Longitudinal/Cross-sectional Study of the Impact of *Mathematics in Context* on Student Performance

Opportunity to Learn with Understanding for 1997-1998

(Technical Report #41)

Mary C. Shafer
Northern Illinois University

Lorene Folgert and Joan Kwako
University of Wisconsin–Madison

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Introduction

The purposes of the longitudinal/cross-sectional study of the impact of *Mathematics in Context* (MiC; National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998) on student performance are (a) to determine the mathematical knowledge, understanding, attitudes, and levels of student performance as a consequence of studying MiC for over three years; and (b) to compare student knowledge, understanding, attitudes, and levels of performance of students using MiC with those using conventional mathematics curricula. The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). For this study, information is being gathered on 14 variables over a 3-year period for three groups of students (those in Grades 5, 6, and 7 in 1997). The variables have been organized in five categories (prior, independent, intervening, outcome, and consequent). (See Figure 1 for variables and hypothesized relationships.)

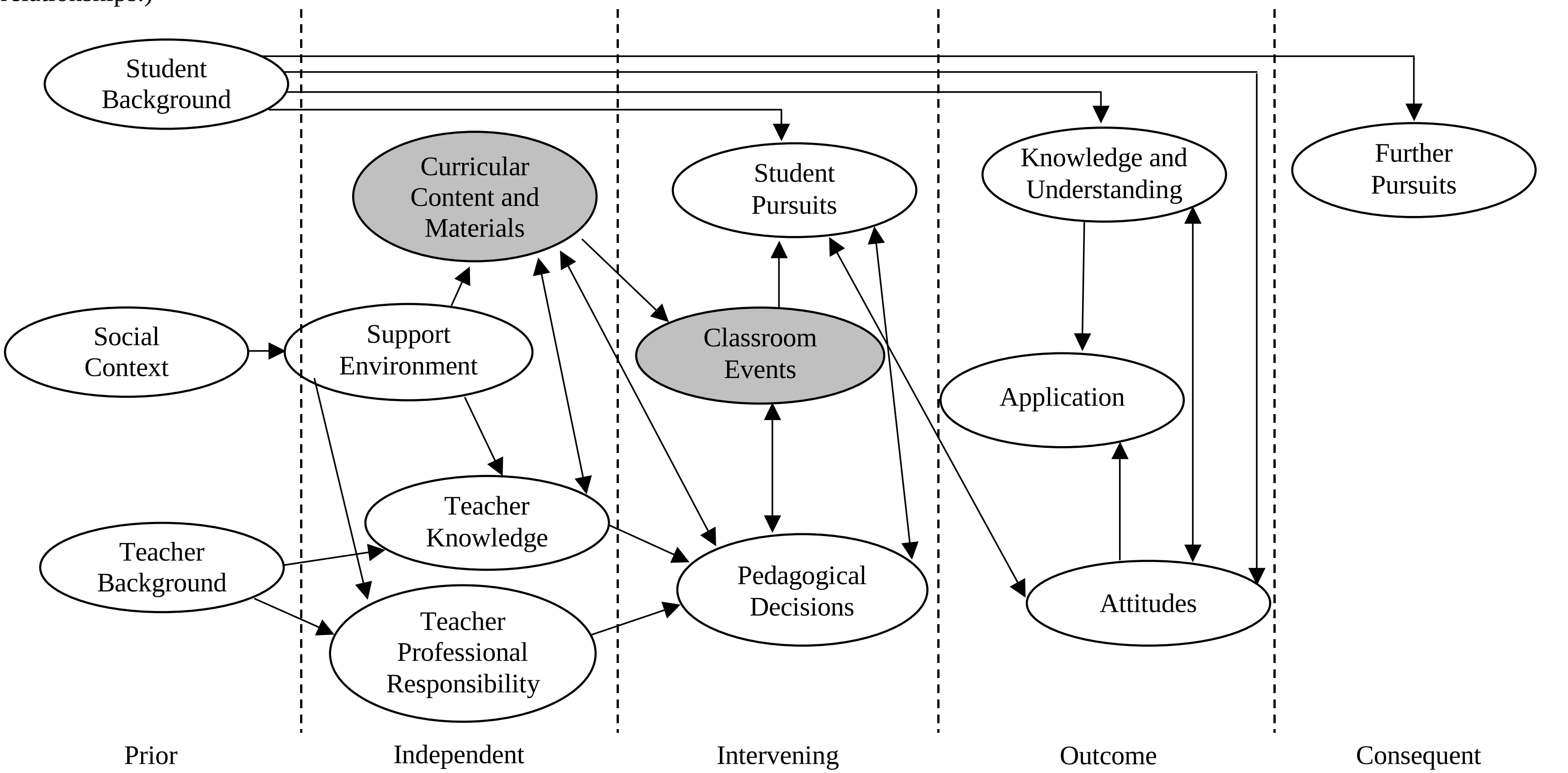

Figure 1. Revised model for the monitoring of school mathematics.

For analytical purposes, although the variation in each set of variables could be examined using structural equations, the number of classes at a given grade level is small, and collinearity across variables poses a serious interpretation problem. For this reason, a simplified research function was developed to make both the cross-sectional and longitudinal comparisons. Variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, can be attributed to variations in opportunity to learn with understanding (OTL_u), preceding achievement (PA), and method of instruction (I). This relationship can be expressed as—

$$CA = OTL_u + PA + I.$$

However, as the research staff worked with the data, a decision was made to distinguish school capacity (SC) from $OTL_{u.}$ Thus, the research function can be expressed as—

$$CA = SC + OTL_u + PA + I.$$

Each of these composite indices is being specified from the variables in the original model. This paper details the analysis of the opportunity to learn with understanding variable.

Overview

The purpose of this working paper is to summarize information for the composite variable *opportunity to learn with understanding* collected during the first year of the longitudinal/cross-section study, the 1997–1998 school year, for fifth-, sixth-, and seventh grade teachers and students. The purpose of gathering this information was to document the variation in opportunity to learn with understanding that study students experienced as they studied either MiC or conventional mathematics curricula. The composite index opportunity to learn with understanding is specified from data gathered on the independent variable curricular content and materials and the intervening variable classroom events in the structural research model. In the simplified research function, opportunity to learn with understanding includes six major aspects: curricular content, modifications in curricular content, conceptual understanding, conjectures, connections within mathematics, and connections between mathematics and students' daily lives (see Figure 2). This information was gathered through Teacher Questionnaire (Shafer, Davis, & Wagner, 1997; see Appendix A), Teacher Logs (Shafer, Wagner, & Davis, 1997; see Appendix B), and the Classroom Observation Instrument (Davis, Wagner, & Shafer, 1998; see Appendix C).

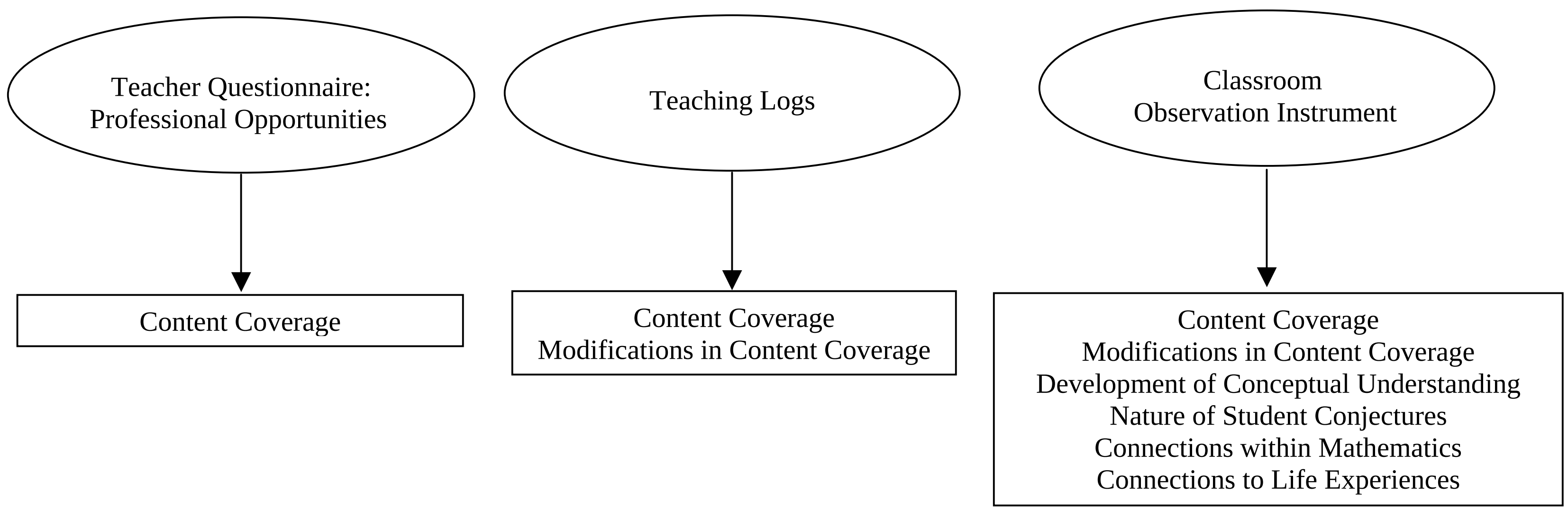

Figure 2. Categories of opportunity to learn with understanding in the longitudinal/cross-sectional study of the impact of *Mathematics in Context* on student performance and their sources.

Thirty-five teachers and students in 63 classes from two school districts participated in the study. Districts are identified by number, and the classes by school and teacher (both pseudonyms). Also noted are the type of materials used (MiC materials or a conventional text).

The Sample

Districts 1 and 2

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. All MiC teachers used commercial versions of the units, which became available during the summer before the study began. Teachers using conventional curricula used the curricula that were already available in place in the schools. The sample in Districts 1 and 2 consisted of 35 fifth-, sixth- and seventh-grade teachers and their classes from diverse schools in Districts 1 and 2. These teachers taught a combined total of 63 classes involving 1561 students (see Tables 1 and 2).

Table 1
Summary of Study Participants, Districts 1 and 2, by Curriculum

Curriculum	**Teachers *(N)***	**Classes *(N)***	**Students *(N)***
District 1			
MiC	12	22	532
Conventional	6	11	240
District 2			
MiC	12	23	584
Conventional	5	7	205
Total	35	63	1561

Table 2
Characteristics of the Student Participants, Districts 1 and 2, by Curriculum

Curriculum	**Gender *(%)***		**Ethnicity *(%)* *(self-identified)***							**Primary Language *(%)* *(self-identified)***	
	Male	**Female**	**African American**	**Native American**	**Asian**	**Hispanic**	**White**	**Multiracial**	**Other[1]**	**English**	**Other[2]**
District 1											
MiC	49	51	21	1	1	7	54	10	5	92	7
Conventional	53	47	25	0	–	3	54	13	3	91	9
District 2											
MiC	47	53	9	1	–	49	17	17	7	85	15
Conventional	56	44	18	1	–	34	14	18	15	84	15

[1]Includes Haitian, Jamaican, other ethnic groups, and unclassifiable responses such as religions and nonresponses.

[2]Includes nonresponses.

District 1 was located in an urban region in the eastern part of the United States. In 13 elementary, 6 middle, and 4 high schools, 1325 teachers were responsible for teaching the district's 15,532 students. Three elementary schools and four middle schools participated in the first year of the study. Six fifth-grade study classes were in self-contained elementary classrooms. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. Approximately 30–40% of the students in the district were eligible for government-funded lunch programs. Professional development to acquaint teachers of mathematics with reform-based curricula was offered in District 1, and monthly meetings were provided for teachers who were implementing such programs. For preliminary teacher certification, 24 credit hours were recommended for fifth- and sixth-grade teachers; 24 credit hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements.

District 2, located in a large urban area in southeastern United States, had 19,352 teachers and 342,996 students housed in 201 elementary schools, 51 middle schools and numerous high schools. Three elementary and three middle schools participated in the first year of the study. In District 2, two of the nine fifth-grade study classes were in self-contained settings in elementary schools. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. The district student population was predominantly minority, with 33% African American students and 52% Hispanic students. Over 50% of the students in the district were eligible for government-funded lunch programs. District 2 provided numerous possibilities for professional development. Each school was given six early-release days for general professional development. In addition, each school received 10 substitute days for professional development in mathematics and/or science, 12–18 days of in-service days in mathematics provided by (USI or Eisenhower) government funding (each involving 2–6 teachers), and 3–5 days of district-wide mathematics in-service. Teachers also had opportunities to participate in five days of paid in-service for mathematics during the summer. District requirements for preparation of mathematics teachers were the same as state requirements. For elementary teachers, preliminary teacher requirements mandated the study of arithmetic for the elementary school. For middle-grade mathematics certification (Grades 5–9), 18 semester hours in mathematics were required; for certification in mathematics for Grades 6–12, 30 semester hours in mathematics were required. Continuing certification required the completion of six semester hours in mathematics or 120 district in-service credits in mathematics every 5 years.

Analysis of opportunity to learn with understanding was not conducted for Districts 3 and 4 because classroom observation and teaching log data were not gathered in those districts.

The Composite Variable for Opportunity to Learn with Understanding

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding (see Figure 3). The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. An index was created for the categories of curricular content and modification of curricular materials, and an index was created for each dimension of teaching for understanding.

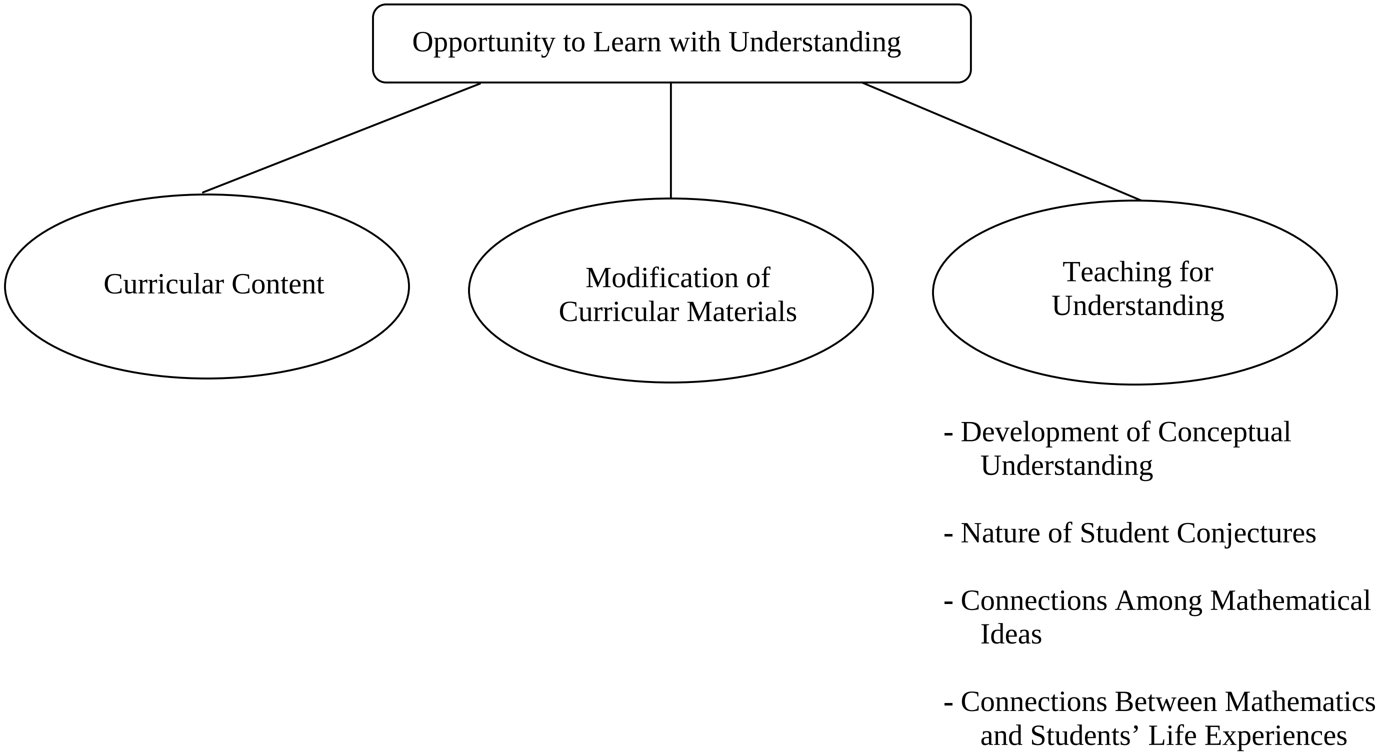

Figure 3. Major dimensions of opportunity to learn with understanding.

A single index, a composite of multiscaled information from each category and each dimension, represents opportunity to learn with understanding in the simplified research function. Graphs of the levels for each dimension of opportunity to learn with understanding and the composite index are contained in Appendix D. Description of the methodology used in the analysis is contained in Appendix E, and description of the theoretical framework that guided the analysis of opportunity to learn with understanding is contained in Appendix F. For complete sets of data, see Appendices G, H, and I for fifth-, sixth-, and seventh-grade teachers, respectively.

Categories of Opportunity to Learn with Understanding

In this study, three categories characterized opportunity to learn with understanding: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Curricular Content

The index for the category of curricular content describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts. Forty percent of the teachers in Districts 1 and 2 were categorized at Level 5, indicating that they presented a comprehensive, integrated curriculum with attention to all content areas. Six percent of the teachers were assigned Level 4, indicating that they taught mathematics concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra). Nine percent of the teachers were at Level 3B, indicating that they used a new curriculum and slow pacing resulted in coverage of only a few topics. Twenty percent of the teachers were categorized at Level 2, signifying that they presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content. Twenty-six percent of the teachers were assigned at Level 1, indicating that they presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms. More fifth-grade teachers were assigned Level 5 (10 of the 13 teachers) than sixth- and seventh-grade teachers who were more frequently assigned Levels 1 and 2 (8 of the 12 sixth-grade and 6 of the 10 seventh-grade teachers). Fourteen of the 24 MiC teachers were assigned Level 5 and 7 were assigned Level 2. In comparison, 9 of the 11 teachers using conventional curricula were at Level 1. A greater number of MiC teachers in District 2 were at Level 2 (5 of the 12 teachers in District 2 compared with 2 of the 12 teachers in District 1). (See Figures D1-D4 in Appendix D and Tables G1, G8, H1, H8, I1, and I8 in Appendices G, H, and I, respectively, for more detail.)

Modification of Curricular Materials

The index for the category of modification of curricular materials measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts. Six percent of the teachers were assigned Level 6B, indicating that they supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives. Fifty-four percent of the teachers were at Level 5A, signifying that they occasionally supplemented the text with activities disconnected from the text. Three percent of the teachers were categorized at Level 5B, indicating that they presented the curriculum as it was written with few, if any, modifications. Fourteen percent of the teachers were assigned Level 4, indicating that they supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum). Nine percent of the teachers were categorized at Level 2A, signifying that they supplemented a reform curriculum with conventional materials to the extent that the supplementary materials subsumed the reform curriculum. Eleven percent of the teachers were assigned Level 2B, indicating that they abandoned the reform curriculum in favor of a conventional curriculum. Three percent of the teachers were at Level 1, indicating that they presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics. More fifth-grade teachers (11 of the 13) were assigned Levels 5 and 6 than sixth-grade (6 of the 12) and seventh-grade teachers (5 of the 10). Half of the 24 MiC teachers were assigned Levels 5 and 6, and 7 were assigned Level 2. In comparison, 10 of the 11 teachers using conventional curricula were at Levels 5 and 6, and one was assigned Level 1. More MiC teachers in District 1 were at Levels 5 and 6 (8 of the 12 teachers) compared to District 2 (4 of the 12 teachers). Teachers using conventional curricula in both districts were comparable (5 of the 6 teachers in District 1 were at Levels 5 and 6 and all 5 teachers in District 2 were at Level 5). (See Figures D5-D8 in Appendix D and Tables G2, G9, H2, H9, I2, and I9 in Appendices G, H, and I, respectively, for more detail.)

Teaching for Understanding

Development of conceptual understanding. The index for the dimension of development of conceptual understanding measures the extent to which the lesson fostered the development of conceptual understanding. Level 4 signifies that the teacher had continual focus of the lesson on building connections or linking procedural knowledge with conceptual knowledge. At Level 3, some lesson questions fostered students' development of conceptual understanding of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning. Level 2 indicates that few questions fostered students' development of conceptual understanding or conceptual understanding was a small part of lesson design. Level 1 indicates that the lesson as presented did not promote conceptual understanding. The overall mean for all teachers in Districts 1 and 2 was 2.41. Fifth-grade teachers had a higher mean (2.74**)**, than the sixth-grade teachers (2.21) and seventh-grade teachers (2.22). MiC teachers had a much higher mean (2.74) than teachers of conventional curricula (1.68). In District 2, both MiC teachers (2.89) and teachers of conventional curricula (1.88) had higher means than teachers in District 1: MiC teachers (2.60), teachers of conventional curricula (1.52). (See Figures D9 and D10 in Appendix D and Tables G3, G10, H3, H10, I3, and I10 in Appendices G, H, and I, respectively, for more detail.)

Nature of student conjectures. The index for the dimension of the nature of student conjectures measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas. At Level 4, students made generalizations about mathematical ideas. For Level 3, observed conjectures consisted mainly of student investigations about the truthfulness of particular statements. At Level 2, observed conjectures consisted mainly of making connections between a new problem and problems previously seen. For Level 1, no conjectures of any type were observed in the lesson. Students were not encouraged to make connections. The overall mean for all teachers in Districts 1 and 2 was 1.91. Again, fifth-grade teachers had a higher mean (2.10) than six-grade teachers (1.83) and seventh-grade teachers (1.78). MiC teachers had a mean of 2.08, whereas the mean for teachers of conventional curricula was 1.56. The means for the two districts were comparable. In District 1, MiC teachers had a mean of 2.09 and teachers of conventional curricula, 1.50. In District 2, MiC teachers had a 2.07 mean, teachers of conventional curricula, 1.63. (See Figures D11 and D12 in Appendix D and Tables G4, G11, H4, H11, I4, and I11 in Appendices G, H, and I, respectively, for more detail.)

Connections among mathematical ideas. The index for the dimension of connections among mathematical ideas measured the extent to which connections within mathematics were explored in the lesson. At Level 4, the mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics. For Level 3, connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher. For Level 2, the teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail. Level 1 indicated that the mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics. The overall mean for all teachers in Districts 1 and 2 was 1.83. Fifth-grade teachers had a slightly higher mean (1.99) compared to sixth-grade teachers (1.71) and seventh-grade teachers (1.79). Again, MiC teachers had a much higher mean (2.06) than teachers of conventional curricula (1.35**)**. MiC teachers in District 2 had a higher mean (2.24) than MiC teachers in District 1 (1.87). Teachers of conventional curricula had similar means in both districts (1.33 in District 1; 1.37 in District 2). (See Figures D13 and D14 in Appendix D and Tables G5, G12, H5, H12, I5, and I12 in Appendices G, H, and I, respectively, for more detail).

