully to others' ideas in order to understand them, building conversation on them, or extending the idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions.[5] In contrast, student exchanges with little or no substantive conversation involve reporting facts or procedures in ways that do not encourage further discussion of ideas.

1. There were no exchanges between peers in small groups or as a formal part of the general discourse within a large-group setting.

2. Student exchanges with peers reflected little or no substantive conversation of mathematical ideas.

3. Most students only asked one another for a clarification of directions given by the teacher or simply accepted someone's answer without an explanation of how it was found. Few students asked how a solution was found or asked for a clarification of another student's answer.

4. Most of the students asked their classmates for a description of how they solved a particular problem, discussed alternative strategies, and/or questioned how classmates arrived at a solution.

[5] Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Substantive Conversation (pp. 35-40).

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D.2. Interactions among students reflected collaborative working relationships.

The collaborative nature of the classroom can be thought of as students working together, exchanging ideas, and finding solutions to the same problem. This includes providing assistance to one another, making sure that everyone understands and is working on the same problem, exchanging ideas, and seeking help from each other when it is needed. Student collaboration can occur in a small-group or large-group setting. If the major focus of the lesson is on providing students with individual work, then N/A should be selected.

N/A. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Few students were sharing ideas or discussing how a problem should be solved in small groups or in a large-group setting. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and at different paces.

3. Some students were exchanging ideas, or providing assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.

4. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

D.3. The overall level of student engagement throughout the lesson was serious.[6]

This scale measures the extent to which students remained on task during the lesson.

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions by many. This was the central characteristic during much of the class.

2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task but not trying very hard.

3. Sporadic or episodic engagement. Most students, some of the time, were engaged in class activities, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

4. Widespread engagement. Most students, most of the time, were on task pursuing the substance of the lesson. Most students seemed to take the work seriously and were trying hard.

[6] Ideas were drawn from Secada & Byrd (pp. 14-15).

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Observation Scale

Observer: _____ Time Lesson Begins: _____
Teacher: _____ Time Lesson Ends: _____
School: _____ Textbook: _____
Grade: _____ Chapter/Unit: _____
Date of Observation: _____ Lesson (pages): _____

A. Pre-observation Interview With Teacher

1. What mathematical concept(s) or important ideas are being conveyed in this lesson?
   _____
   _____

2. Where is this activity generally situated in the development of a unit? (For example, day 1 (introduction) of 5 days needed to complete the unit) _____

B. Lesson Flow

Describe the main activities that occurred during the class period and the amount of time devoted to each activity. For example: warm-up—5 minutes, introduction of concept through context—7 minutes, large group discussion—10 minutes, group activity—25 minutes, summary by teacher—5 minutes.

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For sections C and D please refer to the observation scale descriptors on the attached sheets. Please provide evidence supporting your rating.

C. Classroom Events

		Evidence
1. The lesson provided opportunities for students to make conjectures about mathematical ideas.	1 2 3 4	
2. The lesson fostered the development of conceptual understanding.	1 2 3 4	
3. Connections within mathematics were explored in the lesson.	1 2 3 4	
4. Connections between mathematics and students' daily lives were apparent in the lesson.	1 2 3	
5. Students explained their responses or solution strategies.	1 2 3	
6. Multiple strategies were encouraged and valued.	1 2 3 4	
7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.	1 2 3	
8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.	Yes No N/A	
9. The teacher encouraged students to reflect on the reasonableness of their responses.	1 2 3	

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D. Pupil Pursuits

Evidence

1. Student exchanges with peers reflected substantive conversation of mathematical ideas. 1 2 3 4

2. Interactions among students reflected collaborative working relationships. 1 2 3 4 N/A

3. The overall level of student engagement throughout the lesson was serious. 1 2 3 4

E. Postobservation Interview With Teacher

1. On a scale of 1 to 4, how did the teacher rate the lesson as achieving the intended purpose cited in the pre-observation interview?

Goals Not Met			All Goals Met
1	2	3	4

Comments: _____

_____

_____

2. Did the teacher feel that any incidents occurred which revealed student misunderstanding or provided opportunities to facilitate student understanding in any way? Yes No If yes, please describe and explain if the lesson was modified in any way.

_____

_____

F. Additional Information

1. Please feel free to add any comments or information (on the back) that you think would be of interest in describing the classroom that you observed.

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Appendix D

Theoretical Framework for *Instruction*

Theoretical Framework

The research function used to make both the cross-sectional and longitudinal comparisons attributes variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, to variations in prior achievement (PA), method of instruction (I), opportunity to learn with understanding (OTL*u*), and school capacity (SC). This relationship can be expressed as—

$$CA = PA + I + OTL_u + SC.$$

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice,*[7] and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each (see Figure 1): (a) *consideration of students' prior knowledge during unit planning*; (b) *unit sequence*; (c) *pace of instruction*; (d) *consideration of students' performance in the previous lesson during lesson planning*; (e) *the purpose of the lesson*; (f) *forms of instruction that promote discourse for the purpose of the lesson*; (g) *student activities that promote discussion, problem solving, and reflection on the content of the lesson*; (h) *lesson presentation and development*; (i) *nature of mathematical inquiry during instruction*; (j) *interactive decisions during instruction*; (k) *nature of students' explanations*; (l) *elicitation of multiple strategies*; (m) *lesson reflection, summary, or closure*; (n) evidence sought during classroom *assessment*; (o) *purpose and coherence of feedback given in response to classroom assessment*; (p) *content of feedback provided in response to classroom assessment*; (q) *nature of student–student conversation*; (r) *collaborative working relationships among students*; and (s) *level of student engagement*. A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the research function. The focus of the remaining sections of this paper is the theoretical framework for the composite index *Instruction*.

The instruction envisioned in standards-based classrooms differs in character from conventional pedagogy in its emphasis on lessons that involve mathematical reasoning, conjectures, problem solving, verification of mathematical arguments, and connections among mathematical ideas (NCTM, 1991). When learning mathematics with understanding, students need the time and opportunity to develop relationships among mathematical ideas, extend and apply these ideas in new situations, reflect on and articulate their thinking, and make mathematical knowledge their own (Carpenter & Lehrer, 1999). Understanding mathematics requires reflection, the "conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed" (Battista, 1999, p. 429), and communication. Through reflection, students can look for connections between new mathematical ideas and ones they already know. Teachers can specifically ask students to identify and articulate these relationships or to discuss how a procedure is linked to a particular notation or underlying mathematical concept (Carpenter & Lehrer, 1999). Research in a growing number of studies underscores the importance of teaching mathematics for understanding, which is based on the principles that knowledge is constructed by the learner and is situated in the context of the learner's existing knowledge, skills, and beliefs; that the teacher's role is a guide for facilitating conceptual understanding; that mathematical tasks are nonroutine, accessible to all students, and engage students' thinking about important mathematics; that classrooms are communities of learners; and that mathematical tools are supports for learning (Cohen, McLaughlin, & Talbert, 1993; Fennema & Romberg, 1999; Hiebert et al., 1997). In contrast, when the aim of the

[7] The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

lesson is primarily coverage of content, the emphasis is often on unconnected pieces of information and on the practice of repetitive procedures or heuristics determined by others (Battista, 1999). Such situations reduce cognitive demands on students.

Mathematical Interaction

Classroom discourse, including expressing thinking and representing mathematical ideas, is central to learning mathematics as a domain of inquiry (NCTM, 1991). Substantive discourse promotes shared understanding of mathematical ideas and emphasizes higher order thinking, which requires students to “combine facts and ideas in order to synthesize, generalize, explain, hypothesize or arrive at some conclusion or interpretation” (Newmann, Secada, & Wehlage, 1995, pp. 86–87). When students rethink their ideas in order to clarify their reasoning for others, they engage in “reflective abstract thinking” as they consider their solutions from the perspective of another student (Wood, 1996, p. 102). In addition, students learn through collaborative problem solving when they are confronted with “a surprise outcome (particularly when two alternative procedures lead to the same result), verbalize their mathematical thinking, and resolve conflicting points of view” (Yackel, Cobb, & Wood, 1991, p. 395). With appropriate guidance in large- and small-group settings, students discuss interpretations of problem situations, express their thinking, and react to the different levels and qualities of solution strategies shared in the group (Gravemeijer, 1994). During class discussions, for example, "solutions can be paralleled; ideas exchanged; approaches on different levels of schematizing considered and compared; arguments criticized, refuted, amended, supported . . ." (Streefland, 1992, p. 237). Such conversation is characterized by reciprocity. Students listen carefully to each other’s ideas, build conversation on those ideas, and mutually construct their understanding to a more abstract level (Newmann, Secada, & Wehlage, 1995). In order for students to participate in substantive mathematical discussions, classroom norms are important (Yackel & Cobb, 1996). Social norms form the basis of classroom cultures in which students explain their thinking, justify their approaches to problem solving, and support their mathematical arguments. Sociomathematical norms provide attention to facets of mathematical conversation that are related to students’ own mathematical cognitive activity, for example, “normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom . . . [as well as] what counts as an acceptable mathematical explanation and justification” (Yackel & Cobb, 1996, p. 461). In contrast, student conversation with little or no substantive content involves reporting facts and procedures in ways that do not encourage further discussion of ideas, or focusing primarily on correct answers. These situations can ultimately result in students' appropriating the answers or the solution strategies of the teacher or a dominant peer.

The types of exercises, problems, projects, and investigations used during instruction convey messages about the nature of mathematical activity and shape students' opportunities to learn mathematics with understanding. Tasks can provide ways for students to explore concepts and procedures, develop skills in meaningful contexts, seek connections among mathematical ideas, and apply mathematics in real-world situations. Tasks that promote depth of knowledge require that students "organize, interpret, evaluate, or synthesize complex information," allow students to consider alternative solutions or perspectives, and open opportunities for elaboration of thinking and support of conclusions (Newmann, Secada, & Wehlage, 1995, pp. 81, 83). Tasks are appropriate for students when they are genuinely problematic (i.e., they provide interesting situations in which students can extend and apply their knowledge of mathematics), promote connections to students’ existing knowledge, and allow for thinking about significant mathematics (Hiebert et al., 1997). The role of the teacher is the selection or creation and sequencing of these tasks, which provide opportunities for learning, reflection, and discussion. Tasks, however, can be adjusted to either reduce or encourage cognitive

activity. The difference resides in enabling teachers to "proactively and consistently support students' cognitive activity without reducing the complexity and cognitive demands of the task" (Henningsen & Stein, 1997).

Pedagogical Decisions

The pedagogical decisions teachers make prior to instruction have a direct impact on student learning. These decisions include deliberate advanced planning such as student grouping for instruction, time allotted to specific aspects of lessons, and emphases given during instruction. In order to orchestrate classroom discourse (both oral and written), the planning of various forms of instruction and student activities that promote discussion, problem solving, and reflection is imperative. The planning involved in teaching standards-based curricula requires more time than generally perceived necessary for teaching using conventional methods (Battista, 1999; Bay, Reys, & Reys, 1999). Standards-based curricula present mathematics in ways that may be unfamiliar to teachers and require advanced planning for successful instruction. Romberg (1997) outlined several features of the mathematical content of MiC that were problematic for teachers who used prepublication units. In MiC, topics traditionally reserved for high-school students are introduced to middle-school students in real-world contexts and with emphasis on student reasoning rather than on procedures. The sequence of topics throughout the grade levels in MiC units is also different from that in conventional middle-school mathematics curricula (e.g., concepts related to percent are introduced in fifth and sixth grades in MiC rather than, more conventionally, in eighth grade). In the prepublication study, this change in sequence sometimes meant that teachers had not taught the content previously. Furthermore, although MiC units primarily present mathematical ideas from one content strand, most involve topics from several strands (in order to emphasize the interrelated nature of mathematical ideas) and include some problems that can be approached in multiple ways (opening possibilities for students' access to problems). These changes provided challenges to these teachers, who were accustomed to teaching mathematics as isolated pieces of knowledge. In contrast, lesson planning with conventional curricula is often minimal because lesson content tends to be the same over many years, and lesson material is conveniently parceled into 2-page spreads.

Teachers also draw on pedagogical content knowledge, which includes information about difficulties students might encounter as they learn new topics, typical sequences students go through as they learn the topics, and potential ways of helping students overcome difficulties (Bransford, Brown, & Cocking, 1999). The thought required by teachers is demanding: “The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students” (Shulman, 1987, p. 15, original emphasis). This transformation of teacher knowledge into goals and expectations for student learning during the lesson is a distinguishing factor in planning quality instruction.

When instruction begins, however, teachers' interactive decisions become more important. Teachers' knowledge of mathematics and their own understanding of their students' thinking are also critical influences on teachers' interactive decision-making. As Heaton and Lampert (1993) pointed out:

> Interactive competence and a knowledge of mathematics are required if mathematics is going to be taught in a way that is responsive *both* to the way children think about number and shape *and* to the structure of the mathematical ideas under discussion.

> This knowledge is different from what one needs to know in order to competently teach a textbook lesson oriented toward having students remember how and when to do procedures. . . . The teacher's knowledge of the subject must be used to make connections between what he or she understands and what students bring to the lesson. The teacher's mathematical knowledge must be held in a form that enables it to be reshaped depending on the kinds of ideas students bring up in a discussion or in the activity of trying to solve a problem (p. 54, original emphasis).

The interactive decisions of teachers using prepublication versions of MiC became more complex as students assumed more active roles in their own learning. Clarke (1995) investigated the nature of critical incidents that developed as two teachers used MiC for the first time. In their efforts to understand students' solution strategies at various levels of sophistication, these teachers found it difficult both to structure lessons that allowed for some guidance without limiting opportunities for student thinking and to determine the length of time to let students struggle with a problem before intervening. They also faced challenges of "valuing all genuine attempts at problems, while seeking to move students toward increasingly mathematically elegant methods" (p. 156), and they found that it was "very difficult to be interacting, to be listening in an attentive way, trying to understand students' solutions, and thinking of an appropriate response, many times during a lesson" (p. 163). Thus, the nature of teachers' interactive decision-making changes dramatically as teachers move away from conventional pedagogy toward teaching mathematics with curricula designed for students to learn mathematics with understanding.

Classroom Assessment

Classroom assessment is the process in which teachers gather, synthesize, and interpret information to make instructional decisions (Airasian, 1994). The content of classroom assessment should correspond to the mathematics content standards and emulate what it means to know and do mathematics. Multidimensional assessment practice attends to important instructional goals and the processes through which a student's understanding is developing and is more directly connected to ongoing instruction (Graue, 1993; Shepard, 2000; Wolf, Bixby, Glenn, & Gardner, 1991). Evidence gathered by teachers can be based on a taxonomy of problems and solution strategies such as that for whole number operations in Cognitively Guided Instruction (Carpenter, Fennema, Levi, & Empson, 1999) or particular statements that arise during classroom interaction such as "an interpretation of language or symbols, an assertion about a pattern, a proof that a pattern would continue beyond the observed data, or an interpretation of another student's assertion" (Lampert, 1988, p. 460). Evidence might also include students' plans to test conjectures, flexibility in working with mathematical tools, increased use of mathematical concepts and procedures, alternative approaches to problems, and sophistication of solution strategies (Shafer, 1996). Because each piece of evidence represents a sample of a student's performance at a particular point in time, it is important that the evidence a teacher collects for each student is representative of the student's achievements. Instructionally embedded assessments such as observing students as they work on tasks, listening to students' strategies, responses to teacher or student queries, and written assignments and quizzes provide a range of opportunities for students to demonstrate what they know about mathematics. The quality of such evidence also depends on the tasks in which students engage, the interactions among students and between the students and the teacher, and a learning environment in which social and sociomathematical norms have been developed. Over time, information from multiple assessments provides the foundation for a teacher to develop a broadly based understanding of each student's growth in mathematical domains (Gipps, 1994). Thus, the methods of assessment used and the types of evidence teachers actively seek during instruction affect information available for making instructional decisions.

Another aspect of classroom assessment practice is to providing multiple opportunities for feedback to encourage students' participation in mathematical discussion and to help them judge their own work against established performance standards and criteria. Ideally, verbal or written feedback is responsive to student statements in order to promote mathematical understanding and greater perspective of ideas. Providing feedback is not solely a teacher action. Students should be given the opportunity to share and critique the explanations, arguments, strategies and responses of other students. Substantive feedback provides students with information to improve their mathematical work and communication of their thinking, and it can be an important catalyst for students to extend their thinking to higher levels. Quality feedback includes references to mathematics principles and concepts (Bransford, Brown, & Cocking, 1999; NCTM, 1995). If feedback lacks mathematical substance, it is limited only to correct answers or superficial features of responses such as organization and neatness.

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Appendix E

Methodology for the Composite Index *Instruction*

The composite index *Instruction* is specified from data gathered on the intervening variables (*pedagogical decisions*, *classroom events*, and *student pursuits*) in the structural research model for the longitudinal/cross-sectional study. In the simplified research function, *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice,*[8] and *student pursuits during instruction*. The five categories of *Instruction* were further subdivided into 19 subcategories, and an index was created for each (see Figure E1). In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence; and pace of instruction. Four subcategories characterized lesson planning: consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson;* and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Six subcategories characterized the *mathematical interaction during instruction*: lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of *students' explanations; elicitation of multiple strategies;* and *lesson reflection, summary, or closure*. Three subcategories characterized

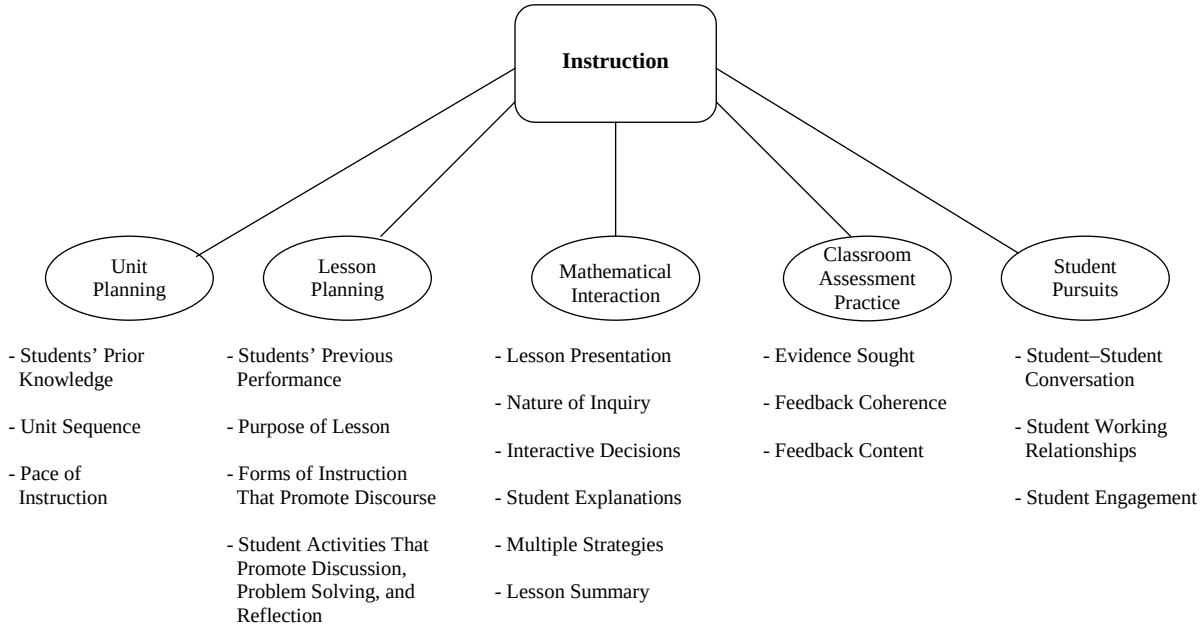

Figure E1. Major categories and subcategories of *Instruction*.

[8] The research team believes that classroom assessment practice should be integrated with instruction. In the research design, however, classroom assessment practice was distinguished from classroom interaction in order to examine its effects on the composite index for instruction.

classroom assessment practice: *evidence sought; purpose and coherence of feedback*; and *content of feedback*. Finally, three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students*; and *level of student engagement*.

A single index, a composite of multiscaled information from each subcategory, represents *Instruction* in the simplified research function. Description of the theoretical framework that guided the analysis of instruction is contained in Appendix D. For complete descriptions of teacher data, see Appendices F, G, and H for fifth-, sixth-, and seventh-grade teachers, respectively.

Sample

Districts 1 and 2

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. Data collection included teacher interviews, classroom observations, and teaching logs. Analysis focused on all five categories of instruction. All MiC teachers used commercial versions of the units, which became available during the summer before the study began. Teachers using conventional curricula used whatever curriculum was already in place in the schools. The sample in Districts 1 and 2 consisted of 35 fifth-, sixth- and seventh-grade teachers and their classes from diverse schools in Districts 1 and 2. These teachers taught a combined total of 63 classes involving 1561 students (see Tables E1 and E2).

Table E1
Summary of Study Participants, Districts 1 and 2, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 1	
MiC	12	22	532
Conventional	6	11	240
		District 2	
MiC	12	23	584
Conventional	5	7	205
Total	35	63	1561

Table E2
Characteristics of the Student Participants, Districts 1 and 2, by Curriculum

Curriculum	Gender *(%)*		Ethnicity *(%)* *(self-idenified)*							Primary Language *(%)* *(self-idenified)*	
	Male	Female	African American	Native American	Asian	Hispanic	White	Multiracial	Other[1]	English	Other[2]
District 1											
MiC	49	51	21	1	1	7	54	10	5	92	7
Conventional	53	47	25	0	–	3	54	13	3	91	9
District 2											
MiC	47	53	9	1	–	49	17	17	7	85	15
Conventional	56	44	18	1	–	34	14	18	15	84	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.
[2]Includes nonresponses.

District 1 was located in an urban region in the eastern part of the United States. In 13 elementary, 6 middle, and 4 high schools, 1325 teachers were responsible for teaching the district's 15,532 students. Three elementary schools and four middle schools participated in the first year of the study. Six fifth-grade study classes were in self-contained elementary classrooms. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. Approximately 30–40% of the students in the district were eligible for government-funded lunch programs. Professional development to acquaint teachers of mathematics with reform-based curricula was offered in District 1, and monthly meetings were provided for teachers who were implementing such programs. For preliminary teacher certification, 24 credit hours were recommended for fifth- and sixth-grade teachers; 24 credit hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements.

District 2, located in a large urban area in southeastern United States, had 19,352 teachers and 342,996 students housed in 201 elementary schools, 51 middle schools and numerous high schools. Three elementary and three middle schools participated in the first year of the study. In District 2, two of the nine fifth-grade study classes were in self-contained settings in elementary schools. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district student population was predominantly minority, with 33% African American students and 52% Hispanic students. Over 50% of the students in the district were eligible for government-funded lunch programs. District 2 provided numerous possibilities for professional development. Each school was given six early-release days for general professional development. In addition, each school received 10 substitute days for professional development in mathematics and/or science, 12–18 days of in-service days in mathematics provided by (USI or Eisenhower) government funding (each involving 2–6 teachers), and 3–5 days of district-wide mathematics in-service. Teachers also had opportunities to participate in five days of paid in-service for mathematics during the summer. District requirements for preparation of mathematics teachers were the same as state requirements. For elementary teachers, preliminary teacher requirements mandated the study of arithmetic for the elementary school. For middle-grade mathematics certification (Grades 5–9), 18 semester hours in mathematics were required; for certification in mathematics for Grades 6–12, 30 semester hours in mathematics were

required. Continuing certification required the completion of six semester hours in mathematics or 120 district in-service credits in mathematics every 5 years.

Districts 3 and 4

Districts 3 and 4 agreed to participate in a modified research design in which all MiC teachers used MiC. Data collection included teacher interviews, but excluded classroom observations and teaching logs. Therefore, data analysis in Districts 3 and 4 was limited to teachers' unit and lesson planning. All teachers used commercial versions of MiC units. The sample in Districts 3 and 4 consisted of 18 fifth-, sixth-, and seventh-grade teachers who taught a combined total of 31 classes involving 665 students (see Tables E3 and E4).

Table E3
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
		District 3	
MiC	13	21	438
		District 4	
MiC	5	10	227
Total	18	31	665

Table E4
Summary of Study Participants, Districts 3 and 4, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-idenified)***							**Primary Language *(%)* *(self-idenified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
					District 3						
MiC	48	52	--	--	--	2	87	9	--	98	5
					District 4						
MiC	49	51	31	--	1	20	3	15	29	85	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.
[2]Includes nonresponses.

District 3 was located in a suburban area in a large western state. In the district's two elementary and one middle school, 73 teachers taught 1480 students. In District 3, the six self-contained 5th grade study classes are in a school for Grades 3–5; Grades 6-8 were housed in a middle school. Three of the middle school study classes were in self-contained classrooms. The remaining middle-school study classes had several subject-matter teachers. Study participants included all fifth-, sixth-, and seventh-grade mathematics classes in the district. The schools' student and teacher

populations were predominately White. Approximately 10–20% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. School administrators provided paid monthly meetings for mathematics teachers who were implementing standards-based curricula for the first time. Teachers often met weekly without pay to prepare mathematics lessons. For preliminary teacher certification, the state mandated single-subject credentials for Grades 7–8. Teachers for Grades K–6 were required to complete a multiple-subjects credential including several mathematics courses. Although the district provided mathematics courses and staff development opportunities, it did not require additional certification and courses for experienced teachers.

District 4 was one of many districts located in a large urban area in the eastern part of the United States. The district's 1075 teachers were responsible for teaching the 20,000 students in 23 elementary schools, seven middle schools, and several high schools. In District 4, Grades 6–8 are contained in middle schools in which students have several subject-matter teachers. Study participants are from one middle school in this district. Because fifth-grade students in District 4 are dispersed among several middle schools, fifth-grade classes were not included in the data collection for District 4. Four sixth-grade and six seventh-grade classes from one middle school participated in the study. The student population was predominately minority with 64% African American students and 28% Hispanic students. Approximately 37% of the teacher population was minority with 31% African American teachers and 6% Hispanic teachers. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. For new mathematics teachers, 36 credits in mathematics or a mathematics major were required by the state, but no specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements. Professional development opportunities were provided to all mathematics teachers at both district and school levels, including personalized discussions with the assistant principal for mathematics and science. These discussions focused on reform recommendations in curriculum, instruction, and assessment; research in mathematics education; and applications of research in classroom practice.

Data Collection

Data were collected through the use of instruments designed to examine instruction in the longitudinal/cross-sectional study. Information on *unit planning*, *lesson planning*, and *classroom assessment practice* was gathered through the Teacher Interview: Instructional Planning and Classroom Interaction (see Appendix A; Shafer, Davis, & Wagner, 1998). Teachers in Districts 1-4 were interviewed twice during the school year. The interviews conducted in the spring semester focused on teachers' instructional planning and classroom interaction. The first interview question asked teachers about their planning for each mathematics unit/chapter. The probes provided specific attention to (a) whether the teacher planned with others; (b) the considerations given in planning at the unit level, specifically, students' prior knowledge, textbook scope and sequence, district curriculum guidelines, state standards, district or state standardized tests, and other resources; and (c) setting the pace for instruction. An additional probe was reserved for teachers who used MiC. The focus of this probe was comparison of planning to teach MiC with planning to teach other mathematics textbooks used in the past. The second interview question was dedicated to planning individual lessons. Probing questions provided attention to (a) considerations of students' performance in the previous lesson, and (b) whether the teacher solved unit/chapter problems or exercises before teaching, and, if so, the effect this preparation had on teaching the lesson. The third interview question was designed to collect information on classroom interaction and provided insight into teachers' classroom assessment practice. In particular, the question addressed the issue of what counts as an acceptable answer. Related probes were (a) how students determine if their answers are acceptable; and (b) how the

teacher determines if a student's answer is acceptable. The third probe elicited information about the ways in which students contribute to classroom discussions. A fourth probe was reserved for teachers who used MiC. The focus of this probe was differences in student participation when using MiC in comparison to student participation when using conventional mathematics curricula in the past. Additional probes for this question provided attention to differences in the types of conversation generated with each type of curriculum such as student conjectures, answers, and explanations.

Information on *mathematical interaction during instruction*, *classroom assessment*, and *student pursuits* was gathered through the Classroom Appendix B; Shafer, Wagner, & Davis, 1997). Each teacher in Districts 1 and 2 was asked to complete a daily teaching log. Three sections of the log are pertinent to data collection on instruction: lesson activities, student activities, and classroom assessment. Data from teacher log sheets, which were completed daily by each study teacher, were used to more fully describe the forms of instruction and the student activities teachers used. One item in the log was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). Another item was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. These data were summarized over all the days a particular study teacher completed a daily log sheet. Information from classroom observations generally supported the information teachers provided in these two items.

Data from teacher logs were also used to more fully describe teachers' use of *classroom assessment*. If teachers assessed students informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered.

Each teacher in Districts 1 and 2 was also asked to reflect on his/her teaching and respond to two journal questions at least once a week. Journal entries were submitted monthly as part of a daily teaching log (see Appendix B). The journal questions focused on emphases given during instruction, modification of curricular materials, and notable classroom events. The first journal question provided information about the interactive decisions teachers made during instruction. The second journal question provided additional insight into classroom interaction, especially when the journal entry was related to a lesson that was observed and teachers' classroom assessment practices. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a

surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them.

Information on *mathematical interaction during instruction*, *classroom assessment*, and *student pursuits* was gathered through the Classroom Observation Instrument (see Appendix C; Davis, Wagner, & Shafer, 1998). Classroom observations were conducted on each study teacher in Districts 1 and 2. Most teachers in District 1 were observed once a month for a total of nine observations per teacher. One teacher in District 1 accepted an administrative position in December; consequently, she was observed three times, and the newly assigned teacher was observed five times. Teachers in District 2 were observed a total of three to seven times each. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. Four teachers from one school in District 2 withdrew from participation in the study during the spring semester; consequently, they were observed only three times. The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. The research staff worked with both observers to establish interrater reliability. Completed observation reports were sent to the research center monthly.

The observation instrument contained six sections, four of which provided data about instruction: lesson activities, ratings on 12 indices about classroom events and student pursuits, postobservation interview, and observer comments. Two sections of the observation instrument were particularly useful in describing the mathematical interaction and student pursuits that transpired during class time. In one section, the observer recorded the flow of the lesson, which was a list of lesson activities along with the time allotted to each. This information illuminated the relative emphases teachers gave to particular forms of instruction and student activities. Another section of the observation instrument was collectively composed of 12 indices for various subcategories of instruction. For each index, the observer circled a rating and provided a statement of support for the given rating. Nine of these indices focused on classroom events, three of which provided particular attention to classroom assessment and teachers' interactive decisions: the use of student inquiries as a guide for instructional mathematics investigation or as a guide to shape the mathematical content of the lesson; encouragement for students to reflect on the reasonableness of their responses; and the use of student statements to build discussion or work toward shared understanding for the class. The remaining three indices focused on student pursuits. Ratings given by observers were checked by research staff for accuracy and were changed when appropriate. For example, on a few occasions, when the evidence provided by the observer strongly indicated another rating than the one circled, the rating was changed to reflect the evidence.

A brief postobservation interview was conducted during which the teacher was asked whether incidents occurred that revealed student misunderstanding or provided opportunities to facilitate student understanding in any way. If such an incident occurred, the teacher was asked to explain whether and in what way the lesson was modified. These data provided insights into teachers' interactive decisions and classroom assessment practices. In the final section of the observation instrument, the observer recorded any additional comments about the lesson. These comments included field notes that the observer deemed important for the research team or descriptions that more fully portrayed the lesson.

Indices

An index was created for each of the 19 subcategories of *Instruction*. The indices used to characterize each subcategory of *Instruction* were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through (a) review of literature that was specific to each subcategory (see Appendix D for a summary of the relevant literature) and (b) review of the teacher and observer data based on Strauss' (1987) system of open, axial, and selective coding, which involved repeated coding of the data for interpretive codes. These codes included both external codes identified prior to reviewing the data (e.g., demonstration of procedures and strategies) and internal codes that emerged from the data (e.g., students' inability to solve problems using a presented procedure). Categories were further refined, and additional categories were integrated for each index as data for fifth-grade study teachers in one district were reviewed. Indices were further revised during review of data from sixth- and seventh-grade teachers and teachers in other districts. As a result, three to five levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts. Indices included in the observation instrument were refined as a consequence of observing nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Before these indices were used in study classrooms, district and school administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms currently implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each subcategory. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the subcategory being scaled. For example, *levels of lesson presentation and development* range from no lesson presentation during instruction to lessons in which the teacher facilitated students' active participation in learning mathematics with understanding. One level in the some indices were subdivided further in order to more adequately describe the variation among teachers who demonstrated procedures or strategies to their students. For example, sublevels in one level of the index for lesson presentation and development categorized situations in which students were unable to complete exercises using the presented procedure and lessons in which students practiced the presented procedure in a rote fashion. In the remainder of this section, the indices for each subcategory of instruction, organized by major category, are described.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge; unit sequence*; and *pace of instruction*. Four levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts.

Students' Prior Knowledge. The following index measured the extent to which the teacher thought about and identified students' prior knowledge while planning to teach a unit:

1. *Little or no consideration of students' prior knowledge*. The teacher planned the unit with little or no consideration of the prior knowledge of students in the current class.
2. *Consideration of student abilities*. The teacher planned the unit based on perceptions of students' needs.
    (a) The teacher bases plans on perceptions of students' mathematics skills and/or reading ability and vocabulary.
    (b) The teacher bases plans on perceptions of students' needs related to the development of concepts.
3. *Informal or formal assessment of students' understanding*. The teacher planned the unit on the basis of information gathered through informal or formal assessment. The teacher might have, for example, planned remedial skill-based activities to address weaknesses or planned extension activities for students ready for such challenges.
4. *Conceptually-based activities planned*. The teacher planned unit activities that were designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence. The following index measured the extent to which the teacher might consider the sequence of instructional units:

1. *Little or no variance from the text sequence*. The teacher followed the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher based decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequenced units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequenced units to support the development of mathematics concepts.

Pace of Instruction. The following index measured the extent to which the teacher might consider the pace for instruction when planning to teach a unit:

1. *Little or no consideration of pacing*.
    (a) The teacher followed the recommendations for pacing in teacher support materials.
    (b) The teacher did not plan unit pacing because the curriculum was unfamiliar.
2. *Adjustment anticipated*. The teacher considered the recommendations for pacing in teacher support materials, but planned to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considered the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher planned substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Lesson Planning

Four subcategories characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. Four levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts.

Students' Performance in the Previous Lesson. The following index measured the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might have noted students' performance, but the lesson was planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might have, for example, extended the previous lesson to complete a task, disregarded time constraints, or added a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might have used the information gathered to allow a more in-depth exploration of the mathematical content or to introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might have, for example, varied problem structure/setting to encourage thinking at higher levels or to emphasize connections with related concepts.

Purpose of the Lesson. This index measured the extent to which particular teachers thought about and identified the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose was to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction was to cover lessons in the textbook or curriculum; thus, no additional planning was deemed necessary.
2. *Overall curriculum plan in mind.*
   (a) The teacher selected lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might have included an additional focus on problem solving, applications of mathematics, or practice.
   (b) The teacher planned lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher made decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might have, for example, planned questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson. The following index measured the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention was focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might have, for example, planned for such work/discussion, but continued to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse was not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction*. The teacher encouraged such participation, but it was not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned.* The teacher might have, for example, planned classroom activities that encouraged students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson. The following index measured the extent to which a particular teacher might have included various student activities that promoted discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis was placed on practicing routine calculations, and little discussion among students was anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might have planned problem investigation or class discussion, but the significance of these activities was not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encouraged students to reflect on or summarize lessons, however, were not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher viewed reflection on or summarization of the lesson as an important element in instruction.

Mathematical Inquiry during Instruction

Because the data collection in Districts 1 and 2 included classroom observations and teacher logs, the remaining 12 indices were completed for only the 35 study teachers in these districts. Six subcategories characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Lesson presentation and development. For *lesson presentation and development*, six levels were defined. Data from classroom observations and teacher journal entries provided multiple sources for which lesson presentation and development could be examined for each teacher. Two members of the research staff independently rated each piece of evidence according to the levels in the index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. An overall rating was then assigned for each teacher. Although variability did occur in the multiple pieces of evidence for each teacher over the course of the school year, each teacher maintained basic elements of pedagogy that permitted the characterization of *lesson presentation and development* at one particular level. (See Shafer (2000b) for a detailed description of this index.)

This index measured the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding:

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.
2. *Emphasis on review*. The lesson was not well developed; consequently, students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.
   (a) A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.
   (b) Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.
3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.
   (a) Students were unable to solve problems using the presented procedure or strategy.
   (b) Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.
4. *Attempt made to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Although students were allowed to find their own solution strategies, they generally used a procedure or strategy presented by the teacher.
5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.
6. *Emphasis on conceptual understanding with active participation by students with teacher support*. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher.

The Nature of Inquiry. The index for the *nature of inquiry* is a composite of four indices that were included in the observation instrument: the development of conceptual understanding, the nature of students' mathematical conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. Three levels were identified for connections between mathematics and students' life experiences; four levels were identified for the other indices. The sum of the ratings given by the observers on these four indices for all

observations of each study teacher was calculated. Cluster analysis permitted the classification of teachers. As a result, four levels were identified to capture the variation among teachers at different grade levels and in different districts.