Connections between mathematics and students' life experiences. The dimension of connections between mathematics and students' life experiences measured the extent to which connections between mathematics and students' daily lives were apparent in the lesson. Level 3 indicated that connections between mathematics and students' daily lives were clearly apparent in the lesson. For a Level 2, connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher. A rating of Level 1 was assigned when connections between mathematics and students' daily lives were not apparent in the lesson. The overall mean for teachers in both districts was 2.04. The mean for this dimension decreases over the grades: fifth-grade teachers had a higher means (2.28**),** than six-grade teachers (2.02) and seventh-grade teachers (1.74**)**. MiC teachers had a much higher mean (2.34) than teachers of conventional curricula (1.37). MiC teachers had similar means in each district: MiC teachers had a mean of 2.32 in District 1 and 2.37 in District 2. Teachers of conventional curricula had means of 1.39 in District 1 and 1.35 in District 2. (See Figures D15 and D16 in Appendix D and Tables G6, G13, H6, H13, I6, and I13 in Appendices G, H, and I, respectively, for more detail.)

Composite Variable for Opportunity to Learn with Understanding

The OTL_u composite variable conceptualized for the longitudinal/cross-sectional study includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences. The composite index for OTL_u was designed to account for differences noted for study teachers. The composite index is composed of four levels, and ratings for both teachers using MiC and teachers using conventional curricula spanned the four levels.

Level 4: High Level of Opportunity to Learn with Understanding

At Level 4, teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few, if any, modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

Level 3: Moderate Level of Opportunity to Learn with Understanding

At Level 3, teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although in the lesson connections between mathematics and students' daily lives were implicit, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

Level 2: Limited Opportunity to Learn with Understanding

At Level 2, teachers covered only a few topics. Because many MiC teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both MiC teachers and teachers using conventional curricula, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

Level 1: Low Level of Opportunity to Learn with Understanding

At Level 1, teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Composite Index for Opportunity to Learn with Understanding

The eight Grade 5 teachers in District 1 varied in the OTL_u composite from Level 2 to Level 4. Four MiC teachers were at Level 4, indicating that students experienced a high level of opportunity to learn mathematics with understanding. Three teachers (two MiC) were at Level 3, indicating that students experienced a moderate level of opportunity to learn with understanding. The remaining teacher (using a conventional curriculum) was at Level 2, indicating that students experienced a limited opportunity to learn with understanding. The five Grade 5 teachers in District 2 varied in the instruction composite from Level 2 to Level 4. All four MiC teachers were at Level 4, and the teacher using a conventional curriculum was at Level 2. (See Tables G7 and G14 in Appendix G for more detail.)

The six Grade 6 teachers in District 1 varied in the OTL_u composite from Level 1 to Level 4. One MiC teacher was at Level 4, and another MiC teacher was at Level 3. Three teachers (two MiC) were at Level 2, and the remaining teacher (using a conventional curriculum) was at Level 1, indicating that students experienced a low level of opportunity to learn mathematics with understanding. The six Grade 6 teachers in District 2 varied in the lesson composite from Level 2 to Level 3. Two MiC teachers were at Level 3. The four remaining teachers (two MiC) were at Level 2. (See Tables H7 and H14 in Appendix H for more detail.)

The four Grade 7 teachers in District 1 varied in the OTL_u composite from Level 1 to Level 4. One MiC teacher was at Level 4, and the other MiC teacher was at Level 2. Both teachers using conventional curricula were at Level 2. The six Grade 7 teachers in District 2 varied in the opportunity to learn with understanding composite from Level 2 to Level 4. One MiC teacher was at Level 4, and two teachers (one MiC) were at Level 3. Three teachers (two MiC) were at Level 2. (See Tables I7 and I14 in Appendix I for more detail.)

The results for all 35 teachers for the OTL_u composite revealed differences by grade level, type of curriculum taught, and by district. Eight fifth-grade teachers, one sixth-grade teacher, and two seventh-grade teachers were at Level 4, indicating that students experienced a high level of opportunity to learn mathematics with understanding. Three fifth-grade teachers, three sixth-grade teachers, and two seventh-grade teachers were at Level 3, indicating that students experienced a moderate level of opportunity to learn with understanding. In other words, more fifth-grade students experienced a moderate or high level of opportunity to learn mathematics with understanding than sixth- or seventh-grade students. Also, seven sixth-grade teachers and six seventh-grade teachers, in comparison to two fifth-grade teachers, were at Level 2, indicating that students experienced a limited opportunity to learn with understanding. Furthermore, one sixth-grade teacher was at Level 1, indicating that students experienced a low level of opportunity to learn mathematics with understanding. Therefore, fifth-grade students were more likely to experience moderate or high levels of opportunity to learn with understanding. An additional pattern of variation was found when the levels of the OTL_u composite were reviewed by curriculum taught. Eleven of the 24 teachers using MiC were at Level 4, in comparison to none of the 11 teachers using conventional curricula. Also, 6 of the 24 MiC teachers were at Level 3 in comparison to two teachers using conventional curricula. In contrast, 8 of the 11 teachers using conventional curricula were at Level 2 compared with 7 of the 24 teachers using MiC. One teacher using a conventional curriculum was at Level 1, but none of the MiC teachers were at that level. When these results were reviewed by district, few differences became apparent among MiC teachers and teachers of conventional curricula. Six of the 12 MiC teachers in District 1 and 5 of the 12 MiC teachers in District 2 were at Level 4. None of the teachers of conventional curricula in either district were rated at Level 4. Also, 3 MiC teachers in District 1 and 3 MiC teachers in District 2 were at Level 3. Only 1 teacher of conventional curricula in each district was at Level 3.Three MiC teachers in District 1 and 4 MiC teachers in District 2 were at Level 2. Four teachers of conventional curricula in District 1 and 4 teachers of conventional curricula in District 2 were at Level 2. One teacher of conventional curricula in District 1 was at Level 1. Therefore, MiC students in both districts were more likely to experience a moderate to high level of opportunity to learn with understanding and students of conventional curricula in both districts were more likely to experience a limited level of opportunity to learn with understanding. (See Figures D17-D20 in Appendix D for more detail.)

Predictors of Opportunity to Learn with Understanding

The OTL_u composite variable conceptualized for the longitudinal/cross-sectional study includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences. To examine the relationships between each of these categories and dimensions, a correlation matrix was calculated. The results suggest that one category and four dimensions—curricular content, development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences—were strongly correlated with each other (see Table 6).

Table 6
Correlation of the Categories and Dimensions of the Opportunity to Learn with Understanding Composite Variable

Dimension	Curricular Content	Modification of Curricular Materials	Teaching for Understanding			
			Conceptual Understanding	Conjectures	Connections within Mathematics	Connections to Students' Daily Lives
Curricular Content	1.000					
Modification of Curricular Materials	0.351*	1.000				
Conceptual Understanding	0.741**	0.233	1.000			
Conjectures	0.626**	0.186	0.711**	1.000		
Connections within Mathematics	0.542**	0.072	0.808**	0.647**	1.000	
Connections to Daily Lives	0.770**	0.120	0.672**	0.625**	0.671**	1.000

*p<.05
**p<.01

Regression analysis was used to compare the relative influence of the categories and dimensions of opportunity to learn with understanding, which were treated as individual variables in the analysis. The results suggest that the most important predictor of the OTL_u composite is curricular content. Modification of curricular materials and development of conceptual understanding were the next most important predictors of the OTL_u composite. Along with curricular content, these dimensions explained 70% of the variance of the OTL_u composite. These results confirm critical dimensions of OTL_u for classrooms in which MiC or conventional curricular materials are used.

Conclusion

The methods used to examine classroom OTL_u have discriminated differences in opportunity to learn mathematics with understanding among 35 study teachers in two districts who used standards-based and conventional curricula. Variation was found by grade level, by curriculum, and by district. Students in the classes of eleven Grade 5 teachers (85%) in comparison to four Grade 6 teachers (33%) and four Grade 7 teachers (40%) experienced a moderate or high level of opportunity to learn mathematics with understanding. On the other hand, students in the classes of two Grade 5 teachers (15%), eight Grade 6 teachers (66%), and six Grade 7 teachers (60%) experienced limited or low opportunity to learn mathematics with understanding. MiC students experienced a moderate or high level of opportunity to learn with understanding (71%) in comparison to students using conventional curricula (18%). By district, MiC students in both districts were more likely to experience a moderate to high level of opportunity to learn with understanding (75% in District 1 and 67% in District 2). Students of conventional curricula in both districts were more likely to experience a low level of opportunity to learn with understanding (83% in District 1 and 80% in District 2). The results also suggest that two categories and one dimension of OTL_u—curricular content, modification of curricular materials, and development of conceptual understanding—account for 70% of the variation in the OTL_u composite among study teachers. These results underscore the importance of teaching curricular content for conceptual understanding. (See Tables D17-D20 for more detail.)

The results suggest that the MiC materials affected students' opportunity to learn mathematics with understanding. The curricular content is comprehensive, with its attention to geometry, algebra, and statistics in addition to number, and is rich in developing connections among mathematical ideas. Contexts in which lessons are situated provide a basis for exploring mathematical ideas and applying mathematics in daily life experiences.

Further research will examine changes in levels of the OTL_u composite index for the same teachers over time as well as the factors that might influence those changes. Further research might involve using the composite index with in-service and pre-service teachers as a reflection tool on students' opportunity to learn comprehensive mathematics content with conceptual understanding.

In a study of the impact of any standards-based curriculum, analysis of classroom OTL_u and the factors that influence OTL_u are important considerations in students' achievement. The findings suggest that variation in students' opportunity to learn mathematics with understanding must be accounted for in the interpretation of the impact of the curriculum on student achievement.

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Appendix A

Teacher Questionnaire: Professional Opportunities

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teacher Questionnaire: Professional Opportunities
(Working Paper #11)

Mary C. Shafer, Jon Davis, and Lesley R. Wagner

Wisconsin Center for Education Research
University of Wisconsin–Madison

Shafer, M. C., Davis, J., & Wagner, L. R. (1997). *Teacher questionnaire: Professional opportunities*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 11). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

The development of this instrument was supported in part by the National Science Foundation #REC-9553889 and the Wisconsin Center for Education Research, School of Education, University of Wisconsin–Madison.

Teacher Questionnaire IV: Professional Opportunities

The purpose of Teacher Questionnaire IV was to gather information on teachers' professional development opportunities. Questionnaire items were adapted from the Center on Organization and Restructuring of Schools (1996) and National Center for Improving Student Learning & Achievement in Mathematics & Science (1997).

In the initial part of the questionnaire, the teacher listed local information, including the school name and address. In Item 1, the teacher indicated the professional literature he/she read from a list of seven items including district and state mathematics frameworks, the NCTM *Standards* documents (1989, 1991, 1995), NCTM journals, and other professional journals. In the second item, the teacher circled the frequency of opportunities to observe and discuss another teacher's teaching; have another teacher observe his/her own teaching; receive feedback on his/her own teaching; and participate in a network of teachers outside of school. In Item 3, the teacher circled the frequency of participation in formal meetings with other mathematics teachers with respect to curriculum, methods, classroom assessment, and evaluation of the mathematics program. In Item 4, the teacher circled the number of college/university courses taken in the past 12-month period.

In Item 5, the teacher described the topics addressed during professional development opportunities through his/her reaction to seven topics. If a particular topic was addressed, the teacher indicated whether the professional development did lead to changes in his/her teaching of mathematics on a 4-point Likert scale that included "strongly disagree," "disagree," "agree," and "strongly agree." If the teacher responded "agree" or "strongly agree," he/she then indicated whether the change was effective in facilitating/enhancing student learning. These ratings were on a continuum from "not effective" to "very effective." In Item 6, the teacher circled the type of support he/she received for attending professional development, including meetings, workshops, and conferences. The choices were release time from teaching, paid expenses, honorarium, continuing education units, none, and other (with a request for description).

In Items 7-10, the teacher described the length, frequency, context, and content of typical formal and informal meetings or planning sessions with other mathematics teachers. In Item 7, the teacher indicated the time reserved for planning daily and weekly during one contractual work week. In Item 8, the teacher indicated the number of days in which he/she spent at least 15 minutes collaborating with other mathematics teachers. In Item 9, the teacher circled the setting in which such planning occurred. In Item 10, the teacher indicated the frequency in which he/she participated in discussions that were characterized in 13 statements. These included instructional materials and activities, instructional methods, assessments, student difficulties, scheduling events or projects, sharing stories about classroom experiences, and discussing professional literature. The teacher circled his/her participation as "never," "sometimes," "frequently," or "always."

In Item 11, the teacher supplied an estimate of the percent of mathematics teachers in the school that were involved in efforts to improve the mathematics program. In Item 12, the teacher characterized his/her own efforts to improve the mathematics program by circling one of the following: "strong opposition," "slight opposition," "slight support," or "strong support."

For the spring administration of this questionnaire, an additional item was included. In Item 13, the teacher circled the specific *Mathematics in Context* units taught during the current school year from a list of all 40 *Mathematics in Context* units listed by grade level.

Teachers completed the questionnaire in the fall of the first year of their participation in the study and in the spring of each year of their participation. For the fall administration, teachers in Districts 1 and 2 completed the questionnaire during the professional development institutes provided by the research team in the August prior to the school year. Each teacher received an honorarium for participating in the August institutes. Teachers in Districts 3 and 4, and teachers in Districts 1 and 2 who did not attend the institutes, completed the questionnaire (along with other teacher questionnaires) at times that were convenient for the them and that did not interfere with classroom instruction, such as during their planning time or before or after school. These teachers received an honorarium of $50 upon receipt of all questionnaires at the research center. For the spring administration in the first and second study years, teachers in all districts completed the questionnaire (along with another questionnaire) during the spring professional development institutes provided by the research team. In the spring of the third study year, teachers in Districts 1, 2, and 4 completed the questionnaire during the spring professional development institutes. Teachers received release time to participate in the spring institutes. Teachers who did not attend the institutes completed the questionnaire (along with another teacher questionnaire) at times that were convenient for them and that did not interfere with classroom instruction. These teachers received an honorarium of $50 upon receipt of all questionnaires at the research center. Ninety-six percent of the teachers completed questionnaires.

References

Center on Organization and Restructuring of Schools. (1996). *Teacher questionnaire*. Madison, WI: University of Wisconsin–Madison.

National Center for Improving Student Learning & Achievement in Mathematics & Science. (1997). *Elementary school teacher questionnaire*. Madison, WI: University of Wisconsin–Madison.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

Teacher Questionnaire IV: Professional Opportunities

Thank you for completing this questionnaire. Your responses will enable the staff of the *Mathematics in Context* longitudinal study to learn more about opportunities for professional development in your school and your participation in professional activities.

Your responses will be kept confidential. Your responses will not be used to evaluate you in any way, and your name will not be mentioned in reports of this research. The information on this sheet will help us ensure that a questionnaire was received from each teacher in the study.

We hope that you will answer every question, but you may skip any questions you do not wish to answer. When you have completed the questionnaire, please look over your responses to see that you have not skipped anything unintentionally.

Last name First name MI

District

School

City State Zip Code

1. Which of the following have you read? (Circle all that apply)

a.	Your school district mathematics framework or curriculum guide	1
b.	Your state mathematics framework or curriculum guide	2
c.	*Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (1989)	3
d.	*Professional Standards for Teaching Mathematics* published by the National Council of Teachers of Mathematics (1991)	4
e.	*Assessment Standards for School Mathematics* published by the National Council of Teachers of Mathematics (1995)	5
f.	Journals specifically related to mathematics teaching and learning such as *Teaching Children Mathematics* (formerly *Arithmetic Teacher*), *Mathematics Teaching in the Middle School*, and *Mathematics Teacher*	6
g.	Journals related to teaching and learning in the elementary and middle school that are not specifically targeted for mathematics	7

2. During the last school year, how often did you do the following? (Circle one response for each statement)

		Number of Times					
a.	Visit another teacher's classroom to observe and discuss his/her mathematics teaching	0	1	2	3-4	5-9	10+
b.	Have another teacher observe your mathematics teaching	0	1	2	3-4	5-9	10+
c.	Receive meaningful feedback on your mathematics teaching from peers or supervisors	0	1	2	3-4	5-9	10+
d.	Participate in a group or network with other mathematics teachers outside of your school	0	1	2	3-4	5-9	10+

3. During the last school year, how often did you participate in formal meetings (e.g., department meetings) with other mathematics teachers in your school related to the following discussions? (Circle one for each statement)

	Number of Times
a. The mathematics curriculum	0 1 2 3-4 5-9 10+
b. Mathematics teaching techniques and student activities	0 1 2 3-4 5-9 10+
c. Ideas for assessing student learning of mathematics	0 1 2 3-4 5-9 10+
d. Evaluation of your mathematics program	0 1 2 3-4 5-9 10+

4. During the past 12 months, how many college or university courses did you take? (Circle one)

0 1 2 3 4 more than 4

5. Answer the following questions for each topic in the left column:
   1. Have you participated in professional development activities during the past 12 months that have addressed that topic? If yes, please answer question 2.
   2. Did that professional development activity lead to changes in your teaching of mathematics? If you agree or strongly agree, please answer question 3.
   3. Did the changes in your teaching enhance your students' learning?

	1. My professional development activities addressed this topic		2. My professional development on this topic led to changes in my teaching of mathematics				3. The changes inspired by this professional development activity were effective in facilitating/enhancing student learning.			
	yes	no	strongly disagree	disagree	agree	strongly agree	not effective			very effective
a. Core ideas of mathematics	1	2	3	4	5	6	7	8	9	10
b. Techniques of classroom discourse	1	2	3	4	5	6	7	8	9	10
c. Direct instruction	1	2	3	4	5	6	7	8	9	10
d. Student reasoning	1	2	3	4	5	6	7	8	9	10
e. Using on-going assessment to guide instruction	1	2	3	4	5	6	7	8	9	10

	1. My professional development activities addressed this topic		2. My professional development on this topic led to changes in my teaching of mathematics				3. The changes inspired by this professional development activity were effective in facilitating/enhancing student learning.			
f. Basing instructional practices on student knowledge	1	2	3	4	5	6	7	8	9	10
g. *Mathematics in Context*	1	2	3	4	5	6	7	8	9	10

6. What type of support did you receive for attending professional development meetings, workshops, and conferences? (Circle all that apply)

Release time from teaching	1	Continuing Education Units	4
Paid travel expenses	2	None	5
Honorarium	3	Other (Please specify)	

7. During the contracted school week, how much planning time do you typically have?

   a. ____________ minutes/day

   b. ____________ minutes/week

8. How often do you spend at least 15 minutes (in formal or informal sessions) planning mathematics lessons, activities, assessments, etc, with other mathematics teachers? (Circle one)

   Number of days: 0 <1 1-3 4-6 > 6

9. When you plan mathematics lessons, activities, assessments, etc, with other mathematics teachers, when does this collaboration take place? (Circle one)

Does not apply	1
During formal meetings	2
During contracted planning time	3
After school on your own time	4

10. In a typical formal and informal meeting or planning session with other mathematics teachers, indicate the number of times you participated in each of the following types of discussion. (Circle one response for each statement)

		Never	Sometimes	Frequently	Always
a.	Decisions about concepts to be emphasized in instruction, guiding instruction, obtaining materials, or including related activities	1	2	3	4
b.	Teaching materials and activities	1	2	3	4
c.	Specific teaching techniques	1	2	3	4
d.	Assessment procedures that reveal how students understand mathematics	1	2	3	4
e.	Problems with specific students and arrangement of appropriate help for them	1	2	3	4
f.	Individual preparation of lessons, tests, or grades	1	2	3	4
g.	Develop course goals or objectives for mathematics	1	2	3	4
h.	Scheduling, student grouping, or planning group events or projects	1	2	3	4
i.	Sharing ideas about mathematics that are interesting to you as an adult	1	2	3	4
j.	Sharing stories about teaching experiences in mathematics	1	2	3	4
k.	Discussing something you have read from professional literature about mathematics	1	2	3	4
l.	Parent issues	1	2	3	4
m.	Other typical activity. Please describe.	1	2	3	4

11. About what percent of the mathematics teachers at your school are involved in efforts to improve the mathematics program? __________%

12. In general, how would you characterize your efforts to improve the mathematics program at your school? (Circle one)

Strong opposition	Slight opposition	Slight support	Strong support
1	2	3	4

13. Which the following *Mathematics in Context* units have you taught in the **current** school year? (Circle all that apply.)

Side Seeing	1	Reallotment	11
Figuring All the Angles	2	Made to Measure	12
Some of the Parts	3	Fraction Times	13
Measure for Measure	4	More or Less	14
Per Sense	5	Ratios and Rates (Smooth Operators)	15
Grasping Sizes	6	Expressions and Formulas	16
Patterns and Symbol	7	Tracking Graphs (Functions of Time)	17
Dry and Wet Numbers	8	Comparing Quantities	18
Picturing Numbers	9	Operations	19
Take a Chance	10	Dealing with Data	20
Packages and Polygons	21	Triangles and Patchwork	31
Ways to Go	22	Digging Numbers	32
Triangles and Beyond	23	Going the Distance	33
Looking at an Angle	24	Reflections on Number	34
Cereal Numbers	25	Graphing Equations	35
Powers of Ten	26	Growth	36
Ups and Downs	27	Get the Most Out of It	37
Building Formulas	28	Patterns and Figures	38
Decision Making	29	Insights into Data	39
Statistics & the Environment	30	Great Expectations	40

Appendix B

Teaching Log

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Teaching Log
(Working Paper #5)

Mary C. Shafer, Lesley R. Wagner, and Jon Davis

Wisconsin Center for Education Research
University of Wisconsin–Madison

Shafer, M. C., Davis, J., & Wagner, L. R. (1997) *Teaching log*. (*Mathematics in Context* Longitudinal/Cross-Sectional Study Working Paper No. 5). Madison, WI: University of Wisconsin, Wisconsin Center for Education Research.

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Description of the Teaching Log

The teaching log compiled by study teachers daily was one of the instruments designed to record information about daily instruction in study classes for the longitudinal/cross-sectional study. Information from the teaching logs was used in the analysis of the content of the actual curriculum, the use and modification of curricular materials, lesson planning, mathematical interaction during instruction, and classroom assessment. The teaching log was pilot-tested with nonstudy teachers during the spring semester prior to the study. Based on feedback from pilot teachers and input from district personnel involved in the study, the log was refined to clarify items and make the format easier for teachers to complete in five to ten minutes daily.

The teaching log consisted of Introductory Information, Daily Logs, and Journal Questions. The purpose of the Introductory Information was to document the unit/chapter taught, changes in class rosters, information about grouping for instruction, and the physical arrangement of the classroom. After indicating their name, the school, city, and date, teachers identified the text and the unit/chapter currently taught. Because the study was longitudinal, teachers noted names of students who were added (Item 1) or dropped from the class (Item 2). Over time, tables were used for teachers to record specific information. For students who were added to the class, teachers noted the approximate date students joined the class and the class periods to which the students were assigned. For students who dropped the class, teachers noted the approximate date students dropped and reasons they left the class. For Item 3, teachers indicated whether they intended for students to work in small groups or pairs during the teaching of the particular unit/chapter. They also described the reason for grouping students in that way and criteria for grouping the students. Finally, teachers sketched the physical arrangement of the classroom. The Introductory Information was completed once a month.

The daily log was printed on both sides of a single sheet of paper. The first side of the log was designed to document content taught, forms of instruction, and student activities. After noting the date, unit/chapter and pages taught on a particular day, teachers indicated if the lesson was a continuation of the previous lesson. If the lesson was continued, teachers were asked to indicate activities that were new to the current lesson. For Item 2, teachers checked whether all students in the class covered the same content. If they did not, they described the ways the content differed and the reasons for these differences. Item 3 was designed to learn about the instructional activities that were used during the class period: warm-up activity, review of previous material, teacher presentation of material, whole-class discussion, small-group or pair work, independent practice, or another activity specified by the teacher. Teachers checked the instructional activities used and circled an emphasis code for each one that ranged from 1 (used for 15% or less of the class period) to 4 (used for more than 75% of the lesson). In Item 4, teachers noted whether the instructional activities precipitated changes in the physical arrangement of the classroom, and they described the reason for such changes. The last item on the first page was designed to learn about the types of activities students engaged in during the class period: listened to teacher or took notes, investigated problems, discussed answers and solution strategies, participated in whole-class discussion, practiced computation, took a quiz or test, reflected on or summarized lesson content, began homework, or another activity specified by the teacher. Teachers checked the student activities and circled an emphasis code for each one from the same scale used for instructional activities. Items 2, 3, and 5 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

The second page of the log was designed to document supplemental materials, classroom assessment, homework, and formal assessment. For Item 6, teachers checked the additional materials used during the lesson: teacher-designed materials, work from text resource materials, work from other resources, quiz, calculators, or another resource specified by the teacher. Teachers were asked to date and attach teacher-designed materials, worksheets from other resources, and quizzes to the daily log page. Item 7 was designed to gather information about teachers' use of classroom assessment. If they assessed students informally during the class period, they completed three sets of questions which addressed (a) what was assessed (students' understanding of particular content or procedure, students' efforts in working as a group, students attitudes toward mathematics, or another item specified by the teacher); (b) the methods of classroom assessment (observation, listening during group work, questioning, checklists, checking student work); and (c) description of changes made in instruction, if any, based on the information gathered. After the first semester of the study, parts (a) and (b) were changed to a checklist format for easier use by the teachers. In Item 8, teachers checked the type of homework assignment, if given: exercises form the text, completion of work begun in class, teacher-designed work, work from text resource materials, exercises from another text, supplementary practice, investigation or project, or other assignment specified by the teacher. After the first semester of the study, Item 8 was revised to be less time-intensive for teachers. In the original log, teachers were asked to list the pages and exercise numbers for text assignments and to attach exercises from supplemental resources and investigations or projects. For the revised item, a more inclusive checklist was used (adding teacher-designed materials, work from text resources materials, and supplemental practice). Teachers were asked to briefly describe the content of teacher-designed and supplemental practice in lieu of attaching copies of such materials, and listing exercise numbers was eliminated. In the final item on the second page, for lessons during which a formal assessment was given, teachers checked the type of formal assessment used: end-of-unit or chapter test, district or state test, student presentation, or student projects. Teachers were asked to date and attach copies of end-of-unit or chapter tests that were representative of below average, average, and above average performance and copies instructions given for student presentations or projects. Items 6 and 8 were adapted from the daily log in Porter, Kirst, Osthoff, Smithson, & Schneider (1993).

Journal questions were printed on one side of a paper, and one sheet was inserted after each daily log sheet. The journal questions were designed to document lesson content that was emphasized or modified and notable classroom events. Each journal question was accompanied by a list of suggestions for reflection. The first journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter. The second journal question focused on notable classroom events. Suggestions for reflection were: a lesson or part of a lesson that went exceptionally well; a surprising event that occurred; content that was particularly difficult for students; an event in which students comprehended content that was previously difficult for them; emergent student misconceptions; an unusual or unexpected strategy brought out by a student; and a student's question that caused a modification in the lesson. Teachers had the option of commenting on other instructional issues of importance to them. In preparation for analysis, journal entries for each teacher were typed and collated by research staff.

Teachers were instructed to complete to complete a daily log sheet for each day of instruction as soon as possible after the lesson and complete at least one set of journal entries per week for the entire school year. If teachers taught multiple mathematics classes, they were asked to complete the log for the class that was observed monthly by the on-site observer. In this way, the information gathered through the log would add the teacher's perspective on the particular lessons for which observation reports were completed, thereby adding a means of triangulating data from observations and teaching logs. Each month the teaching log was a different color for ease in documenting the receipt of teacher logs. A binder was given to each teacher at the beginning of the school year. This binder contained the one-page Introductory Information, daily log sheets and journal questions for each instructional day for one month, a pocket folder for holding supplementary resources, quizzes, and formal assessments used by the teacher during instruction, and a postage-paid envelope for sending the log to the research team. Instructions for completing the teaching log and models of completed logs were reviewed with the teachers each August during the Summer Institute sponsored by the project for study teachers. District contact persons reviewed the instructions with teachers who were unable to attend the Summer Institutes. Subsequent teaching logs with postage-paid envelopes were sent to each teacher monthly. Logs were sent to a contact teacher at each school for distribution. The contact teacher was given an honorarium of $50 per semester for distributing all study instruments to teachers on a timely basis. Teachers received an honorarium of $50 per teaching log upon receipt of the log at the research center. (As a result of negotiation with the teachers, the honorarium was increased to $125 per log during the second and third years of the study.)

Graduate project assistants were liaisons between the research staff and study teachers. Each project assistant read and commented on teacher logs received from one of two research sites (Districts 1 and 2). The numbers of teachers in Districts 1 and 2 who sent logs and journal entries to the research center monthly varied greatly (see Table 1), despite our extensive efforts to collect a full set of teaching logs from each teacher. During the first and second years of data collection, reminders were sent to teachers from the research staff, and graduate research assistants encouraged teachers to continue completing this important source of data through personalized letters of interest in the teachers' work.

Table 1
Number of Teaching Logs Received, by Grade and Year

Grade *(No. of Teachers*)*	Number of Teaching Logs Per Teacher	Percent of Teachers Submitting Teaching Logs			
		0-2 Logs	3-6 Logs	7-8 Logs	9 Logs
1997-1998					
5 *(13)*	0-9	23	15	23	38
6 *(12)*	0-9	33	42	8	17
7 *(10)*	0-9	10	20	30	40
1998-1999					
6 *(12)*	0-9	33	25	0	42
7 *(12)*	0-9	50	17	8	25
8 *(10)*	0-9	30	20	0	50
1999-2000					
7 *(9)*	0-9	44	22	0	33
8 *(9)*	0-9	22	11	11	56

*Includes teachers who taught portions of the school year

Reference:

Porter, A. C., Kirst, M. W., Osthoff, E. J., Smithson, J. L., & Schneider, S. A. (1993). *Reform up close: A classroom analysis*. Madison, WI: University of Wisconsin–Madison.

DAILY LOG PROCEDURES

The daily logs you complete are crucial components of the longitudinal study. These logs are designed to record daily practices in your mathematics classroom. No single instrument can characterize the complexities of classroom life, but the logs are intended to facilitate the general description of your teaching practices and your students' activities in the classroom. Your thoroughness in completing the daily logs is a most vital and appreciated aspect of this study.

INSTRUCTIONS:

At the beginning of month, please complete the introductory information.

The daily log is intended to reflect the character of your mathematics classroom. As its name implies, this log should be completed on a daily basis, as soon after the math class meets as possible. Most questions on the daily log can be completed with a check mark or brief descriptions.

The most crucial components of the daily log, the journal questions, require as thorough description as you can give to accurately reflect the lesson flow and classroom events. The importance of your thoroughness in answering these two questions cannot be underestimated. We would like you to respond to these questions as often as you notice events reflective of the suggested topics in your classroom, but we expect them to be completed at least once a week as these events present themselves.

If you do not have enough room to complete these, or any other questions, please complete them on an additional sheet of paper and attach the paper behind its corresponding entry.

In addition to filling in the daily log, where requested, we would like you to place hand-outs and/or student work in the folder provided and send these items along each month when you send your daily log entries to us. As you make copies of materials for students, please and date an additional copy to include with its corresponding daily log entry.

We estimate that it should take no more than 10 minutes each day to complete the daily log. The journal questions will require additional time to answer each week.

At the end of month, please staple or clip the log together with the introductory information in front, and return the completed log, the requested materials, and student work in the provided envelope.

If at any time you have questions about these procedures or the completion of the logs, please contact Lesley Wagner at 1-800-862-1055 or via e-mail at lrwagner@students.wisc.edu.

Thank you for your invaluable time in completing the daily log.

Please complete this information at the beginning of each month.

INTRODUCTORY INFORMATION

Name ___________________________ Date ____/____/_____
School ___________________________ Text ___________________________
City ___________________________ Unit/Chapter _____________________

1. Please list the names of students added to the class, the date they were added, and the period to which they were added:

Name of Student	Date Added to Class	Class Period

2. Please list the names of students dropped from the class, the date they were dropped, and the reason they were dropped (e.g., transferred to new class, transferred to new school):

Name of Student	Date Dropped from Class	Reason Dropped

3. If students will work in groups or pairs during the teaching of this unit or chapter, please describe how these groups or pairs are chosen and the reason for grouping the students this way.

4. Please sketch the physical arrangement of the classroom.

DAILY LOG

Date ____/___/_____ Pages __________________ Unit/Chapter ________________________

1. __ Check here if today's lesson is a continuation of yesterday's. If so, only check activities which are new to today's lesson.

2. __ All students covered the same content.
   __ Content covered varied by student/group. Please describe how the instruction differed and the reasons for these differences.

3. Please check the instructional formats utilized during the lesson and circle the emphasis each received.

Instructional Formats	Emphasis Received			
__ Warm-up	1	2	3	4
__ Review of previous material	1	2	3	4
__ Teacher presentation of material	1	2	3	4
__ Whole class discussion during lesson	1	2	3	4
__ Small group or pair work	1	2	3	4
__ Independent practice	1	2	3	4
__ Other (please specify) ______________________________	1	2	3	4

Emphasis Scale
4-Primary instructional format, used for more than 75% of the lesson.
3-Significant instructional format, used for 50-75% of the class period.
2-One of several instructional formats used with approximately equal emphasis.
1-An instructional activity used for a short period of time, approximately 15% or less of the class period.

4. If the activities in the lesson necessitated any physical change in the classroom or significant student movement, please describe.

5. Please check the student activities completed during the lesson and circle the emphasis each received.

Student Activities	Emphasis Received			
__ Listened to teacher and/or took notes	1	2	3	4
__ Investigated problem	1	2	3	4
__ Discussed answers and solution strategies	1	2	3	4
__ Participated in whole class discussion	1	2	3	4
__ Practiced computation	1	2	3	4
__ Took a test or quiz	1	2	3	4
__ Reflected on/summarized lesson concepts	1	2	3	4
__ Began homework	1	2	3	4
__ Other (please specify) ______________________________	1	2	3	4

Emphasis Scale
4-Primary student activity, engaged in for more than 75% of the class period.
3-Significant student activity, engaged in for 50-75% of the class period.
2-One of several activity types, engaged in with approximately equal emphasis.
1-An activity engaged in for a short period of time, approximately 15% or less of the class period.

6. Please check the additional materials used during the lesson:

__ Teacher designed materials (Please date and attach)
__ Work from publisher resource materials
__ Worksheets or activities from sources other than the text or unit (Please date and attach)
__ Quiz (Please date and attach)
__ Calculators
__ Other (please specify) ____________________________

7. If you informally assessed students during the class period, please answer the following questions:

a) Please check what you were assessing
__ Students' understanding of _________________________
__ Students' efforts in working as a group
__ Students' attitudes toward math (e.g., confidence, perseverance)
__ Other, please describe________________________

b) Please check way(s) in which you informally assessed students
__ Observation
__ Listening during group work
__ Questioning
__ Checklists
__ Checking their work

c) Did the information you gained affect your instruction? __Yes __No
If yes, please describe.

8. Please check all student homework assignments that apply.
__ Exercises from text/unit
__ Completion of work begun in class
__ Teacher designed, please indicate content_________________________
__ Work from publisher resource materials
__ Exercises from source other than text
__ Supplementary practice, please indicate content __________________________
__ Investigation/Project
  __ related to the unit, please describe____________________________
  __ supplementary to the unit, please describe__________________________
__ Other, ________________________

9. If a formal assessment was part of the lesson, please indicate the type of assessment. Please attach copies of student assessments that are representative of below average, average, and above average performance as well as copies of student papers that show any interesting or unusual work.
__ Test
__ District or state developed test, please specify__________________________________
__ Student presentations (Please date and attach the instructions or options given to students)
__ Student projects (Please date and attach the instructions or options given to students)

****Please remember to reflect on the following aspects of classroom instruction at least once a week.****

Date___/___/___

Journal Questions

1. Please describe the parts of the lesson you emphasized and any modifications you made in the lesson as compared to its presentation in the unit or chapter of the text. Please check and reflect on one or more of the following occurrences:

   ___ particular problems or aspects of the lesson that were emphasized and explain why they were emphasized
   ___ particular problems or aspects of the lesson that were deleted and explain why they were deleted
   ___ additional activities, problems, or procedures that were included in the lesson and explain why they were added
   ___ the order of presentation of lesson activities and/or content as compared to its presentation in the unit or chapter; if you changed the order of presentation, please describe how it was changed and explain why
   ___ other changes, please describe
   ____

2. Please describe any notable classroom event(s) related to the lesson.
   Please check and reflect on one or more of the following events:

   ___ the lesson or part of the lesson went exceptionally well
   ___ something surprising occurred
   ___ an idea was particularly difficult for the students
   ___ students seemed to comprehend an idea that had previously been troublesome
   ___ student misconceptions emerged
   ___ a student offered an unusual or unexpectedly sophisticated strategy
   ___ a student's question caused a modification in the lesson
   ___ other(s), please describe____

Appendix C

Classroom Observation Instrument

A Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Mathematical Performance

Observation Scale
(Working Paper #6)

Mary C. Shafer, Lesley R. Wagner, and Jon Davis

Wisconsin Center for Education Research
University of Wisconsin–Madison

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Description of the Observation Scale

The observation instrument for the longitudinal/cross-sectional study was designed to measure one independent variable (curricular content and materials—the actual curriculum) and the three intervening variables: pedagogical decisions, classroom events, and student pursuits. The observation instrument is composed of seven sections. In the first section, the observer recorded pertinent information related to the teacher and students: the name of the teacher, the school, and the grade level of the students in the class. The observer also recorded information pertinent to the particular lesson: the date of the observation, times the lesson began and ended, text used, unit/chapter taught, and the page numbers taught during the lesson. In the second section of the observation instrument, the observer conducted and recorded notes from a brief preobservation interview of the teacher during which the teacher was asked to identify the mathematical content to be explored or conveyed in the lesson and the location of the lesson with respect to the development of concepts in the instructional unit/chapter. In the third section, the observer recorded the flow of the lesson, which was a list of lesson activities along with the time allotted to each.

The next two sections of the observation instrument were collectively composed of 12 indices for various dimensions of instruction, which addressed the three intervening variables in the research model for the study. Nine of these indices focused on classroom events; the remaining three indices focused on student pursuits. Pedagogical decisions, although not presented in a separate section of the observation instrument, were central to both classroom events and student pursuits.

The indices used to characterize each dimension were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995), Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993), and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through a review of literature that was specific to each dimension. The indices were refined as a result of classroom observations of nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Three to four levels were identified for each dimension in order to identify differences in these variables between conventional and reform-based approaches to teaching and learning mathematics. The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the dimension being scaled. For example, levels of lessons that fostered conceptual understanding range from no attention to conceptual understanding during instruction to lessons in which the continual focus was on building connections among mathematical ideas.

The observation instrument was pilot-tested by project staff numerous times in both MiC and conventional classrooms in order to define and clarify descriptors for each item and to determine ways to achieve interrater reliability. Before the observation instrument was used in study classrooms, administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each dimension. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

In the sixth section of the observation instruction, the observer conducted and recorded notes from a brief postobservation interview of the teacher during which the teacher was asked to rate and comment on the degree to which the teacher felt the lesson achieved the purpose noted in the preobservation interview. The teacher was also asked whether any incidents occurred during the lesson which revealed student misunderstanding or provided opportunities to facilitated student understanding in any way. In this way, teachers had an opportunity to describe and explain modifications made during the lesson. In the final section of the observation instrument, the observer recorded any additional comments about the lesson.

Indices

Classroom Events

The lesson provided opportunities for students to make conjectures about mathematical ideas. In the conceptualization of conjectures in the longitudinal study, three types of student conjectures are described and sought in classroom interaction. First, students can make conjectures that are realizations of the connections between existing knowledge and the application of these concepts in new contexts. That is, students might see a connection between a new problems and problems they have already solved. Second, students may investigate the truthfulness of particular statements. Third, conjecturing may permeate a lesson. Given a pattern, for example, students are asked to devise a formula that captures the essence of the pattern in a concise form, which in turn leads to generalizations. Each type of conjecture is given a specific rating in the index, with an observation of the third type given the highest rating.

The lesson fostered the development of conceptual understanding. Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. Lower ratings in this category describe classrooms in which teaching for conceptual understanding occurs, but is often overshadowed by an emphasis on procedural knowledge. The highest rating describes a lesson in which links between conceptual and procedural understanding are the main emphasis of the instruction.

Connections within mathematics were explored. In this index, mathematical topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios which connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating is meant to reflect both the problems and instruction.

Connections between mathematics and students' daily lives were apparent in the lesson. This index measures whether connections between mathematics and students' daily lives were apparent in text problems or problems presented in class or were discussed by the teacher or students.

Students explained their responses or solution strategies. This index is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

Multiple strategies were encouraged and valued. This index measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. Higher ratings on this index refer to lessons in which discussion of alternative strategies is a frequent, important element of classroom instruction.

The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class. This index is intended to measure the ways in which the teacher uses student responses during instruction. The highest rating is reserved for lessons in which the teacher not only probed individual students' thinking but also encouraged other students to comment on the solution strategies or used students' thinking processes to open discussions that encourage deeper understanding of mathematics.

The teacher used student inquiries as a guide for instructional mathematics investigation or as a guide to shape the mathematical content of the lesson. Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. This index measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

The teacher encouraged students to reflect on the reasonableness of their responses. This index is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

Student Pursuits

Student exchanges with peers reflected substantive conversation of mathematical ideas. Substantive conversation by students is characterized by interaction that is reciprocal, which involves listening carefully to others' ideas in order to understand them, building conversation on others' ideas, or extending an idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions. (Newmann, Secada, and Wehlage, 1995). While other items in this observation scale refer to the role of the teacher in mediating discourse, this item measures student discourse between peers in either large-group or small-group settings.

Interactions among students reflected collaborative working relationships. A low rating is given when students are physically sitting in groups but rarely working together. In contrast, the highest rating denotes a lesson in which students are actively involved in solving problems with their classmates and in which students made sure that all students in the group understood one problem before moving on to the next. N/A is reserved for lessons in which the goal is for students to work on problems independently.

The overall level of student engagement throughout the lesson was serious. This index measures the extent to which students remained on task during the lesson. Engagement is exemplified by behaviors in which students are attentive, complete assigned work, participate by raising questions, contribute to both large-group and small-group discussions, and help their peers (Secada and Byrd, 1993).

Observations

The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. Throughout the class period, the observer continually judged the levels of each dimension of classroom events and student pursuits. During each observation the observer took field notes that pertained to the 12 indices. Immediately after observing a lesson, the observer rated each item and recorded evidence from the lesson (consisting of dialogue or an artifact) to support the given rating. In general, a rating of 1 on a particular item indicated that the dimension was rarely or never seen in the lesson; the highest rating indicated that the dimension received major emphasis in the classroom. In practice, high ratings were rarely attained on every item during one observation. Ratings also varied in different observations of the same teacher.

The number of observations per teacher varied in each district (see Table 1). Most teachers in District 1 were observed once a month for a total of nine observations per teacher. During the first year of data collection, one teacher in District 1 accepted an administrative position in December; consequently, she was observed three times, and the newly assigned teacher was observed five times. During the second and third years of data collection, one eighth-grade control class had three teachers, and two seventh-grade experimental classes had two teachers over the course of the school year. As a result, each teacher was observed only a few times. Teachers in District 2 were observed a total of two to nine times each. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. In addition, four teachers from one school in District 2 withdrew from participation in the study during the spring semester of the first year of data collection; consequently, they were observed only three times. During the third year of data collection, two seventh-grade experimental classes were observed twice because the teacher had been on parental leave.

Table 1
Number of Observations Conducted, by Grade and Year

Grade *(No. of Teachers*)*	Number of Observations Per Teacher	Percent of Teachers Observed			
		1-3 Times	4-6 Times	7-8 Times	9 Times
1997-1998					
5 *(13)*	5-9	0	38	0	62
6 *(12)*	3-9	25	33	8	33
7 *(10)*	3-9	20	40	0	40
1998-1999					
6 *(12)*	5-9	0	33	25	42
7 *(12)*	5-9	0	42	25	33
8 *(10)*	2-9	20	40	30	10
1999-2000					
7 *(9)*	2-9	22	11	11	56
8 *(9)*	8-9	0	0	44	56

*Includes teachers who taught portions of the school year

Interrater Reliability

In the August prior to the study, each observer viewed two videotaped lessons with a graduate project assistant who developed the observation instrument and rated the lessons using the instrument. During these meetings discussions centered on consistency of ratings and descriptions of the types of conjectures observed, the nature of student–student conversation, and instances in which teachers used student inquiries to shape the lesson.

In the fall of 1997, each observer and a project assistant visited five classes in District 1 and nine classes in District 2. During the first few observations at each site, the project assistant's and observer's ratings of several items differed by one point. By the last observation, however, this disagreement had subsided considerably. The first dimension, student conjectures, initially caused difficulty for both observers. For example, observers initially categorized the repetitious practice of problems using a single prescribed algorithm as a first level conjecture, when the first level of conjecture is meant to describe the preponderance of students making conjectures that link concepts they have studied in the past with the same concept set within a new context. Another dimension, students' level of collaboration in the classroom, one observer tended to give the highest rating if students were physically sitting in groups. The project assistant emphasized the importance of circulating around the classroom to determine if students actually worked in groups to support each other's learning. After this training, the observers began observing each study teacher once a month and completed a report for each observation. Completed reports were sent electronically to the research center for analysis. Each observer was compensated an amount per observation as part of a subcontract between the observer and the University of Wisconsin. The amount of payment varied according to the length of the class period observed. In September 1998, both observers worked on interrater reliability with a project assistant during on-site classroom observations in District 1. Because of the lack of funds, on-site work for interrater reliability between observers and a project assistant were not conducted in the fall of 1999.