This index characterizes the nature of mathematical inquiry during instruction:

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking; lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.
2. *Limited attention to conceptual understanding.* Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.
3. *Emphasis on conceptual understanding.* Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.
4. *Emphasis on relationships among mathematical ideas or linking procedural and conceptual knowledge.* The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to make generalizations; and connections between mathematics and students' lives were discussed.

Teachers' interactive decisions. The index for *teachers' interactive decisions* is a composite of three indices, which included *explanation-oriented decisions, task-oriented decisions*, and *shifts in pedagogical approach*. The categories in the three indices are similar to the other indices in that they are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the subcategory being scaled. For example, categories of *shifts in pedagogical approach* range from no change during instruction when warranted to lessons in which the teacher modified a lesson based on a student's statement or inquiry. Two members of the research staff independently rated each piece of evidence according to the levels in each index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. The indices for *teachers' interactive decisions*, however, differ from the others in a significant way. In order to describe variation in the data, the categories in each index for interactive decisions were further classified into decisions that were (a) least aligned with teaching for understanding, (b) reflective of standard pedagogy, or (c) most aligned with teaching for understanding. Patterns of variation among the teachers became more apparent with this additional categorization of *teachers' interactive decisions*. A profile of each *teacher's interactive decisions* was constructed. In the initial phase of this process, the percent of coded items in each category was calculated. *Explanation-oriented decisions, task-oriented decisions*, and *shifts in pedagogical approaches* were set in juxtaposition along a continuum from least aligned with teaching for understanding to most aligned with teaching for understanding in order to look for patterns among the teachers. The percent of coded items in the three subcategories (decisions least aligned with teaching for understanding, reflective of good standard pedagogy, and most aligned with teaching for understanding) were calculated. The profiles formed the basis of the composite index for *teachers' interactive decisions*. Five levels of interactive decisions were identified in this process. (See Shafer (2000a) for a detailed description of this index.)

This index measured the extent to which a teacher's interactive decisions were aligned with teaching mathematics for understanding:

1. *Least aligned with teaching for understanding*. Teachers' interactive decisions were least likely to support students' learning mathematics with understanding. At least 60% of the coded items represented decisions least aligned with teaching mathematics for understanding.

2. *More emphasis on standard pedagogy, but decisions predominantly coded as least aligned with teaching for understanding*. More interactive decisions were classified as reflective of good standard pedagogy and were more aligned with teaching for understanding than at Level 1, although 40–55% of the decisions were still coded as least aligned with teaching for understanding.
3. *Stronger emphasis on standard pedagogy*. Teachers' interactive decisions in general were more reflective of good standard pedagogy and more supportive of teaching for understanding than at Level 2, and decisions least aligned with teaching for understanding were less pronounced (around 30% of the coded interactive decisions).
4. *More emphasis on standard pedagogy and teaching for understanding*. Teachers' interactive decisions were characterized by greater attention to standard pedagogy and teaching for understanding than at Level 3, and decisions least aligned with teaching for understanding were less likely to occur (generally around 20% of the coded decisions).
5. *Most aligned with teaching for understanding*. Teachers' interactive decisions were predominantly aligned with teaching for understanding and were least likely to be coded as least aligned with teaching for understanding (around 10%).

The Nature of Students' Explanations. The index for the *nature of students' explanations* was included in the classroom observation instrument. (See Shafer (2000b) for a detailed description of this index.) The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only*. Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures*. Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes*. Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Elicitation of Multiple Strategies. The index for the *elicitation of multiple strategies* was included in the classroom observation instrument. This index measured the extent to which students were asked to consider different perspectives in approaching the solution to a problem:

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies as a substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Lesson Reflection or Summary. Data gathered through the teacher log regarding student activities that provided opportunities for students to reflect on or summarize the mathematics in a lesson or series of lessons were used to support data gathered from observer comments on the observation reports. Two members of the research staff independently rated each piece of evidence according to the levels in the index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers. An overall rating was then assigned for each teacher. Although variability did occur in the multiple pieces of evidence for each teacher over the course of the school year, each teacher maintained basic elements

of pedagogy that permitted the characterization of lesson reflection or summary at one particular level. This index measured the extent to which the teacher included reflection on or summary of lesson concepts:

1. *Limited opportunities.* Few opportunities, if any, were provided for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. The lesson or series of lessons concluded at the end of the class period without discussion of the primary mathematics content or connections among mathematical ideas.
2. *Some opportunities.* Some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. These opportunities may have included completion of reflection questions in section summaries.
3. *Frequent opportunities.* Students were frequently provided with opportunities to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. These opportunities included teacher-designed writing prompts or questions posed during whole-class discussions.

Classroom Assessment Practice

Three subcategories characterized *classroom assessment practice*: *evidence sought; purpose and coherence of feedback*; and *content of feedback.* Two members of the research staff independently rated each piece of evidence according to the levels in each index. Discrepancies in ratings, if any, were resolved in discussions between the two researchers.

Evidence Sought. This index measured the evidence teacher regarded as indicative of student achievement and understanding:

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.
2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.
3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations were often void of mathematical substance and often promoted without purpose (i.e., to generate some form of communication rather than assess student understanding). Teachers at this level valued student explanations but might have had difficulty eliciting student responses and orchestrating substantive whole class discussions.
4. *Conservative process orientation.* Teachers at this level value were somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures. The teacher might have required students to justify solutions with explanations, but often forced their own strategy and restricted the responses that were shared. Student misconceptions were rarely utilized as opportunities for instruction.
5. *Principled process orientation.* Teacher viewed student explanations as evidence of student learning. While the teacher sought both process and product as evidence, s/he sought demonstration of student learning through verbal or written communication of process. Answers, solutions, and procedures were recognized as inadequate forms of demonstrating understanding. Instruction wais used as an opportunity to gather evidence of student learning. Teachers at this level regularly elicited student (mis)conceptions to guide instruction.

Feedback Coherence and Purpose. This index measured the method and goal orientation of feedback that the teacher makes available to students.

1. *No feedback.* Teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.
2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through teacher-directed practices that involved "more of the same," such as additional instruction and practice sets. Ineffective attempts to promote student-to-student interaction often resulted in reteaching or additional practice. Feedback might have also occurred as individualized responses to specific student questions and procedural errors where the teacher moved from desk to desk responding to student requests for assistance during seatwork. Feedback provided by other students was minimal or non-existent.
3. *Emerging shared responsibility.* Students received feedback from peers through student–student discussions in pairs or groups and through sharing examples of their responses to assigned work. However, student–student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making. Students' role or purpose in sharing their work with others might have been unclear.
4. *Purposeful shared responsibility.* Student interactions were used to promote sense making of tasks, student responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work in progress and examples of refined responses.
5. *Toward student self-assessment.* In addition to the practices described at Level 4, the process and criteria used by the teacher to evaluate mathematical work is revealed to students, and they are invited to assess their own and other students' work. The teacher provides opportunities for students to participate in the creation or modification of performance standards, reflect upon and judge their own work, and revise more complex assignments that require elaborated communication such as projects, reports, and written explanations.

Content of Feedback. This index measured the degree of substantive feedback provided to students, from teachers, students, and available resources:

1. *Feedback withheld and/or misleading.* Teacher feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance. Teacher did not attend to student misconceptions and often promoted or sustains student misconceptions.
2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions. Feedback might have been limited to praise, criticism or student work habits.
3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer (such as simplifying fractions) rather than clarifying explanations or developing student understanding.
4. *Mixed, superficial feedback.* An emerging blend of feedback addressed skills, procedures, and concepts. Feedback was directed toward mathematics although, at times, feedback might have favored problem context over mathematical content. Feedback and guidance given in response to student work was clear and mathematically sound.
5. *Concept-directed feedback.* Feedback was directed towards conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive

discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall. The process and criteria used by the teacher to evaluate mathematical work was revealed to students, and they were invited to assess their own and other students' work. The teacher provided opportunities for students to participate in the creation or modification of performance standards, reflect upon and judge their own work, and revise more complex assignments that require elaborated communication such as projects, reports, and written explanations.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation; collaborative working relationships among students;* and *level of student engagement*. The indices for these subcategories were included in the classroom observation instrument. (See Shafer (2000b) for a detailed description of the index for the nature of student–student conversation.)

Nature of Student–Student Conversation. The index for *student–student conversation* measured the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged*. Conversation among students was not permitted or was social in nature.
2. *Limited conversation*. Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature*. Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation*. Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Student Collaborative Working Relationships. This index measured the extent to which students collaborated with one another during the lesson:

N/A. *Independent work.* The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. *No collaboration among students.* None of the students worked together in small-group or large-group settings. If students worked in small groups, then one student typically gave answers to other members of the group without explanation for the use of certain procedures.
2. *Limited exchange of ideas.* Few students shared ideas or discussed how a problem should be solved in small-groups or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems at different paces.
3. *Uneven participation.* Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.
4. *Substantive collaboration.* Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Student Engagement during the Lesson. This index measured the extent to which students remained on task during the lesson:

1. *Disruptive disengagement*. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. *Passive disengagement*. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. *Sporadic or episodic engagement*. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. *Widespread engagement*. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

The Composite Index *Instruction*

Although teachers in all four research sites completed interviews, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index *Instruction*, therefore, was created only for teachers in Districts 1 and 2 for whom there was a complete set of ratings on all 19 indices. Thirty-four teachers were involved in the analysis for the first year of data collection. One teacher was not included in the analysis because she did not teach a full semester during the study.

The composite index *Instruction* was created in a multiple-step process. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the Instruction Total.[9] Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subcategories and the Instruction Total (see Table E5).

[9] The sum of the weighted results was taken as a measure of the quality of instruction. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of instruction.

Table E5.

Correlation between the Instruction Total and the Subcategories of Instruction

Subcategory	Unit Planning			Lesson Planning				Mathematical Interaction						Classroom Assessment			Student Pursuits		
	SPK	US	PI	SPPL	PL	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE
SPK																			
US	0.092																		
PI	0.321**	0.017																	
SPPL	0.164	0.295**	0.051																
PL	0.136	-0.026	0.147	0.409***															
FIFD	0.038	-0.077	0.229*	0.177	0.326**														
SAPD	0.016	-0.181	0.134	0.118	0.474***	0.640***													
LPD	-0.023	0.104	0.176	0.157	0.312**	0.537***	0.546***												
NI	-0.026	-0.116	0.208	0.217*	0.506***	0.665***	0.549***	0.647***											
ID	-0.033	-0.015	0.285**	0.099	0.309**	0.568***	0.527***	0.676***	0.702***										
SE	-0.008	-0.072	0.141	0.175	0.260*	0.497***	0.468***	0.559***	0.692***	0.660***									
MS	-0.088	-0.060	0.048	0.025	0.227*	0.523***	0.426***	0.552***	0.635***	0.619***	0.788***								
LCS	-0.032	0.000	0.084	0.172	0.291**	0.476***	0.534***	0.556***	0.476***	0.495***	0.445***	0.471***							
ES	0.005	0.000	0.015	0.152	0.257*	0.408***	0.457***	0.650***	0.573***	0.602***	0.635***	0.599***	0.537***						
FCP	-0.036	0.055	0.108	0.079	0.266*	0.442***	0.423***	0.570***	0.520***	0.532***	0.561***	0.523***	0.480***	0.791***					
FC	-0.136	0.030	0.142	0.159	0.228*	0.398***	0.401***	0.624***	0.547***	0.613***	0.551***	0.469***	0.464***	0.695***	0.753***				
SC	0.015	-0.002	0.101	0.189	0.375**	0.493***	0.425***	0.436***	0.624***	0.532***	0.506***	0.475***	0.409***	0.483***	0.503***	0.578***			
SWR	0.073	0.008	0.063	0.109	0.336**	0.437***	0.422***	0.369***	0.532***	0.487***	0.521***	0.480***	0.385***	0.450***	0.486***	0.492***	0.845***		
OSE	-0.049	0.015	0.082	0.220	0.272*	0.322**	0.336**	0.525***	0.562***	0.512***	0.623***	0.572***	0.364***	0.548***	0.489***	0.518***	0.592***	0.593***	
Instr. Total	0.089	0.081	0.272*	0.309**	0.501**	0.697***	0.665***	0.801***	0.833***	0.803***	0.781***	0.721***	0.654***	0.787***	0.748***	0.751***	0.723***	0.667***	0.688***

*p<.05
**p<.01
***p<.001

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction
Instr. Total--Instruction Total

Five subcategories were not well correlated with the Instruction Total and other subcategories: *consideration of students' prior knowledge; unit sequence; pace of instruction; students' performance in the previous lesson*; and *the purpose of the lesson*. To verify these results, a principle component factor analysis was completed using SAS. Factors 1 and 2 accounted for a significant amount of the variance among the subcategories. Fourteen subcategories were included in Factors 1 and 2 (see Table 6). The five subcategories that had weak correlations to the Instruction Total were not influential in Factors 1 and 2 and were important only in the composition of other factors. Consequently, these subcategories were excluded from the analysis. The Instruction Total for each teacher was then recalculated.

Table E6.
Contribution of Subcategories to Principle Component Factors

Subcategory	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
SPK	-18	11	16	72*	9
US	4	-5	7	18	83*
PI	21	-5	-4	85*	7
SPPL	5	9	75*	0	46*
PL	19	21	78*	11	-12
FIFD	60*	15	33	23	-32
SAPD	59*	9	44*	9	-41*
LPD	83*	10	17	7	6
NI	65*	39	33	12	-23
ID	76*	28	8	19	-11
SE	68*	45*	5	5	-10
MS	68*	39	-2	-3	-17
LCS	68*	6	29	-4	-5
ES	79*	30	4	-10	12
FCP	75*	332	-1	-4	15
FC	74*	35	0	-8	17
SC	36	80*	22	6	-6
SWR	29	85*	15	8	-8
OSE	46*	65*	8	-5	9

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

Using the revised Instruction Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into six groups. For each group of teachers, common characteristics from the subcategories of instruction were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each subcategory of *Instruction*. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. Similar to the index for each subcategory, the underlying single dimension of the composite index *Instruction* was teaching mathematics for understanding. The levels of the composite index were on a continuum from least to most reflective of teaching mathematics for understanding. The six levels are summarized in Table 7.

Level 6: Most reflective of teaching for understanding
Level 5: Reflective of teaching for understanding
Level 4: Attempted to teach mathematics for understanding
Level 3: Limited attention to conceptual understanding
Level 2: Focus on procedures
Level 1: Underdeveloped lessons.

Level 6: Most Reflective of Teaching Mathematics for Understanding

Lessons emphasized conceptual understanding, and students actively participated in lessons with the support of their teachers. Lesson presentations set the stage for students to explore the mathematical content of the lessons on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher. Mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge. Students were encouraged to make generalizations about mathematical ideas, and connections between mathematics and students' life experiences were discussed. Students' explanations varied in attention to mathematical reasoning. During some lessons, students explained the processes used in finding solutions; during other lessons, explanations focused on procedures used to determine answers. Interactive decisions were predominantly aligned with teaching for understanding. Teachers frequently asked students to articulate their thinking, discuss mathematical reasoning, and talk about the reasonableness of solutions. Teachers shifted students' attention from small-group work to whole-class discussion, introduced or encouraged students to introduce alternative strategies, or modified a lesson in response to a student's statement or inquiry.

Teachers' classroom assessment practices were characterized by attention to mathematical processes such as justifying approaches to problems, explanations of reasoning, and supporting conclusions. Teachers used students' misconceptions to guide instruction through the use of probing questions, counterexamples, and alternative representations. Feedback promoted making sense of mathematical tasks, students' solutions, and mathematical conventions. Criteria for judging student work were openly discussed, and students were given opportunities to create or modify performance standards. Students assessed their own work and the work of others and revised assignments that required elaborate communication, such as projects and reports.

Conversation among students frequently was characterized by students discussing procedures or asking each other for clarification of a procedure presented in class. Occasionally, conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend the ideas to a new level. During

collaborative group work, students exchanged ideas or provided assistance to their classmates, but all students did not make equal contributions to problem solving. Most students were on task, seriously pursuing the substance of the lesson.

Level 5: Reflective of Teaching Mathematics for Understanding

Lessons emphasized conceptual understanding of the mathematical content, and students and their teachers actively participated in lessons. Student conjectures were characterized by investigating the veracity of particular statements. Teachers and students discussed connections among mathematical ideas, or teachers clearly explained such connections. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Teachers' interactive decisions were characterized by less attention to teaching for understanding than at Level 6. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools such as fraction bars and focused students' attention on pertinent elements of tasks. Their questioning techniques focused on articulation of student thinking, understanding mathematics, or reasonableness of solutions. They added a different context, a review lesson, or explanations that promoted connections among mathematical ideas or between mathematics and students' lives. Teachers were more likely to modify a lesson based on a student's statement or inquiry than at Level 4.

Teachers' classroom assessment practices were characterized by accepting students' explanations as evidence of their learning. Some teachers orchestrated discussions around students' solutions to learn about their procedural competence. Other teachers sought evidence of student learning through students' verbal or written communication of mathematical processes. These teachers used students' misconceptions to guide instruction through the use of probing questions, counterexamples, and alternative representations. Feedback was an ongoing, purposeful, and shared responsibility of both teachers and students. Feedback promoted making sense of mathematical tasks, students' solutions, and mathematical conventions.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Occasionally, students discussed procedures or asked each other for clarification of procedures demonstrated by their teachers. Although seated in groups, students generally worked at different paces on the assigned work. Most students were on task, seriously pursuing the substance of the lesson.

Level 4: Attempt to Teach Mathematics for Understanding

Teachers attempted to teach for conceptual understanding, but the primary focus of lessons was on building students' procedural understanding. On some occasions, students were asked if different strategies were used in solving particular problems, but this was not a primary goal of instruction. Although students were allowed to find their own solution strategies, they generally used a procedure or strategy presented by the teacher. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Interactive decisions were characterized by greater attention to standard pedagogy and teaching for understanding than at Level 3. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools, included additional exercises based on student interest or need for practice, or focused students' attention on pertinent

elements of tasks. They added minilessons on algorithms or procedures, a different context, or a review lesson. Teachers also asked students about the reasonableness of their solutions. At times, interactive decisions involved the introduction of alternative strategies by teachers or students.

Teachers' classroom assessment practices were characterized by gathering evidence from student explanations as evidence of their skills and procedures. Teachers elicited students' solution strategies and orchestrated whole-class discussions around them, but the primary goal of their assessment practice remained demonstration of procedural competence. Student misconceptions were rarely used as opportunities for instruction. Feedback students received from other students rarely went beyond sharing answers and procedures. Teachers' feedback included attention to mathematical concepts or the contexts in which problems were situated.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Although seated in groups, students worked at different paces on the assigned work. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Teachers' planning for teaching instructional units or chapters was consistent with Level 5.

Level 3: Limited Attention to Conceptual Understanding

Inquiry during class provided limited attention to conceptual understanding of the mathematical content. The main focus of lessons was on building students' procedural understanding. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Interactive decisions were more reflective of good standard pedagogy than at Level 2. Teachers emphasized understanding of mathematical vocabulary and at times used questioning techniques that focused on articulation of thinking, understanding of mathematics, or the reasonableness of solutions. Occasionally, interactive decisions involved shifting students' attention from small-group work to whole-class discussion. Teachers at times added a different context or a review lesson, or modified a lesson based on a student's statement or inquiry.

Teachers' classroom assessment practices were characterized by gathering evidence from homework and classwork to substantiate students' procedural competence. Occasionally, teachers included student explanations as evidence of their skills and procedures, but explanations were generally elicited to generate communication in class rather than as an avenue for assessment of student understanding or as a basis for substantive mathematical conversation. Teacher-directed feedback was indirectly responsive to student needs in that it involved additional whole-class instruction using the same presentation method or the assignment of additional exercises of the same type. For some teachers, feedback was directed toward correct procedures and the format of the answers such as simplifying fractions. The feedback of other teachers, however, included attention to mathematical concepts or the contexts in which problems were situated. Their feedback was mathematically sound and clear to students.

Level 2: Focus on Procedures

The focus of lessons was on particular procedures or strategies, and students were expected to use the methods presented by their teachers. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences. Teachers' interactive decisions were predominantly less aligned with teaching for understanding. Limited changes were made in instruction even when teachers or students experienced confusion or misunderstanding.

Teachers' classroom assessment practices were characterized as gathering evidence from homework and classwork to determine students' procedural competence. Teacher-directed feedback was indirectly responsive to student needs and was directed toward correct procedures and the format of the answers. Student–student conversation, students' working relationships, students' engagement in lessons, and teachers' planning for teaching instructional units or chapters were generally consistent with Level 3.

Level 1: Underdeveloped Lessons

Mathematics was presented in ways that gave students only a surface treatment of the content. Inquiry during class was limited to lower order thinking, and lessons did not promote conceptual understanding. Some teachers devoted a major portion of the class period to review of a previous lesson or homework. The subsequent lesson presentation was not well developed; consequently, students began independent work or small-group work with little direction. Teachers assisted individuals or groups of students on a one-to-one basis during independent work or small-group work. Teachers presented particular procedures or strategies, and students were expected to use those methods. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences. Teachers' interactive decisions were least likely to support students' learning mathematics with understanding. Teachers' questions focused on following particular step-by-step procedures, and their own explanations were preferred over student explanations. No changes were made during instruction to address student questions, difficulties, or unexpected strategies.

Teachers' classroom assessment practices were characterized by gathering evidence from homework and classwork to determine students' procedural competence. Teacher-directed feedback was indirectly responsive to student needs. Individualized feedback occurred as teachers responded to questions from specific students during independent seatwork. Feedback provided by students was minimal or nonexistent. For some teachers, feedback was not attentive to student misconceptions, was misleading, or lacked mathematical substance. For other teachers, feedback was directed toward correct procedures and the format of answers rather than to the development of mathematical understanding.

Conversation among students was not encouraged. When students did talk with one another, the conversation was social in nature. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Planning for teaching instructional units or chapters was consistent with Levels 2 and 3.

Analysis of Variance

Analysis of the variance was completed to check for significant differences among teachers by grade level, curriculum taught, and district. The Instruction Total was used as the dependent variable. Results suggest that 71% of the variance in the Instruction Total was accounted for by differences in grade level, curriculum taught, and district (see Table E7). The results show that there is an effect for curriculum, grade, and district.

Table E7.
ANOVA with Instruction Total as the Dependent Variable

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	6425.05	1070.84	33.76	<.0001
Error	80	2537.28	31.71		
Corrected Total	86	8962.33			

R-Square	Coeff Var	Root MSE	Instruction Total Mean
0.71	15.47	5.63	36.38

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Curriculum Taught	2	4317.18	2158.59	68.06	<.0001
Grade Level	3	424.17	141.39	4.46	0.006
District	1	306.15	306.15	9.65	0.002

When the contrasts were examined (see Tables E8 and E9), there were significant differences between the means of teachers using MiC and teachers using conventional curricula, between Grade 5 and the other grade levels, and between districts. The results suggest that students experienced significantly different instruction when studying MiC than when studying conventional curricula, in fifth-grade classrooms than in middle-school classrooms, and in District 2 than in District 1.

Table E8.
Least Squares Means for Curriculum Taught, Grade Level, and District

Least Squares Means	
Curriculum Taught	
MiC	40.61
Conventional	31.20
Grade Level	
Grade 5	41.86
Grade 6	35.07
Grade 7	36.46
Grade 8	35.44
District	
District 1	35.27
District 2	39.15

Table E9.
Contrasts in Means for the Instruction Total by Curriculum Taught, Grade, and District

Parameter	Standard Estimate	Error	t Value	Pr > |t|
		Curriculum Taught		
MiC vs Conventional	9.41	1.86	5.05	<.0001
		Grade Level		
Grade 5 vs Grade 6	6.79	1.98	3.43	0.001
Grade 5 vs Grade 7	5.40	1.90	2.83	0.005
Grade 5 vs Grade 8	6.42	2.05	3.13	0.002
Grade 6 vs Grade 7	-1.39	1.53	-0.91	0.364
Grade 6 vs Grade 8	-0.37	1.74	-0.22	0.830
Grade 7 vs Grade 8	1.02	1.65	0.62	0.538
		District		
District 1 vs District 2	-3.87	1.24	-3.11	0.002

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Appendix F

Instruction for 1999-2000

Grade 7

Grade 7

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the 19 indices, grouped by major categories, and the composite index for instruction, are described and illustrated with evidence from classroom observations, teacher interviews, teaching logs, and teacher journal entries. The ratings assigned to each teacher in Districts 3 and 4 on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in Districts 3 and 4. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 1

In District 1, five Grade 7 teachers participated in the study. Four teachers used MiC, and one teacher used the conventional curriculum already in place in his school. One MiC study class had two teachers over the course of the school year, Lawton and Botkin.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

St. James generally followed the recommended sequence of MiC units, although the sequence of units he taught was influenced by content students needed for eighth grade and for district and state standardized tests. When planning to teach a MiC unit, he read through the unit to become familiar with its content and the number and level of difficulty of the problems. St. James divided the unit into weekly sections and anticipated adjusting the suggested pacing to accommodate students' learning. St. James determined students' prior knowledge in several ways: pretests, questioning, writing in class and his general understanding of the content students had in the past year. Even though St. James thought that the district standardized tests reflected MiC content, he planned review using commercial materials during the week prior to the tests (St. James, Interview 5/8/00).

Table F1
Subcategories of Unit Planning: St. James, Addams Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Data on unit planning for Lawton for 1999-2000 were unavailable. Botkin replaced Lawton for the second semester of the school year. Botkin generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she worked through the unit, organized the needed materials, and made any models that students were to construct. Botkin identified difficult problems, those that were essential for conceptual understanding, and those that could be used for homework. She did not formally assess students' prior knowledge, but observed the students' level of conceptual understanding and skill development as they worked through the unit. When deficiencies arose, Botkin provided instruction, review, and/or practice. She planned to use MiC ancillary materials *(Number Tools)* and student activity sheets that accompanied MiC units. Botkin generally followed the suggested pace of instruction. She thought MiC was well aligned with state standards and that district standardized tests reflected MiC content (Botkin, Interview 5/3/00).

Table F2
Subcategories of Unit Planning: Botkin, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Muldoon generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the entire unit, making special note of the Hints and Comments column. Muldoon gathered information about students' prior mathematics experiences by asking the sixth-grade teachers about the MiC units and other topics they had taught the previous year. She used supplementary activities to provide instruction, review, and/or practice of needed concepts or skills. This was the second year Muldoon taught MiC, but she was not confident in determining unit pacing prior to instruction: "I'm trying to get through as many of these books as possible without going too fast so that [students] are frustrated. I stop where I need to, get something from someplace else, practice whatever they need to, and then go through [the units] as fast as [students] can handle it" (Muldoon, Interview 5/3/00). Muldoon thought MiC was well aligned with the state standards. She did not consider district standardized tests when she planned (Muldoon, Interview 5/3/00).

Table F3
Subcategories of Unit Planning: Muldoon, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Hodge developed a sequence of units based on the NCTM *Standards*, district curriculum guidelines, and district standardized tests. He planned units using conventional textbooks, commercial resource materials, and teacher-made materials. Hodge's unit plans included an overview or a general idea of the topic for students, an outline of specific topics, and a timeline for teaching. He noted how he could help students make connections among math topics and between specific topics and students' lives. He also developed probing questions to help students explain their thinking. Hodge anticipated modifying the unit plan to accommodate students' learning (Hodge, Interview 5/9/00).

Table F4
Subcategories of Unit Planning: Hodge, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

In summary, the four seventh-grade teachers in District 1 varied minimally in two of the three subcategories of *unit planning, prior knowledge* and *unit sequence* (see Figure F1). Three teachers based unit planning on their perceptions of students' mathematics skills and reading ability. One teacher planned units with little or no understanding of the students' prior knowledge.

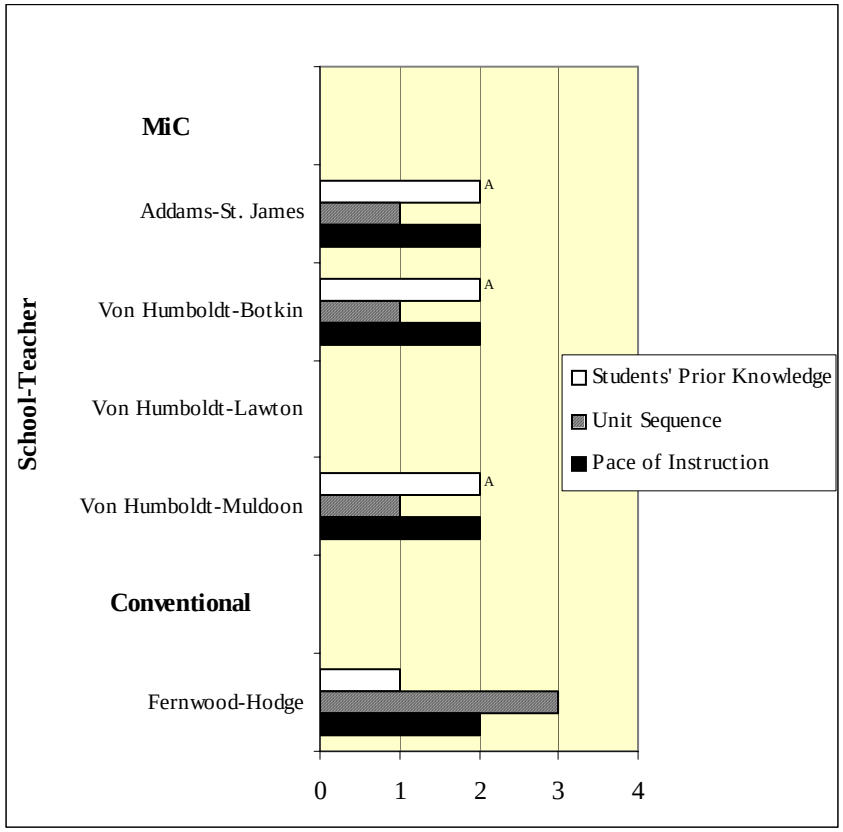


Figure F1. Unit planning, Grade 7, District1.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table F48 in this appendix.

In determining the *sequence of instructional units*, three teachers followed the unit sequence recommended in teacher support materials. One teacher considered content and students interests. In determining the *pace for instruction*, all of the teachers anticipated various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction. No data on unit planning for Lawton were available.

Lesson Planning

In this study, four subcategories of I*nstruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, St. James worked through enough problems to get an idea of the lesson content and anticipated the amount of time needed for each problem. St. James did not refer to the teacher's guide for alternative solution strategies, but often asked his daughter, a high school freshman, his son, a university math-ed student, and his wife, who had studied English, to work through the problems. This process provided different points of view for alternative solutions. He thought of ways to help keep whole-class discussions focused on the mathematics, not just on the context. St. James began each day with a brief review of the previous day's lesson. If he sensed students did not understand the previous lesson, he provided review. Throughout the new lesson, St. James linked new concepts and/or skills to those in previous lessons. St. James thought students benefited from small-group work when the teacher was unavailable and when students might learn different solution strategies from each other. He thought students didn't benefit from small- group work when the group did not stay focused on the mathematics. St. James preferred whole-class direct teaching and students working in pairs, striving to use each format about half of the class time. He explained:

> I like instructing. I like instructing with pairs because if I say something, students can ask the person next to them if they missed it. Unfortunately, I end up changing from pair work a lot, because it looks like to me when I'm observing them, they're not doing what they're supposed to do. They're kind of having a good time. (St. James, Interview 5/8/00)

St. James did not plan for whole-class discussion, but when it happened, it was teacher led (St. James, Interview 5/8/00).

Two instructional formats were important elements in St. James' teaching: whole-class discussion (on 60% of the reported days) and review of previous material (58%). Both were given similar amounts of class time: equal emphasis with other instructional forms on nearly 70% of the reported days and at least half of the class period on about 25% of the reported days. Teacher presentation (36%) and small-group work (28%) were also given equal emphasis with other instructional forms on 70% of the reported days. Warm-up activities (24%) and independent practice (11%) were given equal emphasis with other instructional forms on three fourths of the reported days (St. James, Teacher Log 1999-2000).

St. James' students frequently engaged in five activities: participating in whole-class discussion (63% of the reported days), discussing answers and solution strategies (62%), reflecting on or summarizing lessons (61%), investigating problems (52%), and listening to the teacher or taking notes (47%). These student activities were given equal emphasis with other student activities on nearly 80% of the reported days. In contrast, students infrequently practiced computation (19%), began homework (11%), and took a test or quiz (8%; St. James, Teacher Log 1999-2000). In general, observation reports completed during 1999-2000 classes supported the information St. James reported in his teacher logs. The lesson observed on 5/15/00, for example, included: housekeeping duties (3 minutes); large group, teacher reviewed previous material (5 minutes); individual work (2 minutes); large group, teacher-led discussion (13 minutes); individual work (7 minutes); large group, teacher-led discussion (8 minutes); and large group, homework assignment (3 minutes; St. James, Observation 5/15/00).

Table F5
Subcategories of Lesson Planning: St. James, Addams Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

Data on lesson planning for Lawton for 1999-2000 were unavailable. Botkin replaced Lawton for the second semester of the school year. When planning to teach individual MiC lessons, Botkin sorted lesson problems into those suitable for class work and homework. She adjusted her daily plans if the lesson took longer than anticipated by carrying it over to the next day. She added re-teaching, review, and/or practice if the majority of students exhibited difficulties with the concepts or skills. If part of the class needed to work at a slower pace and with more direction than the rest of the class, Botkin separated the students into two groups. One group went on with the lesson while she worked with the other group. The next day she provided additional related work for the group who completed the previous lesson while the group that had needed more direction completed the lesson. Botkin valued small-group work because it gave students opportunities to talk about their solution strategies and to help each other. She preferred small groups of four because more points of view and more conversation could be generated. Also because there were fewer groups, she would have more time to work with each group. During whole-class teacher-led discussions, only the honors students readily explained their solution strategies or justified their thinking. The rest of the students wanted to give short answers and resisted elaborating (Botkin, Interview 5/3/00).

On approximately one third of the reported days, Botkin incorporated into her teaching three instructional forms: whole-class discussion (on 33% of the reported days), teacher presentation (30%), and review of previous material (30%). Whole-class discussion was given more class time: at least half of the class period on 40% of the reported days, equal emphasis with other instructional forms on 40% of the reported days, and less than 15 % of class time on 20% of the reported days. Teacher presentation was used at least half of the class period on 11% of the reported days, equal emphasis with other instructional forms on 44% of the reported days, and less than 15% of class time on 44% of the reported days. In contrast, review of previous material was given the least amount of class time, frequently less than 15% of class time. Botkin seldom used small-group work (17%), independent practice (10%), and warm-up activities (7%; Botkin, Teacher Log 1999-2000).

Botkin's students participated in whole-class discussion (on 37% of the reported days) more frequently than other student. Whole-class discussions were given equal emphasis with other student activities on 82% of the reported days. On approximately one fourth of the reported days, students were engaged in four student activities: listening to the teacher or taking notes (27%), investigating problems (27%), discussing answers and solution strategies (23%), and reflecting on or summarizing lesson concepts (23%). Of the four activities, discussing answers and solution strategies was given the most class time: at least half of the class period on 29% of the reported days and equal emphasis with other student activities on 71% of the reported days. Investigating problems was given at least half of the class period on 13% of the reported days, equal emphasis with other student activities on 75% of the reported days, and less than 15% of class time on 13% of the reported days. Listening to the teacher or taking notes was given at least half of the class period on 13% of the reported days, equal emphasis with other student activities on 38% of the reported days, and less than 15% of class time on 50% of the reported days. Reflecting on or summarizing lesson concepts was given equal emphasis with other student activities on 86% of the reported days and less than 15% of class time on the remaining days. Students seldom took a test or quiz (7%), practiced computation (3%), and began homework (3%; Botkin, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Botkin reported in her teacher logs. The lesson observed on 5/18/99, for example, included: small-group work (15 minutes); teacher presentation (5 minutes); small-group/pairs work (14 minutes); large-group discussion (8 minutes); teacher presentation (15 minutes); small-group/pairs work (5 minutes); and large group, clean up (2 minutes; Botkin, Observation 5/18/00).

Table F6
Subcategories of Lesson Planning: Botkin, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

Interview data on lesson planning for Muldoon for 1999-2000 were incomplete. The following is a summary of her comments on the 1998-1999 interview with respect to lesson planning. When planning to teach individual MiC lessons, Muldoon looked over the notes she made during unit planning, which included the thrust of the lesson, possible problems with format or language, sections with unclear directions, and concepts that needed to be further developed. She considered the students' performance on the previous day's lesson, noting whether she needed to re-teach or

review a particular concept or skill or skip repetitious activities. Muldoon thought it was valuable for students to work in small groups, especially when the lesson required physical activity. However, she thought small groups were not valuable when “the work is over [students’] heads. They’ll start fooling around. They can’t get it. They’re just not interested. Then the groups are useless at that point” (Muldoon, Interview 4/15/99). When Muldoon planned for group work, she determined the group membership, the directions and expectations, and consequences for not following directions. Muldoon recognized that in order to have successful group work, each group needed a leader, an initiator. She explained, “Then I could put one kid in each group and there would be somebody that could at least initiate an idea” (Muldoon, Interview 4/15/99). Because she taught students with average or below average math skills, she felt that it was difficult to find enough group leaders (Muldoon, Interview 4/15/99).