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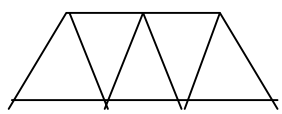
Observation Scale Descriptors

C.1. The lesson provided opportunities for students to make conjectures about mathematical ideas.

This scale measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas. There are three types of conjectures that students might make. One type of conjecture involves the student in making a guess about how to solve a particular problem based on experience solving problems with similar solution strategies. For example, students were solving problems in which they used properties of similar triangles. When asked to determine the height of a tree, students conjectured that an appropriate solution strategy would involve similar triangles. The students made a connection between the new problem and problems that they had previously solved. A second type of conjecture occurs when a student makes a guess about the truthfulness of a particular statement and subsequently plans and conducts an investigation to determine whether the statement is true or false. For example, a 12-year-old student disagreed with a statement that she was half as tall as she is now when she was 6-years old, and proceeded to support her argument by comparing her present height with heights of 6-year-old children. A third type of conjecture is a generalization. A generalization is created by reasoning from specific cases of a particular event, is tested in specific cases, and is logically reasoned to be acceptable for all cases of the event. For example, given that a beam is constructed of rods in the following configuration,



students are asked to describe the relation between the number of rods and the length of the beam[1] (Wijers, Roodhardt, van Reeuwijk, Burrill, Cole, & Pligge, 1998). Using a table to organize their reasoning, students described the pattern that emerged, explained how the pattern fit the given diagram, and generated formulas for the relationship. In this situation, students reasoned from specific cases, tested and supported their ideas with evidence from drawings and the table, and described the relation in a formula.

1. No conjectures of any type were observed in the lesson. Students were not encouraged to make connections between a new problem and problems previously seen, investigate the validity of their own guesses, look for patterns, or make generalizations.

2. Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

3. Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

4. Students made generalizations about mathematical ideas.

1 The length of the beam is the number of rods on the bottom of the beam.

C.2. The lesson fostered the development of conceptual understanding.

Conceptual knowledge is described as the “facts and properties of mathematics that are recognized as being related in some way” (Hiebert & Wearne, 1986, p. 200), or as a network of relationships that link pieces of knowledge (Hiebert & Lefevre, 1986). In the primary grades, for example, students learn the labels for whole-number place-value positions. If this information is stored as isolated pieces of information, the knowledge is not conceptual. If this knowledge, however, is linked with other information about numbers, such as grouping objects into sets of ten or counting by tens or hundreds, then the information becomes conceptual knowledge. The network of relationships about place value grows as other pieces of knowledge related to place value, such as regrouping in subtraction, are recognized. Procedural knowledge, in contrast, is described as having two parts. One category comprises the written mathematical symbols, which are devoid of meaning and are acted upon through knowledge of the syntax of the system. A second category is composed of rules and algorithms for solving mathematics problems, step-by-step procedures that progress from problem statement to solution in a predetermined order. Procedural knowledge is rich in rules and strategies for solving problems, but it is not rich in relationships (Hiebert & Wearne, 1986).

Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students’ attention to linking procedural and conceptual knowledge. In addition and subtraction of decimals, for example, lining up the decimal points should be linked with combining like quantities. Instruction might explicitly bring out the relationships between lining up the decimal point in addition and subtraction and lining up whole numbers on the right side for the same operations (Hiebert & Wearne, 1986).

1. The lesson as presented did not promote conceptual understanding.

2. The lesson asked few questions that fostered students’ conceptual development of mathematical ideas, or conceptual understanding was a small part of lesson design.

3. Some lesson questions fostered students’ conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students’ procedural understanding without meaning.

4. The continual focus of the lesson was on building connections between disparate pieces of information or linking procedural knowledge with conceptual knowledge.

C.3. Connections within mathematics were explored in the lesson.

This scale measures the extent to which instruction addressed mathematical topics thoroughly enough to explore relationships and connections among them.[2] A low rating is given when the mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning, and instruction treated this topic in isolation of other mathematical topics. A high rating is given when the mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. Rather than examining fragmented pieces of information, students looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical topics, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

Topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios, as in the following problem. Students are asked to determine the probability of a frog jumping from a cage and landing on white or black floor tiles and to express this probability as a fraction or percent (Jonker, van Galen, Boswinkel, Wijers, Simon, Burrill, & Middleton, 1998). In solving this problem, students use area, number, and probability concepts. Second, connections can be made among more narrowly defined areas such as a lesson involving the solution of quadratic equations. In this lesson, connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The rating should reflect both the problems and instruction.

1. The mathematical topic of the lesson was covered in ways that gave students only a surface treatment of its meaning. The mathematical topic was presented in isolation of other topics, and the teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

2. Connections among mathematical topics were present in the lesson. The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail by the teacher or the students.

3. Connections among mathematical topics were discussed by teacher and students during the lesson, or connections were clearly explained by the teacher.

4. The mathematical topic of the lesson was explored in enough detail for students to think about relationships and connections among mathematical topics. During instruction, many students did at least one of the following: looked for and discussed relationships among mathematical ideas, expressed understanding of mathematical relationships, or provided explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated.

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2 Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Deep Knowledge (pp. 31-35).

C.4. Connections between mathematics and students' daily lives were apparent in the lesson.

This scale measures whether connections between mathematics and students' daily lives were apparent in text problems or discussed by the teacher or students. Examples of problems that foster such connections are estimating the sale price of an item or determining the amount of ingredients required to serve four people when a recipe serves seven. In contrast, word problems such as "Bart is two years older than Lisa. In five years Bart will be twice as old as Lisa. How old are they now?" are devoid of connections between mathematics and students' lives.

1. Connections between mathematics and students' daily lives were not apparent in the lesson.

2. Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

3. Connections between mathematics and students' daily lives were clearly apparent in the lesson.

C.5. Students explained their responses or solution strategies.

This scale is intended to measure the extent to which students elaborate on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results, rather than simply stating answers.

1. Students simply stated answers to problems. They did not explain their responses or solution strategies orally or in written form.

2. Students explained how they arrived at an answer, but these explanations focused on the execution of procedures for solving problems rather than an elaboration on their thinking and solution path.

3. Students explained their responses or solution strategies. They elaborated on their solutions orally or in written form by justifying their approach to a problem, explaining their thinking, or supporting their results.

C.6. Multiple strategies were encouraged and valued.

This scale measures the extent to which students were asked to consider different perspectives in approaching the solution to a problem. In a classroom where multiple strategies are encouraged and valued, students spend much of their time discussing different strategies in a substantive manner, and this discourse is an important element within the classroom. Multiple strategies might be elicited by the teacher during whole-class or small-group discussion in which students explicitly share their strategies. The task itself might clearly involve students in solving the problem in different ways (e.g., find the discount in another way), or the task may require students to consider alternative approaches for successful completion (e.g., list as many ways as you can to calculate 15 x $1.98).

1. Multiple strategies were not elicited from students.

2. Different problem-solving strategies were rarely elicited from students or only briefly mentioned by the teacher.

3. Students were asked if alternate strategies were used in solving particular problems, but this was not a primary goal of instruction.

4. Discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction.

C.7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.

This scale is intended to measure the ways in which the teacher uses student responses during instruction. Teachers can give credence to students' responses by inviting students to listen carefully to other students, to ask each other questions that clarify meaning, and to compare other students' strategies with their own. Teachers can also use student responses to pose questions that stimulate further discussion, to illustrate a point, or to relate them to other aspects of the lesson.

1. The teacher was interested only in correct answers. The majority of the teacher's remarks about student responses were neutral short comments such as "Okay," "All right," or "Fine." No attempt was made to use students' responses to further discussion.

2. The teacher established a dialogue with the student by asking probing questions in an attempt to elicit a student's thinking processes or solution strategies.

3. The teacher valued students' statements about mathematics by using them to foment discussion or to relate them to the lesson in some way. The teacher opened up discussion about the student response by asking other students questions such as: "Does everyone agree with this?" or "Would anyone like to comment on this response?"

C.8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Occasionally a student's inquiry can be used to introduce the topic of the lesson, supplement a lesson, or connect the lesson to students' lives. In other cases, a student's question or response may provide a starting point for a rich mathematical journey. A student's question about whether the sum of the angles of every triangle is always 180°, for example, might lead to a discussion of non-Euclidean geometry. This scale measures the teacher's responsiveness to student inquiries and the teacher's flexibility in using these inquiries in ways that enhance the lesson.

Circle Yes, if the teacher used students' inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.

Circle No, if a student's comment or question potentially could have led to such a discussion, but the teacher did not pursue it.

Circle N/A, if no such opportunities came about during the lesson.

C.9. The teacher encouraged students to reflect on the reasonableness of their responses.

An unreasonable response refers to a response that is mathematically distant from the correct answer and might even be distant from an answer that students recognize as reasonable in contexts outside the classroom. One explanation for unreasonable responses is that students do not check the reasonableness of their answers. Although this may be true in some cases, unreasonable responses may also be the result of the lack of connections between symbols and their meaning. Evaluating the reasonableness of a solution involves connections between conceptual and procedural knowledge. These connections are especially significant at the end of the problem-solving process. Lining up decimal points when adding or subtracting decimals, for example, without connecting the process to place value concepts, may lead to unreasonable responses. Students might rely on rules or procedures to obtain correct answers and not have the conceptual knowledge to help them evaluate reasonableness of the answer (Hiebert & Wearne, 1986). This scale is intended to measure whether the teacher encouraged students to reflect on the reasonableness of their answers and whether the discussion involved emphasis on conceptual understanding.

1. The teacher rarely asked students whether their answers were reasonable. If a student gave an incorrect response, another student provided or was asked to provide a correct answer.

2. The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding.

3. The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

D.1. Student exchanges with peers reflected substantive conversation of mathematical ideas.

With this scale we are attempting to capture the quality of student communication. Substantive conversation by students is characterized by interaction that is reciprocal, involving listening carefully to others' ideas in order to understand them, building conversation on them, or extending the idea to a new level. Substantive conversation also promotes shared understanding of mathematical ideas and the use of higher order thinking, such as applying ideas, making comparisons, or raising questions.[3] In contrast, student exchanges with little or no substantive conversation involve reporting facts or procedures in ways that do not encourage further discussion of ideas.

1. There were no exchanges between peers in small groups or as a formal part of the general discourse within a large-group setting.

2. Student exchanges with peers reflected little or no substantive conversation of mathematical ideas.

3. Most students only asked one another for a clarification of directions given by the teacher or simply accepted someone's answer without an explanation of how it was found. Few students asked how a solution was found or asked for a clarification of another student's answer.

4. Most of the students asked their classmates for a description of how they solved a particular problem, discussed alternative strategies, and/or questioned how classmates arrived at a solution.

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3 Ideas were drawn from Newmann, Secada, & Wehlage (1995), Chapter 3, Authentic Instruction, Substantive Conversation (pp. 35-40).

D.2. Interactions among students reflected collaborative working relationships.

The collaborative nature of the classroom can be thought of as students working together, exchanging ideas, and finding solutions to the same problem. This includes providing assistance to one another, making sure that everyone understands and is working on the same problem, exchanging ideas, and seeking help from each other when it is needed. Student collaboration can occur in a small-group or large-group setting. If the major focus of the lesson is on providing students with individual work, then N/A should be selected.

N/A. The main purpose of the lesson was to give students needed individual practice, or students spent nearly all of the class period involved in independent work.

1. None of the students were working together in small groups or in a large-group setting. If students were working in small groups, then one student typically gave answers to other members of group without explanation of why certain procedures were used.

2. Few students were sharing ideas or discussing how a problem should be solved in small groups or in a large-group setting. Although students physically sat together, there was little exchange of ideas or assistance. Many of the students in a group were working on different problems and at different paces.

3. Some students were exchanging ideas, or providing assistance to their classmates; however, a few students relied on other members of the group to solve problems. Contributions to solving problems were not made equally by all students.

4. Most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem.

D.3. The overall level of student engagement throughout the lesson was serious.[4]

This scale measures the extent to which students remained on task during the lesson.

1. Disruptive disengagement. Students were frequently off task, as evidenced by gross inattention or serious disruptions by many. This was the central characteristic during much of the class.

2. Passive disengagement. Students appeared lethargic and were only occasionally on task carrying out assigned activities. For substantial portions of time, many students were either clearly off task or nominally on task but not trying very hard.

3. Sporadic or episodic engagement. Most students, some of the time, were engaged in class activities, but this engagement was inconsistent, mildly enthusiastic, or dependent on frequent prodding from the teacher.

4. Widespread engagement. Most students, most of the time, were on task pursuing the substance of the lesson. Most students seemed to take the work seriously and were trying hard.

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4 Ideas were drawn from Secada & Byrd (pp. 14-15).

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Observation Scale

Observer: ________________________________ Time Lesson Begins: ______________________
Teacher: ________________________________ Time Lesson Ends: ______________________
School: ________________________________ Textbook: ________________________________
Grade: ________ Chapter/Unit: ____________________________
Date of Observation: ______________________ Lesson (pages): _________________________

A. Pre-observation Interview With Teacher

1. What mathematical concept(s) or important ideas are being conveyed in this lesson?
   ____
   ____

2. Where is this activity generally situated in the development of a unit? (For example, day 1 (introduction) of 5 days needed to complete the unit)
   ____

B. Lesson Flow

Describe the main activities that occurred during the class period and the amount of time devoted to each activity. For example: warm-up—5 minutes, introduction of concept through context—7 minutes, large group discussion—10 minutes, group activity—25 minutes, summary by teacher—5 minutes.

For sections C and D please refer to the observation scale descriptors on the attached sheets. Please provide evidence supporting your rating.

C. Classroom Events

		Evidence
1. The lesson provided opportunities for students to make conjectures about mathematical ideas.	1 2 3 4	
2. The lesson fostered the development of conceptual understanding.	1 2 3 4	
3. Connections within mathematics were explored in the lesson.	1 2 3 4	
4. Connections between mathematics and students' daily lives were apparent in the lesson.	1 2 3	
5. Students explained their responses or solution strategies.	1 2 3	
6. Multiple strategies were encouraged and valued.	1 2 3 4	
7. The teacher valued students' statements about mathematics and used them to build discussion or work toward shared understanding for the class.	1 2 3	
8. The teacher used student inquiries as a guide for instructional mathematics investigations or as a guide to shape the mathematical content of the lesson.	Yes No N/A	
9. The teacher encouraged students to reflect on the reasonableness of their responses.	1 2 3	

D. Pupil Pursuits

Evidence

1. Student exchanges with peers reflected substantive conversation of mathematical ideas. 1 2 3 4

2. Interactions among students reflected collaborative working relationships. 1 2 3 4 N/A

3. The overall level of student engagement throughout the lesson was serious. 1 2 3 4

E. Postobservation Interview With Teacher

1. On a scale of 1 to 4, how did the teacher rate the lesson as achieving the intended purpose cited in the pre-observation interview?

   Goals Not Met All Goals Met
   1 2 3 4

   Comments: ____
   ____
   ____

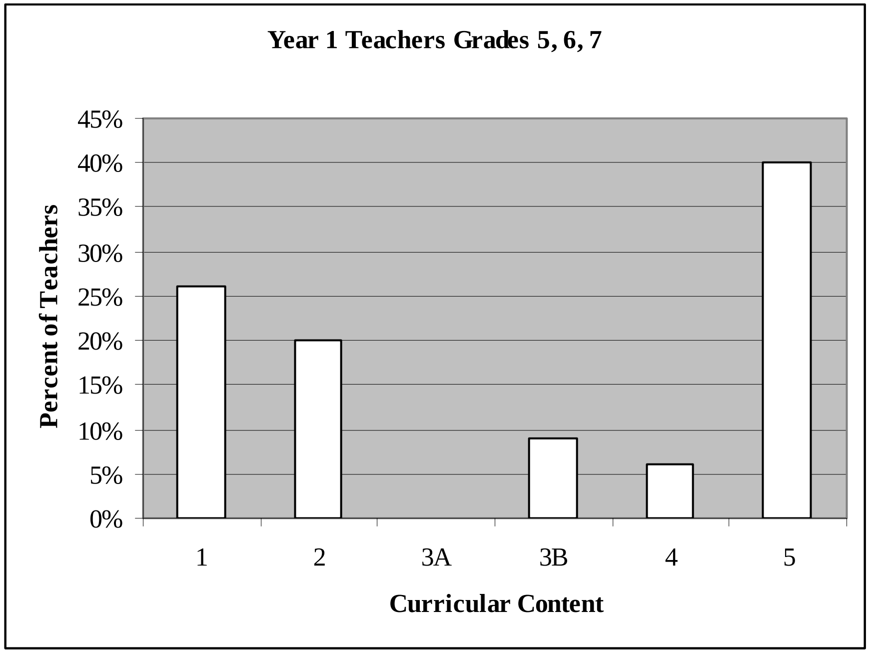
2. Did the teacher feel that any incidents occurred which revealed student misunderstanding or provided opportunities to facilitate student understanding in any way? Yes No If yes, please describe and explain if the lesson was modified in any way.
   ____
   ____

F. Additional Information

1. Please feel free to add any comments or information (on the back) that you think would be of interest in describing the classroom that you observed.

Appendix D

Dimensions of Opportunity to Learn with Understanding



Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Figure D1. Teacher level of curricular content, 1997-1998.

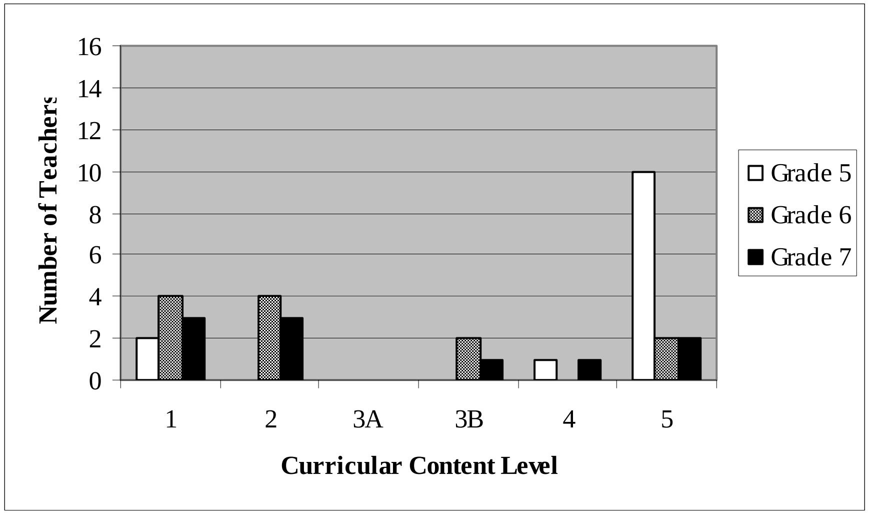


Figure D2. Teacher level of curricular content, by grade, 1997-1998.

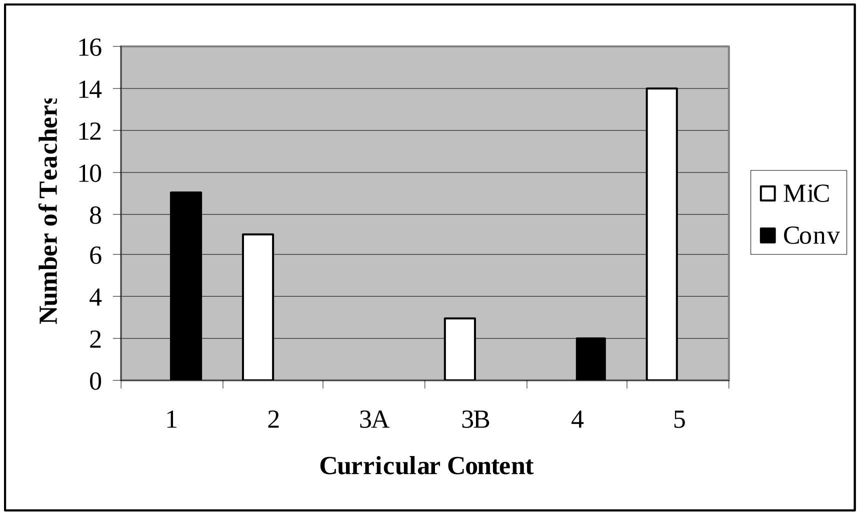


Figure D3. Teacher level of curricular content, by curriculum, 1997-1998.

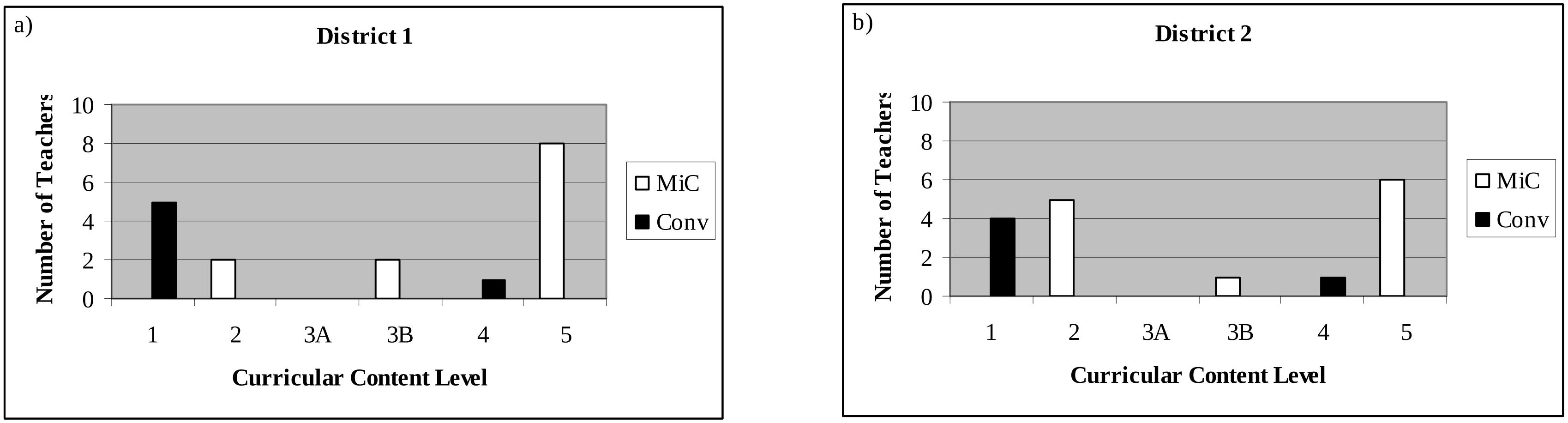
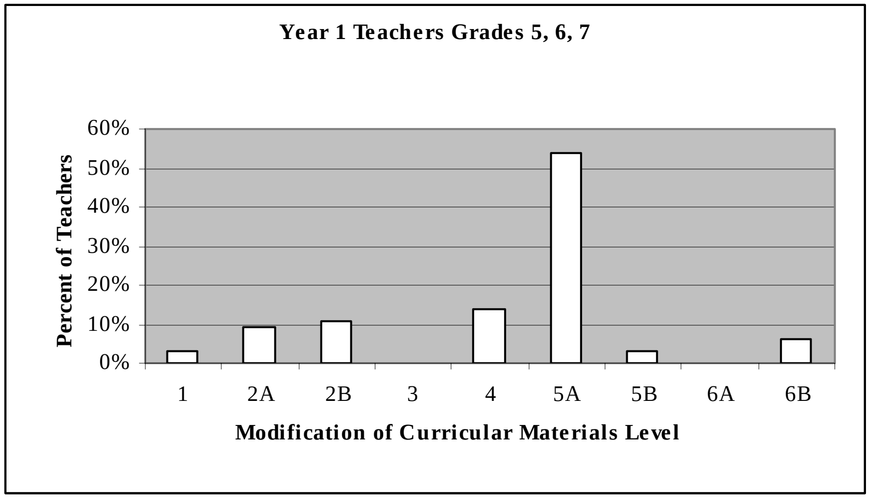


Figure D4. Teacher level of curricular content, by district: a) District 1 and b) District 2; and by curriculum, 1997-1998.



Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6	The teacher modified the curriculum in ways that enhanced conceptual development of the content.
6A	The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.
6B	The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.
5	The teacher followed the curriculum faithfully.
5A	The teacher occasionally supplemented the text with activities disconnected from the text.
5B	The teacher presented the curriculum as it was written with few, if any, modifications.
4	The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).
3	Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.
2	The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.
2A	The teacher supplemented a reform curriculum with conventional materials to the extent that the supplementary materials subsumed the reform curriculum.
2B	The teacher abandoned the reform curriculum in favor of a conventional curriculum.
1	The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Figure D5. Teacher level of modification of curricular materials, 1997-1998.

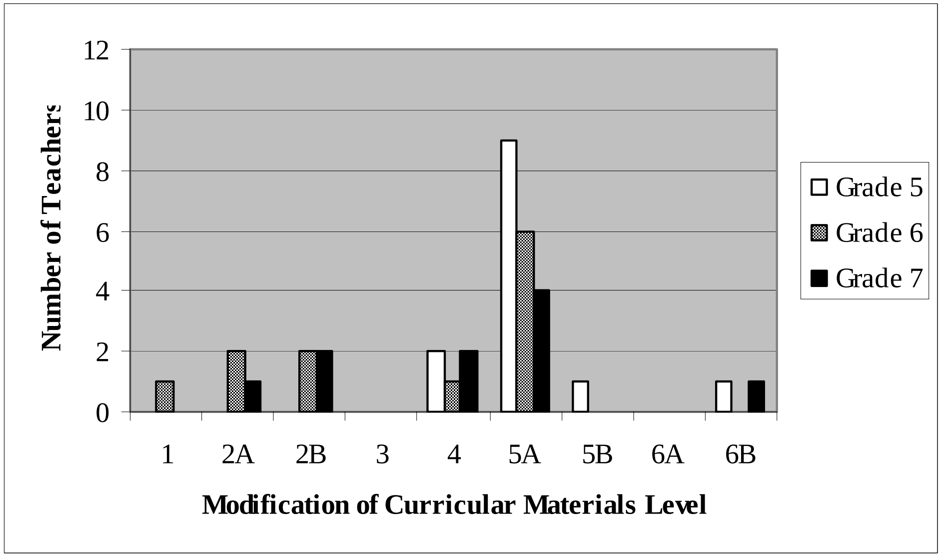


Figure D6. Teacher level of modification of curricular materials, by grade, 1997-1998.

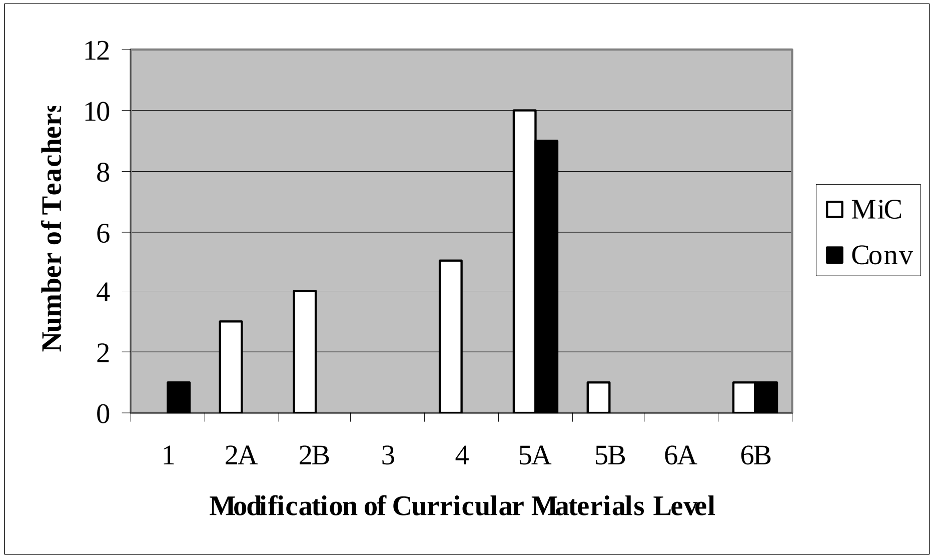


Figure D7. Teacher level of modification of curricular materials, by curriculum, 1997-1998.

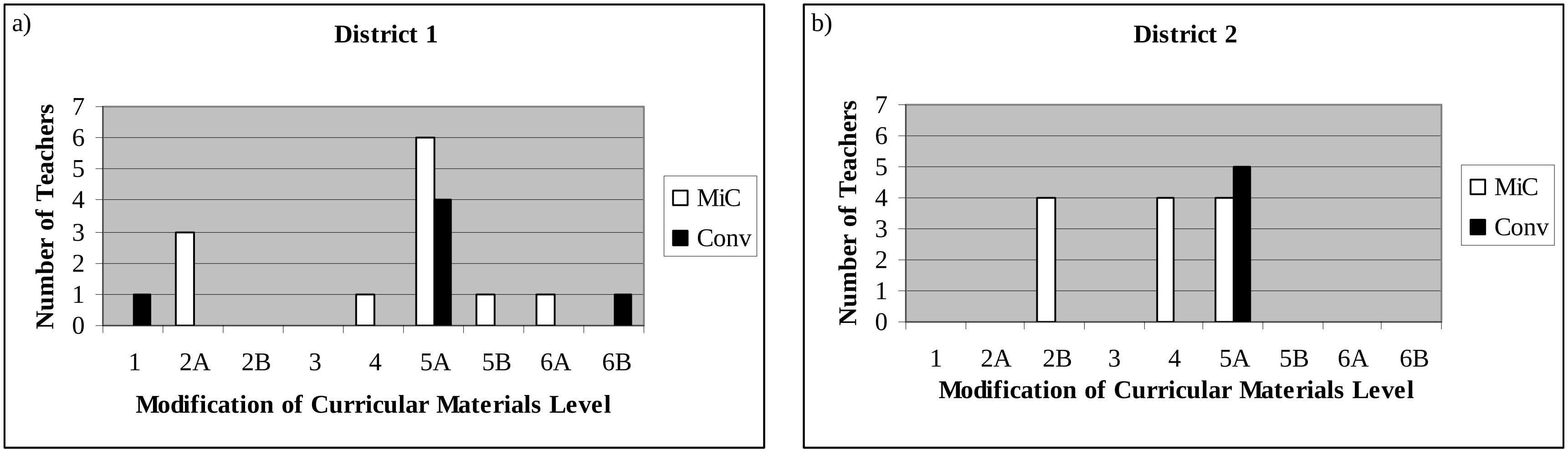
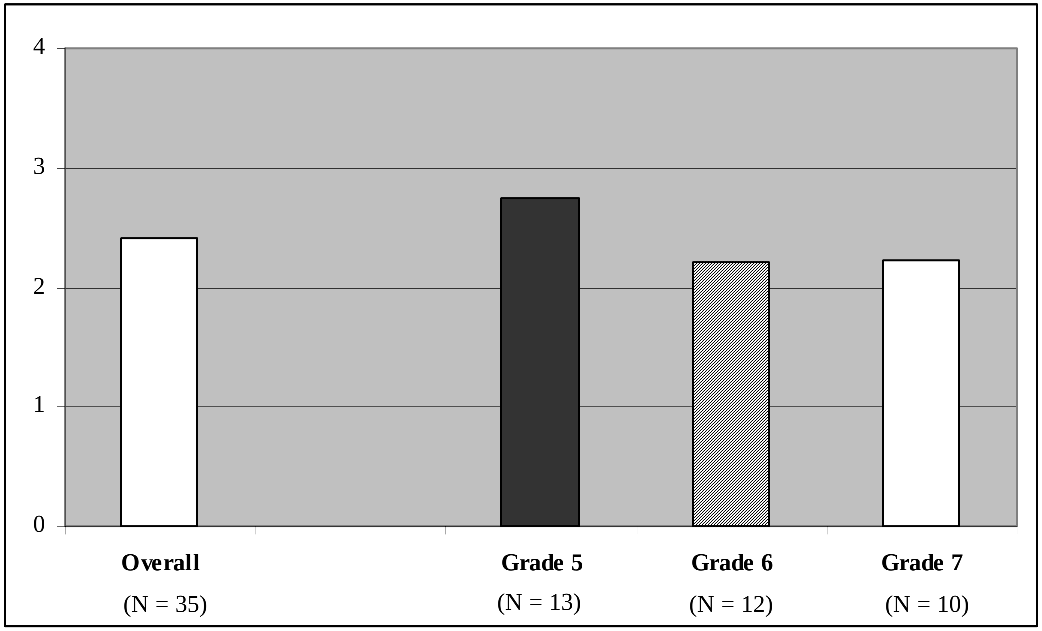


Figure D8. Teacher level of modification of curricular materials, by district: a) District 1and b) District 2; and by curriculum, 1997-1998.



Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.
4. The continual focus of the lesson whas on building connections or linking procedural knowledge with conceptual knowledge.
3. Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.
2. Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.
1. The lesson as presented did not promote conceptual knowledge.

Figure D9. Mean level for teachers of the development of conceptual understanding, overall and by grade, 1997-1998.

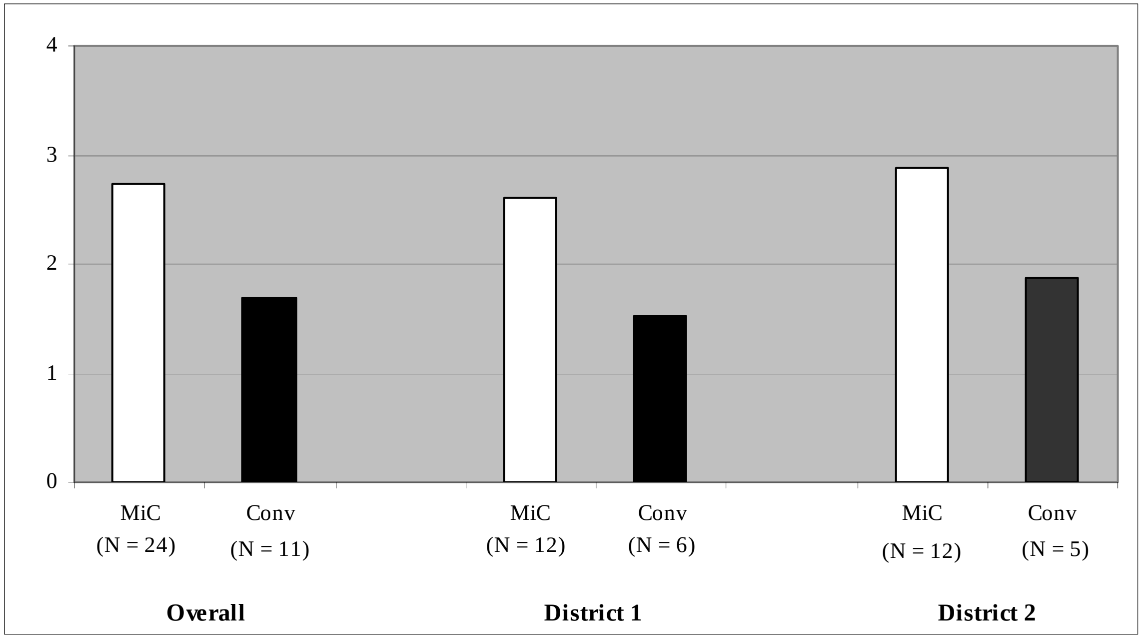
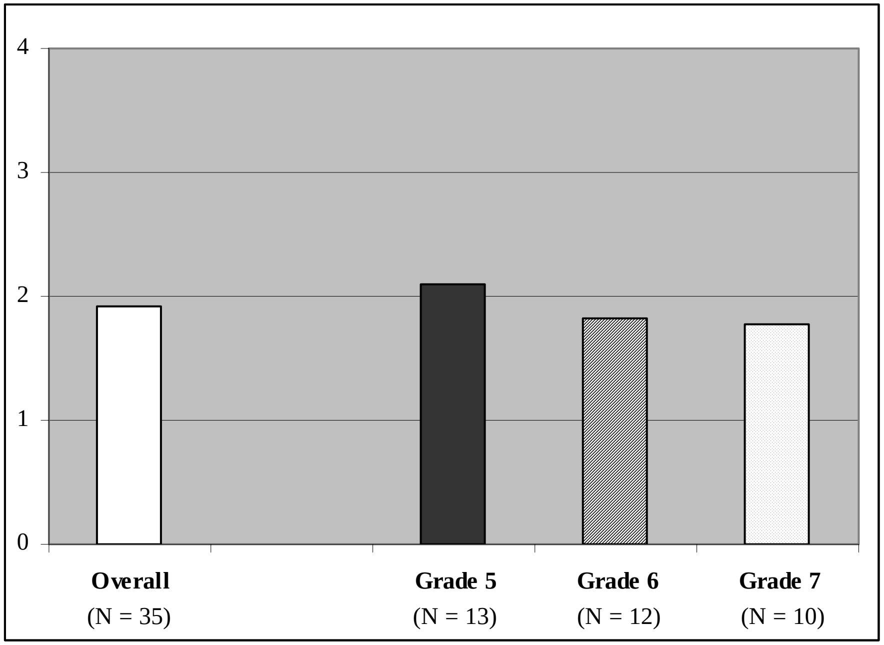


Figure D10. Mean level for teachers of the development of conceptual understanding, by curriculum and by district, 1997-1998



Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas.

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Figure D11. Mean level for teachers of the nature of student conjectures, overall and by grade, 1997-1998.

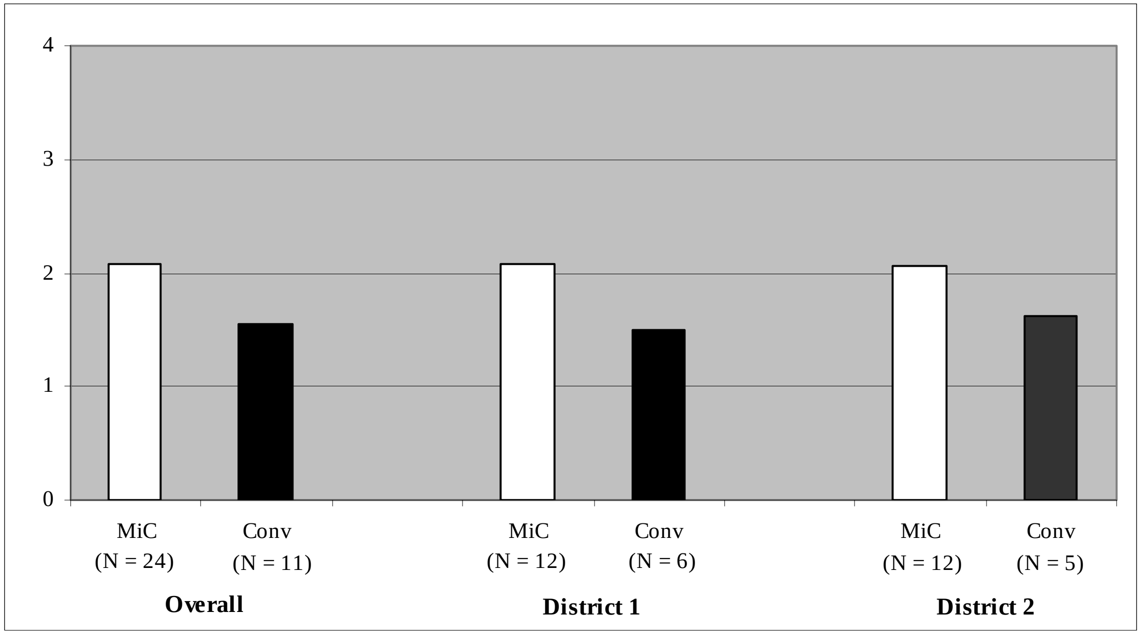
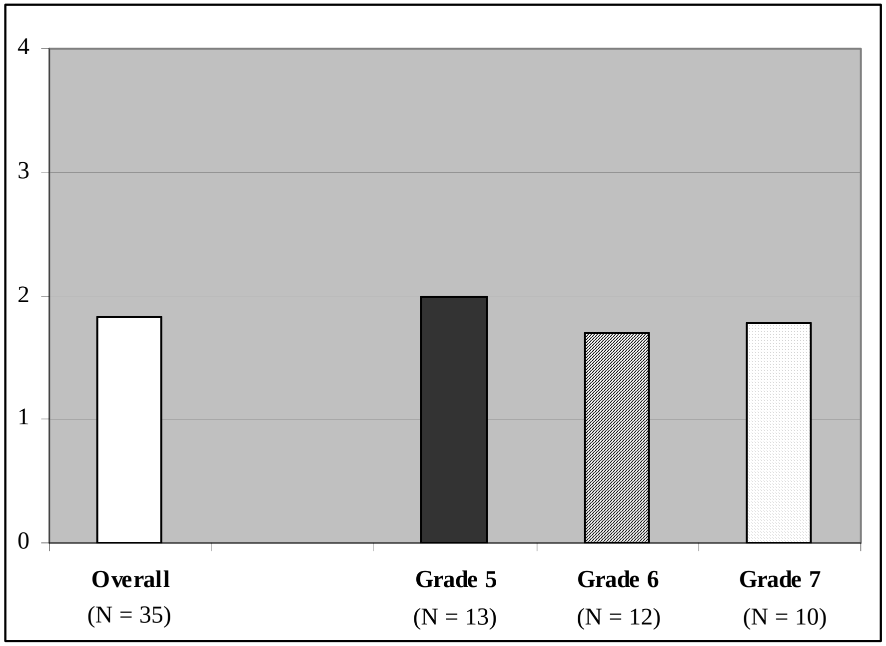


Figure D12. Mean level for teachers of the nature of student conjectures, by curriculum and by district, 1997-1998



Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Figure D13. Mean level for teachers of the nature of mathematical connections, overall and by grade, 1997-1998.

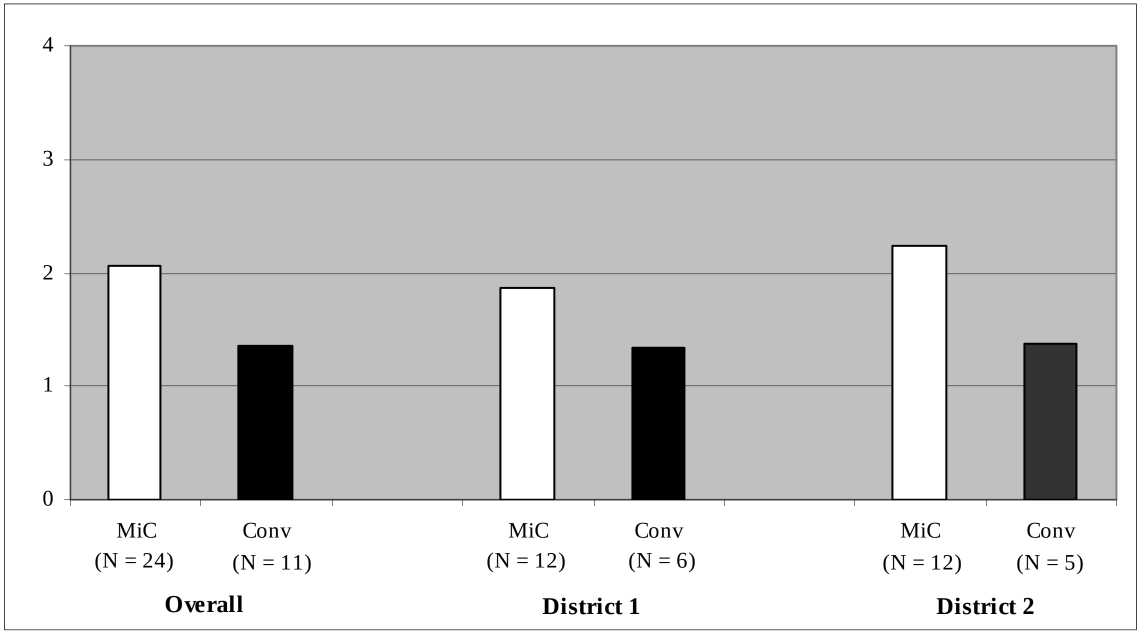
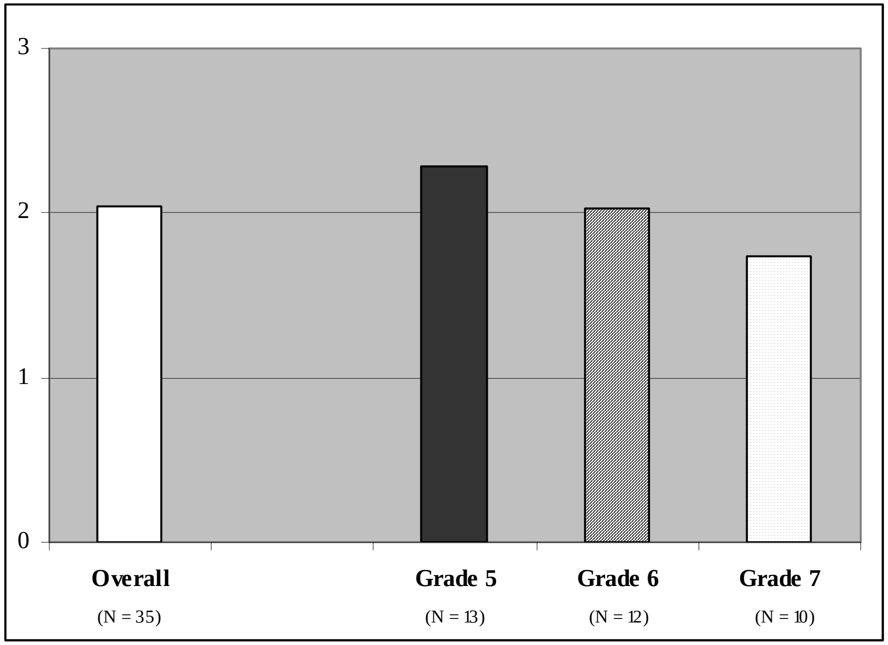


Figure D14. Mean level for teachers of the nature of student mathematical connections, by curriculum and by district, 1997-1998



Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Figure D15. Mean level for teachers of connections to life experiences, overall and by grade, 1997-1998.