Two instructional forms that were often incorporated into Muldoon’s teaching, review of previous materials, 72% of the reported days, and warm-up activities, 67% of the reported days, were given less class time than other instructional forms. Review of previous materials was given equal emphasis with other instructional forms on 23% of the reported days and less than 15% of class time on 77% of the reported days. Warm-up activities were always given less than 15% of class time. Whole-class discussion and teacher presentation were each used on 61% of the days. However, whole-class discussion was given more class time: at least half of the class period on 18% of the reported days, equal emphasis with other instructional forms on 64% of the reported days, and less than 15% of class time on the remaining days. Teacher presentation was usually given less than 15% of class time (73% of the reported days). Independent practice was used on 50% of the days and was given a moderate amount of class time: at least half of the class period on 11% of the reported days, equal emphasis with other instructional forms on 44% of the reported days, and less than 15% of class time on 44% of the reported days. Small-group work was used on only 38% of the days, but was given a significant amount of class time: at least half of the class period on 58% of the reported days, equal emphasis with other instructional strategies on 14% of the reported days, and less than 15% of class time on 29% of the reported days (Muldoon, Teacher Log 1999-2000).

On approximately two thirds of the reported days, Muldoon’s students discussed answers and solution strategies (67%), investigated problems (61%), participated in whole-class discussions (61%), and listened to their teacher or took notes (61%). Of these four student activities, investigating problems was given the most class time: at least half of the class period on 36% of the reported days, equal emphasis with other student activities on 27% of the reported days, and less than 15% of class time on 36% of the reported days. Participating in whole-class discussion was given equal emphasis with other student activities on 64% of the reported days and less than 15% of class time on 36% of the reported days. Discussing answers and solution strategies and listening to the teacher or taking notes were given similar amounts of class time. Discussing answers and solution strategies was given equal emphasis with other student activities on 33% of the reported days and less than 15% of class time on 67% of the reported days. Listening to the teacher or taking notes was given equal emphasis with other student activities on 36% of the reported days and less than 15% of class time on 64% of the reported days. Students reflected on or summarized lesson concepts on 50% of the reported days, but always for less than 15% of class time. Students practiced computation on 28% of the days. This student activity was given equal emphasis with other student activities on 40% of the reported days and less than 15% of class time on 60% of the reported days. Students took tests or quizzes on only 11% of the days but always for at least half of the class period. Students did not begin homework during class time on any of the reported days (Muldoon, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Muldoon reported in her teacher logs. The lesson observed on 9/23/99, for example, included: housekeeping duties (8 minutes); teacher presentation (10 minutes); small-group work (38 minutes); large group, closure (4 minutes); and large group, review and homework assignment (3 minutes; Muldoon, Observation 9/23/99).

Table F7
Subcategories of Lesson Planning: Muldoon, Von Humboldt Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with the content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual lessons from a conventional textbook, Hodge worked through some of the problems to become familiar with the lesson and to determine possible strategies for helping students with problems that involved difficult skills. For example:

> Last week we were doing area, length times width problems. [Students] were fine if they were given a length of 3 and a width of 2. If I gave them a decimal or gave them a fraction, it was another story. So I showed them how to do the calculation with paper and pencil. I showed them how to do it with a calculator. I showed all kinds of different ways for them to actually get it done. (Hodge, Interview 5/9/00)

Hodge considered the students' performance on the previous lesson. He explained: "I'll always reflect back as to how the lesson the day before ended. That's going to be a big determining factor as to where I start, whether I need to start with a review, spend an extra day on something, or if [students] are ready to go onto the next section" (Hodge, Interview 5/9/00). Hodge preferred whole-class direct teaching, but he sometimes moved from whole-class to small-group work during a lesson. For instance, if he saw that some students experienced difficulty with a skill, he found peer tutoring effective, pairing a capable student with a student who was struggling. Hodge valued small-group work because he didn't want students to become dependent on him for their learning. He might have students explain how they solved a problem or defend their strategies to their partners. The type of activity or the topic influenced his choice of instructional format. Hodge thought activities in statistics and probability were ideally suited for small-group work. But when students were learning specific skills, for instance, with fractions and decimals, he thought small-group work was inappropriate (Hodge, Interview 5/9/00).

Hodge usually began each class period with a warm-up activity (on 93% of the reported days), but gave it less than 15% of class time. Hodge frequently used review of previous material (77%), teacher presentation (73%), and whole-class discussion (68%). Teacher presentation was given the most class time: at least half of the class period on 73% of the reported days and equal emphasis with other instructional forms on 25% of the reported days. Whole-class discussion was given at least half of the class period on 57% of the reported days and equal emphasis with other instructional form on 39% of the reported days. Review of previous material was given less class time: at least half of the class period on 17% of the reported days, equal emphasis with other instructional forms on 50% of the reported days, and less than 15% of class time on 33% of the reported days. Small-group work was used on 31% of the days and was given equal emphasis with other instructional forms on 48% of the reported days and less than 15% of class time on 39% of the reported days. Independent practice was used on only 15% of the days (Hodge, Teacher Log 1999-2000).

Three student activities were important elements in Hodge's instruction: students discussed answers and solution strategies (72%), listened to the teacher or took notes (71%), and participated in whole-class discussions (67%). Each student activity was given significant amounts of class time. Students discussed answers and solution strategies at least half of the class period on 43% of the reported days and with equal emphasis as with other student activities on 54% of the reported days. They listened to the teacher or took notes at least half of the class period on 59% of the reported days and with equal emphasis as with other student activities on 38% of the reported days. Students participated in whole-class discussion at least half of the class period on 54% of the reported days and with equal emphasis as with other student activities on 42% of the reported days. On 47% of the reported days, students investigated problems and reflected on or summarized lesson concepts. Investigating problems was given more class time: at least half of the class period on 54% of the reported days, equal emphasis with other student activities on 11% of the reported days, and less than 15% of class time on 34% of the reported days. Reflecting on or summarizing lesson concepts was given equal emphasis with other student activities on 40% of the reported days and less than 15% of class time on 54% of the reported days. On fewer days, students took a test or quiz (12%) and practiced computation (5%). Hodge's students did not begin homework during class time on any of the reported days (Hodge, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Hodge reported in his teacher logs. The lesson observed on 5/2/00, for example, included: warm-up activity (14 minutes); large group, corrected warm-up problem and homework (18 minutes); large group, teacher-led review of integers, coordinate system, and operation rules with integers (29 minutes); and large group, closure (2 minutes; Hodge, Observation 5/24/00).

Table F8
Subcategories of Lesson Planning: Hodge, Fernwood Middle School, District 1, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the four seventh-grade teachers in District 1 varied minimally on two of the four subcategories of *lesson planning*, *teachers' attention to the purpose of the lesson* and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* (see Figure F2). With respect to *students' performance in the previous lesson*, all teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Little variation was seen in teachers' attention to the *purpose of the lesson*. Three teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. One teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.

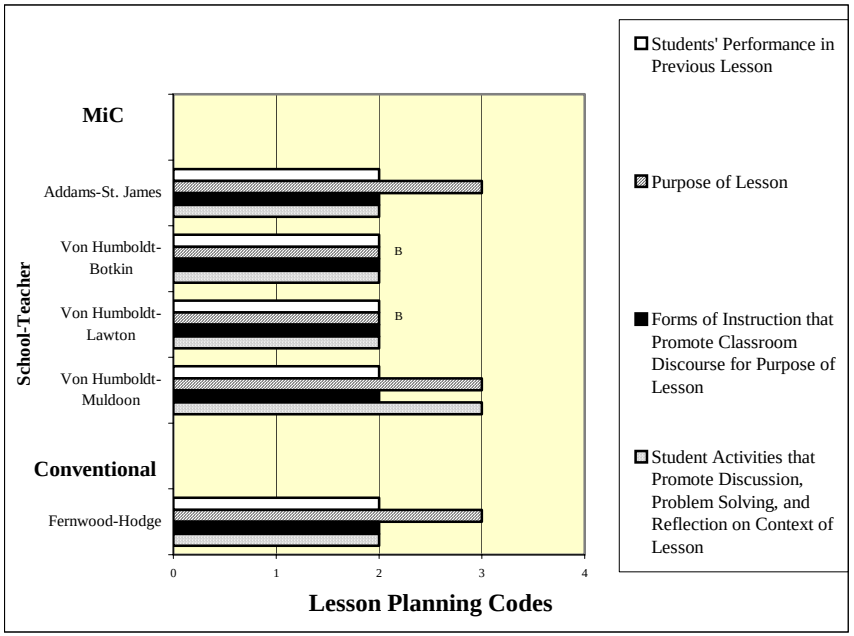


Figure F2. Lesson planning, Grade 7, District 1.

Lesson Planning Codes*

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F49 in this appendix.

Teachers did not vary in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. All teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Teachers varied minimally in the fourth subcategory of *lesson planning, students activities that promote discussion, problem solving, and reflection on he content of the lesson.* For one teacher, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. The other three teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

St. James. The evidence gathered for lesson presentation and development for St. James were all at 2B, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table F50 in this appendix). An overall rating of 1 was assigned for St. James for the nature of inquiry that transpired during instruction, indicating inquiry was limited to lower-level thinking (see Table F51). St. James was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure F9). For St. James, all interactive decisions were coded as least aligned with teaching for understanding.

Across all observations for St. James, the mean rating for the index on students' explanations was 1.00, indicating that students only provided answers (see Table F52). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table F53). A rating of 1 was assigned for St. James on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F9
Overall Ratings for Mathematical Interaction for St. James, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.00	Answers only
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Lawton. The evidence gathered for lesson presentation and development for Lawton ranged from Level 3B to 5. An overall rating of Level 3B was assigned, indicating that she demonstrated strategies or procedures that students practiced in rote fashion (see Table F50 in this appendix). An overall rating of 2 was assigned for Lawton for the nature of inquiry promoted limited attention to conceptual understanding (see Table F51). Lawton was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure F10). For Lawton, 75% of the interactive decisions were coded as most aligned with teaching for understanding, and 25% were least aligned with teaching for understanding.

Across all observations for Lawton, the mean rating for the index on students' explanations was 1.67, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies

(see Table F52). The mean rating for elicitation of multiple strategies was 1.33, indicating that multiple strategies were generally not elicited from students (see Table F53). A rating of 1 was assigned for Lawton on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F10
Overall Ratings for Mathematical Interaction for Lawton, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3b	Demonstration of strategy or procedure
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	1.67	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.33	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Botkin. Botkin replaced Lawton during the second semester of the school year. The evidence gathered for lesson presentation and development for Botkin ranged from Level 2A to 5. An overall rating of Level 4 was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table F50 in this appendix). An overall rating of 3 was assigned for Botkin for the nature of inquiry that transpired during instruction, indicating attention to teaching mathematics for understanding (see Table F51). Botkin was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure F11). For Botkin, 20% of the decisions were coded as reflective of good standard pedagogy, 40% were most aligned with teaching for understanding, and 40% were least aligned with teaching for understanding.

Across all observations for Botkin, the mean rating for the index on students' explanations was 2.00, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F52). The mean rating for elicitation of multiple strategies was 1.50, indicating that multiple strategies were generally not elicited from students (see Table F53). A rating of 2 was assigned for Botkin on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F11
Overall Ratings for Mathematical Interaction for Botkin, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to teach for understanding
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	2.00	Focus on procedures
Elicitation of Multiple Strategies	1.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Muldoon. The evidence gathered for lesson presentation and development for Muldoon ranged from Level 2A to 6. An overall rating of Level 3B was assigned, indicating that during lessons, attempts were made to develop a conceptual basis for the mathematical content (see Table F50 in this appendix). An overall rating of 3 was assigned for Muldoon for the nature of inquiry that transpired during instruction, indicating attention to teaching mathematics for understanding (see Table F51). Muldoon was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure F12). For Muldoon, 51% of the decisions were coded as reflective of good standard pedagogy, 13% were most aligned with teaching for understanding, and 38% were least aligned with teaching for understanding.

Across all observations for Muldoon, the mean rating for the index on students' explanations was 2.33, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F52). The mean rating for elicitation of multiple strategies was 1.67, indicating that multiple strategies were generally not elicited from students (see Table F53). A rating of 1 was assigned for Muldoon on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F12
Overall Ratings for Mathematical Interaction for Muldoon, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	3	Attention to teaching for understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	2.33	Focus on procedures
Elicitation of Multiple Strategies	1.67	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Hodge. The evidence gathered for lesson presentation and development for Hodge ranged from Level 2A to 4. An overall rating of Level 3B was assigned, indicating that strategies or procedures were demonstrated and students practiced them in a rote fashion (see Table F50 in this appendix). An overall rating of 2 was assigned for Hodge for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table F51). Hodge was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure F13). For Hodge, 22% of the decisions were coded as reflective of good standard pedagogy, 34% were most aligned with teaching for understanding, and 45% were least aligned with teaching for understanding.

Across all observations for Hodge, the mean rating for the index on students' explanations was 1.44, indicating that on some occasions students only provided answers and on other occasions their explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F52). The mean rating for elicitation of multiple strategies was 1.33, indicating that multiple strategies were generally not elicited from students (see Table F53). A rating of 1 was assigned for Hodge on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F13
Overall Ratings for Mathematical Interaction for Hodge, District 1, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.44	Answers only or focus on procedures
Elicitation of Multiple Strategies	1.33	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the five Grade 7 teachers in District 1 varied from Level 2B to Level 4 in *lesson presentation and development*. One MiC teacher was assigned Level 4, indicating that attempts were made to develop a conceptual basis for the mathematical content. Three teachers (two MiC) were assigned Level 3B, indicating that students practiced the strategy or procedure presented by their teacher in a rote fashion. The remaining MiC teacher was assigned Level 2B, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident.

Table F14
Nature of Mathematical Interaction, Grade 7 Teachers in District 1, 1999-2000

School-Teacher	**Lesson Presentation and Development**	**Nature of Inquiry**	**Teachers' Interactive Decisions**	**Nature of Student Explanations**	**Elicitation of Multiple Strategies**	**Lesson Closure, Reflection, or Summary**	**Total**
			— MiC —				
Addams-St. James	2B	1	1	1.00	1.00	1	7.00
Von Humboldt-Botkin	4	3	3	2.00	1.50	2	11.50
Von Humboldt-Lawton	3B	2	5	1.67	1.33	1	14.00
Von Humboldt-Muldoon	3B	3	3	2.33	1.67	1	14.00
			— Conventional —				
Fernwood-Hodge	3B	2	3	1.44	1.33	1	11.77

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 3. Two MiC teachers were assigned Level 3, indicating that attention was given to conceptual understanding during instruction. Two teachers (one MiC) were assigned Level 2, indicating that

limited attention was given to conceptual understanding during instruction. The remaining MiC teacher was assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to *teachers' interactive decisions*, teachers varied from Level 1 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding. Three teachers (two MiC) were assigned Level 3, indicating that her interactive decisions provided greater attention to good standard pedagogy. The remaining MiC teacher was assigned Level 1, indicating that his interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.00 to 2.33, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 1.67. In general, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, one MiC teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.The remaining four teachers (three MiC) offered few, if any, such opportunities.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought*, *purpose and coherence of feedback*, and *content of feedback*. The seventh-grade teachers in District 1 varied in all three subcategories of *classroom assessment* (see Tables F15-F18). With respect to the evidence sought during classroom assessment, one MiC teacher maintained an underdeveloped process orientation. Two teachers (one MiC) sought procedural competence of student learning. The remaining MiC teachers sought little evidence of student learning. Feedback in the classes of four teachers (three MiC) was very teacher-directed, and no feedback was provided for students in the classes of one MiC teacher. The teacher using a conventional curriculum provided low-level closed feedback. The four MiC teachers provided feedback that allowed students to know whether answers were correct or incorrect.

Table F15
Evidence Sought through Classroom Assessment Practice, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Addams-St. James	1	Limited evidence
Von Humboldt-Botkint	3	Underdeveloped process orientation
Von Humboldt-Lawton	2	Procedural competence
Von Humboldt-Muldoon	1	Limited evidence
Conventional		
Fernwood-Hodge	2	Procedural competence

Table F16
Feedback Coherence and Purpose, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Addams-St. James	1	No feedback
Von Humboldt-Botkin	2	Teacher-directed feedback
Von Humboldt- Lawton	2	Teacher-directed feedback
Von Humboldt-Muldoon	2	Teacher-directed feedback
Conventional		
Fernwood-Hodge	2	Teacher-directed feedback

Table F17
Feedback Content, Grade 7 Teachers, District 1

	Rating	Description
MiC		
Addams-St. James	2	Answer-only feedback
Von Humboldt-Botkin	2	Answer-only feedback
Von Humboldt- Lawton	2	Answer-only feedback
Von Humboldt- Muldoon	2	Answer-only feedback
Conventional		
Fernwood-Hodge	3	Low-level closed feedback

Table F18
Classroom Assessment, Grade 7, District 1

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Addams-St. James	1	1	2	4
Von Humboldt-Botkin	3	2	2	7
Von Humboldt-Lawton	2	2	2	6
Von Humboldt-Muldoon	1	2	2	5
		— Conventional —		
Fernwood-Hodge	2	2	3	7

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence*. Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence*. Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation*. Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation*. The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation*. The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback*. The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback*. Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility*. Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility*. Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment*. The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading*. The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback*. Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback*. In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback*. An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback*. Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for St. James ranged from Level 1 to Level 2 (see Table F54 in this appendix). The mean rating across observations was 1.13, indicating that student–student conversation was not encouraged. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by St. James are described. The observer noted, "None of the students worked together in small groups or in the structure of the large group. All interaction was between the teacher and one student. The students were seated in pairs but they did not discuss mathematics with each other" (St. James, Observation 5/15/00). In this example, students conversed with the teacher, but student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Lawton were all Level 1 (see Table F54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Lawton are described. The observer noted, "Students did not interact with their peers. They only talked to the teacher one to one" (Lawton, Observation 10/26/99). In this example, students conversed with the teacher, but student–student conversation did not take place.

Botkin replaced Lawton for the second semester of the school year. The index ratings about the nature of student–student conversation for Botkin were all Level 1 (see Table F54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Botkin are described. The observer noted, "The students socialized and watched the teacher do work at the overhead. They did not discuss mathematics with each other" (Botkin, Observation 1/13/00). In this example, student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Muldoon ranged from Level 1 to Level 4 (see Table F54 in this appendix). Although the mean rating across observations was 1.78, on six of the nine observations student–student conversation was rated at Level 1, indicating that student–student conversation usually was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Muldoon are described. The observer noted, "The students did not talk to each other about the mathematics in the large group or in small groups. They only talked to the teacher" (Muldoon, Observation 3/28/00). In this example, students conversed with the teacher, but student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Hodge were all Level 1 (see Table F54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred

during a lesson by Hodge are described. The observer noted, “The students did not discuss mathematics with each other. At times, they answered questions from the teacher” (Hodge, Observation 4/17/00). In this example, students conversed with the teacher, but student–student conversation did not exist.

In summary, the mean ratings for *student–student conversation* for the five Grade 7 teachers in District 1 varied from 1.00 to 1.78. While one MiC teacher had a mean rating near 2.00, a rating of 1.00 was given on the majority of observations. Four teachers (three MiC) had mean ratings at or near 1.00. For all five of the Grade 7 teachers in District 1, student–student conversation was usually not encouraged or did not occur.

Collaborative Working Relationships among Students

The index ratings about the nature of students’ collaboration in the classroom for St. James ranged from Level 1 to Level 2 (see Table F55 in this appendix). The mean rating across observations was 1.13, indicating that on most occasions none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students’ collaboration at Level 1, student collaboration that occurred during a lesson by St. James is described. The observer noted, “Even though students were seated together, they did not work collaboratively on anything” (St. James, Observation 12/8/99). In this example, although students physically sat together, they did not work collaboratively to solve problems.

The index rating about the nature of students’ collaboration in the classroom for Lawton during one lesson was at Level 1 (see Table F55 in this appendix), indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. On two of the three observations, no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. Student collaboration that occurred during the lesson was described. The observer noted, “Even though the students were seated in pairs, they did not work together” (Lawton, Observation 11/15/99). In this example, although students physically sat together, they did not work collaboratively.

Botkin replaced Lawton during the second semester. The index ratings about the nature of students’ collaboration in the classroom for Botkin ranged from Level 1 to Level 3 (see Table F55 in this appendix). The mean rating across observations was 2.00, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students’ collaboration at Level 2, student collaboration that occurred during a lesson by Botkin is described. The observer noted, “In the group presentations at the beginning of class, each student had a role explaining the group’s solution to the class. Most of the students during the rest of the class did not work together to solve problems” (Botkin, Observation 5/18/00). In this example, although students physically sat together, there was little exchange of ideas or assistance.

The index ratings about the nature of students’ collaboration in the classroom for Muldoon ranged from Level 1 to Level 4 (see Table F55 in this appendix). Although the mean rating across observations was 2.00, on three of the nine observations no rating for student collaboration was given

because the main purpose of the lesson was to give students needed individual practice or independent work. On three other observations students' collaboration was rated at Level 1, indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Muldoon is described. The observer noted, "Even though the students were seated in pairs, they did no work together" (Muldoon, Observation 11/17/99). In this example, although students physically sat together, they did not work collaboratively.

The index ratings about the nature of students' collaboration in the classroom for Hodge were all Level 1 (see Table F55 in this appendix), indicating that none of the students were working together in small or large-group settings. If students worked in small groups, then one student typically gave answers to other members of the group without explanation. However, on three of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students needed individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Hodge is described. The observer noted, "No one worked in small groups or together in the large group during the class. There was no individual practice. Students did not exchange ideas or solutions to problems" (Hodge, Observation 11/11/99). In this example, students did not work collaboratively to solve problems.

In summary, the mean ratings for *student collaboration* for the five Grade 7 teachers in District 1 varied from 1.00 to 2.00. One MiC teacher had a mean rating of 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. Another MiC teacher had a mean rating of 2.00, but on a third of the observations no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work, and on a third of the observations a rating of Level 1 was given, indicating that on a few occasions a few students shared ideas or discussed how a problem should be solved. Three teachers (two MiC) had mean ratings at or near 1.00, indicating that none of the students were working collaboratively. On several observations, two of these teachers (one MiC) received no rating for student collaboration because the main purpose of the lesson was to give students individual practice or independent work.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for St. James ranged from Level 1 to Level 3 (see Table F56 in this appendix). The mean rating across observations was 1.78, indicating that usually students appeared lethargic and were only occasionally on task carrying out assigned activities. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by St. James is described. The observer noted, "Students listened to the teacher. They were not disruptive and occasionally wrote down something that [St. James] said" (St. James, Observation 3/20/00). In this example, students appeared lethargic and were only occasionally on task carrying out assigned activities.

The index ratings about the extent to which students were engaged during the lesson for Lawton ranged from Level 3 to Level 4 (see Table F56 in this appendix). The mean rating across observations was 3.67, indicating that usually student engagement was widespread with students on task

pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Lawton is described. The observer noted, "The students were very engaged the entire period and were working diligently. They were also enjoying themselves" (Lawton, Observation 9/20/99). In this example, student engagement was widespread with students putting forth much effort.

Botkin replaced Lawton during the second semester. The index ratings about the extent to which students were engaged during the lesson for Botkin ranged from Level 1 to Level 3 (see Table F56 in this appendix). The mean rating across observations was 3.00, indicating that student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Botkin is described. The observer noted, "Many students worked ahead and did not pay attention to what the teacher was trying to cover. They were restless and did not listen well to directions. They were not very serious. However, most of the students did the work that was required of them during class" (Botkin, Observation 5/18/00). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Muldoon ranged from Level 2 to Level 4 (see Table F56 in this appendix). The mean rating across observations was 2.78, indicating that on some occasions student engagement was often sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Muldoon is described. The observer noted, "Some students needed prodding. Some quit before the end of the period. But for the majority of time, students were very involved and interested in answering the questions that had been posed" (Muldoon, Observation 2/11/00). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and dependent on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Hodge ranged from Level 2 to Level 4 (see Table F56 in this appendix). The mean rating across observations was 1.75, indicating that on some occasions student engagement was often sporadic or episodic and on other occasions it was passive and off task. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Hodge is described. The observer noted, "Students were mildly interested in the lesson. They were more interested in certain parts than in others. Students listened some of the time and were very quiet, not reacting enthusiastically to the lesson" (Hodge, Observation 12/15/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during another lesson by Hodge is described. The observer noted, "Most students acted bored and lethargic. They were really not very interested in the lesson. They took no notes and many had their heads down. They were quiet. There was not much enthusiasm in the room" (Hodge, Observation 4/17/00). In this example, many students were either clearly off task or nominally on task for substantial portions of time. They did not seem to put forth much effort.

In summary, the mean ratings for *student engagement in the lesson* for the five Grade 7 teachers in District 1 varied from 1.88 to 3.67. One MiC teacher had a mean rating near 4.00, indicating that student engagement was widespread with students on task pursuing the substance of the lesson

most of the time. Two MiC teachers had mean ratings near 3.00, indicating that usually student engagement was sporadic or episodic. One teacher using a conventional curriculum had a mean rating greater than 2.00, indicating that on some occasions student engagement was sporadic or episodic and on other occasions passive. The remaining MiC teacher had a mean rating near 2.00, indicating that generally students were passively disengaged, appearing lethargic, and only occasionally on task carrying out assigned activities.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table F19
Teacher level of Instruction, Grade7, District 1

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Addams-St. James	2.5	2.5	2	1	1	1.25	1.25	1.67	1	1	2	1.41	1.41	2.23	22.22	**1**
Von Humboldt-Botkin	2.5	2.5	4	3	3	2.50	1.88	3.33	3	2	2	1.25	2.50	3.75	37.21	**3**
Von Humboldt-Lawton	2.5	2.5	3	2	5	2.78	1.66	1.67	2	2	2	1.25	1.25	4.59	34.20	**2**
Von Humboldt-Muldoon	2.5	3.75	3	3	3	3.88	2.09	1.67	1	2	2	2.23	2.50	3.48	36.10	**3**
— Conventional —																
Fernwood-Hodge	2.5	2.5	3	2	3	2.40	1.66	1.67	2	2	3	1.25	1.25	3.20	31.43	**3**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

District 2

In District 2, four Grade 7 teachers participated in the study. Three teachers used MiC, and one teacher used the conventional curriculum already in place in her school.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge, unit sequence,* and *pace of instruction*.

Broughton followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read through the material, determined the worksheets to use, organized manipulatives, and outlined the best way to present the unit. Broughton taught students who were considered at risk with low abilities, so she assumed they had little if any prior knowledge to bring to the unit. She did not determine the pace prior to instruction, but let the students' learning set the pace of instruction. When students needed review or more practice, Broughton used a conventional textbook for supplementary material. Broughton thought MiC was aligned with the state standards, but planned practice for district standardized tests (Broughton, Interview 5/10/00).

Table F20
Subcategories of Unit Planning: Broughton, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	3	Consideration of the needs of current students

Redling generally followed the recommended sequence of MiC units. When planning to teach a MiC unit, she looked through the unit and noted the main points that were assessed at the end of the unit. Because Redling had taught this same group of students the previous year, she was aware of their prior knowledge and considered that in planning the unit. Redling planned to complete one unit each nine weeks. She explained:

> If it's a difficult unit, then the kids take much longer. Like this graphing unit, I find that the students have a good sense of graphing and that we're moving very quickly through this unit whereas a number strand unit at the beginning of the year was very difficult for them. It went slower. (Redling, Interview 3/27/00)

Redling used a pre-algebra book when she planned for a substitute teacher. She chose sections that included those skills students had mastered and could easily do. Redling thought MiC was well aligned with district and state standards (Redling, Interview 3/27/00).

Table F21
Subcategories of Unit Planning: Redling, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Flader developed her own sequence of units, which was influenced by the content of the state standardized tests and students' scores on various tests during the previous year. When planning to teach a MiC unit, she read through the unit and thought about how she wanted to introduce it to students. She explained:

> Initially I think about how to introduce it. Do I want to use a previous unit to link the information to something new? I think in terms of transition. Do I want to make it fun? Do I want to introduce it using something concrete or something abstract? Do I want to just let the students begin the lesson on their own and use a discovery approach? I try to use several different methods so it will appeal to different students, and I try to make it as interesting as possible so that [students] will want to continue learning more about the unit. (Flader, Interview 5/25/00)

Flader assessed students' prior knowledge through warm-up activities and questioning, which influenced the emphasis Flader gave to the unit introduction. Flader used various supplementary materials, such as activities from the NCTM *Standards*, worksheets from conventional textbooks, and workshop materials. She did not determine the pace prior to instruction, but was guided by students' daily performance. Flader thought MiC was well aligned with the state standards. She followed district guidelines for preparing students for the district standardized tests (Flader, Interview 5/25/00).

Table F22
Subcategories of Unit Planning: Flader, Weir Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Friedman developed and sequenced units to coincide with the district curriculum guide, using resources from several conventional mathematics textbooks and other published materials. Students at Von Steuben Elementary School were placed in homogeneous groups by levels (advanced, intermediate, or beginners). Friedman and other teachers used this information as an indication of students' prior knowledge on which to base instruction. Friedman determined the pace of instruction for each unit based on student performance. She explained:

> I explain something new to [students] usually on Mondays. On Tuesdays I also explain a new concept. Now on Wednesdays, with students working in groups on handouts, I make sure that they understand what I explained Monday and Tuesday. So I kind of like let [the students] lead the way. If they understand, I continue. If they don't, then I have to stop and do some more teaching. (Friedman, Interview 4/26/00)

Table F23
Subcategories of Unit Planning: Friedman, Von Steuben Elementary School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the four seventh-grade teachers in District 2 varied in all three subcategories of *unit planning* (see Figure F3). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Three teachers based unit planning on their perceptions of students' mathematics skills and reading ability without informal or formal assessment.

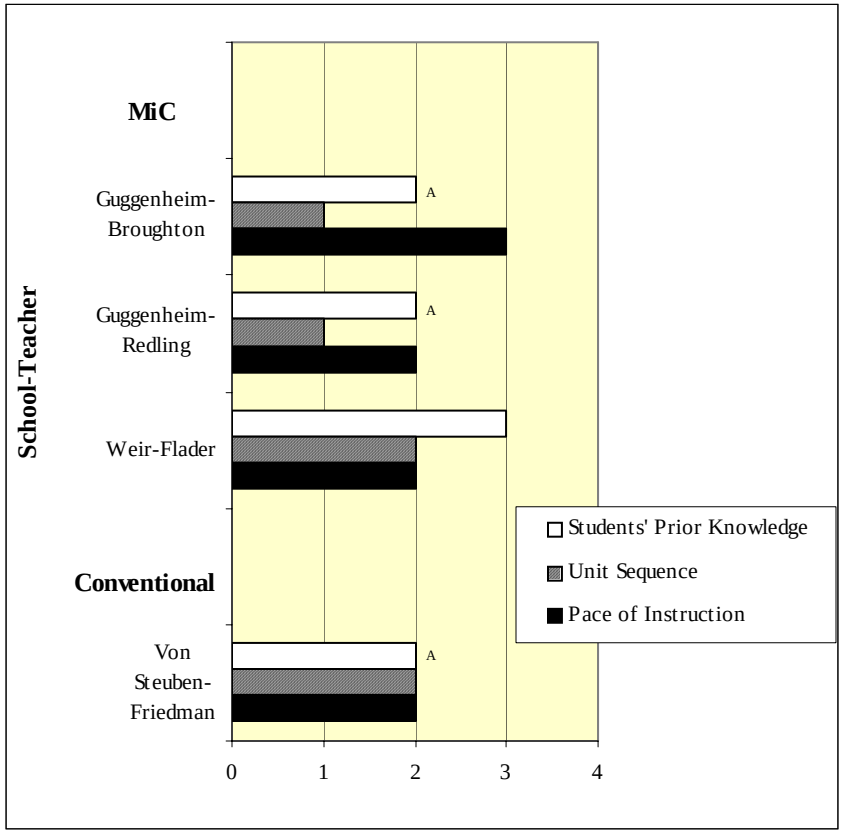


Figure F3. Unit planning, Grade 7, District 2.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table F57 in this appendix.

In determining the *sequence of instructional units*, two teachers considered external factors such as the content of the statewide testing program. Two teachers followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, one teacher considered current students' learning styles and reasoning skills. Three teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

(New grade level for Broughton this year.) When planning to teach individual MiC lessons, Broughton, read over the lesson, the problems, and the solutions. The difficulty and complexity of the problems determined how she divided the section into individual days. She explained:

> Some questions don't require as much as others. When I see a question that requires more thinking into the answer, or more demonstration, more explaining, then I might not ask [students] to do as much. Before I give the assignment, based on how my kids are doing, I decide whether they'd be able to handle much more than a page or two. (Broughton, Interview 5/10/00)

Broughton knew a lot about her students' learning styles and previous math experiences because most of them had been in her mathematics class the previous year. She began each day with a review of the previous day's lesson. Students' performance on the review determined whether she went on with the new lesson or did a more in-depth review. Although Broughton thought students learned from each other when working in small groups, she felt that her students were too playful when they worked together. She commented, "I'm still working with them on that. I worked with them on that last year as sixth graders. I'm seeing a little improvement, but they're still playful" (Broughton, Interview 5/10/00). She didn't plan for small-group work, but allowed it to happen as long as the groups were productive. She found that groups of two or three remained more on task. Some students always chose to work alone. During teacher-led whole-class discussions, few students volunteered to participate, the majority had to be greatly encouraged (Broughton, Interview 5/10/00).

Broughton used various instructional forms: warm-up activities (on 33% of the reported days), teacher presentation (31%), independent practice (30%), small-group work (30%), review of previous material (26%), and whole-class discussion (22%). The amount of class time given to each instructional form varied widely with small-group work and independent practice receiving the most time. Small-group work was given at least half of the class period on 12% of the reported days, equal emphasis with other instructional forms on 73% of the reported days, and less than 15% of class time on 15% of the reported days. Independent practice was generally given equal emphasis with other instructional forms. Teacher presentation, warm-up activities, review of previous material, and whole-class discussions were generally given less than 15% of class time (Broughton, Teacher Log 1999-2000).

Broughton's students investigated problems (on 38% of the reported days) and practiced computation (33%). Practicing computation was given more class time: equal emphasis with other student activities on 86% of the reported days and less than 15% of class time on 11% of the reported days. Investigation of problems was given equal emphasis with other student activities on 58% of the reported days and less than 15% of class time on 36% of the reported days. Broughton's students discussed answers and solution strategies, on 27% of the reported days and listened to the teacher or took notes on 20% of the reported days. Both activities were given less than 15% of class time. Students less frequently participated in whole-class discussions (17%) took a test or quiz (6%), or reflected on or summarized lesson concepts (1%). Broughton's students did not begin homework during class time on any of the reported days (Broughton, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Broughton reported in her teacher logs. The lesson observed on 3/28/00, for example, included: housekeeping and organizational activities (19 minutes); teacher presentation (17 minutes); individual seatwork (20 minutes); large-group discussion (44 minutes); and individual seatwork (12 minutes; Broughton, Observation 3/28/00).

Table F24
Subcategories of Lesson Planning: Broughton, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Redling worked through all of the problems to become familiar with the lesson. Redling felt that she did not need to consider students' performance on the previous lesson because students at Guggenheim Middle School were homogeneously grouped, and Redling's class was composed of advanced mathematics students. Rather, she checked to see how far ahead one of the groups in her class had worked during the class period. Redling valued small-group work because "They get a lot out of helping each other with the problems and it's easier, I think. The students enjoy it and are more motivated to do the lesson too when they get to work in groups. I think they get more out of it by discovering the math themselves rather than the teacher telling them this is how you have to do it" (Redling, Interview 3/27/00).

Because all students were at the same level, Redling didn't plan for small-group work. The physical arrangement of the classroom facilitated students working in small groups. Groups of two or four were most successful and stayed the same all year unless behavior problems developed.

Students' discussions whether in small groups or with the whole class were generally around correct answers. Students explained their strategies when asked and questioned each other if they disagreed (Redling, Interview 3/27/00).

On nearly half of the reported days, Redling used review of previous material (48%), teacher presentation (47%), whole-class discussion (46%), and small-group work (43%). Of these four instructional forms, whole-class discussion and small-group work were given the most class time. Whole-class discussion was given at least half of the class period on 25% of the reported days and equal emphasis with other instructional forms on 75% of the reported days. Small-group work was given at least half of the class period on 30% of the reported days and equal emphasis with other instructional forms on the remaining days. Review of previous material was generally given equal emphasis with other instructional forms. Teacher presentation was given at least half of the class period on 10% of the reported days, equal emphasis with other instructional forms on 66% of the reported days, and less than 15% of class time on 24% of the reported days. Redling used independent practice on 29% of the reported days and for a considerable amount of class time: at least half of the class period on 52% of the days and equal emphasis with other instructional forms on 44% of the days. Redling rarely used warm-up activities (1%; Redling, Teacher Log 1999-2000).

Discussion of answers and solution strategies and participation in whole-class discussions were important elements in Redling's instruction. Students were engaged in each of these activities on nearly half of the reported days: discussion of answers and solution strategies (51%) and participation in whole-class discussion (48%). Each activity was generally given equal emphasis with other student activities. On approximately one fourth of the reported days, students began homework (28%), investigated problems (22%), and listened to the teacher or took notes (22%). Beginning homework was always given equal emphasis with other student activities. Listening to the teacher or taking notes was given at least half of the class period on 31% of the reported days, equal emphasis with other student activities on 42% of the reported days, and less than 15% of class time on 26% of the reported days. Investigating problems was given at least half of the class period on 21% of the reported days and equal emphasis with other student activities on the remaining days. Students less frequently practiced computation (16%), reflected on or summarized lesson concepts (13%), and took a test or quiz (9%; Redling, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Redling reported in her teacher logs. The lesson observed on 10/4/99, for example, included: large group, teacher-led review (9 minutes); small-group/pairs work on summary questions, page 10 (15 minutes); large-group discussion of summary questions (5 minutes); small-group/pairs work, complete page 10 (10 minutes); large-group discussion of small-group work and introduction to next problem (20 minutes); students worked individually on next problem (16 minutes); lunch (41 minutes); and large-group discussion of individual work (28 minutes; Redling, Observation 10/4/99).