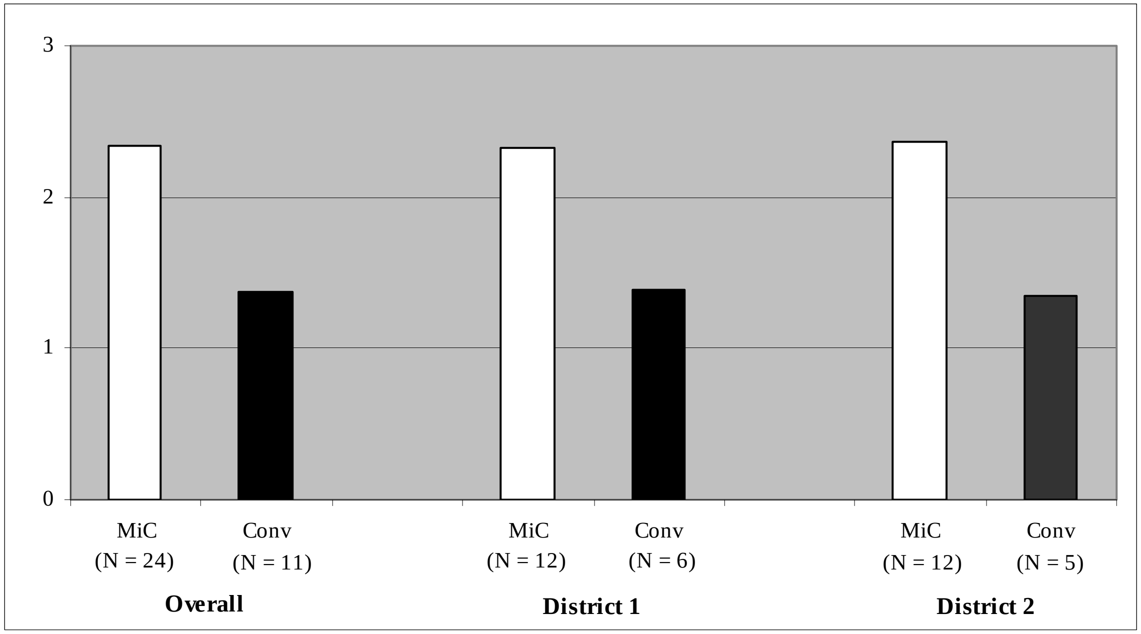
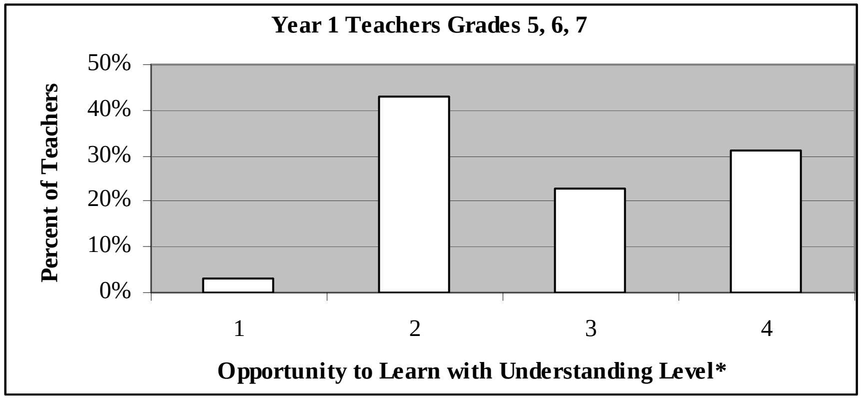


Figure D16. Mean level for teachers of connections to life experiences, by curriculum and by district, 1997-1998



Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 *High Level of Opportunity to Learn with Understanding:* Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 *Moderate Level of Opportunity to Learn with Understanding:* Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 *Limited Opportunity to Learn with Understanding:* Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 *Low Level of Opportunity to Learn with Understanding:* Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Figure D17. Teacher level of opportunity to learn with understanding, 1997-1998

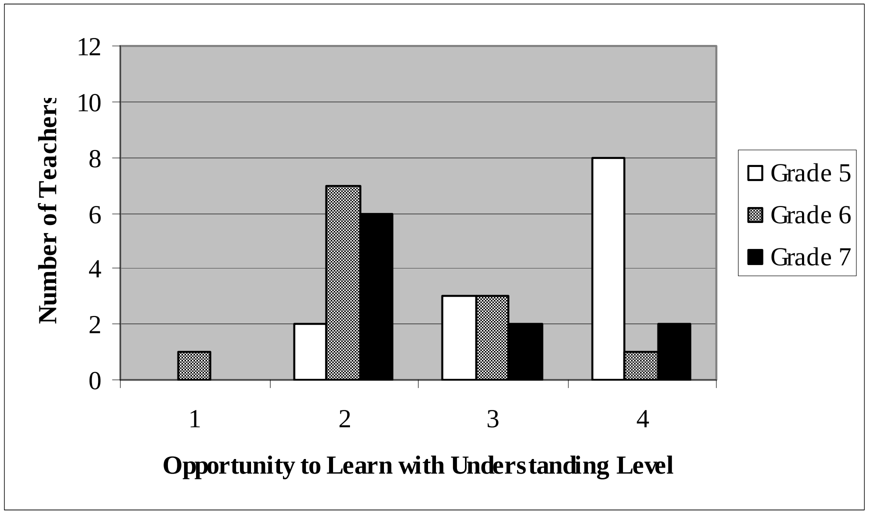


Figure D18. Teacher level of opportunity to learn with understanding, by grade, 1997-1998.

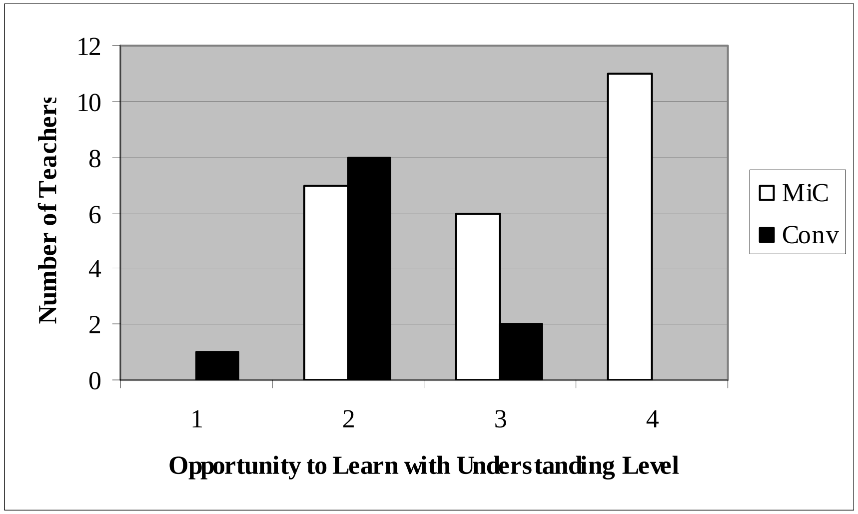


Figure D19. Teacher level opportunity to learn with understanding, by curriculum, 1997-1998.

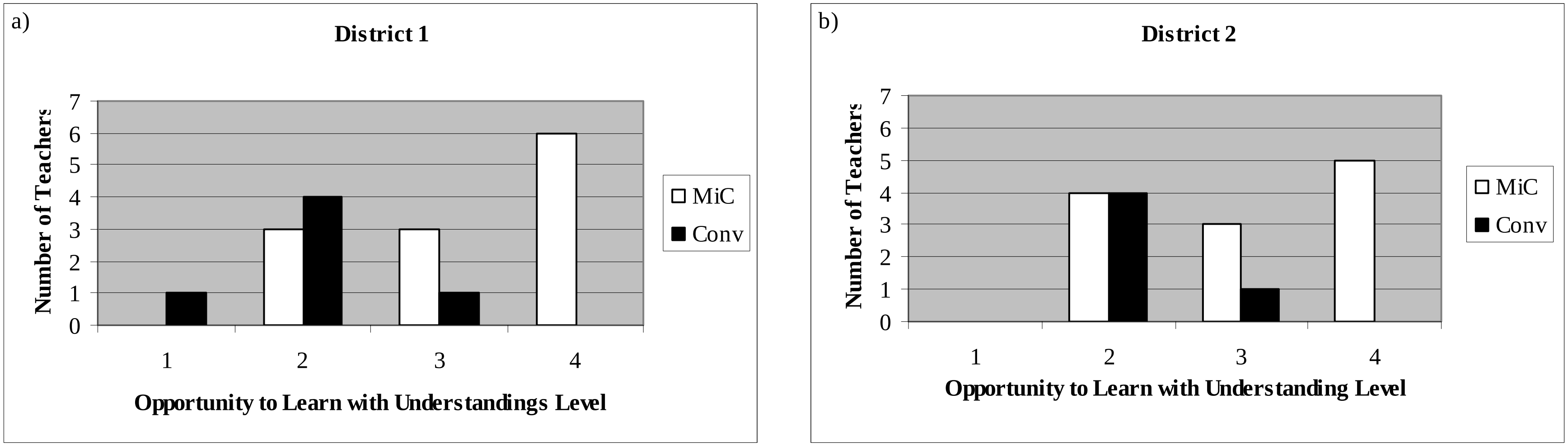


Figure D20. Teacher level opportunity to learn with understanding, by district: a) District 1 and b) District 2; and by curriculum, 1997-1998.

Appendix E

Methodology for the Composite Index Opportunity to Learn with Understanding

For analytical purposes, although the variation in each set of variables could be examined using structural equations, the number of classes at a given grade level is small, and collinearity across variables poses a serious interpretation problem. For this reason, a simplified research function was developed to make both the cross-sectional and longitudinal comparisons. Variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, can be attributed to variations in opportunity to learn with understanding (OTL_u), preceding achievement (PA), and method of instruction (I). This relationship can be expressed as—

$$CA = OTL_u + PA + I.$$

However, as the research staff worked with the data, a decision was made to distinguish school capacity (SC) from $OTL_{u.}$ Thus, the research function can be expressed as—

$$CA = SC + OTL_u + PA + I.$$

Each of these composite indices is being specified from the variables in the original model. This paper details the analysis of the opportunity to learn with understanding variable.

The Composite Variable for Opportunity to Learn with Understanding

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding (see Figure 1). The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. An index was created for the categories of curricular content and modification of curricular materials, and an index was created for each dimension of teaching for understanding.

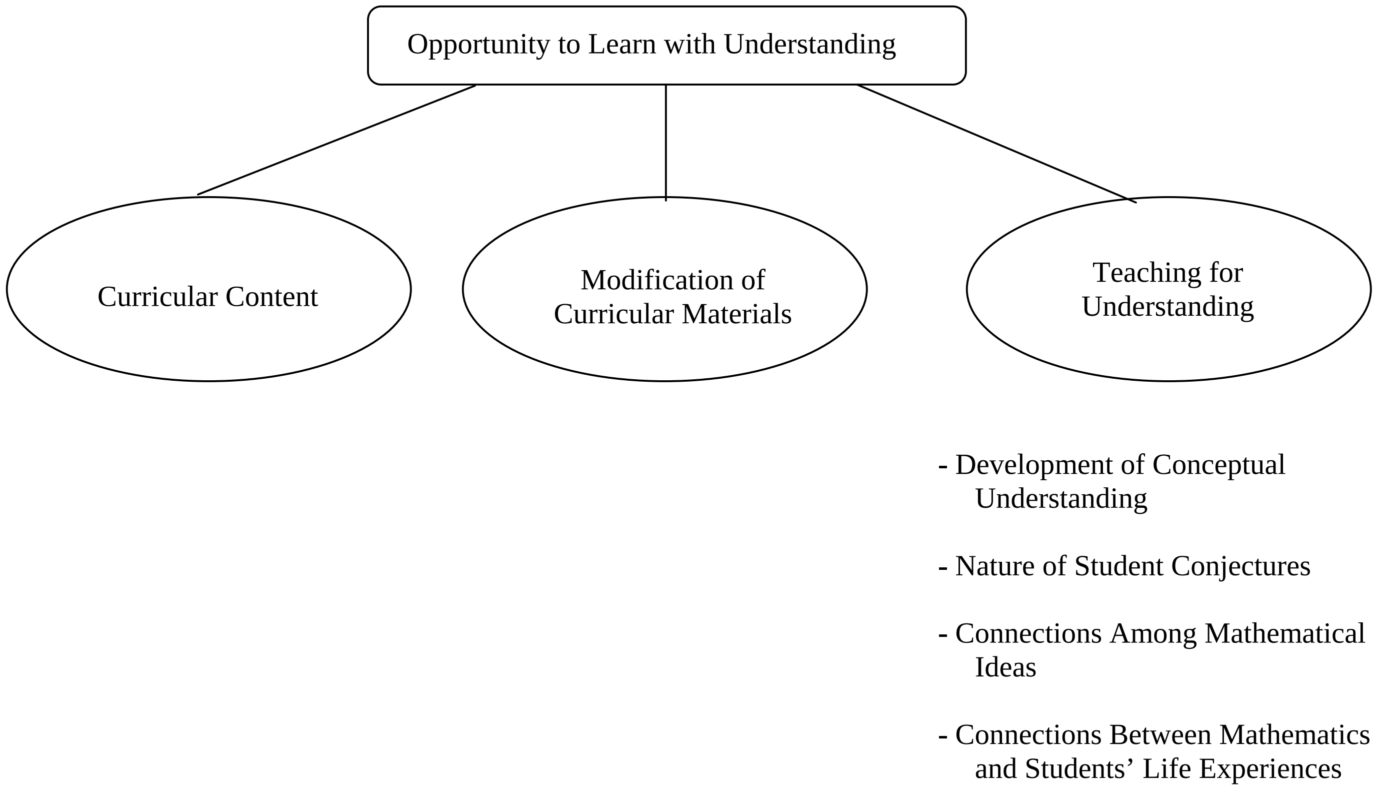


Figure 1. Major dimensions of opportunity to learn with understanding.

Sample

Districts 1 and 2 agreed to participate in a comparative research design which included students who used MiC and students who used conventional curricula. Data collection included teacher interviews, classroom observations, and teaching logs. Analysis focused on all five categories of instruction. All MiC teachers used commercial versions of the units, which became available during the summer before the study began. Teachers using conventional curricula used whatever curriculum was already in place in the schools. Analysis of opportunity to learn with understanding was not conducted for Districts 3 and 4 because classroom observation and teaching log data were not gathered in those districts.

In District 1, 6 fifth-grade study classes were in self-contained elementary classrooms. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. District 1 was located in an urban region in the eastern part of the country. The district had a 45% minority student population with 30% African American students and 12% Hispanic students. Approximately 30–40% of the students in the district were eligible for government-funded lunch programs. Professional development to acquaint teachers of mathematics with reform-based curricula was offered in District 1, and monthly meetings were provided for teachers who were implementing such programs. For preliminary teacher certification, 24 credit hours were recommended for fifth- and sixth-grade teachers; 24 credit hours were required for seventh- and eighth-grade teachers. No specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements.

In District 2, 2 of the 9 fifth-grade study classes were in self-contained settings in elementary schools. The remaining fifth-grade study classes, also in elementary schools, and all middle-school study classes had several subject-matter teachers. District 2, located in a large urban area in the southeastern United States, contained 251 elementary and middle schools and numerous high schools. The district student population was predominantly minority, with 33% African American students and 52% Hispanic students. Over 50% of the students in the district were eligible for government-funded lunch programs. District 2 provided numerous possibilities for professional development. Each school was given six early-release days for general professional development. In addition, each school received 10 substitute days for professional development in mathematics and/or science, 12–18 days of in-service days in mathematics provided by (USI or Eisenhower) government funding (each involving 2–6 teachers), and 3–5 days of districtwide mathematics in-service. Teachers also had opportunities to participate in five days of paid in-service for mathematics during the summer. District requirements for preparation of mathematics teachers were the same as state requirements. For elementary teachers, preliminary teacher requirements mandated the study of arithmetic for the elementary school. For middle-grade mathematics certification (Grades 5–9), 18 semester hours in mathematics were required; for certification in mathematics for Grades 6–12, 30 semester hours in mathematics were required. Continuing certification required the completion of six semester hours in mathematics or 120 district in-service credits in mathematics every 5 years.

District 3 was located in a suburban area in a large western state. In the district’s two elementary and one middle school, 73 teachers taught 1480 students. In District 3, the six self-contained 5th grade study classes are in a school for Grades 3–5; Grades 6-8 were housed in a middle school. Three of the middle school study classes were in self-contained classrooms. The remaining middle-school study classes had several subject-matter teachers. Study participants included all fifth-, sixth-, and seventh-grade mathematics classes in the district. The schools’ student and teacher populations were predominately White. Approximately 10–20% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. School administrators provided paid monthly meetings for mathematics teachers who were implementing standards-based curricula for the first time. Teachers often met weekly without pay to prepare mathematics lessons. For preliminary teacher certification, the state mandated single-subject credentials for Grades 7–8. Teachers for Grades K–6 were required to complete a multiple-subjects credential including several mathematics courses. Although the district provided mathematics courses and staff development opportunities, it did not require additional certification and courses for experienced teachers.

District 4 was one of many districts located in a large urban area in the eastern part of the United States. The district's 1075 teachers were responsible for teaching the 20,000 students in 23 elementary schools, seven middle schools, and several high schools. In District 4, Grades 6–8 are contained in middle schools in which students have several subject-matter teachers. Study participants are from one middle school in this district. Because fifth-grade students in District 4 are dispersed among several middle schools, fifth-grade classes were not included in the data collection for District 4. Four sixth-grade and six seventh-grade classes from one middle school participated in the study. The student population was predominately minority with 64% African American students and 28% Hispanic students. Approximately 37% of the teacher population was minority with 31% African American teachers and 6% Hispanic teachers. More than 50% of the students were eligible for government-funded lunch programs. Fewer than 20% of the students had learned English as a second language. For new mathematics teachers, 36 credits in mathematics or a mathematics major were required by the state, but no specific mathematics requirements were necessary as part of continuing education. District requirements were the same as the state requirements. Professional development opportunities were provided to all mathematics teachers at both district and school levels, including personalized discussions with the assistant principal for mathematics and science. These discussions focused on reform recommendations in curriculum, instruction, and assessment; research in mathematics education; and applications of research in classroom practice.

Data Collection

Data were collected through the use of instruments designed to examine content taught, modifications to curricular materials, and teaching mathematics for understanding in the longitudinal/cross-sectional study. Information on content taught was gathered through the Teacher Log (see Appendix B; Shafer, Wagner, & Davis, 1997). Each teacher in Districts 1 and 2 was asked to complete a daily teaching log and weekly journal entries. Three sections of the log are pertinent to data collection on opportunity to learn with understanding: lessons taught, additional materials used during instruction, and homework assignments. In the Introductory Information for the log, teachers documented the unit/chapter taught. On each daily sheet, after noting the date, unit/chapter and pages taught on a particular day, teachers indicated if the lesson was a continuation of the previous lesson. If the lesson was continued, teachers were asked to indicate activities that were new to the current lesson. For Item 2, teachers checked whether all students in the class covered the same content. If they did not, they described the ways the content differed and the reasons for these differences. For Item 6, teachers checked the additional materials used during the lesson: teacher-designed materials, work from text resource materials, work from other resources, quiz, calculators, or another resource specified by the teacher. Teachers were asked to date and attach teacher-designed materials, worksheets from other resources, and quizzes to the daily log page. In Item 8, teachers checked the type of homework assignment, if given: exercises form the text, completion of work begun in class, teacher-designed work, work from text resource materials, exercises from another text, supplementary practice, investigation or project, or other assignment specified by the teacher. After the first semester of the study, Item 8 was revised to be less time-intensive for teachers. In the original log, teachers were asked to list the pages and exercise numbers for text assignments and to attach exercises from supplemental resources and investigations or projects. For the revised item, a more inclusive checklist was used (adding teacher-designed materials, work from text materials, and supplemental practice). Teachers were asked to briefly describe the content of teacher-designed and supplemental practice in lieu of attaching copies of such materials, and listing exercise numbers was eliminated. Information on content taught for MiC teachers was also gathered through Teacher Questionnaire: Professional Opportunities, which was completed in the spring of each study year. Item 13 asked teachers to circle the MiC units they taught during the current school year.

The first of two journal question focused on parts of the lesson that were emphasized and modifications made in the lesson from its presentation in the unit/chapter taught. Suggestions for reflection were: particular items or aspects of the lesson emphasized (or deleted) and the reasons for the emphasis (or deletion); additional activities, exercises, or procedures included and the reasons for adding them; and changes in the order of the lessons as compared to the order presented in the unit/chapter.

Information on teaching for understanding was gathered through the Classroom Observation Instrument (see Appendix C; Davis, Wagner, & Shafer, 1998). Classroom observations were conducted on each study teacher in Districts 1 and 2. Fewer observations were conducted in District 2 due to differences in school schedules, procedures for assigning students to classes, and preparation for district and state standardized testing. The observers (one each from Districts 1 and 2) were retired teachers with many years of experience teaching mathematics and were selected with district input. The research staff worked with both observers to establish interrater reliability. Completed observation reports were sent to the research center monthly.

Regarding opportunity to learn, the observation instrument documented the unit/chapter and pages numbers of the lesson; lesson content and goals; and time allotted to various instructional activities. The observation instrument also contained twelve indices sections, four of which provided data about teaching for understanding: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students’ life experiences. Ratings given by observers were checked by research staff for accuracy and were changed when appropriate. For example, on a few occasions, when the evidence provided by the observer strongly indicated another rating than the one circled, the rating was changed to reflect the evidence.

Indices

The indices used to characterize each dimension of instruction were based on levels of authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993). Several levels for each index were preliminarily defined by describing each aspect of instruction and identifying differences between conventional approaches to teaching learning mathematics and approaches that were aligned with the NCTM *Standards* (1989, 1991, 1995), authentic instruction (Newmann, Secada, & Wehlage, 1995), and teaching mathematics for understanding (Carpenter & Lehrer, 1999). Further distinctions in the levels were identified through (a) review of literature that was specific to each dimension (see Appendix F for a summary of the relevant literature) and (b) review of the teacher and observer data based on Strauss' (1987) system of open, axial, and selective coding, which involved repeated coding of the data for interpretive codes. These codes included both external codes identified prior to reviewing the data (e.g., few mathematical topics were covered) and internal codes that emerged from the data (e.g., slow pacing resulted in coverage of only a few topics). Categories were further refined, and additional categories were integrated for each index as data for fifth-grade study teachers in one district were reviewed. Indices were further revised during review of data from sixth- and seventh-grade teachers and teachers in other districts. As a result, three to six levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts. Indices included in the observation instrument were refined as a consequence of observing nonstudy teachers who used MiC or conventional curricula during the year prior to the study. Before these indices were used in study classrooms, district and school administrators, on-site observers, and curriculum specialists from anticipated research sites used and commented on a draft of the observation instrument in classrooms currently implementing MiC. As a result, project staff refined descriptions and clarified ratings for the final version of the index for each dimension. In order to maintain interrater reliability between the observers in each district and consistency in rating across all three years of the longitudinal study, these indices were not changed after review of data from study teachers.

The levels in each index are positioned along a continuum from the least appearance of a given characteristic to the most sophisticated implementation of the dimension being scaled. For example, levels of curricular content range from breadth in content with little or no depth to a curriculum in which the teacher nurtured both breadth and depth in content. One level in the some indices was subdivided further in order to more adequately describe the variation among teachers. For example, sublevels in one level of the index for curricular content, few mathematical topics were covered, categorized situations in which the teacher lingered over content until students demonstrated mastery and situations in which the teacher used a new curriculum and slow pacing resulted in coverage of only a few topics. In the remainder of this section, the indices for each dimension of opportunity to learn with understanding, organized by major category, are described.

Curricular Content

As it has been historically conceived, OTL has examined the exposure to content topics that students had received. OTL(u) also characterizes the content taught in terms of the number of units or chapters taught and the specific content presented. For example, some teachers may choose to emphasize particular strands of mathematics, such as number. Other teachers may more equitably teach topics from number, algebra, geometry, probability, and statistics. For curricular content, five levels were identified based on differences in the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts. The levels for curricular content are listed in the following index:

5. *Breadth and depth in content*. The teacher presented a comprehensive, integrated curriculum with attention to all content strands.
4. *Concentration on only one or two mathematical strands*. The teacher taught mathematical concepts in depth but restricted content presented primarily to one or two content strand (e.g., number and algebra).
3. *Few mathematical topics were covered*.
    a. The teacher lingered over content until students demonstrated mastery.
    b. The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.
2. *Combination of curricular approaches*. The teacher presented a combination of conventional and reform curricula, which resulted in a dual

emphasis on basic skills and some conceptual content.

1. *Breadth in content with little or no depth.* The curricular content covered spanned a vast content plane. This content, however, was presented as disparate pieces of knowledge and was heavily laden with vocabulary and prescribed algorithms. This category can be characterized as “a mile wide and an inch deep.”

Modification of Curricular Materials

While examining the content students have experienced is important, a simple inventory of the content covered during lessons or over the course of a year does not provide insight into the opportunities students have to learn that content with understanding (e.g., to articulate their thinking, reflect on or summarize mathematical ideas, extend and apply concepts, and make mathematics one’s own [Carpenter & Lehrer, 1999]). Although MiC is designed to provide students with the opportunity to develop understanding of important conceptual ideas, the mere use of the curriculum does not promise that it will be implemented as its designers intended. Teachers might include modifications such as supplemental materials to help students make connections between the mathematics in the text and their own lives, to practice prescribed algorithms in conjunction with the text, or to use the text as a supplement to a curriculum in which drilling basic skills is the primary focus. Modifications to the curriculum can, therefore, strengthen or undermine the intent of the lessons. Consequently, students’ opportunities to learn the content with understanding may be enhanced or compromised. Thus, it is crucial to investigate and characterize the modifications teachers make to the available curriculum.

For modifications of curricular materials, six levels were identified based on the extent to which modifications supported the development of deep understanding of the concepts covered. The levels for modifications of curricular materials are listed in the following index:

6. *Modifications that enhanced conceptual development*.
   a. The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.
   b. The teacher supplements the text with tasks or multiple models that emphasized connections among concepts and connections to the students’ lives.
5. *Curriculum followed faithfully*.
   a. The teacher occasionally supplemented the curriculum with activities disconnected from the text.
   b. The teacher presented the curriculum as it was written with few, if any, modifications.
4. *Primary curriculum supplemented with materials not aligned with intent of curriculum*. The teacher used materials that undermined the philosophy of the curriculum (e.g., added skill-and-drill worksheets to a reform-based curriculum).
3. *Focus lacking*. Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.
2. *Disorganized treatment of content*. The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.
1. *Retreat from a reform-based curriculum toward a more traditional curriculum*.
   a. The teacher supplemented a reform-based curriculum with conventional materials to the extent that the supplementary materials subsumed the reform-based curriculum.
   b. The teacher abandoned a reform-based curriculum in favor of a conventional curriculum.

Teaching for Understanding

Development of conceptual understanding. This index measures the extent to which the lesson fostered the development of conceptual understanding.

4. The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.
3. Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.
2. Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.
1. The lesson as presented did not promote conceptual understanding.

The nature of student conjectures. This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas.

4. Students made generalizations about mathematical ideas.
3. Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.
2. Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.
1. No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Connections within mathematics. This index measured the extent to which connections within mathematics were explored in the lesson.

4. The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.
3. Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.
2. The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.
1. The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Connections between mathematics and students' life experiences. This index measured the extent to which connections between mathematics and students' daily lives were apparent in the lesson.

3. Connections between mathematics and students' daily lives were clearly apparent in the lesson.
2. Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.
1. Connections between mathematics and students' daily lives were not apparent in the lesson.

The Composite Index for Opportunity to Learn with Understanding

The sum of the levels from the six indices for opportunity to learn with understanding was calculated for each teacher. The sum of the levels from each of the indices was calculated for each teacher. Cluster analysis, which tests the similarity, between two objects, permitted the classification of teachers. Cluster analysis was beneficial for two reasons. First, because it considers the distance between each teacher's grand total means, cluster analysis lessened whatever distortion that might have been caused by relying solely on researchers' intuition. Second, because we looked at the means in every category, the categorization of the teachers is more precise. As a result, four levels were identified to capture the variation in opportunity to learn with understanding among teachers at different grade levels and in different districts. A description of each level in the composite index was created from a review of the teachers at each level. The composite index for opportunity to learn with understanding illustrates such descriptions:

Level 4: High Level of Opportunity to Learn with Understanding

At Level 4, teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few, if any, modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

Level 3: Moderate Level of Opportunity to Learn with Understanding

At Level 3, teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although in the lesson connections between mathematics and students' daily lives were implicit, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

Level 2: Limited Opportunity to Learn with Understanding

At Level 2, teachers covered only a few topics. Because many MiC teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both MiC teachers and teachers using conventional curricula, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

Level 1: Low Level of Opportunity to Learn with Understanding

At Level 1, teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Predictors of Opportunity to Learn with Understanding

The opportunity to learn with understanding composite variable (OTL_u) conceptualized for the longitudinal/cross-sectional study includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences. To examine the relationships between each of the categories and dimensions, a correlation matrix was calculated. In Year 1, the results suggest that one category and four dimensions—curricular content, conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences—were strongly correlated with each other (see Table 1).

Table 3
Correlation of the Categories and Dimensions of the Opportunity to Learn with Understanding Composite Variable

Dimension	Curricular Content	Modification of Curricular Materials	Teaching for Understanding			
			Conceptual Understanding	Conjectures	Connections within Mathematics	Connections to Students' Daily Lives
Curricular Content	1.000					
Modification of Curricular Materials	0.351*	1.000				
Conceptual Understanding	0.741**	0.233	1.000			
Conjectures	0.626**	0.186	0.711**	1.000		
Connections within Mathematics	0.542**	0.072	0.808**	0.647**	1.000	
Connections to Daily Lives	0.770**	0.120	0.672**	0.625**	0.671**	1.000

*p<.05
**p<.01

Regression analysis was used to compare the relative influence of the categories and dimensions of opportunity to learn with understanding, which were treated as individual variables in the analysis. The results suggest that the most important predictor of the OTL_u composite is curricular content. Modification of curricular materials and development of conceptual understanding were the next most important predictors of the OTL_u

composite. Along with curricular content, these dimensions explained 70% of the variance of the OTL_u composite. These results confirm critical dimensions of OTL_u for classrooms in which MiC or conventional curricular materials are used.

In Year 2, the results suggest that the four dimensions—development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences—were strongly correlated with the OTL_u composite and with each other (see Table 2).

Table 2
Correlation of the Categories and Dimensions of the Opportunity to Learn with Understanding Composite Variable

Dimension	Curricular Content	Modification of Curricular Materials	Teaching for Understanding			
			Conceptual Understanding	Conjectures	Connections within Mathematics	Connections to Students' Daily Lives
Curricular Content	1.000					
Modification of Curricular Materials	0.216	1.000				
Conceptual Understanding	0.620**	0.105	1.000			
Conjectures	0.613**	0.279	0.805**	1.000		
Connections within Mathematics	0.606**	0.109	0.869**	0.811**	1.000	
Connections to Students' Daily Lives	0.594**	-0.105	0.777**	0.753**	0.689**	1.000

*p<.05
**p<.01

The results of regression analysis suggest that the most important predictor of the OTL_u composite is curricular content. Development of conceptual understanding and modification of curricular materials were the next most important predictors of the OTL_u composite. Along with curricular content, these dimensions explained 67% of the variance of the OTL_u composite. These results confirm critical dimensions of OTL_u for classrooms in which MiC or conventional curricular materials are used.

In Year 3, the results suggest that the dimension development of conceptual understanding was well correlated with the two categories and the other three dimensions (see Table 3).

Table 3
Correlation of the Categories and Dimensions of the Opportunity to Learn with Understanding Composite Variable

Dimension	Curricular Content	Modification of Curricular Materials	Teaching for Understanding			
			Conceptual Understanding	Conjectures	Connections within Mathematics	Connections to Students' Daily Lives
Curricular Content	1.000					
Modification of Curricular Materials	0.574**	1.000				
Conceptual Understanding	0.505**	0.274	1.000			
Conjectures	0.396	0.272	0.886**	1.000		
Connections within Mathematics	0.450	0.192	0.870**	0.835**	1.000	
Connections to Students' Daily Lives	0.160	0.011	0.557*	0.363	0.569*	1.000

*p<.05
**p<.01

The results of the regression analysis suggest that the most important predictor of the OTL_u composite is curricular content. Modification of curricular materials and development of conceptual understanding were the next most important predictors of the OTL_u composite. Along with curricular content, these dimensions explained 69% of the variance of the OTL_u composite. These results confirm critical dimensions of OTL_u for classrooms in which MiC or conventional curricular materials are used.

Conclusion

The results suggest that the MiC materials affected students' opportunity to learn mathematics with understanding. The curricular content is comprehensive, with its attention to geometry, algebra, and statistics in addition to number, and is rich in developing connections among mathematical ideas. Contexts in which lessons are situated provide a basis for exploring mathematical ideas and applying mathematics in daily life experiences.

Further research will examine changes in levels of the OTL_u composite index for the same teachers over time as well as the factors that might influence those changes. Further research might involve using the composite index with in-service and pre-service teachers as a reflection tool on students' opportunity to learn comprehensive mathematics content with conceptual understanding.

In a study of the impact of any standards-based curriculum, analysis of classroom OTL_u and the factors that influence OTL_u are important considerations in students' achievement. The findings suggest that variation in students' opportunity to learn mathematics with understanding must be accounted for in the interpretation of the impact of the curriculum on student achievement.

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Appendix F

Theoretical Framework for Opportunity to Learn with Understanding

The research function used to make both the cross-sectional and longitudinal comparisons attributes variation in classroom achievement (CA), aggregated by content strand, level of reasoning, or total performance, to variations in opportunity to learn with understanding (OTL_u), school capacity (SC), preceding achievement (PA), and method of instruction (I). This relationship can be expressed as—

$$CA = OTL_u + SC + PA + I.$$

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Theoretical Framework

Historically, the concept of opportunity to learn (OTL) was originally described by Carroll (1963) in his model of school learning. In international assessments of student achievement, OTL was operationalized as a measure to determine whether content had been taught to students prior to assessment for mastery of that content (McKnight et al., 1987). It was used as an explanatory variable that took into consideration differences in curriculum and content coverage across international samples of students. Gau (1997) expanded the construct of OTL to include conditions established in the educational system: teachers' mathematical knowledge (mathematics degrees and professional development activities), content and level of instruction (achievement group, content coverage, amount of instructional time and homework), and availability of educational resources (such as calculators) and extra curricular opportunities. Porter (1991) described OTL as the enacted curriculum experienced by students. He argued that OTL involved more than the content of instruction (concepts, skills, and applications taught) and included the quality of instruction students experienced. Assessing OTL for high school students, Porter and colleagues considered modes of instruction, such as whether instruction involved exposition or field work, and skills such as memorization, understanding concepts, solving routine problems, interpreting data, performing procedures, and developing proofs (Porter, Kirst, Osthaoff, Smithson, & Schneider, 1993). Hiebert (1999) noted that in the Third International Mathematics and Science Study (TIMSS), instruction for U. S. students predominately emphasized computational procedures, and conceptual understanding was given little attention. The mathematics curriculum provided "few opportunities for students to solve challenging problems and to engage in mathematical reasoning, communicating, conjecturing, justifying, and proving" (p. 11). Furthermore, the results suggest that students learned what they had the opportunity to learn—simple calculation, terms, and definitions—rather than solving nonroutine problems and using mathematical processes.

In studying the impact of standards-based curricula, a student's understanding, not merely his or her ability to apply memorized facts and equations, receives substantial emphasis. In simplified research function for the *Mathematics in Context* longitudinal study, OTL has been interpreted more broadly than as a mere gauge of content coverage and is viewed as a student's opportunity to learn mathematics with understanding (OTL_u). When learning mathematics with understanding, students need the time and opportunity to develop relationships among mathematical ideas, extend and apply these ideas in new situations, reflect on and articulate their thinking, and make mathematical knowledge their own (Carpenter & Lehrer, 1999). Research in a growing number of studies underscores the importance of teaching mathematics for understanding, which is based on the principles that knowledge is constructed by the learner and is situated in the context of the learner's existing knowledge, skills, and beliefs; that the teacher's role is a guide for facilitating conceptual understanding; that mathematical tasks are nonroutine, accessible to all students, and engage students' thinking about important mathematics; that classrooms are communities of learners; and that mathematical tools are supports for learning (Cohen, McLaughlin, & Talbert, 1993; Fennema & Romberg, 1999; Hiebert et al., 1997). In contrast, when the aim of the lesson is primarily coverage of content, the emphasis is often on unconnected pieces of information and on the practice of repetitive procedures or heuristics determined by others (Battista, 1999). Such situations reduce cognitive demands on students. Teaching for understanding must support students in their efforts to make connections between mathematical concepts as well as between mathematical concepts and situations they encounter outside of the classroom, apply these concepts in novel situations, reflect on and communicate their thinking, and make new knowledge their own.

As conceived in this study, OTL_u consists of three overarching categories: curricular content, modification of curricular materials, and teaching for understanding. The teaching for understanding category of OTLu attempts to capture how well a lesson enables students to pursue these goals. In our model, teaching for understanding consists of four dimensions: the extent to which the lesson fosters the development of conceptual understanding, opportunities the lesson provides for students to make conjectures about mathematical ideas, opportunities for connections within mathematics to be explored in the lesson, and opportunities in the lesson for students to forge connections between mathematics and their daily lives.

Curricular Content

The category curricular content stemmed from traditional studies of OTL and examines the exposure to content topics that students have received. It remains critical to examine content, as all students should have access to varied and challenging content that will allow them to achieve at high levels and ultimately not restrict career options (O’Day & Smith, 1993). As such, OTLu also characterizes the content taught in terms of the number of units or chapters taught and the specific content presented. For example, some teachers may choose to emphasize particular strands of mathematics, such as number. Other teachers may more equitably teach topics from number, algebra, geometry, probability, and statistics.

Modification of Curricular Materials

Although examining the content students have experienced is important, a simple inventory of the content covered during lessons or over the course of a year does not provide insight into the opportunities students have to learn that content with understanding (e.g., to articulate their thinking, reflect on or summarize mathematical ideas, extend and apply concepts, and mathematize problem contexts [Carpenter & Lehrer, 1999]). Although *Mathematics in Context* is designed to provide students with the opportunity to develop understanding of important conceptual ideas, the mere use of the curriculum does not promise that it will be implemented as its designers intended. Teachers might include modifications such as supplemental materials to help students make connections between the mathematics in the text and their own lives, to practice prescribed algorithms in conjunction with the text, or to use the text as a supplement to a curriculum in which drilling basic skills is the primary focus. Modifications to the curriculum can, therefore, strengthen or undermine the intent of the lessons and potentially students’ opportunities to learn the content with understanding may be enhanced or compromised. Thus, it is crucial to investigate and characterize the modifications teachers make to the available curriculum.

Teaching Mathematics for Understanding

Although content coverage is an important indicator of students' OTL, it is not sufficient to determine the opportunity a student has to learn that content with understanding. Four dimensions characterize teaching for understanding in this study: the extent to which the lesson fosters the development of conceptual understanding, opportunities the lesson provides for students to make conjectures about mathematical ideas, opportunities for connections within mathematics to be explored in the lesson, and opportunities in the lesson for students to forge connections between mathematics and their daily lives.

The development of conceptual understanding. Conceptual knowledge is described as the "facts and properties of mathematics that are recognized as being related in some way" (Hiebert & Wearne, 1986, p. 200), or as a network of relationships that link pieces of knowledge (Hiebert & Lefevre, 1986). In the primary grades, for example, students learn the labels for whole-number place-value positions. If this information is stored as isolated pieces of information, the knowledge is not conceptual. If this knowledge, however, is linked with other information about numbers, such as grouping objects into sets of ten or counting by tens or hundreds, then the information becomes conceptual knowledge. The network of relationships about place value grows as other pieces of knowledge related to place value, such as regrouping in subtraction, are recognized. Procedural knowledge, in contrast, is described as having two parts. One category comprises the written mathematical symbols, which are devoid of meaning and are acted upon through knowledge of the syntax of the system. A second category is composed of rules and algorithms for solving mathematics problems, step-by-step procedures that progress from problem statement to solution in a predetermined order. Procedural knowledge is rich in rules and strategies for solving problems, but it is not rich in relationships (Hiebert & Wearne, 1986).

Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in text materials may direct students' attention to linking procedural and conceptual knowledge. In addition and subtraction of decimals, for example, lining up the decimal points should be linked with combining like quantities. Instruction might explicitly bring out the relationships between lining up the decimal point in addition and subtraction and lining up whole numbers on the right side for the same operations (Hiebert & Wearne, 1986). Instruction that fosters the development of conceptual understanding provides students with the opportunity to learn with understanding.

The nature of student conjectures. Conjectures about mathematical ideas help students recognize connections among topics, investigate patterns, and form generalizations about mathematical ideas that are applicable across content strands. There are three types of conjectures that students might make. One type of conjecture involves the student in making a guess about how to solve a particular problem based on experience solving problems with similar solution strategies. For example, students were solving problems in which they used properties of similar triangles. When asked to determine the height of a tree, students conjectured that an appropriate solution strategy would involve similar triangles. The students made a connection between the new problem and problems that they had previously solved. A second type of conjecture occurs when a student makes a guess about the truthfulness of a particular statement and subsequently plans and conducts an investigation to determine whether the statement is true or false. For example, a 12-year-old student disagreed with a statement that she was half as tall as she is now when she was 6-

years old, and proceeded to support her argument by comparing her present height with heights of 6-year-old children. A third type of conjecture is a generalization. A generalization is created by reasoning from specific cases of a particular event, is tested in specific cases, and is logically reasoned to be acceptable for all cases of the event. For example, given that a beam is constructed of rods in Figure 1, students are asked to describe the relation between the number of rods and the length of the beam (Wijers et al., 1998). Using a table to organize their reasoning, students described the pattern that emerged, explained how the pattern fit the given diagram, and generated formulas for the relationship. In this situation, students reasoned from specific cases, tested and supported their ideas with evidence from drawings and the table, and described the relation in a formula.

The length of the beam is the number of rods along the underside.

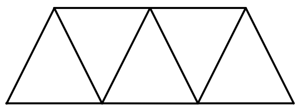

Figure 1. The beams context from *Building Formulas* (Wijers, et al., p. 10).

Connections within mathematics. Traditionally, mathematics has been taught as a series of discrete skills to be memorized rather than as a sense-making endeavor. Ideally, instruction addresses mathematical topics thoroughly enough to explore relationships and connections among them. When connections are drawn between and among mathematical ideas, students can link procedural and conceptual knowledge, recognize relationships between representations, and recognize the interconnectedness of mathematical topics (NCTM, 1989). Students look for and discuss relationships among mathematical ideas, express understanding of mathematical topics, or provide explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated (Newmann, Secada, & Wehlage, 1995). When these connections are promoted, students are provided with an opportunity to learn mathematics with understanding. Sometimes, however, the mathematical topic of the lesson is covered in ways that gave students only a surface treatment of its meaning, and instruction treated this topic in isolation of other mathematical topics.

Topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios, as in the following problem. Students are asked to determine the probability of a frog jumping from a cage and landing on white or black floor tiles and to express this probability as a fraction or percent (Jonker, et al., 1997). In solving this problem, students use area, number, and probability concepts. Second, connections can be made among more narrowly defined areas such as a lesson involving the solution of quadratic equations. In this lesson, connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. This dimension reflects both the problems and instruction.

Connections between mathematics and students' life experiences. This scale measures whether connections between mathematics and students' daily lives were apparent in text problems or discussed by the teacher or students. Examples of problems that foster such connections are estimating the sale price of an item or determining the amount of ingredients required to serve four people when a recipe serves seven. In contrast, word problems such as “Bart is two years older than Lisa. In five years Bart will be twice as old as Lisa. How old are they now?” are devoid of connections between mathematics and students’ lives. When connections are made between mathematics and students’ daily lives, they will see its usefulness and tend not to view mathematics as an academic pursuit confined within the four walls of the classroom. Rather, mathematics can be viewed as a problem solving endeavor in which informal procedures can be used efficiently, a subject that they should and can learn with understanding since it will be used as part of their daily lives.

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Appendix G

Opportunity to Learn with Understanding

GRADE 5

GRADE 5

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. An index was created for the categories of curricular content and modification of curricular materials, and an index was created for each dimension of teaching for understanding. A single index, a composite of multiscaled information from each category and dimension, represents opportunity to learn with understanding in the simplified research function. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the indices and the composite index for opportunity to learn with understanding are displayed in the tables of this appendix.