Table F25
Subcategories of Lesson Planning: Redling, Guggenheim Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	2B	Overall curriculum in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Flader worked through all of the problems in the lesson and sorted them into those suitable for class work and homework. Flader did not follow the recommended sequence of units. Consequently, students did not always have the prerequisite concepts, skills, and experiences with MiC that the text assumed. As she worked the problems Flader identified the needed prerequisite concepts and skills for each problem. She omitted those problems that required a lot of pre-teaching and noted the ones that needed more introductory preparation. She also anticipated difficulties students might have with the assigned problems and identified strategies for helping students avoid these difficulties. Flader considered students' performance on previous lessons. She explained:

> If I knew there were problems in another unit that built on the previous unit and [students] didn't do well with it, I needed to spend more time reviewing so they didn't get frustrated. A good example is teaching how to solve equations with my eighth graders. I knew they needed to know how to add, subtract, multiply, and divide integers. If they didn't do that well, if they didn't understand that skill, they were going to have problems solving equations. So when I introduced the lessons, I also spent time reviewing adding, subtracting, multiplying and dividing integers over and over again to reinforce those skills. I knew that, even though they understood the concept of how to solve an equation, the computation was going to be wrong. (Flader, Interview 5/25/00)

Flader used whole-class direct instruction when introducing a lesson. Whole-class discussions were teacher led and usually between teacher and student. Flader thought small-group work was very effective with high-ability classes because students could help each other and they could cope with the noise level that accompanied small-group work. She thought lower ability classes had behavior problems, difficulty staying on task, and problems concentrating because of the noise level. She established a rule that students asked each member of the group any questions they might have before coming to her, but that did not work with lower ability classes. When students worked in small groups of four or five, they usually self-selected and reorganized the physical layout of the room (Flader, Interview 5/25/00). Teacher Log data for Flader for 1999-2000 were unavailable.

As an example of a lesson observed in Flader's class, the lesson on 4/27/00 included: housekeeping duties (5 minutes); small-group measurement activity (35 minutes; Flader, Observation 4/27/00).

Table F26
Subcategories of Lesson Planning: Flader, Weir Middle School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual lessons from a conventional textbook, Friedman did not work through the problems because she was very familiar with the curriculum materials. Her lesson planning process was similar to her unit planning process, she chose the topic and divided it into daily lessons, chose and prepared the supplementary materials she wanted to use, and prepared a student overview sheet. If students exhibited difficulties with the concepts or skills during a lesson, Friedman taught previous lessons again or gave students more practice and review work during class. Friedman planned for small-group work at least twice a week because students liked to work in small groups and because students helped each other. She used small groups of two, three, or four students for projects, peer tutoring for students who had been absent, and lesson summary worksheets. Whole-class discussions were usually teacher led and the interactions were usually between teacher and student. Friedman commented: "[Students] will raise their hands and sometimes even go to the board or the overhead projector to do their own drawings. [Students] really like to explain things beyond what they have to do. It's really good" (Friedman, Interview 4/26/00).

Friedman reviewed previous material on 32% of the reported days and independent practice on 29% of the reported days. Review of previous material was generally given equal emphasis with other instructional forms. Independent practice was given at least half of the class period on 42% of the reported days and equal emphasis with other instructional forms on 50% of the reported days. Friedman used three other forms of instruction on a similar number of reported days: whole-class discussion (20%), teacher presentation (20%), and small-group work (17%). Whole-class discussion was given at least half of the class period on 38% of the days and equal emphasis with other instructional forms on the remaining days. Teacher presentation was generally given equal emphasis with other instructional forms. Small-group work was given at least half of the class period on 57% of the days and equal emphasis with other instructional forms on the remaining days. Friedman began class with warm-up activities on 12% of the reported days (Friedman, Teacher Log 1999-2000).

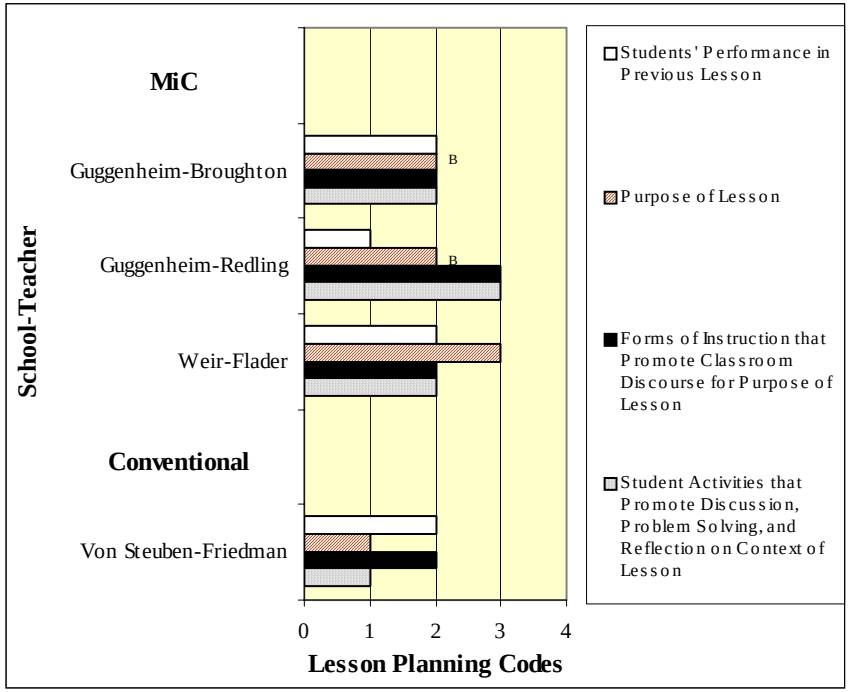
On approximately one fourth of the reported days, Friedman's students listened to their teacher or took notes (27%), discussed answers and solution strategies (24%), participated in whole-class discussions (24%), and took a test or quiz (22%). Of these four student activities, taking tests or quizzes was given the most class time: at least half of the class period on 55% of the reported days and equal emphasis with other student activities on the remaining days. Participation in whole-class discussion was given equal emphasis with other student activities on 70% of the reported days. The other two activities were given similar amounts of class time; generally equal emphasis with other student activities. Students seldom began homework (15%), reflected on or summarized lesson concepts (12%), and practiced computation (5%). Each activity was always given equal emphasis with other student activities. Friedman's students did not investigate problems (Friedman, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Friedman reported in her teacher logs. The lesson observed on 12/3/99, for example, included: housekeeping duties (8 minutes); large-group discussion, 3 problems on previous test (20 minutes); and individual seatwork, test (26 minutes; Friedman, Observation 12/3/99).

Table F27
Subcategories of Lesson Planning: Friedman, Von Steuben Elementary School, District 2, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	1	Little or no planning to teach the specific lesson
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	1	Investigation of problems and discussion of answers and solution strategies seldom in the lesson plan

In summary, the four seventh-grade teachers in District 2 varied in all four subcategories of *lesson planning* (see Figure F4). With respect to *students' performance in the previous lesson*, three teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The other teacher planned in the same manner daily without considering students' previous performance. More variation was seen in teachers' attention to the *purpose of the lesson*. One teacher went beyond checking her own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Two teachers planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context. When the remaining teacher planned, the purpose was to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction was to cover lessons in the textbook or curriculum; thus, no additional planning was deemed necessary.



*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table F58 in this appendix.

Figure F4. Lesson planning, Grade 7, District 2.

Teachers varied minimally in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The other three teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Teachers varied more in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan. Two teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered. The remaining teacher seldom planned for investigation of problems and discussion of mathematical ideas.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Broughton. The evidence gathered for lesson presentation and development for Broughton ranged from Level 2A to Level 5. An overall rating of Level 2B was assigned, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table F59 in this appendix). An overall rating of 3 was assigned for Broughton for the nature of inquiry, indicating attention to conceptual understanding (see Table F60). Broughton was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure F14). For Broughton, 42% of the interactive decisions were coded as reflective of good standard pedagogy, 41% were most aligned with teaching for understanding, and 19% were least aligned for teaching for understanding.

Across all observations for Broughton, the mean rating for the index on students' explanations was2.00, indicating that student explanations were focused on procedures (see Table F61). The mean rating for elicitation of multiple strategies was 2.22, indicating that multiple strategies were rarely elicited from students (see Table F62). A rating of 1 was assigned for Broughton on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F28
Overall Ratings for Mathematical Interaction for Broughton, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2A	Emphasis on review
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	2.00	Focus on procedures
Elicitation of Multiple Strategies	2.22	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Redling. The evidence gathered for lesson presentation and development for Redling ranged from Level 2A to Level 5. An overall rating of Level 5 was assigned, indicating emphasis on conceptual understanding with active participation by students and teacher (see Table F59 in this appendix). An overall rating of 3 was assigned for Redling for the nature of inquiry promoted conceptual understanding (see Table F60). Redling was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure F15). For Redling, 49% of the interactive decisions were coded as most aligned with teaching for understanding, 42% were reflective of good standard pedagogy, and 8% were least aligned with teaching for understanding.

Across all observations for Redling, the mean rating for the index on students' explanations was 2.25, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F61). The mean rating for elicitation of multiple strategies was 2.22, indicating that multiple strategies were generally not elicited from students (see Table F62). A rating of 1 was assigned for Redling on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F29
Overall Ratings for Mathematical Interaction for Redling, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.25	Focus on procedures
Elicitation of Multiple Strategies	2.22	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Flader. The evidence gathered for lesson presentation and development for Flader ranged from Level 2B to 4. An overall rating of Level 2B was assigned, indicating that limited introduction to the lesson, vague direction, or lack of appropriate planning was evident during lessons (see Table F59 in this appendix). An overall rating of 2 was assigned for Flader for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table F60). Flader was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy than at Level 2 (see Figure F16). For Flader, all interactive decisions were coded as reflective of good standard pedagogy.

Across all observations for Flader, the mean rating for the index on students' explanations was 1.00, indicating that student generally provided only answers (see Table F61). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table F62). A rating of 1 was assigned for Flader on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F30
Overall Ratings for Mathematical Interaction for Flader, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	2	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.00	Focus on procedures
Elicitation of Multiple Strategies	1.00	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Friedman. The evidence gathered for lesson presentation and development for Friedman ranged from Level 2A to Level 3B. An overall rating of Level 2A was assigned, indicating that a major portion of the class period was devoted to review (see Table F59 in this appendix). An overall rating of 3 was assigned for Friedman for the nature of inquiry that transpired during instruction, indicating attention to teaching mathematics for understanding (see Table F60). Friedman was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure F17). For Freidman, 18% of the decisions were coded as reflective of good standard pedagogy, 45% were most aligned with teaching for understanding, and 38% were least aligned with teaching for understanding.

Across all observations for Friedman, the mean rating for the index on students' explanations was 2.33, indicating that student explanations were focused on procedures rather than on elaboration of reasoning or strategies (see Table F61). The mean rating for elicitation of multiple strategies was 1.67, indicating that multiple strategies were generally not elicited from students (see Table F62). A rating of 1 was assigned for Friedman on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table F31
Overall Ratings for Mathematical Interaction for Friedman, District 2, Grade 7

Subcategory	Rating	Description
Lesson Presentation and Development	2A	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited attention to teaching for understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.75	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.38	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the four Grade 7 teachers in District 2 varied from Level 2A to Level 5 in *lesson presentation and development* (see Table F32). One MiC teacher was assigned Level 5, indicating that their lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. Two teachers (one MiC) were assigned Level 2A, indicating that a major portion of the class period was devoted to review. The remaining MiC teacher was assigned Level 2B, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident.

Table F32
Nature of Mathematical Interaction, Grade 7 Teachers in District 2, 1999-2000

School-Teacher	Lesson Presentation and Development	Nature of Inquiry	Teachers' Interactive Decisions	Nature of Student Explanations	Elicitation of Multiple Strategies	Lesson Closure, Reflection, or Summary	Total
			— MiC —				
Guggenheim-Broughton	2A	3	4	2.00	2.22	1	14.22
Guggenheim-Redling	5	3	5	2.25	2.22	2	19.47
Weir-Flader	2B	2	3	1.00	1.00	1	10.00
			— Conventional —				
Von Steuben-Friedman	2A	1	3	1.75	1.38	1	10.13

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 3. Two MiC teachers were assigned Level 3, indicating that attention was given to conceptual understanding during instruction. One MiC teacher was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction. The teacher using a conventional curriculum was assigned Level 1, indicating that inquiry was limited to lower order thinking.

With respect to *teachers' interactive decisions*, teachers varied from Level 3 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding, and one MiC teacher was assigned Level 4, indicating that her interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding. Two teachers (one MiC) were assigned Level 3, indicating that their interactive decisions provided greater attention to good standard pedagogy.

The overall means for the *nature of student explanations* ranged from 1.00 to 2.25, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 2.22. In general, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, one MiC teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons. The remaining three teachers (two MiC) offered few, if any, such opportunities.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought*, *purpose and coherence of feedback*, and *content of feedback*. The seventh-grade teachers in District 2 varied in all three subcategories of classroom assessment (see Tables F33-36). With respect to the evidence sought during classroom assessment, two MiC teachers maintained an underdeveloped process orientation. Two teachers (one MiC) sought little evidence of student learning. Students in classes of one MiC teacher experienced emerging shared responsibility for feedback. Feedback in the classes of three teachers (two MiC) was very teacher-directed. One MiC teacher provided mixed, superficial feedback, and two MiC teachers provided low-level closed feedback. The teacher using a conventional curriculum provided feedback that allowed students to know whether answers were correct or incorrect.

Table F33
Evidence Sought through Classroom Assessment Practice, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	3	Underdeveloped process orientation
Guggenheim-Redling	3	Underdeveloped process orientation
Weir-Flader	1	Limited evidence
Conventional		
Fernwood-Hodge	1	Limited evidence

Table F34
Feedback Coherence and Purpose, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	2	Teacher-directed feedback
Guggenheim-Redling	3	Emerging shared responsibility
Weir-Flader	2	Teacher-directed feedback
Conventional		
Von Steuben-Friedman	2	Teacher-directed feedback

Table F35
Feedback Content, Grade 7 Teachers, District 2

	Rating	Description
MiC		
Guggenheim-Broughton	3	Low-level closed feedback
Guggenheim-Redling	4	Mixed, superficial feedback
Weir-Flader	3	Low-level closed feedback
Conventional		
Von Steuben-Friedman	2	Answer-only feedback

Table F36
Classroom Assessment, Grade 7, District 2

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Guggenheim-Broughton	3	2	3	8
Guggenheim-Redling	3	3	4	10
Weir-Flader	1	2	3	6
		— Conventional —		
Von Steuben-Friedman	1	2	2	5

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Broughton ranged from Level 1 to Level 3 (see Table F63 in this appendix). The mean rating across observations was 1.22, indicating that student–student conversation usually was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Broughton are described. The observer noted, "Students were told to work individually on seatwork. During the large-group lessons, students only volunteered to respond to the teacher" (Broughton, Observation 3/28/00). In this example, students conversed with the teacher, but student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Redling ranged from Level 1 to Level 3 (see Table F63 in this appendix). Although the mean rating across observations was 1.67, on six of the nine observations student–student conversation was rated at Level 1, indicating that student–student conversation frequently was not encouraged or did not exist and on the remaining observations it was rated at Level 3, indicating when student–student conversation did occur, it was not substantive in nature. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Redling are described. The observer noted, "A lot of teacher–student conversation happened, but not much student–student interchange" (Redling, Observation 12/6/99). To illustrate a rating at Level 3, student–student conversations that occurred during a lesson by Redling are described. The observer noted, " During small-group time, students discussed and shared ideas. During large-group time, students shared ideas with the class as they volunteered answers to the teacher's questions" (Redling, Observation 3/28/00). In these examples, students on some occasions conversed with the teacher, but student–student conversation did not exist and on other occasions student–student conversation occurred frequently but did not approach substantive discussion of mathematical ideas.

The index ratings about the nature of student–student conversation for Flader ranged from Level 3 to Level 4 (see Table F63 in this appendix). The mean rating across observations was 3.50, indicating that student–student conversation was frequent but was not substantive in nature. To illustrate a rating at Level 3, student–student conversations that occurred during a lesson by Flader are described. The observer noted, "Students did talk in small groups about some of the non-standard measures and about how to use the tape measures" (Flader, Observation 4/27/00). In this example, student–student conversation was not substantive in nature.

The index ratings about the nature of student–student conversation for Friedman ranged from Level 1 to Level 4 (see Table F63 in this appendix). Although the mean rating across observations was 2.00, on five of the eight observations student–student conversation was rated at Level 1, indicating that student–student conversation usually was not encouraged or did not exist. To illustrate a rating at Level 1, student–student

conversations that occurred during a lesson by Friedman are described. The observer noted, “There were teacher–student exchanges only” (Friedman, Observation 2/7/00). In this example, students conversed with the teacher, but student–student conversation did not occur.

In summary, the mean ratings for *student–student conversation* for the four Grade 7 teachers in District 2 varied from 1.22 to 3.50. One MiC teacher had a mean rating greater than 3.00, indicating that student–student conversation occurred frequently but did not approach substantive discussion of mathematical ideas. While two teachers (one MiC) had mean ratings at or near 2.00, the majority of their ratings for student–student conversation were at Level 1, indicating that student–student conversation was not encouraged or did not exist. One MiC teacher had a mean rating near 1.00.

Collaborative Working Relationships among Students

The index ratings about the nature of students’ collaboration in the classroom for Broughton ranged from Level 1 to Level 2 (see Table F64 in this appendix). The mean rating across observations was 1.67, indicating that often few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems at different paces. However, on six of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work. To illustrate a rating for students’ collaboration at Level 2, student collaboration that occurred during a lesson by Broughton is described. The observer noted, “Some students worked together, but most did not. There was no responsibility taken for seeing that other group members were on-task” (Broughton, Observation 12/6/99). In this example, a few students shared ideas, but many of them worked at a different pace.

The index ratings about the nature of students’ collaboration in the classroom for Redling ranged from Level 1 to Level 4 (see Table F64 in this appendix). The mean rating across observations was 2.13, indicating that usually few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems and at different paces. To illustrate a rating for students’ collaboration at Level 2, student collaboration that occurred during a lesson by Redling is described. The observer noted, “Students sat so they could work together, but on the whole, they worked individually except to share materials” (Redling, Observation 9/13/99). In this example, although students physically sat together, few students shared ideas or discussed how a problem should be solved.

The index ratings about the nature of students’ collaboration in the classroom for Flader ranged from Level 3 to Level 4 (see Table F64 in this appendix). The mean rating across observations was 3.50, indicating that on some occasions most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem. On other occasions some students exchanged ideas or provided peer assistance, but all students did not make contributions to problem solving equally. To illustrate a rating for students’ collaboration at Level 4, student collaboration that occurred during a lesson by Flader is described. The observer noted, “Most groups appeared to cooperatively work out problems” (Flader, Observation 4/14/00). In this example, most students were involved with their classmates in solving problems.

The index ratings about the nature of students' collaboration in the classroom for Friedman ranged from Level 1 to Level 4 (see Table F64 in this appendix). The mean rating across observations was 2.60, indicating that on most occasions some students exchanged ideas, or provided assistance to their classmates, but contributions to solving problems were not made equally by all students. However, on three of the eight observations no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work. To illustrate a rating for students' collaboration at Level 3, student collaboration that occurred during a lesson by Friedman is described. The observer noted, "In the last 18-minute segment of the class period, students moved into groups so that someone could help each student who had been absent from class yesterday. Students did help and share" (Friedman, Observation 4/26/00). In this example, some students provided peer assistance.

In summary, the mean ratings for *student collaboration* for the four Grade 7 teachers in District 2 varied from 1.67 to 3.50. One MiC teacher had a mean rating greater than 3.00, indicating that students were involved in solving problems with their classmates. One teacher using a conventional curriculum had a mean rating approaching 3.00, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. However, on three of the eight observations no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work. Two MiC teachers had mean ratings near 2.00, indicating that few students shared ideas or discussed how a problem should be solved, or that many of the students in a group worked on different problems at different paces. However, for one of them, on six of the nine observations no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Broughton ranged from Level 3 to Level 4 (see Table F65 in this appendix). The mean rating across observations was 3.11, indicating that on most occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Broughton is described. The observer noted, "Prodding was used to keep students on task. Most of them finished the worksheet from the first lesson on algebraic expressions but didn't do well on the problem assigned for the second lesson (MiC lesson on justifying formulas for finding total number of rods in a beam)" (Broughton, Observation 3/28/00). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent and dependent on frequent prodding from the teacher.

The index ratings about the extent to which students were engaged during the lesson for Redling ranged from Level 3 to Level 4 (see Table F65 in this appendix). The mean rating across observations was 3.89, indicating that usually student engagement was widespread with students on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Redling is described. The observer noted, "Students were on task and contributed to class discussions" (Redling, Observation 11/2/99). In this example, students were on task pursuing the substance of the lesson most of the time.

The index ratings about the extent to which students were engaged during the lesson for Flader ranged from Level 3 to Level 4 (see Table F65 in this appendix). The mean rating across observations was 3.50, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time, and on other occasions sporadic or episodic. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Flader is described. The observer noted, "Most students were on task. Snags caused progress to slow down before [Flader] wrote the definitions of fathom and pace on the board" (Flader, Observation 4/14/00). In this example, student engagement was widespread. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during aother lesson by Flader is described. The observer noted, "Most students had been on task,but progress was minimal" (Flader, Observation 4/27/00). In this example, student engagement was sporadic or episodic.

The index ratings about the extent to which students were engaged during the lesson for Friedman ranged from Level 3 to Level 4 (see Table F65 in this appendix). The mean rating across observations was 3.88, indicating that usually student engagement was widespread with students on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Friedman is described. The observer noted, "Students were so busy that they really were not aware when the period had ended. The school counselor had to bring it to their attention" (Friedman, Observation 4/11/00). In this example, student engagement was widespread.

In summary, the mean ratings for *student engagement in the lesson* for the four Grade 7 teachers in District 2 varied from 3.11 to 3.89. Two teachers (one MiC) had mean ratings near 4.00, indicating that student engagement was widespread with students on task pursuing the substance of the lesson most of the time (see Figure X). Two MiC teachers had mean ratings greater than 3.00, indicating that student engagement was sometimes widespread but generally sporadic or episodic and often dependent on frequent prodding from the teacher.

***Instruction* Composite Variable**

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table F37
Teacher Level of Instruction, Grade7, District 2

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Guggenheim-Broughton	2.5	2.5	2	3	4	3.33	2.78	1.67	3	2	3	1.53	2.09	3.89	37.29	**3**
Guggenheim-Redling	3.75	3.75	5	3	5	3.75	2.78	3.33	3	3	4	2.09	2.66	4.86	49.97	**5**
Weir-Flader	2.5	2.5	2	2	3	1.25	1.25	1.67	1	2	3	4.38	4.38	4.38	35.31	**3**
— Conventional —																
Von Steuben-Friedman	2.5	1.25	2	1	3	2.92	1.73	1.67	1	2	2	2.50	3.25	4.85	31.67	**2**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

The ratings assigned to each teacher in this district on each of the indices related to *unit* and *lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 3

In District 3, two Grade 7 teachers participated in the study. Both teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Perry followed the recommended sequence of MiC units. However, she was currently considering reorganizing the sequence to allow some topics to be addressed more fully before the state standardized tests. When planning to teach a MiC unit, she reviewed the unit focus and her notes from the previous three years. Before teaching the unit, Perry prepared students by briefly explaining the prerequisite skills and content they needed and the new things they might encounter. She introduced them to and got them involved with the context. During this discussion, Perry got a general impression of the students' prior knowledge. She provided review, practice, or a new lesson to compensate for any missing concepts or skills. For example: "I was going to teach *Cereal Numbers* when I realized that the kids had not had *More Or Less*. So I had to go back and do *More Or Less* before I did *Cereal Numbers"* (Perry, Interview 5/9/00). Perry was aware of the suggested pace for instruction, but she allowed the majority of the students to set the pace. Students who needed more time or more help attended RAP (remediation and practice class), which was held once a week. Perry thought that MiC was fairly well aligned with the state standards but not with the changes in state standardized tests (Perry, Interview 5/9/00).

Table F38
Subcategories of Unit Planning: Perry, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	3	Consideration of the needs of current students

Schroeder was aware of the recommended sequence of MiC units, but modified it to meet the needs of her students who were in special education: "I have to decide first of all what units [students] do. They don't necessarily do the grade level that the students are in. If they haven't had the previous units, we might go back and do a sixth-grade unit" (Schroeder, Interview 6/19/00). When planning to teach a MiC unit, Schroeder reviewed the unit, especially the section reviews, to become familiar with the content of the unit, unit goals, vocabulary, and kinds of problems. She planned a unit introduction that gave students an overview and practice with problems from MiC ancillary materials (*Number Tools*) that were similar to those in the unit. Schroeder did not determine the pace prior to instruction. Because students' understanding was the priority, students set the pace (Schroeder, Interview 6/19/00).

Table F39
Subcategories of Unit Planning: Schroeder, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	4	Conceptually-based activities planned
Unit Sequence	4	Consideration of content and students interests
Pace of Instruction	3	Consideration of the development of mathematics concepts

In summary, the two seventh-grade teachers in District 3 varied in two of the three subcategories of *unit planning, prior knowledge* and *unit sequence* (see Figure F5). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. The other teacher, a teacher of students in special education, planned unit activities that were designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the context presented in the unit.

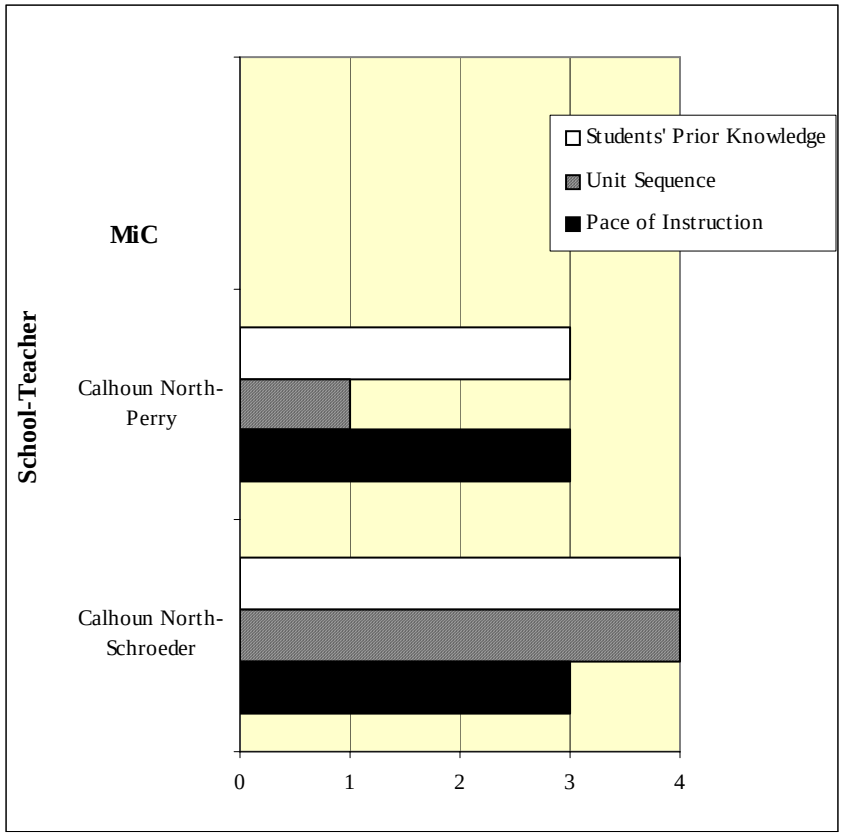


Figure F5. Unit planning, Grade 7, District 3.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

In determining the *sequence of instructional units*, one teacher followed the unit sequence recommended in teacher support materials. The other teacher sequenced units to support the development of mathematics concepts. In determining the *pace for instruction*, both teachers considered students' learning styles and reasoning skills when planning to teach a unit.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Perry read through the lesson and her notes from previous years. She considered students' performance on the previous lessons and determined the problems students should do individually and the ones they should do with partners or in small groups. Perry studied the summary questions, pinpointed certain ones she wanted to stress and/or review, and selected ones that would be worthwhile for students to share with the whole class. Perry valued small-group work. Students in her class knew they could always discuss the problems with a partner. Perry tried to facilitate students working with different partners to get different points of view by frequently changing seats, placing students she thought would be effective partners next to each other. Students had the option of moving to work with a previous partner. Perry thought working with a partner especially benefited students who had lower math abilities. She explained:

> [Students] share ideas and maybe a lower achiever might come up with some brilliant idea and impress somebody else. That raises [his/her] confidence level. I also think it's good for kids to see other kids struggle, to realize that struggling is okay and that we all make mistakes. Also, what [one student] can't think of maybe another can. [Small-group discussions] allow [students] to dig deeper into problems, to think beyond their initial ideas, and to continue this process with other kids. (Perry, Interview 5/9/00)

Perry encouraged student-student interactions during whole-class discussions by asking probing questions and not giving answers (Perry, Interview 5/9/00).

Table F40

Subcategories of Lesson Planning: Perry, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focuses on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

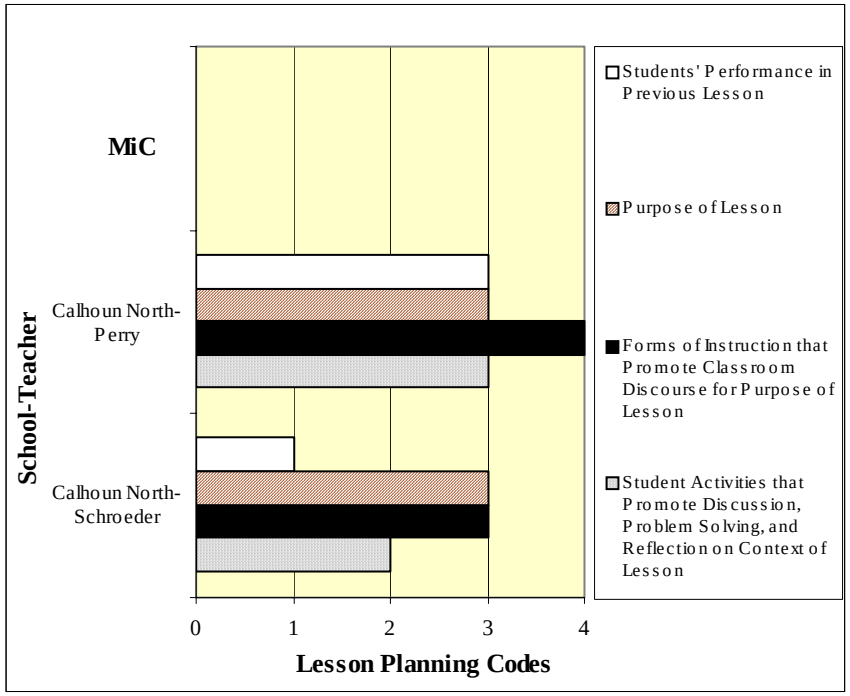
When planning to teach individual MiC lessons, Schroeder, who taught special education students, worked through all of the problems to become familiar with the content of the lesson. Since most of Schroeder's students had difficulties with language and reading, she especially noted the vocabulary and sentence structure and thought of ways to restate the problems without changing their meaning. To help give students a frame of reference, Schroeder prepared a problem that was very similar to the problems in the lesson, which she worked through with them prior to the lesson. To provide students with more practice, Schroeder used problems from MiC ancillary materials (*Number Tools*). Schroeder valued small-group work because it gave students the opportunity to talk about how to do the problem, and it gave her the opportunity to listen to their small-group discussions. She commented: "I can keep track of what [students] are actually doing, because there are only four of them [in a group]. I was observing them every single day, seeing what they were writing, and listening to them" (Schroeder, Interview 6/19/00). Schroeder chose to introduce lessons using whole-class, direct teaching form of instruction. Student participation in the whole-class discussion was limited to their asking questions of the teacher (Schroeder, Interview 6/19/00).

Table F41

Subcategories of Lesson Planning: Schroeder, Calhoun North Middle School, District 3, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the two seventh-grade teachers in District 3 varied in three of the four subcategories of *lesson planning, students' performance in previous lesson, forms of instruction that promote classroom discourse for the purpose of the lesson*, and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* (See Figure F6). With respect to *students' performance in the previous lesson*, one teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. The other teacher planned in the same manner daily without considering students' previous performance. No variation was seen in teachers' attention to the *purpose of the lesson*. Both teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels.



*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure F6. Lesson planning, Grade 7, District 3.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. One teacher planned forms of instruction that promoted substantive conversation, encouraged students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation. The other teacher planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The teachers also varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect or summarize lesson concepts were not included in the lesson plan. The other teacher included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

The ratings assigned to each teacher in this district on each of the indices related to *unit* and *lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 4

In District 4, three Grade 7 teachers participated in the study. All three teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Kane sequenced MiC units by looking for connections among them: "I tried to look at how the units are connected so that I could emphasize connections in class. For example, in *Operations* when we compared the areas of the small figures to the enlarged figures, I told the students to look at it as a ratio. That way when we got into *Ratios and Rates*, they would have heard the language, they would be familiar with it" (Kane, Interview 5/7/99). When planning to teach a MiC unit, Kane read through the unit to become familiar with its content. She then asked students probing questions to determine their prior knowledge about the unit topic, concepts, and skills. She often found that she needed to provide extra skill practice before starting the unit. Students' prior knowledge and the complexity of the unit influenced the pace Kane set prior to instruction. Kane included questions that were worded similarly to those on standardized tests in class discussions, homework, and assessments so the language of the tests would not be foreign to the students (Kane, Interview 6/1/00).

Table F42
Subcategories of Unit Planning: Kane, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Prior knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	3	Consideration of the needs of current students

Lux followed the recommended sequence of MiC units. When planning to teach a MiC unit, she worked through the entire unit, identified the state standards addressed, and prepared an introduction that involved students in the context. Lux generally determined students' prior knowledge during previous lessons and planned to review the skills before beginning the unit. Lux was aware of the suggested pace for instruction, but found

that following a preset pace was difficult: “When I first started using [MiC], I think I was going too fast. When I gave [students] a test, they didn’t know what they were doing. I had to slow down. It depends on the class, too” (Lux, Interview 6/1/00).

Table F43
Subcategories of Unit Planning: Lux, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students’ Prior Knowledge	1	Little or no consideration of students’ prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Woodward generally followed the recommended sequence of MiC units, but modified the sequence to teach content needed for district standardized tests (Woodward, Interview 6/17/99). When planning to teach a MiC unit, Woodward read through the unit to become familiar with the content. He thought being aware of students’ prior knowledge was helpful when planning a unit, but he did not think it was essential information: “It is a concern because obviously [awareness of student’s prior knowledge] does help in knowing whether a child can understand the given content. But it is something that’s not of utmost importance. The child can still get through [the unit] without prior knowledge” (Woodward, Interview 6/19/00). Woodward was aware of the suggested pace for instruction. However, his previous experience teaching MiC helped him determine a more realistic pace. His experience also helped him sort problems into two groups, those that needed to be done during class time and those that could be used for homework. Students in Grade 7 at Kelvyn Park Middle School were required to take many district standardized tests, and Woodward thought supplementary materials were necessary to prepare students for those tests (Woodward, Interview 6/19/00).

Table F44
Subcategories of Unit Planning: Woodward, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students’ Prior Knowledge	1	Little or no consideration of students’ prior knowledge
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

In summary, the three seventh-grade teachers in District 4 varied in all three subcategories of *unit planning* (see Figure F7). With respect to *students' prior knowledge when planning to teach a unit*, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Two teachers planned units with little or no understanding of the students' prior knowledge.

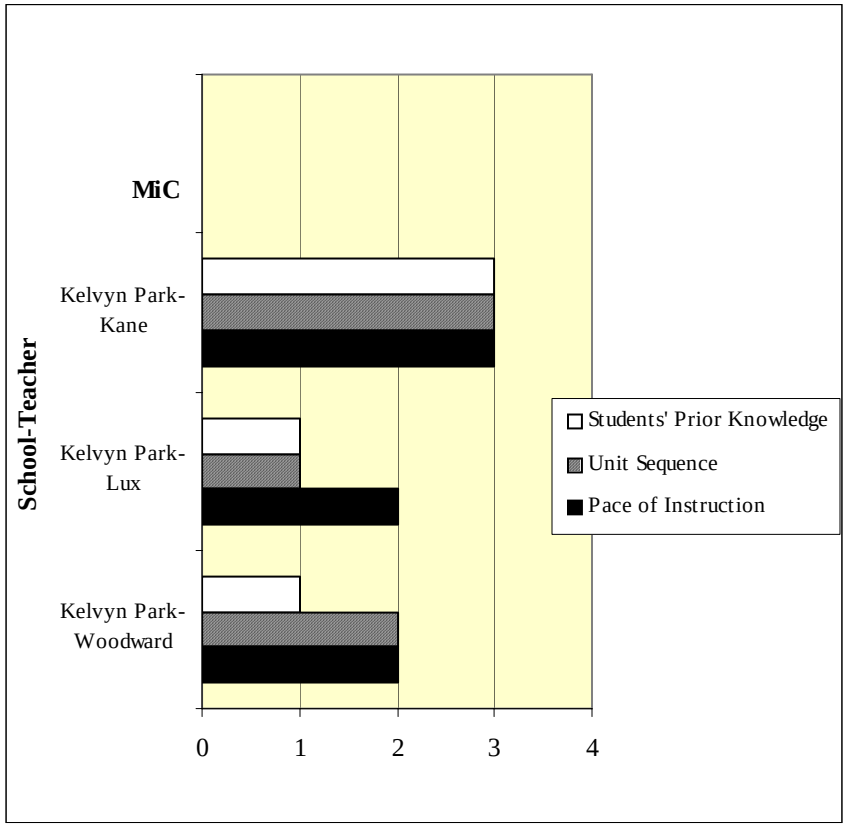


Figure F7. Unit planning, Grade 7, District 4.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

In determining the *sequence of instructional units*, one teacher considered linkages across units of the same content strand. Another teacher considered external factors such as the content of the statewide testing program. The third teacher followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, one teacher considered students' learning styles and reasoning skills. Two

teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of Instruction characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

When planning to teach individual MiC lessons, Kane worked through all of the problems so that she was familiar with the lesson and could anticipate where students might have difficulties. She commented:

> I have to do every problem myself; otherwise, I have no clue what troubles [students are] going to encounter. When I do the problem, I may say, "Oh, [students] may not know how to do that." I'll do a problem as the motivation for the lesson, so I can see whether or not they know it, or maybe if they do, they just need to be refreshed. (Kane, Interview 6/1/00)

Kane considered not only students' performance on previous lessons, but also what she knew about their prior knowledge of a topic or concept. She planned for additional practice if students' performance or prior knowledge indicated weaknesses. She combined or eliminated activities if students' performance or prior knowledge indicated strengths. Kane valued small-group work. She explained:

> Students learn so much more from their peers than they do from the teacher. Sometimes they don't like the teacher. That could be a personality conflict, so they won't allow themselves to be open to an idea or a topic. But, if they're working in small groups, they'll listen to their peers. Second of all, it gives the teacher an opportunity to work with those individuals who need a little bit more explaining or those who are too shy to ask questions or say, "I don't understand what's going on." And I find that it makes teaching much more fun. It makes the whole experience fun. (Kane, Interview 6/1/00)

Kane changed the small groups often because she felt that "You don't really grow if you don't hear other peoples' ideas" (Kane, Interview 6/1/00). She found that three to six students worked well in a group. Kane also led whole-class discussions, especially when introducing a lesson and

following small-group work. The small groups might present their work or give a summary of what they learned or what observations they made. At times, Kane also summarized the lesson (Kane, Interview 6/1/00).

Table F45

Subcategories of Lesson Planning: Kane, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focuses on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	4	Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan

When planning to teach individual MiC lessons, Lux worked through all of the problems in the lesson to anticipate ways students might think about each problem and mistakes they might make, and to think about strategies to deal with mistakes. She also considered whether the class generally needed more practice or if they caught on to new ideas quickly. The type of questions or activities influenced which form of instruction Lux chose. She thought students were less distracted when working in small groups of three or four than when working as a large group. They often strayed from the point when Lux tried whole-class discussions. She planned for small-group work when students needed each other's support to solve the problems. On other occasions, she wanted students to work independently. She commented, "They need to work on their own. That's one of the things that they lack. When they don't work on their own they become too dependent. I'm pretty new to teaching, so I'm trying to figure it all out myself" (Lux, Interview 6/1/00).

Table F46

Subcategories of Lesson Planning: Lux, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

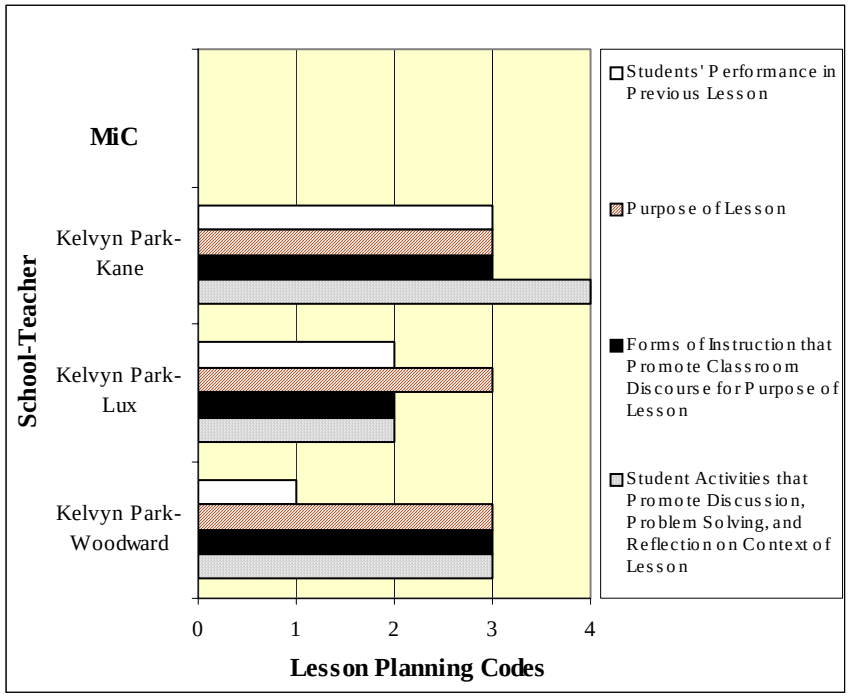
When planning to teach individual MiC lessons, Woodward worked through all of the problems in the lesson in order to understand how the lesson fit within the larger unit and how it was related to previous lessons. He also sorted the problems into two categories, those to be done during class and those that could be used for homework. Woodward planned for small-group work whenever the activity involved an investigation, which was about three or four lessons each week. He thought small-group work was valuable because students learned from each other, were more willing to listen to each other than to the teacher, and benefited from hearing other points of view. He planned heterogeneous small groups of three or four which changed over time. Although each member had a particular responsibility within the group, each member was also responsible for everyone participating and understanding the group's process well enough to be able to report the group's observations and findings to the whole class (Woodward, Interview 6/19/00).

Table F47

Subcategories of Lesson Planning: Woodward, Kelvyn Park Middle School, District 4, Grade 7

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Lesson planning with little or no regard for students' performance on the previous lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

In summary, the three seventh-grade teachers in District 4 varied on three of the four subcategories of *lesson planning, students' performance on previous lesson, forms of instruction that promote classroom discourse for the purpose of the lesson*, and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* (see Figure F8). With respect to *students' performance in the previous lesson*, one teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. Another teacher made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. The remaining teacher planned in the same manner daily without considering students' previous performance. No variation was seen in teachers' attention to the *purpose of the lesson*. All of the teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels.



*Lesson Planning Codes**

Purpose of the Lesson

1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson

1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse

1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection

1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure F8. Lesson planning, Grade 7, District 4.

Little variation was seen in the choice of instructional formats planned to encourage classroom discourse with respect to the purpose of the lesson. Two teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The other teacher included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. The teachers varied more in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution strategies were dominant in the lesson plan. Providing opportunities for students to reflect on or summarize lesson concepts was in important element in instruction. For another teacher, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, questions or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. The remaining teacher included whole-class discussions and small-group work in lesson planning, but the significance of these students activities in learning mathematics with understanding was not considered.

Table F48
Unit Planning, District 1, Grade 7, 1999-2000

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Addams-St. James	2A	1	2	5
Von Humboldt-Botkin	2A	1	2	5
Von Humboldt-Lawton	2A	1	2	5
Von Humboldt-Muldoon	2A	1	2	5
		— Conventional —		
Fernwood-Hodge	1	3	2	6

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
  A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
  B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
  A. The teacher follows the recommendations for pacing in teacher support materials.
  B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table F49

Lesson Planning, District 1, Grade 7, 1999-2000

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Addams-St. James	2	3	2	2	9
Von Humboldt-Botkin	2	2B	2	2	8
Von Humboldt-Lawton	2	2B	2	2	8
Von Humboldt-Muldoon	2	3	2	3	10
			— Conventional —		
Fernwood-Hodge	2	3	2	2	9

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:

1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.

2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.

3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.

4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.

Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:

1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.

2. *Overall curriculum plan in mind.*

A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.

B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).

3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).

4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.

Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:

1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.

2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)

3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.

4. *Forms of instruction that promote substantive conversation planned.* The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.

Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:

1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.

2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.

3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.

4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table F50

Teacher Level of Lesson Presentation and Development, Grade 7, District 1

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Addams-St. James	8	0	0	8	0	0	0	0	0	2B
Von Humboldt-Botkin	4	0	0	1	0	1	1	1	0	4
Von Humboldt-Lawton	3	0	0	0	0	2	0	1	0	3B
Von Humboldt-Muldoon	8	0	1	1	0	3	2	0	1	3B
Conventional										
Fernwood-Hodge	8	0	3	0	0	4	1	0	0	3B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation.* Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy.* A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding.* During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher.* The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking.* Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding.* Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding.* Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics.* The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table F51

Nature of Mathematical Inquiry, Grade 7, District 1

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																							
Addams-St. James	2	1	1	1	2	1	1	N/A	1	1	1	1	1	1	1	1	N/A	1	1	1	1	1	1	1	1	N/A	1	1	1	1	1	3	1	1	N/A	1	*1.13*	***1***
Von Humboldt-Botkin	1	2	4	N/A	4					1	1	2	N/A	4					2	3	2	N/A	4					1	2	2	N/A	1					*2.25*	***3***
Von Humboldt-Lawton	4	2	1							4	1	1							3	1	2							1	1	1							*1.83*	***2***
Von Humboldt-Muldoon	4	4	1	1	4	2	1	2	4	4	4	1	1	1	2	3	1	2	4	2	1	1	3	1	2	1	2	3	2	1	1	1	1	2	1	3	*2.06*	***3***
Conventional																																							
Fernwood-Hodge	4	1	4	4	1	1	1	1	1	2	1	1	1	1	1	1	1	1	2	1	2	1	2	1	1	1	1	2	1	4	1	4	2	1	1	1	*1.58*	***2***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table F52

Nature of Students' Explanations, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-St. James	1	1	1	1	1	1	1	N/A	1	*1.00*
Von Humboldt-Botkin	1	2	3	N/A	2					*2.00*
Von Humboldt-Lawton	2	2	1							*1.67*
Von Humboldt-Muldoon	3	3	3	1	3	1	3	2	2	*2.33*
Conventional										
Fernwood-Hodge	3	2	1	1	1	1	1	1	2	*1.44*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table F53

Elicitation of Multiple Strategies, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-St. James	1	1	1	1	1	1	1	N/A	1	*1.00*
Von Humboldt-Botkin	1	3	1	N/A	1					*1.50*
Von Humboldt-Lawton	2	1	1							*1.33*
Von Humboldt-Muldoon	1	4	2	1	2	1	1	2	1	*1.67*
Conventional										
Fernwood-Hodge	4	1	1	1	1	1	1	1	1	*1.33*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.

2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

4. Strategies substantive element of instruction. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table F54

Nature of Student–Student Conversation, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-St. James	2	1	1	1	1	1	1	N/A	1	*1.13*
Von Humboldt-Botkin	1	1	1	N/A	1					*1.00*
Von Humboldt-Lawton	1	1	1							*1.00*
Von Humboldt-Muldoon	4	3	1	1	1	3	1	1	1	*1.78*
Conventional										
Fernwood-Hodge	1	1	1	1	1	1	1	1	1	*1.00*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table F55

Students' Collaborative Working Relationships, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-St. James	2	1	1	1	1	1	1	N/A	1	*1.13*
Von Humboldt-Botkin	1	2	3	N/A	2					*2.00*
Von Humboldt-Lawton	N/A	N/A	1							*1.00*
Von Humboldt-Muldoon	4	3	1	1	1	2	NA	N/A	N/A	*2.00*
Conventional										
Fernwood-Hodge	N/A	1	1	1	N/A	N/A	1	1	1	*1.00*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table F56

Student Engagement during Instruction, Grade 7, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Addams-St. James	1	2	2	3	2	2	2	1	1	*1.78*
Von Humboldt-Botkin	3	2	3	4	3					*3.00*
Von Humboldt-Lawton	4	4	3							*3.67*
Von Humboldt-Muldoon	4	3	3	2	2	3	4	2	2	*2.78*
Conventional										
Fernwood-Hodge	2	1	3	3	1	1	2	1		*1.75*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table F57
Unit Planning, District 2, Grade 7, 1999-2000

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Guggenheim-Broughton	2A	1	3	6
Guggenheim-Redling	2A	1	2	5
Weir-Flader	3	2	2	7
		— Conventional —		
Von Steuben-Friedman	2A	2	2	6

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table F58
Lesson Planning, District 2, Grade 7, 1999-2000

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Guggenheim-Broughton	2	2B	2	2	8
Guggenheim-Redling	1	2B	3	3	9
Weir-Flader	2	3	2	2	9
			— Conventional —		
Von Steuben-Friedman	2	1	2	1	6

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:
1. *Lesson planning with little or no regard for students' performance on the previous lesson.* The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.
Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:
1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.
2. *Overall curriculum plan in mind.*
A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.
Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:
1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction.* The teacher encourages such participation, but it is still not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned.* The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.
Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:
1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.* Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table F59

Teacher Level of Lesson Presentation and Development, Grade 7, District 2

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Broughton	9	0	5	1	0	0	1	2	0	2A
Guggenheim-Redling	7	0	2	0	0	0	1	4	0	5
Weir-Flader	2	0	0	1	0	0	1	0	0	2B
Conventional										
Von Steuben-Friedman	8	0	7	0	0	1	0	0	0	2A

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support*. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table F60

Nature of Mathematical Inquiry, Grade 7, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																						
Guggenheim-Broughton	2	2	1	3	3	2	4	4	3	1	2	1	3	4	1	3	4	2	1	2	1	2	2	1	3	3	3	3	2	1	2	3	2	2	3	2	*2.31*	***3***
Guggenheim-Redling	3	2	3	4	2	1	4	1	4	2	2	2	3	1	1	4	1	4	1	2	3	3	1	1	4	1	3	3	3	3	1	1	2	3	3	3	*2.36*	***3***
Weir-Flader	2	1								1	1								2	2								3	3								*1.88*	***2***
Conventional																																						
Von Steuben-Friedman	1	1	2	3	1	2	1	1		1	2	1	2	1	1	1	1		1	1	1	2	1	2	1	1		1	1	1	1	2	1	3	1		*1.34*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table F61

Nature of Students' Explanations, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	2	1	2	3	2	2	3	2	*2.00*
Guggenheim-Redling	2	2	2	3	2	1	3	N/A	3	*2.25*
Weir-Flader	1	1								*1.00*
Conventional										
Von Steuben-Friedman	1	1	2	2	2	2	2	2		*1.75*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table F62

Elicitation of Multiple Strategies, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	1	1	2	4	1	4	2	4	*2.22*
Guggenheim-Redling	3	4	2	1	1	1	3	2	3	*2.22*
Weir-Flader	1	1								*1.00*
Conventional										
Von Steuben-Friedman	1	1	2	3	1	1	1	1		*1.38*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table F63

Nature of Student–Student Conversation, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	2	2	1	1	1	1	1	1	*1.22*
Guggenheim-Redling	1	1	1	1	1	1	3	3	3	*1.67*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	1	1	1	1	1	4	4	3		*2.00*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table F64

Students' Collaborative Working Relationships, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	N/A	2	N/A	2	N/A	1	N/A	N/A	N/A	*1.67*
Guggenheim-Redling	2	2	1	1	1	N/A	3	4	3	*2.13*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	N/A	N/A	1	1	N/A	4	4	3		*2.60*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table F65

Student Engagement during Instruction, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	3	3	3	3	4	3	3	3	3	*3.11*
Guggenheim-Redling	3	4	4	4	4	4	4	4	4	*3.89*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	4	4	4	4	4	4	4	3		*3.88*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table F59

Teacher Level of Lesson Presentation and Development, Grade 7, District 2

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Broughton	9	0	5	1	0	0	1	2	0	2A
Guggenheim-Redling	7	0	2	0	0	0	1	4	0	5
Weir-Flader	2	0	0	1	0	0	1	0	0	2B
Conventional										
Von Steuben-Friedman	8	0	7	0	0	1	0	0	0	2A

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table F60

Nature of Mathematical Inquiry, Grade 7, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation									*Mean*	***Rating***
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9		
MiC																																						
Guggenheim-Broughton	2	2	1	3	3	2	4	4	3	1	2	1	3	4	1	3	4	2	1	2	1	2	2	1	3	3	3	3	2	1	2	3	2	2	3	2	*2.31*	***3***
Guggenheim-Redling	3	2	3	4	2	1	4	1	4	2	2	2	3	1	1	4	1	4	1	2	3	3	1	1	4	1	3	3	3	3	1	1	2	3	3	3	*2.36*	***3***
Weir-Flader	2	1								1	1								2	2								3	3								*1.88*	***2***
Conventional																																						
Von Steuben-Friedman	1	1	2	3	1	2	1	1		1	2	1	2	1	1	1	1		1	1	1	2	1	2	1	1		1	1	1	1	2	1	3	1		*1.34*	***1***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table F61

Nature of Students' Explanations, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	2	1	2	3	2	2	3	2	*2.00*
Guggenheim-Redling	2	2	2	3	2	1	3	N/A	3	*2.25*
Weir-Flader	1	1								*1.00*
Conventional										
Von Steuben-Friedman	1	1	2	2	2	2	2	2		*1.75*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table F62

Elicitation of Multiple Strategies, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	1	1	2	4	1	4	2	4	*2.22*
Guggenheim-Redling	3	4	2	1	1	1	3	2	3	*2.22*
Weir-Flader	1	1								*1.00*
Conventional										
Von Steuben-Friedman	1	1	2	3	1	1	1	1		*1.38*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table F63

Nature of Student–Student Conversation, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	1	2	2	1	1	1	1	1	1	*1.22*
Guggenheim-Redling	1	1	1	1	1	1	3	3	3	*1.67*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	1	1	1	1	1	4	4	3		*2.00*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table F64

Students' Collaborative Working Relationships, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	N/A	2	N/A	2	N/A	1	N/A	N/A	N/A	*1.67*
Guggenheim-Redling	2	2	1	1	1	N/A	3	4	3	*2.13*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	N/A	N/A	1	1	N/A	4	4	3		*2.60*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table F65

Student Engagement during Instruction, Grade 7, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Broughton	3	3	3	3	4	3	3	3	3	*3.11*
Guggenheim-Redling	3	4	4	4	4	4	4	4	4	*3.89*
Weir-Flader	4	3								*3.50*
Conventional										
Von Steuben-Friedman	4	4	4	4	4	4	4	3		*3.88*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Explanation-Oriented	1 *50%*	2	3	4 *30%*	5		6	7	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1 *20%*	2			3	4	5	6	7	8
(10 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F9. Interactive Decisions, St. James, Addams Middle School.

Explanation-Oriented	1	2	3	4 *25%*	5		6	7 *75%*	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(4 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F10. Interactive Decisions, Lawton, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4 *20%*	5		6	7	8	
Task-Oriented					1	2 *20%*	3	4		
Shift in Pedagogical Approach	1	2 *20%*			3	4	5	6 *20%*	7 *20%*	8
(5 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F11. Interactive Decisions, Botkin, Von Humboldt Middle School.

Explanation-Oriented	1 *25%*	2	3	4	5 *13%*		6	7	8	
Task-Oriented					1	2 *38%*	3 *13%*	4		
Shift in Pedagogical Approach	1 *13%*	2			3	4	5	6	7	8
(8 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F12. Interactive Decisions, Muldoon, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4 *17%*	5		6	7 *22%*	8	
Task-Oriented					1	2 *22%*	3 *6%*	4		
Shift in Pedagogical Approach	1	2 *28%*			3	4	5	6 *6%*	7	8
(18 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F13. Interactive Decisions, Hodge, Fernwood Middle School.

Explanation-Oriented	1	2	3	4 *14%*	5 *5%*		6 *9%*	7 *18%*	8 *9%*	
Task-Oriented					1	2 *23%*	3 *5%*	4		
Shift in Pedagogical Approach	1 *5%*	2			3	4 *14%*	5	6	7	8
(22 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F14. Interactive Decisions, Broughton, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
							8%	*33%*	*8%*	
Task-Oriented					1	2	3	4		
						25%				
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
		8%				*17%*				
(12 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F15. Interactive Decisions, Redling, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
					100%					
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(2 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F16. Interactive Decisions, Flader, Weir Middle School.

Explanation-Oriented	1	2	3	4	5		6	7	8	
		9%		*27%*				*36%*		
Task-Oriented					1	2	3	4		
					18%					
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
								9%		
(11 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure F17. Interactive Decisions, Friedman, Von Steuben Middle School.

Appendix G

Instruction

Grade 8

Grade 8

The composite variable *Instruction* includes five major categories: *unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice*, and *student pursuits during instruction*. These categories were further subdivided into 19 subcategories, and an index was created for each. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the 19 indices, grouped by major categories, and the composite index for instruction, are described and illustrated with evidence from classroom observations, teacher interviews, teaching logs, and teacher journal entries. The ratings assigned to each teacher in Districts 3 and 4 on each of the indices related to unit and lesson planning are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in Districts 3 and 4. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 1

In District 1, five Grade 8 teachers participated in the study. Three teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge, unit sequence*, and *pace of instruction*.

Dunn generally followed the recommended sequence of MiC units, but modified the sequence to provide students exposure to or review of concepts and skills on district standardized tests. When planning to teach a MiC unit, Dunn read through the unit to become familiar with the scope of its content, worked through the problems to identify the concepts and skills needed, and decided how to present the material to students: "I kind of break it down to their level basically and maybe word it differently than what the question might be so that [students] can better understand what the question is asking" (Dunn, Interview 5/2/00). Before beginning the unit, Dunn planned to review and/or practice prerequisite concepts and skills. She explained:

> Sometimes the book assumes that [students] already know things and I know that they don't. So I'll usually introduce a concept before we get into [the unit]. Like in *Triangles and Patchwork*, the unit went into equilateral triangles and isosceles triangles. Some of [the students] either knew [the vocabulary] but forgot it or never knew it. I just reviewed those terms so when [students] saw those words in a sentence they remembered what they meant. (Dunn, Interview 5/2/00)

Dunn tried to follow the suggested pace for instruction. To help her stay on that pace, she often used one of the unit problems as a warm-up activity and assigned problems not completed in class for homework. Dunn thought MiC was well aligned with the state standards (Dunn, Interview 5/2/00).

Table G1
Subcategories of Unit Planning: Dunn, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Reichers followed the recommended sequence of MiC units. When planning to teach a MiC unit, she read the teacher guide to get an overview of the unit. She explained: "I look at the front of the teacher's manual at the NCTM *Standards*, the mathematical topics that are covered. [I look at the suggested] pacing. I look at how often the [authors] think there should be assessments, what kind of assessments there are, what the main ideas are, and, of course, all of the equipment I'm going to need" (Reichers, Interview 5/9/00). Reichers knew her students' math experiences varied widely because they came from different schools as a result of parental school choice. Reichers did not formally or informally assess students' prior knowledge before she planned a unit. Instead, she informally assessed as students worked through the unit, taking time to provide instruction on missing concepts, skills, and procedures. She felt that a problem inherent with most of the eighth-grade units was the lack of program continuity. Because MiC was designed with a spiral format (mastery over time), the development of the concepts studied in eighth-grade units had begun years earlier. Students whose mathematics instruction was not consistently MiC or had not included all of the units for each year from Grades 5–7 might not have prerequisite conceptual understanding or skill in procedures unique to MiC such as using a ratio table for the division of fractions or decimals. Supplemental instruction influenced her inability to stay on the pace suggested in teacher guides. Reichers did not assign unfinished class work for homework because she thought students needed to complete most of the problems in a class setting where they could discuss and solve problems together. She used self-correcting, skill-based commercial worksheets for homework. Reichers thought MiC was well aligned with the state standards. She felt that the problem with using MiC was the lack of time to address deficiencies and complete the program (Reichers, Interview 5/9/00).

Table G2
Subcategories of Unit Planning: Reichers, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Waters followed the recommended sequence of MiC units. When planning to teach a MiC unit that she had previously taught, Waters went over the entire unit and read the notes she had previously made in the teacher guide reminding her of possible difficulties, questions, and/or successes. Waters thought assessing students' prior knowledge was difficult because "every year is different. Every year students seem to come with different knowledge" (Waters, Interview 5/9/00). She chose to informally assess students as they worked through the unit. Waters interrupted group work when students did not have the needed prerequisite skills and provided whole-class instruction at that time. Waters thought the district was trying to help simplify the problem of disparate mathematics experiences among eighth-grade students. The district had pilot-tested a pretest, linked to the state standards, that provided the opportunity for teachers to identify at the beginning of the school year the standards each student needed to strengthen. Waters thought MiC was well aligned with the state standards and district standardized tests if students completed all of the MiC units at the grade level. However, even when she tried to follow the suggested pacing for instruction, Waters found that interruptions slowed her pace and made it impossible to complete all units. She assigned some problems not completed during class time for homework.

Table G3
Subcategories of Unit Planning: Waters, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

Wolfe generally followed the recommended sequence of chapters in the conventional textbook she used, but reorganized the chapters in preparation for district standardized tests in October and the state standardized tests in the spring. When planning to teach a chapter, Wolfe looked through the whole chapter to become familiar with the content and to consider how to present it to her class. Before beginning a unit, she informally assessed students' prior knowledge during a teacher-led, whole-class discussion in which she asked pertinent questions. Wolfe outlined the unit and indicated a pace of instruction based on the difficulty of the unit, the students' prior understanding of the topic, and the number of planned interruptions as indicated on the school calendar. She found it difficult to maintain the pace suggested in teacher support materials. Wolfe included weekly quizzes in her unit plans (Wolfe, Interview 5/8/00).

Table G4
Subcategories of Unit Planning: Wolfe, Addams Middle School, District 1, Grade 8

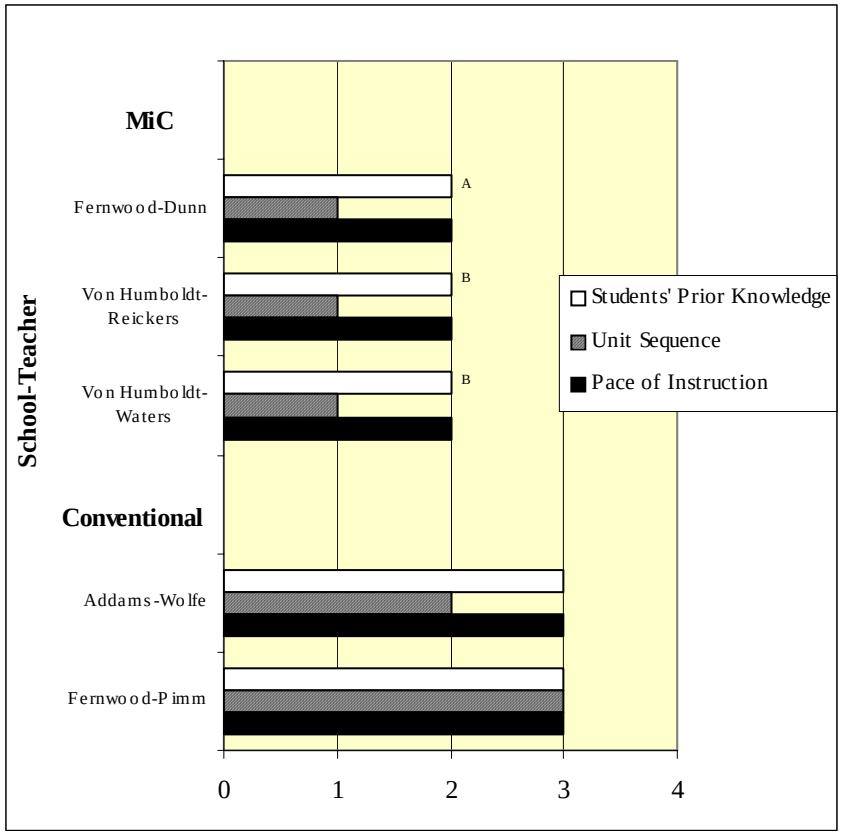
Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	2	Consideration of external factors
Pace of Instruction	3	Consideration of the needs of current students

Pimm did not use a published mathematics curriculum. Instead, she focused on one content strand at a time and searched for good problems to develop specific concepts that were outlined for eighth grade in the district curriculum guidelines and state standards. Pimm found problems in NCTM journals and other teacher resource materials. She prepared students for district tests, using the Key Links materials, two weeks prior to the testing period. Pimm gave frequent tests to assess students' prior knowledge and student progress. She did not determine the pacing prior to instruction; rather Pimm used intuition while teaching the unit to determine the pace of instruction (Pimm, Interview 5/2/00).

Table G5
Subcategories of Unit Planning: Pimm, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	3	Consideration of the needs of current students

In summary, the five eighth-grade teachers in District 1 varied in all three subcategories of *unit planning* (see Figure G1). With respect to *students' prior knowledge* when planning to teach a unit, two teachers assessed students' understanding of skills or concepts needed in a particular unit. One teacher based unit plans on her perceptions of students' mathematics skills and reading ability without formally or informally assessing. Two teachers based unit plans on their perceptions of students' needs related to the development of concepts and procedures.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table G48 in this appendix.

Figure G1. Unit planning, Grade 8, District 1.

In determining the *sequence of instructional units*, one teacher considered linkages across units of the same content stand. Another teacher considered external factors such as the content of the statewide testing program when planning to teach a unit. Three teachers followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, two teachers considered the students' learning styles and reasoning skills. Three teachers anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson.*

When planning to teach individual MiC lessons, Dunn usually worked through the majority of the problems in the lesson and determined the best instructional format for each problem, whether whole-group, small-group, or individual work. Dunn explained, "For instance, we just did a problem with random sampling. We did it together. I had them do their random sampling. Then I had them find who didn't live in the city. They told me their numbers. I knew it was 20, so I guided their thinking" (Dunn, Interview 5/2/00). Dunn adjusted her daily lesson plans based on whether the class completed the previous lesson and the number of students who experienced difficulty with the concepts or skills. If the majority of the students were lost, she did not move on to another problem or lesson. Dunn valued small-group work because it gave students an opportunity to interact. Students rarely participated in whole-class discussions, but when they worked in small groups, they shared their solution strategies and helped those students who where having difficulties. Dunn monitored small groups by walking around the room, listening to the groups' discussions, and asking questions. She found groups of three or four with at least one strong math student in each to be the most effective. Groups or individual students were not responsible for handing in evidence of the groups' work or their conclusions. When Dunn wanted to learn about individual student's understanding, she had students work independently (Dunn, Interview 5/2/00).

Four instructional forms were important components of Dunn's teaching: review of previous material (on 73% of the reported days), small-group work (67%), warm-up activities (66%), and teacher presentation (63%). Warm-up activities were generally given less than 15% of class time. Small-group work was generally given at least half of the class period. Review of previous material was generally given equal emphasis with other instructional forms. Dunn seldom used independent practice (15%) and whole-class discussion (8%; Dunn, Teacher Log 1999-2000).

Investigating problems was an important element in Dunn's instruction. Her students were engaged in this activity on 80% of the reported days, generally with equal emphasis with other student activities. Dunn's students listened to their teacher or took notes on 74% of the reported days, but generally for less than 15% of class time. They discussed answers and solution strategies on 60% of the reported days and practiced computation on 58% of the reported days. Practicing computation was given at least half of the class period on 60% of the reported days and equal emphasis with other student activities on the remaining days. Discussing answers and solution strategies was generally given equal emphasis with other student activities. Students took a test or quiz on 23% of the reported days. Dunn's students seldom participated in whole-class discussions (9%), reflected on or summarized lesson concepts (3%), or began homework (2%; Dunn, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Dunn reported in her teacher logs. The lesson observed on 12/15/99, for example, included: warm-up (5 minutes); teacher presentation (4 minutes); large group, teacher-led discussion (7 minutes); large group, teacher directed activity (22 minutes); small groups, problem 7a (6 minutes); large group, teacher answered problem 7a (1 minute); small group, problem 7b (8 minutes); large group, teacher answered problem 7b (1 minute); small groups, problem 8 (7 minutes); large group, student answered problem 8 (1 minute); and large group, began homework (1 minute; Dunn, Observation 12/15/99).

Table G6
Subcategories of Lesson Planning: Dunn, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	1	Little or no planning to teach the specific lesson
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction .
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important element in the lesson plan

When planning to teach individual MiC lessons, Reichers worked through all of the problems in the lesson and read the accompanying material in the teacher's guide. She noted any struggles she had, the mathematics involved, and possible questions she could ask to lead students' thinking about the mathematics. Reichers considered how students performed during the previous lesson and planned a warm-up activity or oral questioning to determine what they had learned. Reichers valued small-group work because it gave students an opportunity to talk about mathematics. She wanted each student to internalize the concepts and to be accountable for her/his own learning. She explained: "I think the purpose of talking about it is to help [students] internalize it" (Reichers, Interview 5/9/00). Reichers found that small-group work was most successful when students chose their groups and individual groups self-determined their size. She usually followed the suggestions in the teacher's guide regarding instructional format (Reichers, Interview 5/9/00).

On half of the reported days Reichers used three instructional forms: review of previous material (50%), whole-class discussion, (48%), and small-group work (46%). Of these three instructional forms, review of previous material was given the least amount of class time: equal emphasis with other instructional forms on 46% of the reported days and less than 15% of class time on 51% of the days. The other two instructional forms were given similar amounts of class time. Whole-class discussion was given at least half of the class period on 55% of the days, equal emphasis with other instructional forms on 33% of the days, and less than 15% of class time on 13% of the days. Small-group work was given at least half of the class period on 44% of the days, equal emphasis with other instructional forms on 42% of the days, and less than 15% of class time on 14% of the reported days. On approximately one third of the reported days, Reichers used independent practice (30%), teacher presentation (33%), and warm-up activities (35%). Of the three instructional forms, independent practice was generally given at least half of the class period on 74% of the days. Warm-up activities were given equal emphasis with other instructional forms on 64% of the days and less than 15% of class time on 29% of the reported days. Teacher presentation was given at least half of the class period on 11% of the days, equal emphasis with other instructional forms on 55% of the days, and less than 15% of class time on 35% of the days (Reichers, Teacher Log 1999-2000).

Discussing answers and solution strategies (64% of the reported days) and participating in whole-class discussions (51%) were important elements in Reichers' instruction. Each of these student activities was given a considerable amount of class time. Discussion of answers and solution

strategies was given at least half of the class period on 42% of the days, equal emphasis with other student activities on 51% of the days, and less than 15% of class time on the remaining days. Participation in whole-class discussion was given at least half of the class period on 50% of the days, equal emphasis with other student activities on 40% of the days, and less than 15% of class time on the remaining days. Reichers' students listened to their teacher or took notes on 52% of the reported days and reflected on or summarized lesson concepts on 42% of the reported days. However these student activities were given less class time. Listening to the teacher or taking notes was given equal emphasis with other student activities on 43% of the days and less then 15% of class time on 50% of the days. Reflecting on or summarizing lesson concepts was given equal emphasis with other student activities on 70% of the days, and less than 15% of class time on 24% of the days. Students investigated problems on 38% of the reported days. This student activity was given at least half of the class period on 32% of the days, equal emphasis with other student activities on 59% of the days, and less than 15% of class time on 10% of the days. Students less frequently practiced computation (19%), took a test or quiz (11%), and began homework (3%; Reichers, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Reichers reported in her teacher logs. The lesson observed on 5/25/00, for example, included: large group, review of page 16, problem 8 (11 minutes); teacher presentation of page 17 (3 minutes); small groups worked on pages 17 and 18, problems 1 and 2 (19 minutes); large-group discussion of solutions to problem 1 and procedures for solving problem 2 (4 minutes); small groups, continued assignment (14 minutes); large group, teacher-led discussion of problem solutions (10 minutes); and large group, assignment and directions for homework (3 minutes; Reichers, Observation 5/25/00).

Table G7
Subcategories of Lesson Planning: Reichers, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Waters worked through all of the problems in the lesson to get an overall view of the context, concepts, and skills. She studied the notes she had made when she previously taught the lesson and considered leading questions she could ask students to help clarify their thinking. Waters planned two or three lessons at a time: "I always have to be prepared ahead because I might get through more in a day than I think I'm going to. So I would say I'd probably have planned two or three days in advance so that if I end up covering more than one day, I can keep moving" (Waters, Interview 5/9/00). Waters valued small-group work. She explained, "students learn from each other. They have to learn to justify. They have to learn to be able to explain their reasoning. A lot of kids say they just know it and then when

they have to really think it through, to be able to verbalize it, then that makes it more a part of them" (Waters, Interview 5/9/00). Waters thought small-group work was helpful when the problems could be solved using multiple strategies "because the kids will come up with different strategies and they can talk about them" (Waters, Interview 5/9/00). Waters found that the most effective form of instruction with the current group of students was whole-class direct teaching with students sitting in rows. She also found that during whole-class discussions, students sometimes talked with each other, but usually they discussed just with her (Waters, Interview 5/9/00).

Waters used warm-up activities (on 38% of the reported days), teacher presentation (35%), whole-class discussion (27%), and review of previous material (25%). Warm-up activities were generally given less than 15% of class time. In contrast, teacher presentation was given at least half of the class period on 31% of the days, equal emphasis with other instructional forms on 57% of the days, and less than 15% of class time on 11% of the days. Whole-class discussion was given at least half of the class period on 44% of the days and equal emphasis with other instructional forms on 56% of the days. Review of previous material was given at least half of the class time on 39% of the days, equal emphasis with other instructional forms on 52% of the days, and less than 15% of class time on the remaining days. Waters less frequently used small-group work (15% of the reported days) and independent practice (15%). However, when used they were given considerable amounts of class time. Independent practice was given at least half of the class period on 66% of the days and equal emphasis with other instructional forms on 35% of the days. Small-group work was given at least half of the class period on 42% of the days, and equal emphasis with other instructional forms on 58% of the days (Waters, Teacher Log 1999-2000).

Waters' students listened to their teacher or took notes more frequently than they participated in other student activities (42% of the reported days). This student activity was given at least half of the class period on 25% of the days, equal emphasis with other student activities on 48% of the days, and less than 15% of class time on 27% of the days. Students discussed answers and solution strategies (31%), participated in whole-class discussion (27%), and investigated problems (26%). Each of these student activities was given considerable class time. Discussion of answers and solution strategies and participation in whole-class discussion were given at least half of the class period on 35% of the days and equal emphasis with other student activities on about 60% of the days. Investigation of problems was given at least half of the class period on 24% of the days and equal emphasis with other student activities on 73% of the days. Students less frequently took a test or quiz (13%), practiced computation (9%), reflected on or summarized lesson concepts (2%), and began homework during class time (2%; Waters, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Waters reported in her teacher logs. The lesson observed on 3/21/00, for example, included: individual warm-up (9 minutes); large group, teacher led students through the guess-and-check procedure for solving warm-up problems (7 minutes); teacher presentation of page 21 (3 minutes); small groups, problem 1 on page 21 (5 minutes); teacher presentation of answer to problem 1 on page 21 and introduced page 22 (7 minutes); small groups, problems 2a and 2b on page 22 (10 minutes); teacher presentation of solutions to problems 2a and 2b and assignment of homework (11 minutes); and individual, began homework (9 minutes; Waters, Observation 3/21/00).

Table G8
Subcategories of Lesson Planning: Waters, Von Humboldt Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual lessons from a conventional textbook, Wolfe worked through the word problems in the lesson and all of the homework problems. This process enables her to clearly verbalize to students the steps she used to solve the problems. She completed a weekly lesson plan outline form she had developed, indicating such things as the objectives of the lessons, needed materials, and the state standards the lesson addressed. Students' performance each day influenced the following lesson. She explained, "Sometimes I think a concept is just going to be very simple to get through, and it turns into something that I really should have planned for more than just one day" (Wolfe, Interview 5/8/00). Wolfe saw value in small-group work because students could often explain things more clearly to each other, but she was not confident in her skills to facilitate small groups. She tried to incorporate small-group work into her instruction when the activity involved practicing a procedure previously taught. Wolfe preferred teacher-led, whole-class presentation, with students working on problems as a whole group as she wrote the step-by-step procedures as provided by students on the board (Wolfe, Interview 5/8/00). Teacher Log data for Wolfe for 1999-2000 were unavailable.

As an example of a lesson observed in Wolfe's class, the lesson on 11/10/99 included: individual warm-up activity, simplifying fractions (14 minutes); large group, teacher showed solutions to warm-up problems on the board (11 minutes); teacher presentation and student took notes on least common multiple (8 minutes); large group, student listed multiples of 5 and 8 on the board, circled 40 while class watched (2 minutes); large group, students copied rule on how to use prime factorization to find the least common multiple (4 minutes); large group, student dictated how to apply the rule for numbers 18 and 32 as the teacher wrote it on the board (3 minutes); and large group, began assignment (1 minute; Wolfe, Observation 11/10/99).

Table G9
Subcategories of Lesson Planning: Wolfe, Addams Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Students' discourse in the classroom seldom, if at all, planned as part of the lesson
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	1	Investigation of problems and discussion of mathematical ideas seldom planned for the lesson

When planning to teach individual lessons, Pimm selected a series of problems in a content strand that students would work during one week. She sequenced the problems so that procedurally or conceptually they were related and one built upon the structure of the previous one. If students' performance on a lesson indicated they needed more practice or review; the following day's lesson was altered to include the practice or review. Pimm valued small-group work when group membership was structured and when the expectations were clearly understood by each member of the group. Pimm preferred a whole-class, direct teaching introduction with individual students focusing on the problem, which was followed by small groups of four working to solve the problem. Whole-class discussions were teacher led and involved teacher–student interactions, not student–student interactions. She commented:

> I really struggle with the fact that [students] just make eye contact with me when we try to have student discussions. Which is why sometimes I'll have a student come to the front of the room and I'll move back to where the students are so that at least [the presenter] is looking in that direction. If [the presenter] stands there and explains it to me, I try to remind them that I know how to do it. I want them to explain it to their peers. It's very difficult for them. But I think they've come a long way since September and October. (Pimm, Interview 5/2/00)

Small-group work was an important component of Pimm's instruction as was used on 71% of the reported days. Small-group work was given at least half of the class period on 41% of the days, equal emphasis with other instructional forms on 48% of the days, and less than 15% of class time on 11% of the days. Three other instructional forms were used on approximately half of the reported days: whole-class discussion (54%), review of previous material (50%), and teacher presentation (46%). Whole-class discussion was given more class time: at least half of the class period on 44% of the days, equal emphasis with other instructional forms on 36% of the days, and less than 15% of class time on 20% of the days. Review of previous material was given at least half of the class period on 13% of the days, equal emphasis with other instructional forms on 52% of the days, and less than 15% of class time on 35% of the days. Teacher presentation was given at least half of the class period on 10% of the

days, equal emphasis with other instructional forms on 43% of the days, and less than 15% of class time on 48% of the days. Pimm also used independent practice (38%) and warm-up activities (32%). Warm-up activities were generally given less than 15% of class time, whereas independent practice was generally given equal emphasis with other instructional forms (Pimm, Teacher Log 1999-2000).

Pimm's students investigated problems (on 68% of the reported days), discussed answers and solution strategies (64%), participated in whole-class discussion (54%), and listened to the teacher or took notes (49%). Investigating problems and discussing answers and solution strategies were given the most class time. Investigating problems was given at least half of the class period on 52% of the days, equal emphasis with other student activities on 37% of the days, and less than 15% of class time on 11% of the days. Discussing answers and solution strategies was given at least half of the class period on 39% of the days, equal emphasis with other student activities on 49% of the days, and less than 15% of class time on 12% of the days. Participation in whole-class discussion was given at least half of the class period on 34% of the days, equal emphasis with other student activities on 42% of the days, and less than 15% of class time on 24% of the days. Listening to the teacher or taking notes was given equal emphasis with other student activities on 38% of the days and less than 15% of class time on 53% of the days. On fewer days, students reflected on or summarized lesson concepts (39%) and practiced computation (27%). Reflection on or summarization of lesson concepts was given equal emphasis with other student activities on 50% of the days and less than 15% of class time on 44% of the days. Practicing computation was generally given equal emphasis with other student activities. Students seldom began homework during class time (10%) or took a test or quiz (5%; Pimm, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Pimm reported in her teacher logs. The lesson observed on 4/20/00, for example, included: teacher presentation of problem 7, terms sphere and diameter, and how to use the needed tools (21 minutes); small groups, worked on 2 assigned problems (11 minutes); large group, teacher clarified the question asked in the problem (4 minutes);

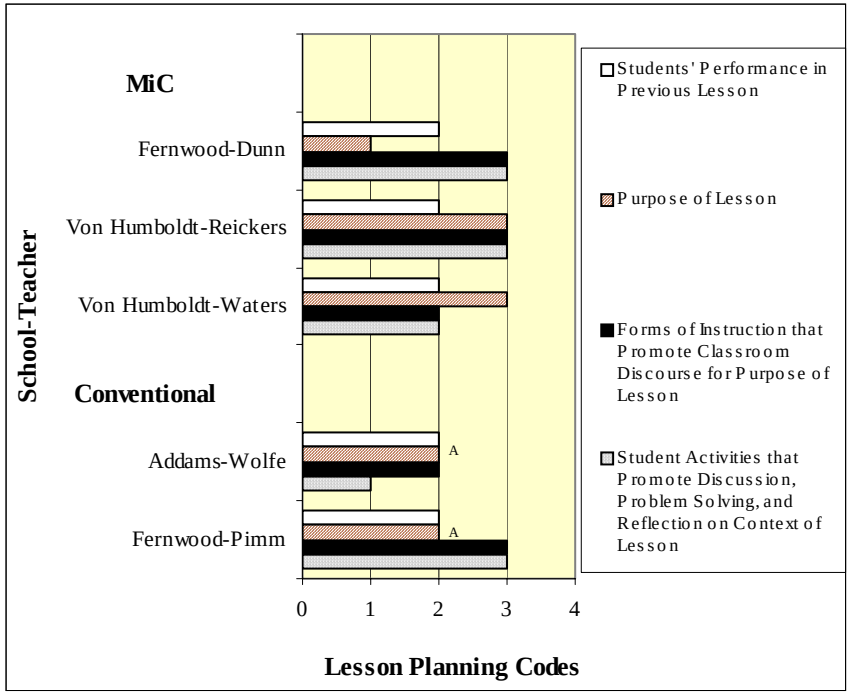
small groups, continued to work on assigned problems (22 minutes); large group, administrative interruption (4 minutes); and large group, students hand in work (1 minute; Pimm, Observation 4/20/00).

Table G10

Subcategories of Lesson Planning: Pimm, Fernwood Middle School, District 1, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2A	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

In summary, the five eighth-grade teachers varied on three of the four subcategories of *lesson planning*, teachers' attention to the *purpose of the lesson, forms of instruction that promote classroom discourse of the purpose for the lesson,* and *student activities that promote discussion, problem solving, and reflection on the content of the lesson* (see Figure G2). With respect to *students' performance in the previous lesson*, all of the teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Variation was seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. Two other teachers selected lesson content to reflect a continuity of mathematical ideas, integrating lesson materials from various resources. When the remaining teacher planned, the purpose was to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction was to cover lessons in the textbook or curriculum; thus, no additional planning was deemed necessary.



*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table G49 in this appendix.

Figure G2. Lesson planning, Grade 8, District 1.

Teachers varied minimally in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Three teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The remaining two teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing

substantive conversation. Teachers varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection on the content of the lesson*. For three teachers, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, question or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plans. One teacher included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered. The remaining teacher seldom planned for investigation of problems and discussion of mathematical ideas.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Dunn. The evidence gathered for lesson presentation and development for Dunn ranged from Level 2B to Level 3B. An overall rating of Level 3B was assigned, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table G50 in this appendix). An overall rating of 1 was assigned for Dunn for the nature of inquiry, indicating that inquiry was limited to lower-order thinking (see Table G51). Dunn was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure G9). For Dunn, 56% of the interactive decisions were coded as least aligned with teaching for understanding, 34% were reflective of good standard pedagogy, and 12% were most aligned with teaching for understanding.

Across all observations for Dunn, the mean rating for the index on students' explanations was 1.33, indicating that students provided only answers or their explanations were focused on procedures (see Table G52). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table G53). A rating of 1 was assigned for Dunn on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G11
Overall Ratings for Mathematical Interaction for Dunn, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Emphasis on review
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	2	More emphasis on standard pedagogy
Nature of Student Explanations	1.33	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Reichers. The evidence gathered for lesson presentation and development for Reichers ranged from Level 3B to Level 5. An overall rating of Level 5 was assigned, indicating that lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher (see Table G50 in this appendix). An overall rating of 4 was assigned for Reichers for the nature of inquiry promoted in-depth exploration of mathematics (see Table G51). Reichers was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure G10). For Reichers, 55% of the interactive decisions were coded as most aligned with teaching for understanding, 25% were reflective of good standard pedagogy, and 16% were least aligned with teaching for understanding.

Across all observations for Reichers, the mean rating for the index on students' explanations was 2.11, indicating that student explanations were focused on procedures (see Table G52). The mean rating for elicitation of multiple strategies was 1.67, indicating that multiple strategies were generally rarely from students (see Table G53). A rating of 1 was assigned for Reichers on the index for reflection or summary, indicating that fe, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G12
Overall Ratings for Mathematical Interaction for Reichers, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.11	Focus on procedures
Elicitation of Multiple Strategies	1.67	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Waters. The evidence gathered for lesson presentation and development for Waters was all at Level 3B, indicating that lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher (see Table G50 in this appendix). An overall rating of 1 was assigned for Waters for the nature of inquiry that transpired during instruction was limited to lower-order thinking (see Table G51). Waters was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure G11). For Waters, 55% of the interactive decisions were coded as least aligned with teaching for understanding, 41% were reflective of good standard pedagogy, and 5% were most aligned with teaching for understanding.

Across all observations for Waters, the mean rating for the index on students' explanations was 1.25, indicating that students provided answers only or their explanations were focused on procedures (see Table G52). The mean rating for elicitation of multiple strategies was 1.13, indicating that multiple strategies were generally not elicited from students (see Table G53). A rating of 1 was assigned for Waters on the index for reflection

or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G13
Overall Ratings for Mathematical Interaction for Waters, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	2	More emphasis on standard pedagogy
Nature of Student Explanations	1.25	Answers only
Elicitation of Multiple Strategies	1.13	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Wolfe. The evidence gathered for lesson presentation and development for Wolfe was all at Level 3B, indicating that lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher (see Table G50 in this appendix). An overall rating of 1 was assigned for Wolfe for the nature of inquiry that transpired during instruction, indicating inquiry was limited to lower-order thinking (see Table G51). Wolfe was assigned Level 1 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly least aligned with teaching for understanding (see Figure G12). For Wolfe, all interactive decisions were coded as least aligned with teaching for understanding.

Across all observations for Wolfe, the mean rating for the index on students' explanations was 1.63, indicating that students provided answers only on some occasions and their explanations were focused on procedures on other occasions (see Table G52). The mean rating for elicitation of multiple strategies was 1.00, indicating that multiple strategies were not elicited from students (see Table G53). A rating of 1 was assigned for Wolfe on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G14
Overall Ratings for Mathematical Interaction for Wolfe, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	1	Limited to lower-order thinking
Interactive Decisions	1	Least aligned with teaching for understanding
Nature of Student Explanations	1.63	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.00	Strategies not elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Pimm. The evidence gathered for lesson presentation and development for Pimm ranged from Level 2A to Level 5. An overall rating of Level 5 was assigned, indicating that lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher (see Table G50 in this appendix). An overall rating of 2 was assigned for Pimm for the nature of inquiry provided limited attention to conceptual understanding (see Table G51). Pimm was assigned Level 4 on the composite index for interactive decision-making, indicating that interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding (see Figure G13). For Pimm, 33% of the interactive decisions were coded as reflective of good standard pedagogy, 40% were most aligned with teaching for understanding, and 27% were least aligned for teaching for understanding.

Across all observations for Pimm, the mean rating for the index on students' explanations was 1.44, indicating that students provided answers only or their explanations were focused on procedures (see Table G52). The mean rating for elicitation of multiple strategies was 1.56, indicating that multiple strategies were generally rarely from students (see Table G53). A rating of 1 was assigned for Pimm on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G15
Overall Ratings for Mathematical Interaction for Pimm, District 1, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	2	Limited attention to conceptual understanding
Interactive Decisions	4	More emphasis on standard pedagogy and teaching for understanding
Nature of Student Explanations	1.44	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.56	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the five Grade 8 teachers in District 1 varied from Level 3B to Level 5 in *lesson presentation and development* (see Table G16). Two teachers (one MiC) were assigned Level 5, indicating that their lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. The three remaining teachers (two MiC) were assigned Level 3B, indicating that students practiced strategies or procedures presented by their teacher in rote fashion.

Table G16
Nature of Mathematical Interaction, Grade 8 Teachers in District 1, 1999-2000

School-Teacher	Lesson Presentation and Development	Nature of Inquiry	Teachers' Interactive Decisions	Nature of Student Explanations	Elicitation of Multiple Strategies	Lesson Closure, Reflection, or Summary	Total
			— MiC —				
Fernwood-Dunn	3B	1	2	1.33	1.00	1	9.33
Von Humboldt-Reichers	5	4	5	2.11	1.63	1	18.74
Von Humboldt-Waters	3B	1	2	1.25	1.13	1	9.38
			— Conventional —				
Addams-Wolfe	3B	1	1	1.63	1.00	1	8.63
Fernwood-Pimm	5	2	4	1.44	1.56	1	15.00

With respect to the *nature of inquiry during instruction*, teachers varied from Level 1 to Level 4. One MiC teacher was assigned Level 4, indicating that she promoted in-depth explorations of mathematics. One teacher using a conventional curriculum was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction. Three teachers (two MiC) were assigned Level 1, indicating that inquiry was limited to lower-order thinking.

With respect to *teachers' interactive decisions*, teachers varied from Level 1 to Level 5. One MiC teacher was assigned Level 5, indicating that her interactive decisions were most aligned with teaching for understanding, and one teacher using a conventional curriculum was assigned Level 4, indicating that her interactive decisions were reflective of good standard pedagogy and teaching mathematics for understanding. Two MiC teachers were assigned Level 2, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding. The remaining teacher using a conventional curriculum was assigned Level 1, indicating that her interactive decisions were least aligned with teaching mathematics for understanding.

The overall means for the *nature of student explanations* ranged from 1.25 to 2.11, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.00 to 1.63. In general, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, the teachers offered few, if any, opportunities for students to reflect on the mathematics in a lesson or in a series of lessons.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought*, *purpose and coherence of feedback*, and *content of feedback*. The eighth-grade teachers in District 1 varied in all three subcategories of *classroom assessment* (see Tables G17-G20). With respect to the *evidence sought* during classroom assessment, two teachers (one MiC) maintained an underdeveloped process orientation. One MiC teacher sought procedural competence of student learning. The remaining two teachers (one MiC) sought little evidence of student learning. *Feedback* in classes of one teacher using a conventional curriculum was purposeful and shared by teacher and students. Feedback in the classes of four teachers (three MiC) was very teacher-directed. Two teachers (one MiC) provided mixed, superficial feedback. One MiC teacher provided low-level closed feedback. The remaining two teachers (one MiC) provided feedback that allowed students to know whether answers were correct or incorrect.

Table G17
Evidence Sought through Classroom Assessment Practice, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Dunn	2	Procedural competence
Von Humboldt-Reichers	3	Underdeveloped process orientation
Von Humboldt-Waters	1	Limited evidence
Conventional		
Addams-Wolfe	1	Limited evidence
Fernwood-Pimm	3	Underdeveloped process orientation

Table G18
Feedback Coherence and Purpose, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Dunn	2	Teacher-directed feedback
Von Humboldt-Reichers	2	Teacher-directed feedback
Von Humboldt-Waters	2	Teacher-directed feedback
Conventional		
Addams-Wolfe	2	Teacher-directed feedback
Fernwood-Pimm	4	Purposeful shared responsibility

Table G19
Feedback Content, Grade 8, District 1

	Rating	Description
MiC		
Fernwood-Dunn	2	Answer-only feedback
Von Humboldt-Reichers	4	Mixed, superficial feedback
Von Humboldt-Waters	3	Low-level closed feedback
Conventional		
Addams-Wolfe	2	Answer-only feedback
Fernwood-Pimm	4	Mixed, superficial feedback

Table G20
Classroom Assessment, Grade 8, District 1

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Fernwood-Dunn	2	2	2	6
Von Humboldt-Reichers	3	2	4	9
Von Humboldt-Waters	1	2	3	6
		— Conventional —		
Addams-Wolfe	1	2	2	5
Fernwood-Pimm	3	4	4	11

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence.* Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation.* The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility.* Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility.* Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading.* The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback.* An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Dunn were all Level 1 (see Table G54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Dunn are described. The observer noted, "In small groups students tended to ask the teacher when they had a question instead of asking each other. If one student got an answer, the rest of the group relied on that person's solution" (Dunn, Observation 9/21/99). In this example, students conversed with the teacher and student–student conversation was limited to reporting answers.

The index ratings about the nature of student–student conversation for Reichers ranged from Level 1 to Level 4 (see Table G54 in this appendix). The mean rating across observations was 2.33, indicating that student–student conversation usually was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Reichers are described. The observer noted, "During the time that students worked on pages 20–22, some got answers from each other but they did not really discuss the mathematics with each other" (Reichers, Observation 1/13/00). In this example student–student conversation was limited and consisted of sharing answers.

The index ratings about the nature of student–student conversation for Waters were all Level 1 (see Table G54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Waters are described. The observer noted, "There were no exchanges between peers in pairs or in the large-group setting. All exchanges were between the teacher and one student at a time" (Waters, Observation 11/15/99). In this example, students conversed with the teacher, but student–student conversation did not occur.

The index ratings about the nature of student–student conversation for Wolfe were all Level 1 (see Table G54 in this appendix), indicating that student–student conversation was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Wolfe are described. The observer noted, "Students do not share ideas with each other about mathematics. Every so often they did ask the teacher a question" (Wolfe, Observation 3/20/00). In this example, students sometimes conversed with the teacher, but student–student conversation did not take place.

The index ratings about the nature of student–student conversation for Pimm ranged from Level 1 to Level 2 (see Table G54 in this appendix). The mean rating across observations was 1.22, indicating that student–student conversation usually was not encouraged or did not exist. To illustrate a rating at Level 1, student–student conversations that occurred during a lesson by Pimm are described. The observer noted, "There were no

exchanges between peers in small groups or in the large group. Students only talked to the teacher to answer questions" (Pimm, Observation 12/15/99). In this example, students conversed with the teacher, but student–student conversation did not exist.

In summary, the mean ratings for *student–student conversation* for the five Grade 8 teachers in District 1 varied from 1.00 to 2.33. One MiC teacher had a mean rating near 2.00, indicating that student–student conversation was usually limited, consisted of sharing answers, or focused on procedures. Four teachers (two MiC) had mean ratings at or near 1.00, indicating that student–student conversation was not encouraged or did not occur.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Dunn ranged from Level 1 to Level 3 (see Table G55 in this appendix). The mean rating across observations was 1.22, indicating that on many occasions none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Dunn is described. The observer noted, "No one worked together in their groups. Sometimes one student gave another student an answer, but they never discussed strategy or mathematics" (Dunn, Observation 1/18/00). In this example, although students physically sat together, they did not work collaboratively.

The index ratings about the nature of students' collaboration in the classroom for Reichers ranged from Level 1 to Level 3 (see Table G55 in this appendix). The mean rating across observations was 2.13, indicating that few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Reichers is described. The observer noted, "Many students were working on different problems. There was little exchange of ideas. Many students in each group relied on another member to give them the answer and the explanation so they could write it down in their notebooks" (Reichers, Observation 5/25/00). In this example, a few students shared ideas, but many of them worked on different problems at a different pace.

The index ratings about the nature of students' collaboration in the classroom for Waters were all Level 1 (see Table G55 in this appendix), indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Waters is described. The observer noted, "The students were seated in groups of four but they did not work with each other collaboratively. Occasionally, one student would copy an answer from another student" (Waters, Observation 5/25/00). In this example, although students physically sat together, they did not work collaboratively.

The index ratings about the nature of students' collaboration in the classroom for Wolfe were all Level 1 (see Table G55 in this appendix), indicating that none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. However, on two of the eight observations no rating for student

collaboration was given because the main purpose of the lesson was to give students individual practice or independent work. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Wolfe is described. The observer noted, "All mathematical discussion was between the teacher and one student at a time. Students did not work collaboratively. There was no individual practice" (Wolfe, Observation 11/10/99). In this example, students did not work collaboratively.

The index ratings about the nature of students' collaboration in the classroom for Pimm ranged from Level 1 to Level 2 (see Table G55 in this appendix). The mean rating across observations was 1.13, indicating that on most occasions none of the students worked together in small or large-group settings. If students did work in small groups, then one student typically gave answers to other members of the group without explanation. To illustrate a rating for students' collaboration at Level 1, student collaboration that occurred during a lesson by Pimm is described. The observer noted, "If someone in the group could solve the problem, the other members of the group copied that student's answer and their work. They really did not solve the problems as a group" (Pimm, Observation 4/20/00). In this example, one student gave the answers to others in the group without explanation; no one worked collaboratively.

In summary, the mean ratings for *student collaboration* for the five Grade 8 teachers in District 1 varied from 1.00 to 2.13. One MiC teacher had a mean rating near 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved. Four teachers (two MiC) had mean ratings at or near 1.00, indicating that on most occasions none of the students worked together in small or large-group settings, or if students did work in groups, then one student typically gave answers to other members of the group without explanation. On several observations for one of these teachers, no rating for student collaboration was given because the main purpose of the lesson was to give students individual practice or independent work.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Dunn ranged from Level 1 to Level 4 (see Table G56 in this appendix). The mean rating across observations was 2.11, indicating that on most occasions student engagement was passive. To illustrate a rating for student engagement at Level 2, students' on-task behavior that occurred during a lesson by Dunn is described. The observer noted, "Most students did not take notes when the teacher gave the solutions on the board. At least half of the class did not do the problems that were assigned" (Dunn, Observation 3/24/00). In this example, most students were passively disengaged, only occasionally on task carrying out assigned activities, and appearing to not put forth much effort.

The index ratings about the extent to which students were engaged during the lesson for Reichers ranged from Level 2 to Level 4 (see Table G56 in this appendix). The mean rating across observations was 3.44, indicating that on some occasions student engagement was sporadic or episodic, but on other occasions widespread with most students on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Reichers is described. The observer noted, "Most students were very engaged in the task. They each had to turn in their own solution to the problem. They listened to the explanations given at the end of the period by the other students" (Reichers, Observation 4/18/00). In this example, most students were on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during another lesson by

Reichers is described. The observer noted, “Students were alert and listening to the teacher. They responded well to her questions and discussed the situations. When students were asked to work individually, they were not as focused as they were in the large group” (Reichers, Observation 9/20/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Waters ranged from Level 1 to Level 3 (see Table G56 in this appendix). The mean rating across observations was 1.75, indicating that on some occasions students were passively disengaged, and on other occasions disengaged because of disruptive or gross inattentive behavior. To illustrate a rating for student engagement at Level 2, students’ on-task behavior that occurred during a lesson by Waters is described. The observer noted, “The students were lethargic and seem bored. They waited for the teacher to do the work for them” (Waters, Observation 9/20/99). In this example, students appeared lethargic and were only occasionally on task carrying out assigned activities. To illustrate a rating for student engagement at Level 1, students’ on-task behavior that occurred during another lesson by Waters is described. The observer noted, “No one in the class listened when students read. They wrote a lot of notes to each other. Some students did a little work while many did no work. The students were very inattentive and frequently off task, but there were no serious disruptions” (Waters, Observation 10/26/99). In this example, students were frequently off task, as evidenced by their gross inattention.

The index ratings about the extent to which students were engaged during the lesson for Wolfe ranged from Level 1 to Level 3 (see Table G56 in this appendix). The mean rating across observations was 1.50, indicating that on some occasions students were passively disengaged, and on other occasions disengaged because of disruptive or gross inattentive behavior. To illustrate a rating for student engagement at Level 2, students’ on-task behavior that occurred during a lesson by Wolfe is described. The observer noted, “Most of the students were not involved in the lesson in any way except as passive observers. Many students did not listen or take notes. Most students talked quietly among themselves, but not about mathematics” (Wolfe, Observation 5/15/00). In this example, many students were clearly off task or nominally on task. They did not seem to put forth much effort. To illustrate a rating for student engagement at Level 1, students’ on-task behavior that occurred during another lesson by Wolfe is described. The observer noted, “There was constant murmuring during the class. Many were not involved in the lesson at all and most were not even following along. [Wolfe] tended to talk to one student while the others did nothing. Several girls were using their graph paper to draw hearts. These students were not serious about the lesson. There were no major disruptions, however” (Wolfe, Observation 12/7/99). In this example, students were frequently off task, as evidenced by their gross inattention.

The index ratings about the extent to which students were engaged during the lesson for Pimm ranged from Level 2 to Level 4 (see Table G56 in this appendix). The mean rating across observations was 3.33, indicating that on most occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students’ on-task behavior that occurred during a lesson by Pimm is described. The observer noted, “There was sporadic engagement from the students. Most of them participated in the activities. They fooled around sometimes, but the involvement was pretty good” (Pimm, Observation 9/21/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

In summary, the mean ratings for *student engagement in the lesson* for the five Grade 8 teachers in District 1 varied from 1.50 to 3.44. Two teachers (one MiC) had mean ratings greater than 3.00, indicating that on some occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time, and on other occasions, sporadic and episodic. Two MiC teachers had mean ratings near

2.00, indicating that for substantial portions of time, many students were either clearly off task or nominally on task. The remaining teacher using a conventional curriculum had a mean rating greater than 1.00, indicating that on some occasions students were passively disengaged and on other occasions grossly inattentive.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table G21
Teacher Level of Instruction, Grade8, District 1

School-Teacher	**Lesson Planning**		**Mathematical Interaction**						**Classroom Assessment**			**Student Pursuits**			**Weighted Sum**	**Composite Level**
	FIPD	**SAPD**	**LPD**	**NI**	**ID**	**SE**	**MS**	**LCS**	**ES**	**FCP**	**FC**	**SC**	**SWR**	**OSE**		
— MiC —																
Fernwood-Dunn	3.75	3.75	3	1	2	2.22	1.25	1.67	2	2	2	1.25	1.53	2.64	30.06	**2**
Von Humboldt-Reichers	3.75	3.75	5	5	5	3.52	2.04	1.67	3	2	4	2.91	2.66	4.30	48.60	**5**
Von Humboldt-Waters	2.5	2.5	3	1	2	2.08	1.41	1.67	1	2	3	1.25	1.25	2.19	26.85	**2**
— Conventional —																
Addams-Wolfe	2.5	1.25	3	1	1	2.72	1.25	1.67	1	2	2	1.25	1.25	1.88	23.77	2
Fernwood-Pimm	3.75	3.75	5	2	4	2.40	1.95	1.67	3	4	4	1.53	1.41	4.16	42.62	**5**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

District 2

In District 2, four Grade 8 teachers participated in the study. All four teachers used MiC.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Carlson generally followed the recommended sequence of MiC units except for twenty minutes each day and the six-week period she devoted to preparing students for the state standardized tests. When planning to teach a MiC unit, Carlson worked through the entire unit to become familiar with its content. She read the teacher's guide to check whether she had given the intended meaning to the problems and to see if she understood the purpose and direction of the unit. Carlson's experience teaching eighth-grade mathematics led her to assume that students did not have adequate prior knowledge. She included additional skill review and practice using supplementary materials. Carlson was aware of the suggested pace of instruction, but thought her students needed more time to acquire the missing prerequisite skills (Carlson, Interview 3/27/00).

Table G22
Subcategories of Unit Planning: Carlson, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	3	Consideration of the needs of current students

Dillard recognized the way in which the MiC units were sequenced to provide students with continuity in skill and concept development. Since he had taught most of these students in Grades 6 and 7, he was aware of their experiences with MiC. Part of his unit planning included checking to see if the new students to his program had worked through the prerequisite units. If not, he amended his plans to accommodate the conceptual or skill gaps. He used commercial materials to help provide students with additional practice or review. Dillard sequenced MiC units to emphasize algebra. Because his students would be taking Algebra I next year, he made sure they completed all three of the eighth-grade algebra units and the listed prerequisite units. During the last nine weeks of the school year, he used a conventional algebra textbook to give students exposure to a conventional format, including assignments of a specific number of pages and regular quizzes and tests. When planning to teach a MiC unit,

Dillard timed himself as he worked through the entire unit. His previous experience teaching MiC led him to conclude that it took students about twice as much time to complete a problem as it took him. He used this information to set his pace of instruction (Dillard, Interview 4/20/00).

Table G23
Subcategories of Unit Planning: Dillard, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2B	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	3	Consideration of the needs of current students

Gallardo followed the district curriculum guidelines and state standards to determine the order of MiC units. When planning to teach a MiC unit, he read through the unit to identify the state standards it addressed and to become familiar with how the content was presented. Gallardo anticipated the difficulties students might have based on their performance on the schoolwide pretest of grade-level state standards' expectations that was given at the beginning of the school year. He then generated possible strategies for helping students including review and practice lessons. Students' prior knowledge and the amount of exposure students had to MiC influenced the pace for instruction. (Gallardo, Interview 4/12/00).

Table G24
Subcategories of Unit Planning: Gallardo, Weir Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	2	Consideration of external factors
Pace of Instruction	3	Consideration of the needs of current students

Shepard followed the recommended sequence of MiC units. When planning to teach a MiC unit, she worked through the unit to become familiar with its expectations and to determine the best instructional format for engaging students. Prior to beginning the unit, Shepard used conventional textbook materials to provide students with review and practice of the needed prerequisite concepts and skills. Homework assignments provided students with more practice and review opportunities. Shepard had a general rule for setting the pace for instruction. Because Weir Middle School was on a block schedule (two-hour classes that met every other day), she determined that students should be able to complete one MiC unit during two class periods. She anticipated adjusting the pace to accommodate the complexity of the unit and student learning. She explained further: "I use

the state standards to make sure that I'm teaching what I'm supposed to. And that's how I can eliminate something out of the unit, if it isn't covered by the state standards" (Shepard, Interview 4/27/00).

Table G25
Subcategories of Unit Planning: Shepard, Weir Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	1	Little or no consideration of students' prior knowledge
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the four eighth-grade teachers in District 2 varied in all three subcategories of *unit planning* (see Figure G3). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. One teacher considered her perceptions of students' mathematics skill and reading ability. Another teacher considered his perceptions of students' needs related to the development of concepts and procedures. One teacher planned units with little or no understanding of students' prior knowledge.

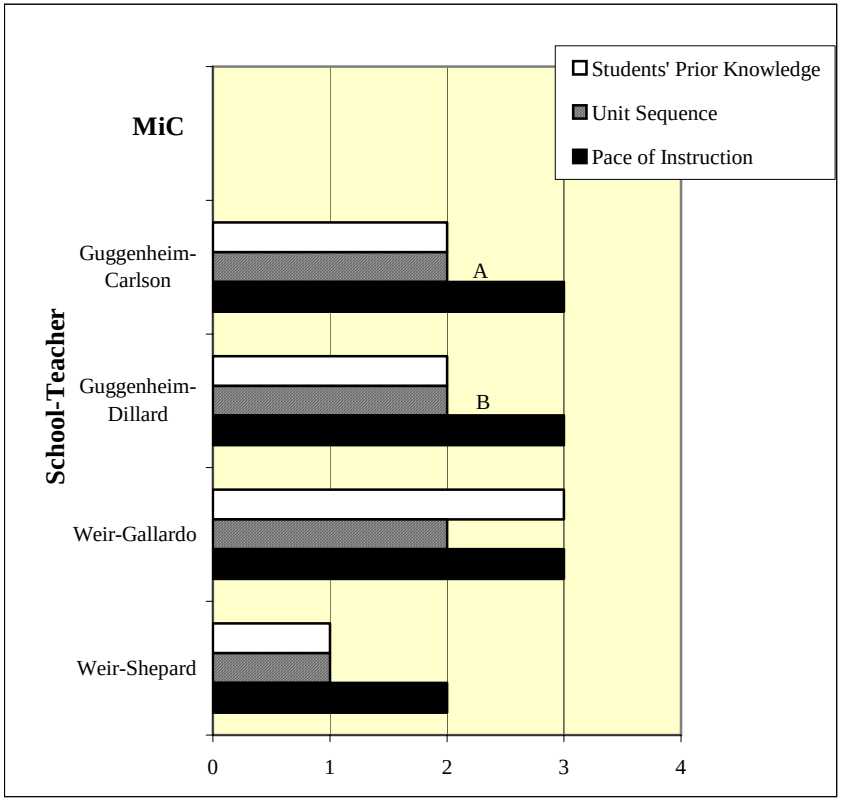


Figure G3. Unit planning, Grade 8, District 2.

UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

For detailed description of Unit Planning Codes, see Table G57 in this appendix.

In determining the *sequence of instructional units*, three teachers considered external factors such as the content of the statewide testing program. One teacher followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, three teachers considered students' learning styles and reasoning skills. One teacher anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. When planning to teach individual MiC lessons, Carlson worked through all of the problems in the lesson and read the teacher guide to check whether she had given the intended meaning to the problems. Carlson commented: "A lot of times the way that I would have done it is different from the way that [the teacher guide] wants it done. So I get to see a different point of view. I have a couple of things to offer to the kids, the book way, and my way. Then I ask them if they can do it" (Carlson, Interview 3/27/00). Carlson considered everything she knew about the students, such as number of years experience with MiC, content of MiC units not covered, and performance on past lessons that showed skill strengths and weaknesses, as she determined if concepts and skills needed to be taught, reviewed, or practiced before beginning the new lesson. Carlson valued small-group work because it gave students opportunities to talk about their solution strategies, to pool their understanding of different parts of the problem, and to ask questions. Students were randomly assigned to tables at the beginning of the year and remained with that group for the year unless behavior problems merited reconsideration. Carlson used a combination of instructional forms, whole-class, direct teaching, individual work, and small-group work (Carlson, Interview 3/27/00). Teacher Log data for Carlson for 1999-2000 were unavailable.

As an example of a lesson observed in Carlson's class, the lesson on 3/1/00 included: large group, teacher-led discussion on previous homework assignment (22 minutes); large group, teacher-led discussion on inequalities using page 3 problem 1 in an Algebra 1 textbook (27 minutes); seatwork, individual or small groups, on page 3 problem 2 (11 minutes); large group, teacher-led discussion on page 4 problem 2 (14 minutes); seatwork, individual or small groups, on page 5 (22 minutes); and large group, teacher-led discussion on using the distributive property (18 minutes; Carlson, Observation 3/1/00).

Table G26
Subcategories of Lesson Planning: Carlson, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Dillard looked for natural breaks in the unit sections to determine the lesson for a particular day. He used a whole-class presentation to introduce, discuss, and work through the first part of the lesson. When Dillard was confident the majority of students understood the ideas, he had them work through the second half of the lesson in small groups. Dillard valued small-group work because it provided opportunities for students to ask questions, to hear a peer's explanation that was often clearer to a student than the teacher's explanation, to explain their understanding of the problem, and to listen to each other. He explained: "We want [students] to teach each other. If [a student] can explain it then [that student has] a better understanding. Both people benefit from it, not just the first student receiving the help, but the person giving the help benefits from it as well" (Dillard, Interview 4/20/00). Dillard usually made heterogeneous groups of four. On some occasions students chose their own groups with guidance from Dillard. For instance, he expected the groups to be working groups, not social groups and that the members were capable of helping each other. Since Dillard had taught many of the students when they were in Grades 6 and 7, he could anticipate how they would perform on lessons and would make appropriate modifications (Dillard, Interview 4/20/00).

Dillard frequently used whole-class discussion (on 68% of the days), and small-group work (58%). Both were generally given equal emphasis with other instructional forms. Dillard used teacher presentation (41%), independent practice (37%), and review of previous material (31%) less frequently. Independent practice was given more class time: at least half of the class period on 31% of the days, equal emphasis with other instructional forms on 62% of the days, and less than 15% of class time on the remaining days. Teacher presentation was generally given equal emphasis with other instructional forms whereas review of previous material was generally given less than 15% of class time. Dillard used warm-up activities on 21% of the reported days (Dillard, Teacher Log 1999-2000).

Four student activities were important elements in Dillard's instruction: discussion of answers and solution strategies (on 71% of the reported days), participation in whole-class discussion (69%), investigation of problems (62%), and listening to the teacher or taking notes (58%). Investigation of problems was given the most class time; generally equal emphasis with other student activities. Listening to the teacher or taking notes was given equal emphasis with other student activities on 42% of the days and less than 15% of class time on the remaining days. Discussion of answers and solution strategies and participation in whole-class discussion were generally given equal emphasis with other student activities. Students seldom practiced computation (15%), took a text or quiz (15%), or began homework (1%). Students did not reflect on or summarize lesson concepts (Dillard, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Dillard reported in his teacher logs. The lesson observed on 5/2/00, for example, included: housekeeping duties (6 minutes); large group, teacher-led discussion introducing section C, problems 1 and 2 (22 minutes); seatwork, small groups or individual, pages 15-16, problems 1-6 (24 minutes); large-group discussion of problem 2 (4 minutes); seatwork, small group or individual, continued (20 minutes); large group, discussion of problem 3 (7 minutes); seatwork, small group or individual, continued (13 minutes); large-group discussion of problems 4 and 5 and homework assignment (15 minutes; Dillard, Observation 5/2/00).

Table G27
Subcategories of Lesson Planning: Dillard, Guggenheim Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	3	Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan

When planning to teach individual MiC lessons, Gallardo usually worked through all of the problems in the lesson so he could anticipate difficulties students might encounter and develop strategies for helping students either avoid the difficulties or work through them. Gallardo not only considered students' performance on previous lessons, he also considered the extent of their prior knowledge. Since MiC was designed with a spiral format (notion of mastery over time), he planned to review critical prerequisite concepts that all students had not mastered during previous lessons. An example of a prerequisite concept for the graphing lessons was scaling of the axes. Gallardo made sure students understood scales before starting any lessons on graphing. Gallardo valued small-group work when the activity did not involve new concepts and/or procedures. He explained, "The students have to have the proper background to complete the assignment in order to work in small groups. At least one person in the group has to be prepared to do it. And students have to be trained to do [small-group work]" (Gallardo, Interview 4/12/00). Gallardo also thought whole-class direct teaching and working individually were important instructional forms. During large-group discussions Gallardo expected students to explain their strategies and give justifications for their answers (Gallardo, Interview 4/12/00). Teacher Log data for Gallardo for 1999-2000 were unavailable.

As an example of a lesson observed in Gallardo's class, the lesson on 4/12/00 included: housekeeping duties (16 minutes); large group, review of previous lesson (29 minutes); large group, teacher-led discussion/lesson on optimization using constraints and feasible areas (23 minutes); seatwork, individual or small group, page 32, problem 2 (16 minutes); large-group discussion of the jogging-swimming line constraints (6 minutes); seatwork, individual or small group, continued (8 minutes); large-group discussion and homework assignment (6 minutes; Gallardo, Observation 4/12/00).

Table G28
Subcategories of Lesson Planning: Gallardo, Weir Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	3	Students' participation and collaboration planned for during instruction
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

When planning to teach individual MiC lessons, Shepard usually worked through the problems in the lesson to get an idea of the lesson expectations. She frequently planned just one day in advance so that she could consider students' performance on the previous lesson. Shepard valued small-group work because:

> A lot of times [students] don't really understand what all the questions are asking them. Maybe the way I'm explaining it or the way the book does it, one student interprets it one way and another interprets it a different way. [In small-group discussions] they get to see the other student's point of view on things. [Together they] maybe come up with the more logical interpretation. It clears up when they work together. (Shepard, Interview 4/27/00)

Shepard found small-group work was more effective when students chose their own groups and groups were not larger than four. Pairs seemed to work best of all in her class. Students asked questions and discussed their solution strategies during whole-class discussions, but the interaction was usually between the teacher and student, not between student and student (Shepard, Interview 4/27/00).

The two instructional forms that Shepard used most often were review of previous material and warm-up activities, each on 41% of the reported days. However, the amount of class time given to each varied widely. Review of previous material was given at least half of the class period on 19% of the days, equal emphasis with other instructional forms on 56% of the days, and less than 15% of class time on the remaining days. Warm-up activities were always given less than 15% of class time. Shepard's students participated in small-group work on 31% of the reported days and always for at least half of the class period. On 26% of the reported days Shepard used teacher presentation and independent practice. Independent practice was given at least half of the class period on 40% of the days and equal emphasis with other instructional forms on 60% of the days. Teacher presentation was given at least half of the class period on 40% of the days, equal emphasis with other instructional forms on 40% of the days, and less than 15% of class time on 20% of the days. Although Shepard used whole-class discussion on fewer days than other instructional forms (21%), it was generally given at least half of the class period (Shepard, Teacher Log 1999-200).

On 46% of the reported days, Shepard's students discussed answers and solution strategies or listened to their teacher or took notes. However, students were given more class time to discuss answers and solution strategies: at least half of the class period on 66% of the days equal emphasis with other student activities on 28% of the days. Listening to the teacher was given at least half of the class period on 17% of the days, equal emphasis with other student activities on 28% of the days, and less than 15% of class time on 56% of the days. On approximately a third of the reported days, students reflected on or summarized lesson concepts (36%) or took a test or quiz (33%). Reflection on or summarization of lesson concepts was given more class time: at least half of the class period on 64% of the days. On 26% of the reported days, students investigated problems or participated in whole-class discussion. These student activities were given similar amounts of class time: at least half of the class period or equal emphasis with other student activities. Students seldom practiced computation (15%) or began homework during class time (3%; Shepard, Teacher Log 1999-2000).

In general, observation reports completed during 1999-2000 classes supported the information Shepard reported in her teacher logs. The lesson observed on 3/29/00, for example, included: individual student seatwork to correct homework papers on Section B, ask questions, and complete unfinished problems (25 minutes); large group, teacher-led discussion of Section B homework (35 minutes); and seatwork, individual or small groups, 2 problems on page 44, a follow-up to Section B (38 minutes; Shepard, Observation 3/29/00).

Table G29
Subcategories of Lesson Planning: Shepard, Weir Middle School, District 2, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	1	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the four eighth-grade teachers in District 2 varied in all four subcategories of *lesson planning* (see Figure G4). With respect to *students' performance in the previous lesson*, one teacher's decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. The other three teachers made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Little variation was seen in teachers' attention to the *purpose of the lesson*. Three teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. The remaining teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.

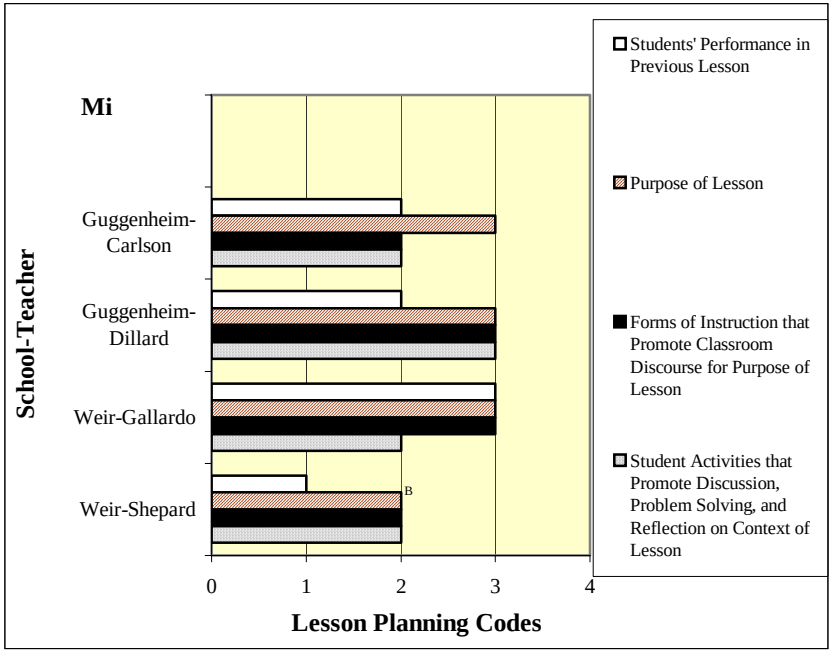


Figure G4. Lesson planning, Grade 8, District 2.

*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

* Detailed description of Lesson Planning Codes in Table G58 in this appendix.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Two teachers planned for students' participation and collaboration, but it was not the primary focus of the lesson plan. The other two teachers included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Teachers varied little in the fourth subcategory of *lesson planning, student activities that promote discussion, problem solving, and reflection as the content of the lesson*. For one teacher, investigation of problems and discussion of answers and solution strategies were important elements in the lesson plan. However, question or activities that encouraged students to reflect on or summarize lesson concepts were not included in the lesson plan. The other three teachers included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Mathematical Interaction

Six subcategories of *Instruction* characterized the *mathematical interaction during instruction*: *lesson presentation and development; nature of mathematical inquiry during instruction; interactive decisions during instruction; nature of students' explanations; elicitation of multiple strategies*; and *lesson reflection, summary, or closure*.

Carlson. The evidence gathered for lesson presentation and development for Carlson ranged from Level 2A to Level 3B. An overall rating of Level 3B was assigned, indicating that students practiced the procedure or strategy presented by their teacher in a rote fashion (see Table G59 in this appendix). An overall rating of 3 was assigned for Carlson for the nature of inquiry, indicating attention to conceptual understanding (see Table G60). Carlson was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure G14). For Carlson, 50% of the decisions were coded as reflective of good standard pedagogy, 12% were most aligned with teaching for understanding, and 38% were least aligned with teaching for understanding.

Across all observations for Carlson, the mean rating for the index on students' explanations was 1.67, indicating that students provided only answers or their explanations were focused on procedures (see Table G61). The mean rating for elicitation of multiple strategies was 1.78, indicating that multiple strategies were rarely elicited from students (see Table G62). A rating of 1 was assigned for Carlson on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G30
Overall Ratings for Mathematical Interaction for Carlson, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	3B	Demonstration of strategy or procedure
Nature of Inquiry	3	Attention to conceptual understanding
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.67	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.78	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Dillard. The evidence gathered for lesson presentation and development for Dillard ranged from Level 2A to Level 5. An overall rating of Level 4 was assigned, indicating attempt to develop conceptual understanding (see Table G59 in this appendix). An overall rating of 4 was assigned for Dillard for the nature of inquiry promoted in-depth exploration of mathematics (see Table G60). Dillard was assigned Level 3 on the composite index for interactive decision-making, indicating that interactive decisions were more reflective of good standard pedagogy and decisions least aligned with teaching for understanding were less pronounced than at Level 2 (see Figure G15). For Dillard, 47% of the decisions were coded as

reflective of good standard pedagogy, 26% were most aligned with teaching for understanding, and 26% were least aligned with teaching for understanding.

Across all observations for Dillard, the mean rating for the index on students' explanations was 1.78, indicating that students provided only answers or their explanations were focused on procedures (see Table G61). The mean rating for elicitation of multiple strategies was 2.11, indicating that multiple strategies were generally not elicited from students (see Table G62). A rating of 2 was assigned for Dillard on the index for reflection or summary, indicating that some opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G31
Overall Ratings for Mathematical Interaction for Dillard, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	4	Attempt to develop conceptual understanding
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	3	Stronger emphasis on standard pedagogy
Nature of Student Explanations	1.78	Answers only and focus on procedures
Elicitation of Multiple Strategies	2.11	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	2	Some opportunities

Gallardo. The evidence gathered for lesson presentation and development for Gallardo ranged from Level 2A to 5. An overall rating of Level 5 was assigned, indicating that lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher (see Table G59 in this appendix). An overall rating of 4 was assigned for Gallardo for the nature of inquiry that transpired during instruction promoted in-depth exploration of mathematics (see Table G60). Gallardo was assigned Level 5 on the composite index for interactive decision-making, indicating that interactive decisions were predominantly most aligned with teaching for understanding (see Figure G16). For Gallardo, 70% of the interactive decisions were coded as most aligned with teaching for understanding, 20 % were reflective of good standard pedagogy, and 10% were least aligned with teaching for understanding.

Across all observations for Gallardo, the mean rating for the index on students' explanations was 2.50, indicating that student explanations were focused on procedures (see Table G61). The mean rating for elicitation of multiple strategies was 2.50, indicating that multiple strategies were rarely elicited from students (see Table G62). A rating of 1 was assigned for Gallardo on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G32
Overall Ratings for Mathematical Interaction for Gallardo, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	5	Emphasis on conceptual understanding with active participation by students and teacher
Nature of Inquiry	4	In-depth exploration of mathematics
Interactive Decisions	5	Most aligned with teaching for understanding
Nature of Student Explanations	2.50	Focus on procedures
Elicitation of Multiple Strategies	2.50	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

Shepard. The evidence gathered for lesson presentation and development for Shepard ranged from Level 2A to Level 4. An overall rating of Level 2B was assigned, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident (see Table G59 in this appendix). An overall rating of 2 was assigned for Shepard for the nature of inquiry that transpired during instruction, indicating limited attention to teaching mathematics for understanding (see Table G60). Shepard was assigned Level 2 on the composite index for interactive decision-making, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding (see Figure G17). For Shepard, 50% of the interactive decisions were coded as least aligned with teaching for understanding, 33% were reflective of good standard pedagogy, and 17% were most aligned with teaching for understanding.

Across all observations for Shepard, the mean rating for the index on students' explanations was 1.75, indicating that students provided answers only on some occasions and their explanations were focused on procedures on other occasions (see Table G61). The mean rating for elicitation of multiple strategies was 1.63, indicating that multiple strategies were rarely elicited from students (see Table G62). A rating of 1 was assigned for Shepard on the index for reflection or summary, indicating that few, if any, opportunities were provided each week for students to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson.

Table G33
Overall Ratings for Mathematical Interaction for Shepard, District 2, Grade 8

Subcategory	Rating	Description
Lesson Presentation and Development	2B	Emphasis on review
Nature of Inquiry	2	Limited attention to teaching for understanding
Interactive Decisions	2	More emphasis on standard pedagogy
Nature of Student Explanations	1.75	Answers only and focus on procedures
Elicitation of Multiple Strategies	1.63	Strategies rarely elicited
Lesson Reflection, Summary, or Closure	1	Limited opportunities

In summary, the four Grade 8 teachers in District 2 (all MiC) varied from Level 2B to Level 5 in *lesson presentation and development* (see Table G34). One teacher was assigned Level 5, indicating that his lessons featured a conceptual basis for the mathematical content and the mathematical work was shared by students and teacher. One teacher was assigned Level 4, indicating that the teacher attempted to teach for conceptual understanding. One teacher was assigned Level 3B, indicating that students practiced strategies or procedures presented by their teacher in rote fashion. The remaining teacher was assigned Level 2B, indicating that limited introduction to the lesson, vague directions, or lack of appropriate planning was evident.

Table G34
Nature of Mathematical Interaction, Grade 8 Teachers in District 2, 1999-2000

School-Teacher	Lesson Presentation and Development	Nature of Inquiry	Teachers' Interactive Decisions	Nature of Student Explanations	Elicitation of Multiple Strategies	Lesson Closure, Reflection, or Summary	Total
			— MiC —				
Guggenheim-Carlson	3B	3	3	1.67	1.22	1	12.89
Guggenheim-Dillard	4	4	3	1.78	2.11	2	16.89
Weir-Gallardo	5	4	5	2.50	2.50	1	20.00
Weir-Shepard	2B	2	2	1.75	1.63	1	10.38

With respect to the *nature of inquiry during instruction*, teachers varied from Level 2 to Level 4. Two teachers were assigned Level 4, indicating that they promoted in-depth explorations of mathematics. One teacher was assigned Level 3, indicating that attention was given to conceptual understanding during instruction. One teacher was assigned Level 2, indicating that limited attention was given to conceptual understanding during instruction.

With respect to *teachers' interactive decisions*, teachers varied from Level 2 to Level 5. One MiC teacher was assigned Level 5, indicating that his interactive decisions were most aligned with teaching for understanding. Two MiC teachers were assigned Level 3, indicating that their interactive decisions provided greater attention to good standard pedagogy. The remaining MiC teacher was assigned Level 2, indicating that, although some emphasis was given to standard pedagogy, interactive decisions were predominantly least aligned with teaching for understanding.

The overall means for the *nature of student explanations* ranged from 1.67 to 2.50, indicating that at some occasions student explanations were limited to answers only and on other occasions explanations were focused on procedures. The overall means for the *elicitation of multiple strategies* ranged from 1.22 to 1.50. In general, teachers rarely elicited multiple strategies. With respect to *lesson reflection, summary, or closure*, one teacher offered some opportunities for students to reflect on the mathematics in a lesson or in a series of lessons. The remaining three teachers offered few, if any, such opportunities.

Classroom Assessment

Three subcategories of *Instruction* characterized *classroom assessment practice*: *evidence sought*, *purpose and coherence of feedback*, and *content of feedback*. The eighth-grade teachers (all MiC) in District 2 varied in all three subcategories of *classroom assessment* (see Tables G35-G38). With respect to the *evidence sought* during classroom assessment, one teacher maintained a conservative process orientation. The remaining three teachers sought little evidence of student learning. *Feedback* in classes of one teacher was purposeful and shared by teacher and students. Feedback in the classes of the other three teachers was very teacher-directed. One teacher provided mixed, superficial feedback, and one provided low-level closed feedback. The remaining two teachers provided feedback that allowed students to know whether answers were correct or incorrect.

Table G35
Evidence Sought through Classroom Assessment Practice, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	1	Limited evidence
Guggenheim-Dillard	1	Limited evidence
Weir-Gallardo	4	Conservative process orientation
Weir-Shepard	1	Limited evidence

Table G36
Feedback Coherence and Purpose, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	2	Teacher-directed feedback
Guggenheim-Dillard	2	Teacher-directed feedback
Weir-Gallardo	4	Purposeful shared responsibility
Weir-Shepard	2	Teacher-directed feedback

Table G37
Feedback Content, Grade 8, District 2

	Rating	Description
MiC		
Guggenheim-Carlson	2	Answer-only feedback
Guggenheim-Dillard	3	Low-level closed feedback
Weir-Gallardo	4	Mixed, superficial feedback
Weir-Shepard	2	Answer-only feedback

Table G38
Classroom Assessment, Grade 8, District 2

School-Teacher	**Evidence Sought**	**Feedback Coherence and Purpose**	**Feedback Content**	**Total**
		— MiC —		
Guggenheim-Carlson	1	2	2	5
Guggenheim-Dillard	1	2	3	6
Weir-Gallardo	4	4	4	12
Weir-Shepard	1	2	2	5

See index next page

Classroom Assessment

Evidence Sought: The following index measures the evidence teacher regarded as indicative of student achievement and understanding.

1. *Limited evidence.* Evidence of student learning was limited to correct answers. Lessons were often tightly scripted and student responses were not recognized as a necessary part of instruction.

2. *Procedural competence*. Evidence of student learning included procedural competence. Greater attention was given to student homework and classwork for instructional decision-making.

3. *Undeveloped process orientation.* Evidence of student learning included student explanations in addition to procedural competence and answers. However, student explanations validated by the teacher were often void of mathematical substance.

4. *Conservative process orientation*. The teacher was somewhat effective at eliciting student responses and orchestrating substantive whole class discussions. However, the overriding focus of classroom practice was directed toward demonstration of student learning through correct answers and procedures.

5. *Principled process orientation.* The teacher viewed student explanations as evidence of student learning. The teacher sought both process and product as evidence and valued demonstration of student learning through verbal or written communication of process.

Feedback Coherence and Purpose: The following index measures the method and goal orientation of feedback that the teacher provided for students.

1. *No feedback.* The teacher did not provide feedback or guidance to students. Classroom practices were not responsive to student needs or misconceptions.

2. *Teacher-directed feedback.* Feedback was indirectly responsive to student needs through whole class, teacher-directed practices that involved "more of the same," such as additional instruction and practice sets.

3. *Emerging shared responsibility*. Students received feedback from peers through student-student discussions in pairs or groups and sharing examples of their responses to assigned work. However, student-student interactions rarely went beyond sharing answers or procedures and were not orchestrated to promote sense making.

4. *Purposeful shared responsibility*. Student interactions were used to promote making sense of tasks, responses to tasks, and mathematical conventions. Feedback was ongoing and offered in multiple ways, through verbal and written modes, from teacher and students, through sharing work-in-progress and examples of refined responses.

5. *Toward student self-assessment.* The process and criteria used by the teacher to evaluate mathematical work was revealed to students and they are invited to assess their own and other students' work.

Content of Feedback: The following index measures the degree of substantive feedback provided to students, from teachers and students.

1. *Feedback withheld and/or misleading*. The teacher's feedback and guidance was not coherent or logical. Feedback was consistently misleading and lacked mathematical substance.

2. *Answer-only feedback.* Feedback was limited to checking correct-incorrect answers. Feedback seldom addressed student misconceptions.

3. *Low-level, closed feedback.* In addition to checking answers, feedback was directed towards skills and procedures. However, feedback rarely addressed the meaning of procedures or related mathematical concepts. Feedback was often directed toward the format of the answer rather than clarifying explanations or developing student understanding.

4. *Mixed, superficial feedback*. An emerging blend of feedback addressing skills, procedures, and concepts was evident. Feedback was directed towards mathematics although, at times, feedback favored problem context over mathematical content. Feedback was clear and mathematically sound.

5. *Concept-directed feedback.* Feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall.

Student Pursuits

Three subcategories characterized *students' pursuits during instruction*: *nature of student–student conversation, collaborative working relationships among students*, and *level of student engagement*.

Nature of Student–Student Conversation

The index ratings about the nature of student–student conversation for Carlson ranged from Level 1 to Level 3 (see Table G63 in this appendix). The mean rating across observations was 1.89, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Carlson are described. The observer noted, "Some discussion was overheard related to the fees that were recorded on the table/chart" (Carlson, Observation 4/10/00). In this example, student–student conversation was limited and consisted of sharing answers.

The index ratings about the nature of student–student conversation for Dillard ranged from Level 1 to Level 4 (see Table G63 in this appendix). The mean rating across observations was 1.89, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Dillard are described. The observer noted, "There was not much student–student discourse related to the mathematics" (Dillard, Observation 11/2/99). In this example student–student conversation was limited.

The index ratings about the nature of student–student conversation for Gallardo ranged from Level 1 to Level 4 (see Table G63 in this appendix). The mean rating across observations was 2.25, indicating that student–student conversation was limited, consisted of sharing answers or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Galllardo are described. The observer noted, "Occasionally students asked or argued with each other about problems, for example, the value of 3^3 and the number of days between Sunday and Wednesday" (Gallardo, Observation 1/25/00). In this example, student–student conversation was limited.

The index ratings about the nature of student–student conversation for Shepard ranged from Level 1 to Level 3 (see Table G63 in this appendix). The mean rating across observations was 2.13, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures. To illustrate a rating at Level 2, student–student conversations that occurred during a lesson by Shepard are described. The observer noted, "No student–student conversation occurred in the large group. I couldn't hear what was going on during the last 14 minutes of the class period, yet, from the body language, it appeared that some students were sharing efforts in the assignment" (Shepard, Observation 10/20/99). In this example, student–student conversation was limited and focused on procedures.

In summary, the mean ratings for *student–student conversation* for the four Grade 8 teachers in District 2 varied from 1.89 to 2.25. All four MiC teachers had mean ratings near 2.00, indicating that student–student conversation was limited, consisted of sharing answers, or focused on procedures.

Collaborative Working Relationships among Students

The index ratings about the nature of students' collaboration in the classroom for Carlson ranged from Level 1 to Level 4 (see Table G64 in this appendix). The mean rating across observations was 2.00, indicating that few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Carlson is described. The observer noted, "Many students were working individually while others worked together, not always in a supportive way unless urged to do so" (Carlson, Observation 9/15/99). In this example, few students shared ideas or discussed how a problem should be solved.

The index ratings about the nature of students' collaboration in the classroom for Dillard ranged from Level 1 to Level 3 (see Table G64 in this appendix). The mean rating across observations was 1.89, indicating that usually few students shared ideas or discussed how a problem should be solved or that many of the students in a group worked on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Dillard is described. The observer noted, "Most students worked individually in their small groups. It was evident that they were working on different problems; individuals were using the graphing calculators at different times" (Dillard, Observation 10/4/99). In this example, although students physically sat together, there was little exchange of ideas or assistance. Many students in a group worked on different problems at different paces.

The index ratings about the nature of students' collaboration in the classroom for Gallardo ranged from Level 1 to Level 3 (see Table G64 in this appendix). The mean rating across observations was 1.88, indicating that usually few students shared ideas or discussed how a problem should be solved or that many of the students in a group working on different problems at different paces. To illustrate a rating for students' collaboration at Level 2, student collaboration that occurred during a lesson by Gallardo is described. The observer noted, "There was no small group work today. Several students informally worked together when the teacher periodically gave them time to work individually" (Gallardo, Observation 3/13/00). In this example, few students shared ideas or discussed how a problem should be solved.

The index ratings about the nature of students' collaboration in the classroom for Shepard ranged from Level 1 to Level 3 (see Table G64 in this appendix). Although the mean rating across observations was 2.29, on four of the seven observations students' collaboration was rated at Level 3, indicating that while some students exchanged ideas or provided peer assistance, contributions to solving problems were not made equally by all students. To illustrate a rating for students' collaboration at Level 3, student collaboration that occurred during a lesson by Shepard is described. The observer noted, "Even though the students were not seated in groups, about half of them did participate in informal working relationships with those seated close to them" (Shepard, Observation 3/17/00). In this example, some students exchanged ideas or provided peer assistance.

In summary, the mean ratings for *student collaboration* for the four Grade 8 teachers in District 2 varied from 1.88 to 2.29. One MiC teacher had a mean rating just greater than 2.00, but on the majority of observations students collaboration was rated at Level 3, indicating that some students exchanged ideas or provided peer assistance. Three MiC teachers had mean ratings at or near 2.00, indicating that on some occasions some students shared ideas or discussed how a problem should be solved.

Student Engagement during Instruction

The index ratings about the extent to which students were engaged during the lesson for Carlson ranged from Level 2 to Level 3 (see Table G65 in this appendix). The mean rating across observations was 2.89, indicating that on most occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Carlson is described. The observer noted, "Students were attentive during the large-group lesson but tended to be off task during the 54-minute and 35-minute periods allotted for group work" (Carlson, Observation 4/10/00). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Dillard ranged from Level 3 to Level 4 (see Table G65 in this appendix). The mean rating across observations was 3.11, indicating that on most occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Dillard is described. The observer noted, "Progress that was observed was varied among students during seatwork time. Yet most students were attentive during the large-group teacher-led discussion time" (Dillard, Observation 3/2/00). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

The index ratings about the extent to which students were engaged during the lesson for Gallardo ranged from Level 3 to Level 4 (see Table G65 in this appendix). The mean rating across observations was 3.88, indicating that on most occasions student engagement was widespread with students on task pursuing the substance of the lesson most of the time. To illustrate a rating for student engagement at Level 4, students' on-task behavior that occurred during a lesson by Gallardo is described. The observer noted, "Most students were on task most of the time. Students were always so enthusiastic and ready to volunteer. If not, [Gallardo] got them to volunteer anyway" (Gallardo, Observation /25/00). In this example, most students were on task pursuing the substance of the lesson.

The index ratings about the extent to which students were engaged during the lesson for Shepard ranged from Level 1 to Level 4 (see Table G65 in this appendix). The mean rating across observations was 2.75, indicating that on many occasions student engagement was sporadic or episodic. To illustrate a rating for student engagement at Level 3, students' on-task behavior that occurred during a lesson by Shepard is described. The observer noted, "About half of the class was on task most of the time. The other half was sometimes on task. At one time, 5 students were talking and not listening to [Shepard] while she was leading a lesson" (Shepard, Observation 12/15/99). In this example, most students were engaged in class activities some of the time, but this engagement was inconsistent.

In summary, the mean ratings for *student engagement in the lesson* for the four Grade 8 teachers in District 2 varied from 2.75 to 3.88. One MiC teacher had a mean rating near 4.00, indicating that student engagement was widespread with students on task pursuing the substance of the lesson most of the time. Three MiC teachers had mean ratings near 3.00, indicating that most students were engaged in class activities some of the time, but this engagement was inconsistent.

Instruction Composite Variable

A single index, a composite of multiscaled information from each subcategory of considered in the *Instruction* composite variable, represents Instruction in the simplified research function. The following table summarizes the weighted ratings for each subcategory for each teacher and indicates the level on the composite index *Instruction* for each teacher.

Table G39
Teacher Level of Instruction, Grade 8, District 2

School-Teacher	Lesson Planning		Mathematical Interaction						Classroom Assessment			Student Pursuits			Weighted Sum	Composite Level
	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE		
— MiC —																
Guggenheim-Carlson	2.5	2.5	3	3	3	2.78	1.53	1.67	1	2	2	2.36	2.50	3.61	33.45	**3**
Guggenheim-Dillard	3.75	3.75	4	4	3	2.97	2.64	3.33	1	2	3	2.36	2.36	3.89	42.05	**4**
Weir-Gallardo	3.75	2.5	5	5	5	4.17	3.13	1.67	4	4	4	2.81	2.35	4.85	52.23	**6**
Weir-Shepard	2.5	2.5	2	2	2	2.92	2.04	1.67	1	2	2	2.66	2.86	3.44	31.59	**2**

Key

FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

The ratings assigned to each teacher in this district on each of the indices related to *unit* and *lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for he remaining indices, and the composite variable instruction was not calculated for these teachers.

District 3

In District 3, two Grade 8 teachers participated in the study. Both used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*. For description of Schroeder's unit planning, see Appendix F for Grade 7.

Data on unit planning for Wells for 1999-2000 were unavailable. The following is a summary of her 1998-1999 comments with respect to unit planning. Wells generally followed the recommended sequence of MiC units. She felt that a problem inherent with most of the eighth-grade units was the lack of program continuity. Because MiC was designed with a spiral format (mastery over time), the development of the concepts studied in eighth-grade units had begun years earlier. Students whose mathematics instruction was not consistently MiC or had not included all of the units for each year from Grades 5–7 might not have prerequisite conceptual understanding or skill in procedures unique to MiC such as using a ratio table for the division of fractions or decimals. She explained:

> For example, I usually start with *Graphing Equations*, but I felt that students would have a really difficult time with *Graphing Equations* without being familiar with *Building Formulas*. *Building Formulas*, a seventh-grade unit, is also a cornerstone unit for algebra and the algebra strand. Since our seventh-grade teachers didn't get to it or to *Ups and Downs* last year, that's the unit I started with this year. (Wells, Interview 9/28/99)

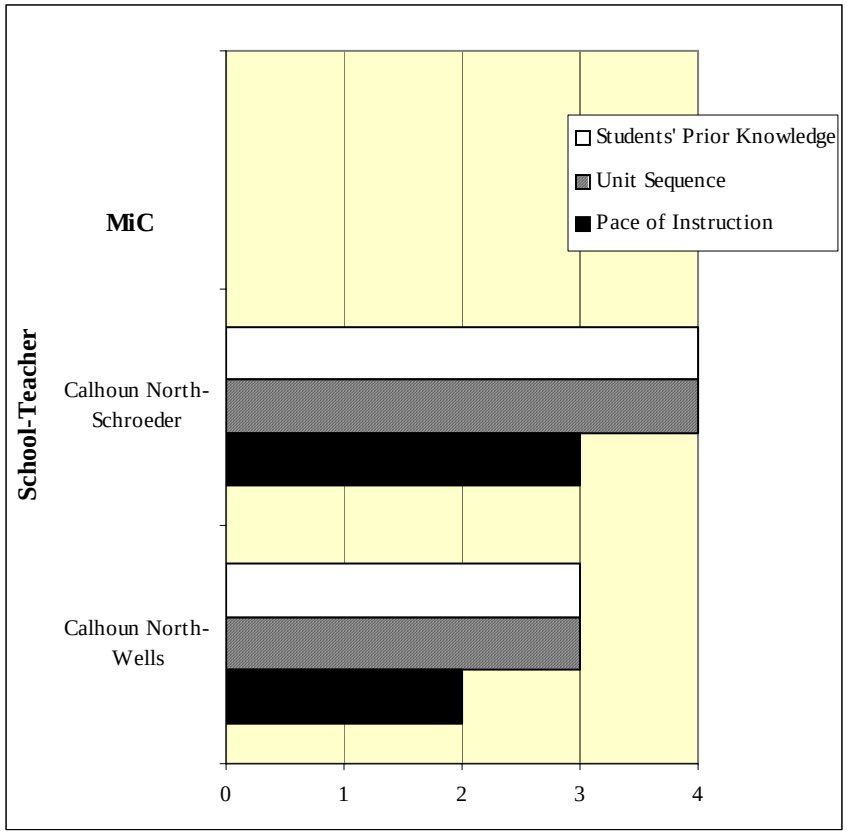
When planning to teach a MiC unit, she read through the unit and assessed students' prior knowledge of the concepts and skills. Wells used that information to determine the questions to discuss with the whole class and those they could work through with a partner. Wells did not determine the pace of instruction prior to teaching the unit. She anticipated the pace of instruction would vary from class to class, based on students' learning.

Wells thought that MiC was well aligned with the district mathematics guidelines because they were based on MiC and that MiC exceeded the state standards (Wells, Interview 9/28/99).

Table G40
Subcategories of Unit Planning: Wells, Calhoun North Middle School, District 3, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	3	Consideration of content and student interests
Pace of Instruction	2	Adjustment anticipated

In summary, the two eighth-grade teachers in District 3 varied in all three subcategories of *unit planning* (see Figure G5). With respect to *students' prior knowledge* when planning to teach a unit, one teacher planned conceptually based activities to bridge the gap between students' prior knowledge and the skills needed for the unit. One teacher formally or informally assessed students' understanding of skills or concepts needed in a particular unit.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure G5. Unit planning, Grade 8, District 3.

In determining the *sequence of instructional units*, one teacher considered the development of mathematics concepts. The other teacher considered linkages across units of the same content strand. In determining the *pace for instruction*, one teacher considered the learning styles and reasoning skills of the students. The other teacher anticipated that various factors such as needed prerequisite skills or difficulty of content would necessitate adjusting the recommended pace for instruction.

The ratings assigned to each teacher in this district on each of the indices related to *unit* and *lesson planning* are described and illustrated with evidence from teacher interviews. In the modified research design, data from classroom observations and teaching logs were not gathered from teachers in this district. As a result, ratings were not assigned for the remaining indices, and the composite variable instruction was not calculated for these teachers.

District 4

In District 4, three Grade 8 teachers participated in the study. All three teachers used MiC. Classroom observation and teacher log data were not collected from these teachers.

Unit Planning

In this study, three subcategories characterized *unit planning*: *consideration of students' prior knowledge*, *unit sequence*, and *pace of instruction*.

Downer followed the sequence of MiC units developed by the eighth-grade mathematics teachers at Kelvyn Park Middle School. This sequence emphasized concepts needed for the district and state standardized tests. When planning to teach a MiC unit, Downer read through the unit to become familiar with its content and organized the needed materials. She sorted the problems into three groups: those to be done individually in class, those to be done in small groups during class, and those to be done as homework. Before beginning to teach the unit, Downer reviewed the prerequisite concepts and skills with the students so that they would have the expected prior knowledge to be successful with the unit. Downer generally thought that each unit needed one month to complete, but anticipated adjusting the pace of instruction to accommodate the ability level of each class. She explained:

> Some books go quickly with the brighter classes. The slower classes take a little longer. I ask the same questions actually to both the slow class and the bright class. The brighter class is able to follow along and get the answers. We move quickly through the unit. But with the slower class it takes a mighty long time. With them I have to do more hands-on [activities]. I might not get the answer from [them] without putting a picture on the board or something when I ask the question. (Downer, Interview 6/13/00)

Table G42
Subcategories of Unit Planning: Downer, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	3	Informal or formal assessment of students' understanding
Unit Sequence	2	Consideration of external factors
Pace of Instruction	3	Consideration of the needs of current students

Novak was aware of the recommended sequence of MiC units, but used input from seventh-grade teachers to determine the sequence of units. He explained:

> In September I speak to the seventh-grade teachers and they tell me where they left off [the previous year]. Sometimes they leave off with different units and I try to pick that up, kind of refresh [the students] where they left off in June. I see what deficiencies there are in the beginning of the year and what the high points are. September and October are more or less an evaluation of where [students] left off in seventh grade and where we are now. (Novak, Interview 6/14/00)

When planning to teach a MiC unit, Novak outlined the unit, organized the needed materials, and created a game or some other activity to motivate students. He did not determine the pace prior to instruction, but adapted the pace of instruction as students worked through the unit. Novak was aware of the state standards and district mathematics guidelines, but did not use them when planning to teach a unit (Novak, Interview 6/14/00).

Table G43
Subcategories of Unit Planning: Novak, Kelvyn Park Middle School, District 4, Grade 8

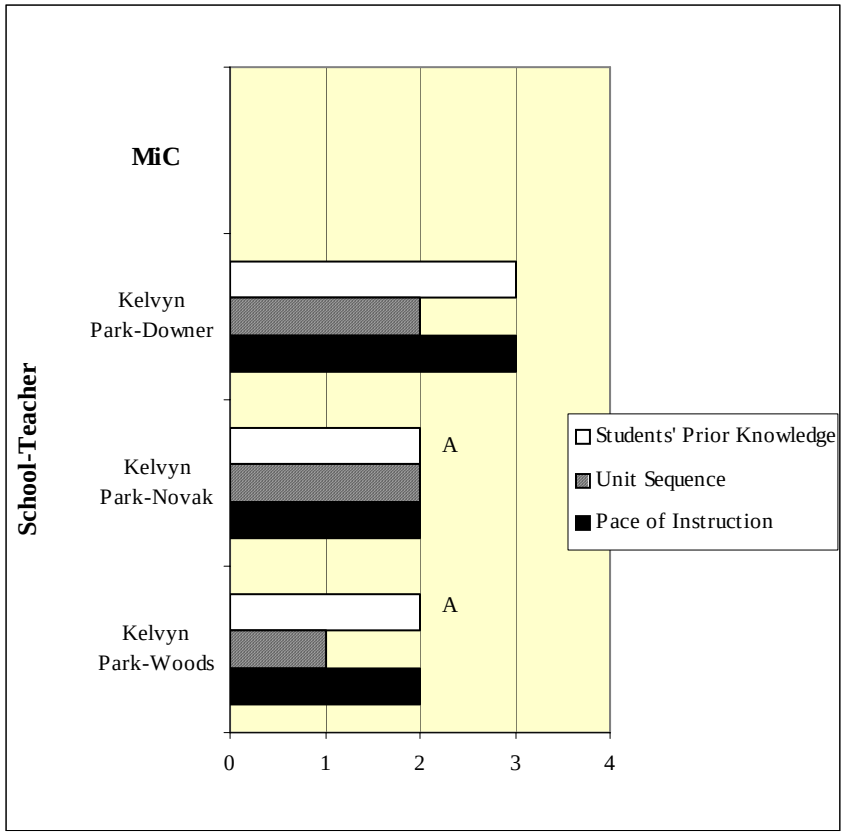
Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	2	Consideration of external factors
Pace of Instruction	2	Adjustment anticipated

Woods followed the recommended sequence of MiC units. When planning to teach a MiC unit, he read through the whole unit to become familiar with the scope of its content. Woods found that he often needed to modify units because students lacked prior knowledge and to provide additional skill practice, concept review, and additional information. He also kept in mind the skills students needed to succeed with district standardized testing and in the real world. Woods was aware of the suggested pace for instruction, but found it unrealistic for him: "The suggested times are somewhat unrealistic. I just try to go with the flow" (Woods, Interview 6/1/00). As he taught the unit, Woods adjusted the pace to meet the needs of the students (Woods, Interview 6/1/00).

Table G44
Subcategories of Unit Planning: Woods, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Prior Knowledge	2A	Consideration of student abilities
Unit Sequence	1	Little or no variance from the text sequence
Pace of Instruction	2	Adjustment anticipated

In summary, the three eighth-grade teachers in District 4 varied in all three subcategories of *unit planning* (see Figure G7). With respect to *students' prior knowledge* when planning to teach a unit, one teacher assessed students' understanding of skills or concepts needed in a particular unit. Two teachers based unit planning on their perceptions of students' mathematics skills and reading ability without informal or formal assessment.



UNIT PLANNING CODES*

Students' Prior Knowledge. The following index scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.

1. Little or no consideration of students' prior knowledge.
2. Consideration of student abilities.
A. The teacher bases plans on perceptions of students' mathematics skills.
B. The teacher bases plans on perceptions of students' needs related to the development of concepts and procedures.
3. Informal or formal assessment of students' understanding.
4. Conceptually-based activities planned.

Unit Sequence. The following index measures the extent to which the teacher might consider the sequence of instructional units.

1. Little or no variance from the text sequence.
2. Consideration of external factors.
3. Consideration of content and student interests.
4. Consideration of the development of mathematics concepts.

Pace of Instruction. The following index measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.

1. Little or no consideration of pacing.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. Adjustment anticipated.
3. Consideration of the needs of current students.
4. Supplemental activities anticipated.

Figure G7. Unit planning, Grade 8, District 4.

In determining the *sequence of instructional units*, two teachers considered external factors such as needed prerequisite skills or difficulty of content. One teacher followed the unit sequence recommended in teacher support materials. In determining the *pace for instruction*, one teacher considered students' learning styles and reasoning skills. Two teachers anticipated that various factors such as needed prerequisite skills or difficulty of the content would necessitate adjusting the recommended pace for instruction.

Lesson Planning

In this study, four subcategories of *Instruction* characterized *lesson planning*: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson*; and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*.

When planning to teach individual MiC lessons, Downer considered the amount of content covered in the previous lesson and the students' performance in that lesson. If students did not complete the previous lesson, she determined if the problems not completed could be done as homework or if the material could be skipped without causing conceptual misunderstandings. Downer worked through all of the problems in the new lesson making note of possible solutions, possible questions students might have, and probing questions she could ask to help students expand their thinking. She also noted the activities such as cutting out shapes that could be assigned homework to save class time. She found that if she gave students the same credit for completing such activities as she would for a take-home quiz, most students returned them completed on time. Downer physically organized her classroom so that students sat in groups of five or six. If a lesson had several problems, Downer assigned different problems to different groups. Each group member was responsible for reporting at least a portion of the group's findings to the whole class. If there were differences in group's findings, the whole class looked for reasons and justifications. Although Downer thought small-group work was generally a good form of instruction because it allowed the opportunity for each student to participate and succeed, she realized that not all students worked well in groups. Some students could work with a partner and some needed to work alone (Downer, Interview 6/13/00).

Table G45
Subcategories of Lesson Planning: Downer, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	3	Changes focused on students' understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	4	Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan

When planning to teach individual MiC lessons, Novak considered the difficulty of the lesson, difficulties that occurred when he taught the lesson previously, ways other teachers had approached the lesson, and how students performed on previous lessons. Novak did not use conventional forms of instruction. He commented, "With the standard lecture type method, [students] fall asleep. They get bored. With question-and-answer, direct response, they tend to waver off, to look out the window" (Novak, Interview 6/14/00). During whole-class discussions, Novak assured

student participation by rewarding them for finding any errors that he made: “I say basically if you can find any mistake that I say or do, I’ll give you some extra credit coupons. Oh, they look for any error and by that I have their attention. They like to catch me. So we do have quite an aggressive communication” (Novak, Interview 6/14/00). Novak valued small-group work because each student had the opportunity to contribute to a better understanding of the topic and students learned from their peers. He selected groups that would be the most effective for the particular activity or lesson. He explained:

> Every class has its own potential. We have certain students who excel in [certain] areas and certain students who are slightly slower. So I try to prepare the group in a way that students can all contribute in their own special talent. Some mixes don’t work, so I plan not to put those three or four together again. I try to think of attitudes, character, personalities, and where they’re coming from when setting up a group situation. (Novak, Interview 6/14/00)

Groups presented their findings to the whole class. However, each group also completed a group sheet that listed each individual name, each person’s responsibility for that particular activity, and each person’s contribution. Each member of the group had to sign that they agreed with the group’s conclusions. Novak commented: “It’s an individual assessment as well as a group assessment. Each feels that [he/she] has had a part in contributing to the group, but they worked as a group, as a whole” (Novak, Interview 6/14/00).

Table G46
Subcategories of Lesson Planning: Novak, Kelvyn Park Middle School, District 4, Grade 8

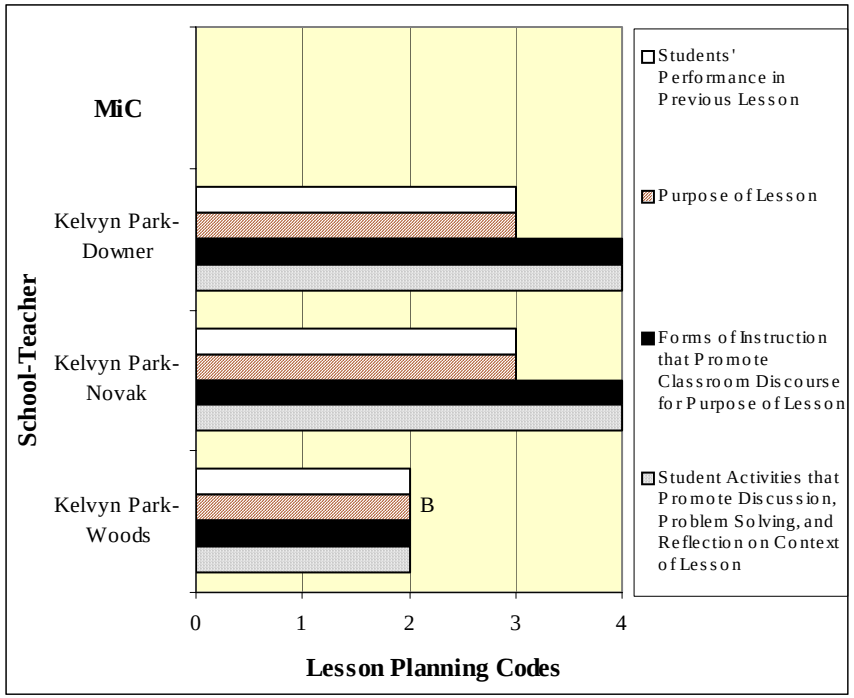
Subcategory	Rating	Description of Rating
Students’ Performance in Previous Lesson	3	Changes focused on students’ understanding of the mathematical content of the lesson
Purpose of the Lesson	3	Planning beyond familiarity with content, presentation, and context
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	4	Forms of instruction that promote substantive conversation planned
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	4	Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan

When planning to teach individual MiC lessons, Woods determined the length of the lesson and then “I just try to go with the flow” (Woods, Interview 6/1/00). Woods classified lessons into two types: those acquiring the tools and those applying the tools. He generally preferred whole-class direct teaching followed by individual practice when students were learning the concepts and skills needed to solve problems. Although the whole-class discussions were at times very interactive, they were teacher led. Woods used small-group work when students applied mathematical tools to a project or an investigation. He determined group membership mainly on behavioral criteria. Although he felt some pressure to conform to the school’s suggestion of four in a group, he did not assign roles to each group member. He explained: “Everybody writes, everybody thinks, everybody talks, everybody does. We just sit together” (Woods, Interview 6/1/00).

Table G47
Subcategories of Lesson Planning: Woods, Kelvyn Park Middle School, District 4, Grade 8

Subcategory	Rating	Description of Rating
Students' Performance in Previous Lesson	2	Changes in the next day's plans because of students' performance
Purpose of the Lesson	2B	Overall curriculum plan in mind
Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	2	Whole-class discussion and small-group or pair work anticipated
Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	2	Investigation of problems and discussion of answers and solution strategies included in the lesson plan

In summary, the three eighth-grade teachers in District 4 varied in all four subcategories of *lesson planning* (see Figure G8). With respect to *students' performance in the previous lesson*, two teachers' decisions focused on students' understanding of the mathematical content of the lesson and using that information to allow more in-depth exploration of the mathematical content or to introduce another approach to encourage students' thinking. The remaining teacher made decisions about extending the lesson to complete a task, adding review, or accounting for individual differences. Little variation was seen in teachers' attention to the *purpose of the lesson*. Two teachers went beyond checking their own understanding of lesson content and presentation to make decisions about student learning such as thinking about questions students might raise, misunderstandings that might emerge, or accommodations for various ability levels. The remaining teacher planned lessons to become familiar with the mathematics, the presentation of the mathematics, and the lesson context.



*Lesson Planning Codes**

Purpose of the Lesson
1. Little or no planning to teach the specific lesson.
2. Overall curriculum plan in mind.
   A. The teacher selects lesson content to reflect a continuity of mathematics content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
   B. The teacher plans lesson from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. Planning beyond familiarity with the content, presentation, and context.
4. Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding.

Students' Performance in the Previous Lesson
1. Lesson planning with little or no regard for students' performance on the previous lesson.
2. Changes in the next day's plans because of students' performance.
3. Changes focused on students' understanding of the mathematical content of the lesson.
4. Changes focused on encouraging thinking at higher levels.

Forms of Instruction That Promote Classroom Discourse
1. Students' discourse in the classroom seldom, if at all, planned as part of the lesson.
2. Whole-class discussion and small-group or pair work anticipated.
3. Students' participation and collaboration planned for during instruction.
4. Forms of instruction that promote substantive conversation planned.

Student Activities That Promote Discussion, Problem Solving, and Reflection
1. Investigation of problems and discussion of mathematical ideas seldom planned for the lesson.
2. Investigation of problems and discussion of answers and solution strategies (whether during small-group or whole-class discussions) included in the lesson plan.
3. Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.
4. Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.

Figure G8. Lesson planning, Grade 8, District 4.

Teachers varied in the choice of *instructional formats* planned to encourage classroom discourse with respect to the purpose of the lesson. Two teachers planned forms of instruction that promoted substantive conversation, encouraging students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others and build substantive conversation. The remaining teacher included the use of whole-class discussion and small-group work, but the focus was on completing tasks rather than on developing substantive conversation. Teachers also varied in the fourth subcategory of *lesson planning, student activities that promote discussion, problems solving, and reflection on the content of the lesson*. For two teachers, investigation of problems and discussion of answers and solution strategies were dominant in the lesson plan. The opportunity for student to reflect on or summarize lesson concepts was an important element in instruction. The other teacher included whole-class discussions and small-group work in lesson planning, but the significance of these student activities in learning mathematics with understanding was not considered.

Table G48
Unit Planning, District 1, Grade 8, 1998-2000

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Fernwood-Dunn	2A	1	2	5
Von Humboldt-Reichers	2B	1	2	5
Von Humboldt-Waters	2B	1	2	5
		— Conventional —		
Addams-Wolfe	3	2	3	8
Fernwood-Pimm	3	3	3	9

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table G49
Lesson Planning, District 1, Grade 8, 1999-2000

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Fernwood-Dunn	2	1	3	3	9
Von Humboldt-Reichers	2	3	3	3	11
Von Humboldt-Waters	2	3	2	2	9
			— Conventional —		
Addams-Wolfe	2	2A	2	1	7
Fernwood-Pimm	2	2A	3	3	10

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:
1. *Lesson planning with little or no regard for students' performance on the previous lesson*. The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.
Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:
1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.
2. *Overall curriculum plan in mind.*
A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.
Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:
1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned*. The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.
Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:
1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table G50

Teacher Level of Lesson Presentation and Development, Grade 8, District 1

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Fernwood-Dunn	9	0	0	1	0	8	0	0	0	3B
Von Humboldt-Reichers	7	0	0	0	0	3	1	3	0	5
Von Humboldt-Waters	8	0	0	1	0	7	0	0	0	3B
Conventional										
Addams-Wolfe	6	0	0	0	0	6	0	0	0	3B
Fernwood-Pimm	8	0	1	0	0	3	1	3	0	5

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review.* The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support.* The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table G51

Nature of Mathematical Inquiry, Grade 8, District 1

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation										
(No. of Observations)	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	*Mean*	***Rating***
MiC																																						
Fernwood-Dunn	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	*1.11*	***1***
Von Humboldt-Reichers	2	4	2	1	4	4	4	4	4	3	2	4	1	4	2	4	2	2	2	3	2	4	4	2	2	4	4	3	2	1	3	3	1	1	3	3	*2.78*	***4***
Von Humboldt-Waters	1	1	2	3	1	1	1	1		1	1	4	2	1	1	1	1		2	1	2	2	2	1	1	1		2	1	1	1	1	2	1	1		*1.41*	***1***
Conventional																																						
Addams-Wolfe	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1		2	1	1	1	1	1	1	1		*1.11*	***1***
Fernwood-Pimm	1	4	4	1	1	4	1	2	2	1	2	1	1	1	4	1	1	1	2	3	1	1	1	1	1	1	1	3	3	3	1	1	2	1	1	1	*1.69*	***2***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table G52

Nature of Students' Explanations, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Dunn	2	2	1	1	1	1	2	1	1	*1.33*
Von Humboldt-Reichers	2	3	1	1	3	1	3	3	2	*2.11*
Von Humboldt-Waters	2	2	1	1	1	1	1	1		*1.25*
Conventional										
Addams-Wolfe	2	2	2	2	1	2	1	1		*1.63*
Fernwood-Pimm	2	1	2	1	1	2	1	1	2	*1.44*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table G53

Elicitation of Multiple Strategies, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Dunn	1	1	1	1	1	1	1	1	1	*1.00*
Von Humboldt-Reichers	1	1	1	1	1	2	4	2	2	*1.67*
Von Humboldt-Waters	1	1	1	2	1	1	1	1		*1.13*
Conventional										
Addams-Wolfe	1	1	1	1	1	1	1	1		*1.00*
Fernwood-Pimm	1	1	1	1	4	1	1	2	2	*1.56*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.
2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.
3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.
4. *Strategies substantive element of instruction.* Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table G54

Nature of Student–Student Conversation, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Dunn	1	1	1	1	1	1	1	1	1	*1.00*
Von Humboldt-Reichers	2	1	1	1	2	4	4	3	3	*2.33*
Von Humboldt-Waters	1	1	1	1	1	1	1	1		*1.00*
Conventional										
Addams-Wolfe	1	1	1	1	1	1	1	1		*1.00*
Fernwood-Pimm	2	1	1	1	1	1	1	2	1	*1.22*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table G55

Students' Collaborative Working Relationships, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Dunn	3	1	1	1	1	1	1	1	1	*1.22*
Von Humboldt-Reichers	2	1	1	N/A	2	3	3	3	2	*2.13*
Von Humboldt-Waters	1	1	1	1	1	1	1	1		*1.00*
Conventional										
Addams-Wolfe	1	1	1	1	N/A	1	N/A	1		*1.00*
Fernwood-Pimm	2	1	1	1	N/A	1	1	1	1	*1.13*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table G56

Student Engagement during Instruction, Grade 8, District 1

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Fernwood-Dunn	4	3	1	2	2	2	2	1	2	*2.11*
Von Humboldt-Reichers	3	4	2	4	3	4	4	4	3	*3.44*
Von Humboldt-Waters	2	1	3	3	1	1	2	1		*1.75*
Conventional										
Addams-Wolfe	1	1	2	1	3	1	1	2		*1.50*
Fernwood-Pimm	3	4	4	4	4	4	3	2	2	*3.33*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Table G57
Unit Planning, District 2, Grade 8, 1999-2000

School-Teacher	Students' Prior Knowledge	Unit Sequence	Pace of Instruction	Total
		— MiC —		
Guggenheim-Carlson	2A	2	3	7
Guggenheim-Dillard	2B	2	3	7
Weir-Gallardo	3	2	3	8
Weir-Shepard	1	1	2	4

Instructional Planning: Unit Planning, District 1, Grade 7, 1997-1998 Unit Planning Codes

Students' Prior Knowledge: The following scale measures the extent to which the teacher thinks about and identifies students' prior knowledge while planning to teach a unit.
1. *Little or no consideration of students' prior knowledge*. The teacher plans the unit with little or no understanding of the prior knowledge of students in the current class.
2. Consideration of student abilities.
A. The teacher planned the unit based on perceptions of students' reading ability and vocabulary.
B. The teacher planned the unit based on perceptions of students' mathematics skills.
3. *Informal or formal assessment of students' understanding*. The teacher plans the unit on the basis of information gathered through informal or formal assessment. The teacher might, for example, plan remedial skill-based activities to address weaknesses or plan extension activities for students who might be ready for such challenges.
4. *Conceptually-based activities planned*. The teacher plans unit activities that are designed to bridge the gap between students' prior knowledge and prerequisite skills for the unit or to familiarize students with the contexts presented in the unit.

Unit Sequence: The following scale measures the extent to which the teacher might consider the sequence of instructional units.
1. *Little or no variance from the text sequence*. The teacher follows the unit sequence recommended in teacher support materials.
2. *Consideration of external factors*. The teacher bases decisions about unit sequence, for example, on the content and dates of district or state standardized testing or on various calendar events.
3. *Consideration of content and student interests*. The teacher sequences units based on one or more of the following: variety of mathematical content; integration of mathematics with other subjects; linkages across units of the same content strand; and students' interests.
4. *Consideration of the development of mathematics concepts*. The teacher sequences units to support the development of mathematics concepts.

Pace of Instruction: The following scale measures the extent to which the teacher might consider the pace for instruction when planning to teach a unit.
1. *Little or no consideration of pacing*.
A. The teacher follows the recommendations for pacing in teacher support materials.
B. The teacher does not plan unit pacing because the curriculum is unfamiliar.
2. *Adjustment anticipated*. The teacher considers the recommendations for pacing in teacher support materials, but plans to adjust the pace as the unit develops or as a result of collaboration with other teachers.
3. *Consideration of the needs of current students*. The teacher considers the learning styles and reasoning skills of current students when planning the pace of instruction.
4. *Supplemental activities anticipated*. The teacher plans substantive supplemental activities for students who complete the lesson in advance of most students in the class.

Table G58
Lesson Planning, District 2, Grade 8, 1999-2000

School-Teacher	Students' Performance in the Previous Lesson	Purpose of the Lesson	Forms of Instruction that Promote Classroom Discourse for the Purpose of the Lesson	Student Activities that Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson	Total
			— MiC —		
Guggenheim-Carlson	2	3	2	2	9
Guggenheim-Dillard	2	3	3	3	11
Weir-Gallardo	3	3	3	2	11
Weir-Shepard	1	2B	2	2	7

Students' Performance in the Previous Lesson: The following scale measures the extent to which a particular teacher might consider students' performance on the previous lesson when planning to teach the subsequent lesson:
1. *Lesson planning with little or no regard for students' performance on the previous lesson*. The teacher might note students' performance, but the lesson is planned in the same way as the previous lesson.
2. *Changes in the next day's plans because of students' performance.* The teacher might, for example, extend the previous lesson to complete a task, disregard time constraints, or add a review.
3. *Changes focused on students' understanding of the mathematical content of the lesson.* The teacher might use the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding.
4. *Changes focused on encouraging thinking at higher levels.* The teacher might, for example, vary problem structure/setting to encourage thinking at higher levels or emphasize connections with related concepts.
Purpose of the Lesson: This scale measures the extent to which particular teachers might think about and identify the purpose of the lesson prior to instruction:
1. *Little or no planning to teach the specific lesson.* When such planning does occur, the purpose is to identify unit/chapter pages to be taught over a period of days and to copy worksheets or quizzes for students. The aim of instruction is to cover lessons in the textbook or curriculum; thus, no additional planning is deemed necessary.
2. *Overall curriculum plan in mind.*
A. The teacher selects lesson content to reflect a continuity of mathematical content, integrating lesson materials from various resources. The selected materials might include an additional focus on problem solving, applications of mathematics, or practice.
B. The teacher plans lessons from unit or chapter materials with emphasis given to becoming familiar with the mathematical content of the lesson, the presentation of the mathematics in the materials, and the context in which the lesson was couched (if any).
3. *Planning beyond familiarity with the content, presentation, and context.* The teacher makes decisions for student learning (e.g., potential student questions, possible misunderstandings, anticipation of various solution strategies, accommodation of various ability levels, or conceptual development within a unit).
4. *Expectation for student learning in the lesson emphasizes higher order thinking, depth of knowledge, and/or understanding*. The teacher might, for example, plan questions that engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons.
Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson: The following scale measures the extent to which a particular teacher might plan the various forms of instruction that promote classroom discourse for a lesson:
1. *Students' discourse in the classroom seldom, if at all, planned as part of the lesson.* Attention is focused, for example, on factual information or presentation of algorithms and procedures.
2. *Whole-class discussion and small-group or pair work anticipated.* The teacher might, for example, plan for such work/discussion, but continue to focus primarily on completing tasks rather than on facilitating or encouraging substantive conversation of mathematics concepts. (The significance of classroom discourse is not considered in the lesson plan.)
3. *Students' participation and collaboration planned for during instruction*. The teacher encourages such participation, but it is still not the primary focus of the lesson plan.
4. *Forms of instruction that promote substantive conversation planned*. The teacher might, for example, plan classroom activities that encourage students to contribute to discussion, evaluate other's ideas, interpret their own ideas in terms of comments from others, and build substantive conversation.
Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson: The following scale measures the extent to which a particular teacher might include various student activities that promote discussion, problem solving, and reflection in lesson plans:
1. *Investigation of problems and discussion of mathematical ideas seldom planned for the lesson*. Emphasis is placed on practicing routine calculations, and little discussion among students is anticipated.
2. *Investigation of problems and discussion of answers and solution strategies (whether during small-group work or whole-class discussions) included in the lesson plan.* The teacher might plan problem investigation or class discussion, but the significance of these activities is not considered in the lesson plan.
3. *Investigation of problems and discussion of answers and solution strategies important elements in the lesson plan.* Questions or activities that encourage students to reflect on or summarize lessons, however, are not included in the lesson plan.
4. *Investigation of problems and discussion of answers and solution strategies dominant in the lesson plan.* The teacher views reflection on or summarization of the lesson as an important element in instruction.

Table G59

Teacher Level of Lesson Presentation and Development, Grade 8, District 2

School-Teacher	Total Cases	Cases at Level 1	2A	2B	3A	3B	4	5	6	Level Assigned
MiC										
Guggenheim-Carlson	8		1		2	5				3B
Guggenheim-Dillard	8		1			1	5	1		4
Weir-Gallardo	8		1				4	3		5
Weir-Shepard	8		3	3		1	1			2B

Lesson Presentation and Development: The following index measures the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding.

1. *No formal presentation*. Students were assigned work to do, but the content was not discussed prior to the assignment. Students attempted to solve problems by themselves but lacked the support needed to understand the mathematical content on their own. The teacher might have assisted individuals or small groups on a one-to-one basis.

2. *Emphasis on review*. The lesson presentation was not well developed; consequently students began independent or small-group work with little direction. The teacher might have assisted individuals or small groups on a one-to-one basis during independent or small-group work.

A. A major portion of the class period was devoted to review of a previous lesson, homework, or a warm-up activity.

B. Limited introduction to the lesson, vague directions, or lack of appropriate planning was evident. Students were left in a state of confusion.

3. *Demonstration of procedure or strategy*. A particular procedure or strategy was demonstrated by the teacher, and students were expected to use the method.

A. Students were unable to solve problems using the presented procedure or strategy.

B. Although students solved problems during independent or small group work, they practiced the presented procedure or strategy in a rote fashion.

4. *Attempt to develop conceptual understanding*. During the lesson, an attempt was made to develop a conceptual basis for the mathematical content. Students generally used a procedure or strategy presented by the teacher although they were allowed to find their own solution strategies.

5. *Emphasis on conceptual understanding with active participation by students and teacher*. The lesson presentation featured a conceptual basis for the mathematical content, and the mathematical work was shared by students and teacher.

6. *Emphasis on conceptual understanding with active participation by students with teacher support*. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared.

Nature of Inquiry: The following index measures the extent to which the nature of inquiry during instruction supported learning mathematics with understanding.

1. *Limited to lower order thinking*. Inquiry during the lesson was limited to lower order thinking. The lesson did not promote conceptual understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

2. *Limited attention to conceptual understanding*. Inquiry during class included limited attention to conceptual understanding. Student conjectures consisted of making connections between a new problem and previous problems. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

3. *Attention to conceptual understanding*. Inquiry during class emphasized conceptual understanding of the mathematical content. Student conjectures were characterized by investigating the veracity of particular statements. Connections among mathematical ideas were explained.

4. *In-depth exploration of mathematics*. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking

Table G60

Nature of Mathematical Inquiry, Grade 8, District 2

School-Teacher	Conceptual Understanding Observation									Conjectures Observation									Mathematical Connections Observation									Connections to Life Experiences Observation										
(No. of Observations)	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	*Mean*	***Rating***
MiC																																						
Guggenheim-Carlson	2	2	1	3	3	3	4	3	1	1	1	1	2	2	1	3	2	1	2	2	1	2	2	3	3	3	1	1	2	2	3	2	3	3	2	1	*2.06*	***3***
Guggenheim-Dillard	4	3	3	3	2	4	3	4	4	2	2	2	2	1	2	2	2	3	4	2	3	2	1	3	2	2	4	3	1	2	3	2	3	3	3	3	*2.61*	***4***
Weir-Gallardo	4	4	3	4	2	4	4	3		4	2	2	4	1	2	2	2		4	2	1	4	2	3	3	4		3	3	3	1	1	3	3	2		*2.78*	***4***
Weir-Shepard	2	2	3	1	1	2	3	3		1	2	2	1	1	1	2	2		2	2	1	1	1	1	2	3		2	2	2	1	2	1	1	3		*1.75*	***2***

Level of Nature of Inquiry

Level 1. Inquiry during the lesson was limited to lower order thinking, that lessons did not promote conceptual understanding; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 2. Inquiry during class included limited attention to conceptual understanding; student conjectures consisted of making connections between a new problem and previous problems; and connections among mathematical ideas and connections between mathematics and students' lives were not discussed.

Level 3. Inquiry during class emphasized conceptual understanding of the mathematical content; student conjectures were characterized by investigating the veracity of particular statements; and connections among mathematical ideas were explained.

Level 4. The mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or linking procedural and conceptual knowledge; students were encouraged to made generalizations; and connections between mathematics and students' lives were discussed.

Table G61

Nature of Students' Explanations, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	1	1	2	2	1	2	3	2	1	*1.67*
Guggenheim-Dillard	2	2	1	2	1	1	2	2	3	*1.78*
Weir-Gallardo	3	3	3	3	2	2	2	2		*2.50*
Weir-Shepard	2	1	2	1	2	2	2	2		*1.75*

Nature of Students' Explanations

The index for the nature of student explanation is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. *Answers only.* Students stated answers and were not expected to elaborate on their reasoning or solution strategies.
2. *Focus on procedures.* Explanations were focused on procedures rather than on elaboration of reasoning or solution strategies.
3. *Focus on mathematical processes.* Explanations were focused on mathematical processes such as justifying the approach to the problem, explaining the reasoning used, or supporting the results.

Table G62

Elicitation of Multiple Strategies, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	1	1	1	1	3	1	1	1	1	*1.22*
Guggenheim-Dillard	2	1	1	3	1	1	4	3	3	*2.11*
Weir-Gallardo	4	3	2	3	2	3	1	2		*2.50*
Weir-Shepard	3	1	1	1	1	1	1	4		*1.63*

Elicitation of Multiple Strategies

This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem.

1. *Strategies not elicited.* Multiple strategies were not elicited from students.

2. *Strategies rarely elicited.* Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

3. *Strategies not primary emphasis.* Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

4. Strategies substantive element of instruction. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

Table G63

Nature of Student–Student Conversation, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	3	3	1	3	1	2	2	1	1	*1.89*
Guggenheim-Dillard	3	2	2	1	1	1	1	2	4	*1.89*
Weir-Gallardo	4	1	1	3	2	2	2	3		*2.25*
Weir-Shepard	1	2	3	1	1	3	3	3		*2.13*

Nature of Student–Student Conversation

The index for student–student conversation measures the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas:

1. *Conversation not encouraged.* Conversation among students was not permitted or was social in nature.
2. *Limited conversation.* Student–student conversation occurred on a limited basis and usually consisted of sharing answers.
3. *Conversation not substantive in nature.* Conversation among students was characterized by students discussing procedures or asking each other for clarification of a procedure demonstrated by the teacher.
4. *Substantive conversation.* Conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level.

Table G64

Students' Collaborative Working Relationships, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	2	2	1	4	1	1	3	2	N/A	*2.00*
Guggenheim-Dillard	3	2	1	2	1	1	2	2	3	*1.89*
Weir-Gallardo	3	1	1	1	1	2	3	3		*1.88*
Weir-Shepard	2	1	3	N/A	1	3	3	3		*2.29*

Students' Collaborative Working Relationships

This scale measures the extent to which interactions among students reflected collaborative working relationships:

NA. Independent work. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. No collaboration among students. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Limited exchange of ideas. Few students were sharing ideas or discussing how a problem should be solved in small-group or large-group settings. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and different paces.

3. Uneven participation. Some students exchanged ideas or provided assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to problem solving were not equally made by all students.

4. Substantive collaboration. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

Table G65

Student Engagement during Instruction, Grade 8, District 2

	Observation									
School-Teacher	1	2	3	4	5	6	7	8	9	*Mean*
MiC										
Guggenheim-Carlson	3	3	3	3	3	2	3	3	3	*2.89*
Guggenheim-Dillard	4	3	3	3	3	3	3	3	3	*3.11*
Weir-Gallardo	4	4	4	4	4	4	3	4		*3.88*
Weir-Shepard	2	1	4	3	1	3	4	4		*2.75*

Student Engagement during Instruction

This index measures the extent to which students remained on task during the lesson:

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions.
2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task. They did not seem to put forth much effort.
3. Sporadic or episodic engagement. Most students were engaged in class activities some of the time, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.
4. Widespread engagement. Most students were on task pursuing the substance of the lesson most of the time. Most students seemed to take the work seriously and put forth much effort.

Explanation-Oriented	1 *6%*	2	3	4 *33%*	5 *6%*		6 *6%*	7	8 *6%*	
Task-Oriented					1	2 *22%*	3	4		
Shift in Pedagogical Approach	1 *11%*	2 *6%*			3	4 *6%*	5	6	7	8
(18 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G9. Interactive Decisions, Dunn, Fernwood Middle School.

Explanation-Oriented	1 *4%*	2	3 *4%*	4 *4%*	5		6 *4%*	7 *26%*	8 *4%*	
Task-Oriented					1	2 *17%*	3 *4%*	4		
Shift in Pedagogical Approach	1	2 *4%*			3	4 *9%*	5	6	7 *13%*	8 *4%*
(23 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G10. Interactive Decisions, Reichers, Von Humboldt Middle School.

Explanation-Oriented	1 *27%*	2	3	4 *23%*	5		6	7	8	
Task-Oriented					1	2 *23%*	3	4		
Shift in Pedagogical Approach	1	2 *5%*			3 *9%*	4 *9%*	5 *5%*	6	7	8
(22 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G11. Interactive Decisions, Waters, Von Humboldt Middle School.

Explanation-Oriented	1	2	3	4 *67%*	5		6	7	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1	2 *33%*			3	4	5	6	7	8
(6 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G12. Interactive Decisions, Wolfe, Addams Middle School.

Explanation-Oriented	1	2	3	4 *7%*	5		6 *7%*	7 *33%*	8	
Task-Oriented					1	2	3	4		
Shift in Pedagogical Approach	1 *7%*	2 *13%*			3 *13%*	4 *20%*	5	6	7	8
(15 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G13. Interactive Decisions, Pimm, Fernwood Middle School.

Explanation-Oriented	1 *17%*	2	3	4 *13%*	5 *4%*		6	7 *8%*	8	
Task-Oriented					1 *4%*	2 *25%*	3	4		
Shift in Pedagogical Approach	1	2 *8%*			3 *4%*	4 *13%*	5	6 *4%*	7	8
(24 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G14. Interactive Decisions, Carlson, Guggenheim Middle School.

Explanation-Oriented	1 5%	2	3	4 *21%*	5 *5%*		6 5%	7 16%	8 5%	
Task-Oriented					1	2 *26%*	3	4		
Shift in Pedagogical Approach	1	2			3	4 *16%*	5	6	7	8
(19 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G15. Interactive Decisions, Dillard, Guggenheim Middle School.

Explanation-Oriented	1	2	3	4 *10%*	5		6	7 *30%*	8 *10%*	
Task-Oriented					1	2 *10%*	3	4		
Shift in Pedagogical Approach	1	2			3	4 *10%*	5	6 *20%*	7 *10%*	8
(10 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G16. Interactive Decisions, Gallardo, Weir Middle School.

Explanation-Oriented	1	2	3	4 *50%*	5		6 17%	7	8	
Task-Oriented					1	2 33%	3	4		
Shift in Pedagogical Approach	1	2			3	4	5	6	7	8
(6 items coded)	Least			Teaching Mathematics for Understanding						Most

Figure G17. Interactive Decisions, Shepard, Weir Middle School.