District 1

In District 1, eight Grade 5 teachers participated in the study. Six teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Curricular Content

Table G1

Curricular Content, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Curricular Content Level**
— MiC —	
Banneker-Greene (9)	5
Beethoven-Kipling (9)	5
Beethoven-LaSalle (9)	5
Beethoven-Linne (9)	5
Dewey-Hamilton (9)	5
Dewey-Mitchell (9)	5
— Conventional —	
Dewey-Kershaw (9)	1
River Forest-Fulton (9)	4

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Table G2

Modification of Curricular Materials, Grade 5 Teachers in District 1, 1997-1998

School-Teacher	**Modification of Curricular Materials Level**
— MiC —	
Banneker-Greene (9)	4
Beethoven-Kipling (9)	5A
Beethoven-LaSalle (9)	5B
Beethoven-Linne (9)	5A
Dewey-Hamilton (9)	5A
Dewey-Mitchell (9)	5A
— Conventional —	
Dewey-Kershaw (9)	5A
River Forest-Fulton (9)	6B

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, the category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table G3

Conceptual Understanding, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Banneker-Greene (9)	2	1	3	4	4	3	2	3	3	2.78
Beethoven-Kipling (9)	4	4	4	4	1	2	1	2	3	2.78
Beethoven-LaSalle (9)	4	4	4	4	4	2	3	3	3	3.44
Beethoven-Linne (9)	4	2	4	1	3	2	1	2	1	2.22
Dewey-Hamilton (9)	4	1	4	4	4	1	2	1	3	2.67
Dewey-Mitchell (9)	1	2	3	4	4	3	2	2	3	2.67
— Conventional —										
Dewey-Kershaw (9)	1	1	1	1	3	1	1	1	1	1.22
River Forest-Fulton (9)	3	4	3	4	1	1	2	4	4	2.89

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table G4

Conjectures, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conjectures Level**
— MiC —										
Banneker-Greene (9)	2	1	3	2	4	2	1	2	2	2.11
Beethoven-Kipling (9)	2	4	4	4	2	2	1	2	2	2.56
Beethoven-LaSalle (9)	4	4	4	4	4	3	2	2	2	3.22
Beethoven-Linne (9)	2	1	1	1	3	1	1	2	1	1.44
Dewey-Hamilton (9)	2	1	1	2	2	2	3	3	4	2.22
Dewey-Mitchell (9)	1	2	3	4	3	2	2	3	2	2.44
— Conventional —										
Dewey-Kershaw (9)	1	1	2	1	1	1	2	1	1	1.22
River Forest-Fulton (9)	3	2	4	2	2	2	2	3	2	2.44

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table G5

Mathematical Connections, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Banneker-Greene (9)	1	1	1	1	1	3	1	2	1	1.33
Beethoven-Kipling (9)	1	2	2	1	2	4	1	2	3	2.00
Beethoven-LaSalle (9)	4	4	4	4	3	3	2	3	4	3.44
Beethoven-Linne (9)	2	1	1	1	2	2	1	2	1	1.44
Dewey-Hamilton (9)	1	2	1	1	3	1	2	1	2	1.56
Dewey-Mitchell (9)	1	1	3	1	1	3	1	2	2	1.67
— Conventional —										
Dewey-Kershaw (9)	1	1	1	1	2	1	1	1	1	1.11
River Forest-Fulton (9)	1	1	2	2	2	1	2	4	1	1.78

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table G6

Connections to Life Experiences, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Connections to Life Level**
— MiC —										
Banneker-Greene (9)	2	1	1	3	3	1	2	1	3	1.89
Beethoven-Kipling (9)	2	2	1	3	3	2	1	3	3	2.22
Beethoven-LaSalle (9)	3	1	3	3	3	3	3	3	3	2.78
Beethoven-Linne (9)	3	2	1	3	3	1	3	3	1	2.22
Dewey-Hamilton (9)	3	3	3	3	3	3	3	2	3	2.89
Dewey-Mitchell (9)	3	1	2	3	3	3	1	3	3	2.44
— Conventional —										
Dewey-Kershaw (9)	3	1	1	1	3	1	1	1	1	1.44
River Forest-Fulton (9)	1	1	1	3	1	1	3	3	1	1.67

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table G7

Opportunity to Learn with Understanding, Grade 5 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Banneker-Greene (9)	5	4	2.78	2.11	1.33	1.89	17.11	**3**
Beethoven-Kipling (9)	5	5A	2.78	2.56	2.00	2.22	19.55	**4**
Beethoven-LaSalle (9)	5	5B	3.44	3.22	3.44	2.78	22.89	**4**
Beethoven-Linne (9)	5	5A	2.22	1.44	1.44	2.22	17.33	**3**
Dewey-Hamilton (9)	5	5A	2.67	2.22	1.56	2.89	19.33	**4**
Dewey-Mitchell (9)	5	5A	2.67	2.44	1.67	2.44	19.22	**4**
— Conventional —								
Dewey-Kershaw (9)	1	5A	1.22	1.22	1.11	1.44	11.00	**2**
River Forest-Fulton (9)	4	6B	2.89	2.44	1.78	1.67	18.78	**3**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

District 2

In District 2, five Grade 5 teachers participated in the study. Four teachers used MiC, and one teacher used the conventional curricula already in place in her school.

Curricular Content

Table G8

Curricular Content, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Curricular Content Level**
— MiC —	
Armstrong-Murphy (6)	5
Armstrong-Nash (6)	5
Ogden-Fiske (5)	5
Ogden-Piccolo (5)	5
— Conventional —	
Von Steuben-Gant (6)	1

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Table G9

Modification of Curricular Materials, Grade 5 Teachers in District 2, 1997-1998

School-Teacher	Modification of Curricular Materials Level
— MiC —	
Armstrong-Murphy (6)	5A
Armstrong-Nash (6)	5A
Ogden-Fiske (5)	4
Ogden Piccolo (5)	5A
— Conventional —	
VonSteuben-Gant (6)	5A

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table G10

Conceptual Understanding, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Armstrong-Murphy (6)	4	3	2	2	4	4				3.17
Armstrong-Nash (6)	2	2	3	4	4	4				3.17
Ogden-Fiske (5)	2	4	4	4	2					3.20
Ogden Piccolo (5)	3	3	3	4	4					3.40
— Conventional —										
VonSteuben-Gant (6)	3	1	3	1	2	2				2.00

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table G11

Conjectures, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Conjectures Level
— MiC —										
Armstrong-Murphy (6)	2	1	1	2	2	2				1.67
Armstrong-Nash (6)	1	1	1	2	2	4				1.83
Ogden-Fiske (5)	1	2	4	2	4					2.60
Ogden Piccolo (5)	2	2	2	2	3					2.20
— Conventional —										
VonSteuben-Gant (6)	2	1	2	1	1	1				1.33

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table G12

Mathematical Connections, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Armstrong-Murphy (6)	2	2	2	2	3	2				2.17
Armstrong-Nash (6)	1	2	2	4	3	4				2.67
Ogden-Fiske (5)	2	3	3	3	1					2.40
Ogden Piccolo (5)	2	2	2	3	4					2.60
— Conventional —										
VonSteuben-Gant (6)	2	2	1	1	2	2				1.67

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table G13

Connections to Life Experiences, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Connections to Life Level**
— MiC —										
Armstrong-Murphy (6)	3	3	3	3	3	3				3.00
Armstrong-Nash (6)	1	1	2	3	3	3				2.17
Ogden-Fiske (5)	1	3	3	3	3					2.60
Ogden Piccolo (5)	3	3	3	3	3					3.00
— Conventional —										
VonSteuben-Gant (6)	1	1	1	1	1	3				1.33

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

Table G14

Opportunity to Learn with Understanding, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Armstrong-Murphy (6)	5	5A	3.17	1.67	2.17	3.00	20.00	**4**
Armstrong-Nash (6)	5	5A	3.17	1.83	2.67	2.17	19.83	**4**
Ogden-Fiske (5)	5	4	3.20	2.60	2.40	2.60	19.80	**4**
Ogden-Piccolo (5)	5	5A	3.40	2.20	2.60	3.00	21.20	**4**
— Conventional —								
Von Steuben-Gant (6)	1	5A	2.00	1.33	1.67	1.33	12.33	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table G14

Opportunity to Learn with Understanding, Grade 5 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Armstrong-Murphy (6)	5	5A	3.17	1.67	2.17	3.00	20.00	**4**
Armstrong-Nash (6)	5	5A	3.17	1.83	2.67	2.17	19.83	**4**
Ogden-Fiske (5)	5	4	3.20	2.60	2.40	2.60	19.80	**4**
Ogden-Piccolo (5)	5	5A	3.40	2.20	2.60	3.00	21.20	**4**
— Conventional —								
Von Steuben-Gant (6)	1	5A	2.00	1.33	1.67	1.33	12.33	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Appendix H

Opportunity to Learn with Understanding

GRADE 6

GRADE 6

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. An index was created for the categories of curricular content and modification of curricular materials, and an index was created for each dimension of teaching for understanding. A single index, a composite of multiscaled information from each category and dimension, represents opportunity to learn with understanding in the simplified research function. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the indices and the composite index for opportunity to learn with understanding are displayed in the tables of this appendix.

District 1

In District 1, six Grade 6 teachers participated in the study. Four teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Curricular Content

Table H1

Curricular Content, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Curricular Content Level**
— *MiC* —	
Fernwood-Lee (3)	3B
Fernwood-Weatherspoon (5)	5
Von Humboldt-Brown (9)	2
Von Humboldt-Harvey (9)	3B
— *Conventional* —	
Addams-Tallackson (9)	1
Wacker-Krittendon (9)	1

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Table H2

Modification of Curricular Materials, Grade 6 Teachers in District 1, 1997-1998

School-Teacher	Modification of Curricular Materials Level
— MiC —	
Fernwood-Lee (3)	5A
Fernwood-Weatherspoon (5)	5A
VonHumboldt-Brown (9)	2A
VonHumboldt-Harvey (9)	2A
— Conventional —	
Addams-Tallackson (9)	1
Wacker-Krittendon (9)	5A

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, the category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table H3

Conceptual Understanding, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Fernwood-Lee (3)	4	4	1							3.00
Fernwood-Weatherspoon (5)	4	3	3	3	3					3.20
VonHumboldt-Brown (9)	2	NA	3	4	3	1	1	1	1	2.00
VonHumboldt-Harvey (9)	2	1	1	4	3	4	2	2	1	2.22
— Conventional —										
Addams-Tallackson (9)	1	1	1	1	3	1	1	1	1	1.22
Wacker-Krittendon (9)	1	1	1	1	2	1	1	1	4	1.44

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table H4

Conjectures, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Conjectures Level
— MiC —										
Fernwood-Lee (3)	2	4	1							2.33
Fernwood-Weatherspoon (5)	1	3	2	2	4					2.40
VonHumboldt-Brown (9)	1	NA	1	NA	2	2	1	2	1	1.43
VonHumboldt-Harvey (9)	2	1	2	2	2	1	2	2	1	1.67
— Conventional —										
Addams-Tallackson (9)	1	1	1	1	1	1	1	1	1	1.00
Wacker-Krittendon (9)	1	2	1	1	2	3	1	2	2	1.67

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table H5

Mathematical Connections, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Fernwood-Lee (3)	3	2	2							2.33
Fernwood-Weatherspoon (5)	1	3	3	3	2					2.40
VonHumboldt-Brown (9)	1	NA	2	NA	1	1	1	2	2	1.43
VonHumboldt-Harvey (9)	2	1	2	1	2	2	1	2	1	1.56
— Conventional —										
Addams-Tallackson (9)	2	1	1	1	1	1	2	1	1	1.22
Wacker-Krittendon (9)	1	2	1	1	1	1	1	1	3	1.33

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table H6

Connections to Life Experiences, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Connections to Life Level**
					— MiC —					
Fernwood-Lee (3)	2	2	3							2.33
Fernwood-Weatherspoon (5)	3	3	3	3	3					3.00
VonHumboldt-Brown (9)	1	2	3	3	3	1	3	3	2	2.33
VonHumboldt-Harvey (9)	3	1	2	3	3	3	3	1	1	2.22
					— Conventional —					
Addams-Tallackson (9)	2	1	1	1	1	1	1	1	1	1.11
Wacker-Krittendon (9)	1	1	1	1	1	1	1	1	3	1.22

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table H7

Opportunity to Learn with Understanding, Grade 6 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Fernwood-Lee (3)	3B	5A	3.00	2.33	2.33	2.33	17.99	**3**
Fernwood-Weatherspoon (5)	5	5A	3.20	2.40	2.40	3.00	21.00	**4**
Von Humboldt-Brown (9)	2	2A	2.00	1.43	1.43	2.33	11.19	**2**
Von Humboldt-Harvey (9)	3B	2A	2.22	1.67	1.56	2.22	12.67	**2**
— Conventional —								
Addams-Tallackson (9)	1	1	1.22	1.00	1.22	1.11	6.56	**1**
Wacker-Krittendon (9)	1	5A	1.44	1.67	1.33	1.22	11.66	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

District 2

In District 2, six Grade 5 teachers participated in the study. Four teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Curricular Content

Table H8

Curricular Content, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Curricular Content Level
— MiC —	
Guggenheim-Broughton (6)	2
Guggenheim-Dillard (7)	5
Hirsch Metro-Davenport (3)	2
Hirsch Metro-Holland (3)	2
— Conventional —	
Newberry-Renlund (5)	1
Newberry-Rhaney (5)	1

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Table H9

Modification of Curricular Materials, Grade 6 Teachers in District 2, 1997-1998

School-Teacher	**Modification of Curricular Materials Level**
— MiC —	
Guggenheim-Broughton (6)	4
Guggenheim-Dillard (7)	5A
HirschMetro-Davenport (3)	2B
HirschMetro-Holland (3)	2B
— Conventional —	
Newberry-Renlund (5)	5A
Newberry-Rhaney (5)	5A

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table H10

Conceptual Understanding, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Guggenheim-Broughton (6)	2	2	2	2	3	2				2.17
Guggenheim-Dillard (7)	3	3	1	2	3	4	4			2.86
HirschMetro-Davenport (3)	4	3	1							2.67
HirschMetro-Holland (3)	3	3	1							2.33
— Conventional —										
Newberry-Renlund (5)	1	2	1	2	3					1.80
Newberry-Rhaney (5)	1	1	1	3	2					1.60

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table H11

Conjectures, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conjectures Level**
— MiC —										
Guggenheim-Broughton (6)	2	1	1	2	2	2				1.67
Guggenheim-Dillard (7)	2	2	1	2	2	2	2			1.86
HirschMetro-Davenport (3)	4	2	1							2.33
HirschMetro-Holland (3)	3	2	1							2.00
— Conventional —										
Newberry-Renlund (5)	1	3	1	2	3					2.00
Newberry-Rhaney (5)	1	1	1	4	1					1.60

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table H12

Mathematical Connections, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Guggenheim-Broughton (6)	1	1	2	1	2	2				1.50
Guggenheim-Dillard (7)	2	2	1	1	3	2	2			1.86
HirschMetro-Davenport (3)	3	3	2							2.67
HirschMetro-Holland (3)	2	2	2							2.00
— Conventional —										
Newberry-Renlund (5)	1	1	1	1	1					1.00
Newberry-Rhaney (5)	1	1	1	2	1					1.20

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table H13

Connections to Life Experiences, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Connections to Life Level
— MiC —										
Guggenheim-Broughton (6)	1	3	3	1	3	2				2.17
Guggenheim-Dillard (7)	3	2	1	1	3	3	3			2.29
HirschMetro-Davenport (3)	3	3	2							2.67
HirschMetro-Holland (3)	2	2	3							2.33
— Conventional —										
Newberry-Renlund (5)	1	1	1	1	2					1.20
Newberry-Rhaney (5)	1	1	1	3	1					1.40

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table H14

Opportunity to Learn with Understanding, Grade 6 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Guggenheim-Broughton (6)	2	4	2.17	1.67	1.50	2.17	13.50	**2**
Guggenheim-Dillard (7)	5	5A	2.86	1.86	1.86	2.29	18.86	**3**
Hirsch Metro-Davenport (3)	2	2B	2.67	2.33	2.67	2.67	14.33	**3**
Hirsch Metro-Holland (3)	2	2B	2.33	2.00	2.00	2.33	12.66	**2**
— Conventional —								
Newberry-Renlund (5)	1	5A	1.80	2.00	1.00	1.20	12.00	**2**
Newberry-Rhaney (5)	1	5A	1.60	1.60	1.20	1.40	11.80	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Appendix I

Opportunity to Learn with Understanding

GRADE 7

GRADE 7

The composite variable opportunity to learn with understanding includes three major categories: curricular content, modification of curricular materials, and teaching for understanding. The category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences. An index was created for the categories of curricular content and modification of curricular materials, and an index was created for each dimension of teaching for understanding. A single index, a composite of multiscaled information from each category and dimension, represents opportunity to learn with understanding in the simplified research function. In this appendix, the ratings for each teacher in Districts 1 and 2 on each of the indices and the composite index for opportunity to learn with understanding are displayed in the tables of this appendix.

District 1

In District 1, four Grade 7 teachers participated in the study. Two teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Curricular Content

Table I1

Curricular Content, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Curricular Content Level**
— MiC —	
Fernwood-Heath (9)	5
Von Humboldt-Donnely (9)	2
— Conventional —	
Addams-St. James (9)	1
Wacker-McLaughlin (9)	1

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorith

Table I2

Modification of Curricular Materials, Grade 7 Teachers in District 1, 1997-1998

School-Teacher	**Modification of Curricular Materials Level**
— MiC —	
Fernwood-Heath (9)	6B
VonHumboldt-Donnely (9)	2A
— Conventional —	
Addams-St.James (9)	5A
Wacker-McLaughlin (9)	5A

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table I3

Conceptual Understanding, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Fernwood-Heath (9)	3	3	3	3	3	3	2	2	1	2.56
VonHumboldt-Donnely (9)	1	1	4	1	1	3	2	1	1	1.67
— Conventional —										
Addams-St.James (9)	1	1	1	1	1	1	1	1	1	1.00
Wacker-McLaughlin (9)	2	1	1	2	1	1	2	1	1	1.33

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table I4

Conjectures, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conjectures Level**
— MiC —										
Fernwood-Heath (9)	2	2	2	2	1	3	2	1	1	1.78
VonHumboldt-Donnely (9)	1	1	4	1	1	1	1	2	1	1.44
— Conventional —										
Addams-St.James (9)	1	1	1	1	1	1	1	2	1	1.11
Wacker-McLaughlin (9)	1	3	1	1	3	1	2	1	1	1.56

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table I5

Mathematical Connections, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Fernwood-Heath (9)	3	1	3	2	3	3	3	1	1	2.22
VonHumboldt-Donnely (9)	1	1	1	1	1	2	1	1	1	1.11
— Conventional —										
Addams-St.James (9)	1	1	2	1	1	1	1	2	2	1.33
Wacker-McLaughlin (9)	1	1	1	1	2	1	2	1	1	1.22

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table I6

Connections to Life Experiences, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Connections to Life Level**
— MiC —										
Fernwood-Heath (9)	3	3	3	2	3	2	3	1	1	2.33
VonHumboldt-Donnely (9)	1	1	1	1	1	1	2	1	2	1.22
— Conventional —										
Addams-St.James (9)	1	1	3	1	1	1	1	2	2	1.44
Wacker-McLaughlin (9)	1	3	1	1	3	1	1	1	1	1.44

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table I7

Opportunity to Learn with Understanding, Grade 7 Teachers in District 1, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— *MiC* —								
Fernwood-Heath (9)	5	6B	2.56	1.78	2.22	2.33	19.88	**4**
Von Humboldt-Donnely (9)	2	2A	1.67	1.44	1.11	1.22	9.44	**2**
— *Conventional* —								
Addams-St. James (9)	1	5A	1.00	1.11	1.33	1.44	10.88	**2**
Wacker-McLaughlin (9)	1	5A	1.33	1.56	1.22	1.44	11.55	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

District 2

In District 2, six Grade 7 teachers participated in the study. Four teachers used MiC, and two teachers used the conventional curricula already in place in their schools.

Curricular Content

Table I8

Curricular Content, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Curricular Content Level**
— *MiC* —	
Guggenheim-Keeton (6)	5
Guggenheim-Teague (6)	3B
Hirsch Metro-Draski (3)	2
Hirsch Metro-McFadden (3)	2
— *Conventional* —	
Newberry-Cunningham (5)	4
Newberry-Stark (5)	1

Level of Curricular Content: This index describes the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts.

5 The teacher presented a comprehensive, integrated curriculum with attention to all content areas.

4 The teacher taught mathematical concepts in depth but restricted content primarily to one or two content strands (e.g., number and algebra).

3 The teacher covered only a few topics.

3A The teacher lingered over content until students demonstrated mastery.

3B The teacher used a new curriculum and slow pacing resulted in coverage of only a few topics.

2 The teacher presented a combination of conventional and reform curricula, which resulted in a dual emphasis on basic skills and some conceptual content.

1 The teacher presented vast content as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms.

Modification of Curricular Materials

Table I9

Modification of Curricular Materials, Grade 7 Teachers in District 2, 1997-1998

School-Teacher	Modification of Curricular Materials Level
— MiC —	
Guggenheim-Keeton (6)	4
Guggenheim-Teague (6)	4
HirschMetro-Draski (3)	2B
HirschMetro-McFadden (3)	2B
— Conventional —	
Newberry-Cunningham (5)	5A
Newberry-Stark (5)	5A

Level of Modification of Curricular Materials: This index measures the extent to which modifications of curricular materials supported the development of deep understanding of the covered concepts.

6 The teacher modified the curriculum in ways that enhanced conceptual development of the content.

6A The teacher regularly supplemented the text with tasks that promoted understanding of concepts; the text was used primarily for practice.

6B The teacher supplemented the text with tasks or multiple models that emphasize connections among concepts and connections to students' lives.

5 The teacher followed the curriculum faithfully.

5A The teacher occasionally supplemented the text with activities disconnected from the text.

5B The teacher presented the curriculum as it was written with few, if any, modifications.

4 The teacher supplemented the text with materials not aligned with the intent of the curriculum (e.g., added skill-and-drill worksheets to reform curriculum).

3 Lack of teacher preparation, materials, and/or student participation undermined the intent of the curriculum.

2 The teacher retreated from using a reform curriculum and subsequently used a conventional curriculum.

2A The teacher supplemented a reform curriculum with conventional materials to the

2B The teacher abandoned the reform curriculum in favor of a conventional curriculum.

1 The teacher presented the curriculum in a haphazard way that did not adhere to a text and did not emphasize connections among topics.

Teaching for Understanding

In this study, category teaching for understanding is characterized by four dimensions: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and students' life experiences.

Table I10

Conceptual Understanding, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	**Observation 1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Conceptual Understanding Level**
— MiC —										
Guggenheim-Keeton (6)	2	4	3	4	4	4				3.50
Guggenheim-Teague (6)	3	4	4	4	3	3				3.50
HirschMetro-Draski (3)	2	4	2							2.67
HirschMetro-McFadden (3)	2	3	1							2.00
— Conventional —										
Newberry-Cunningham (5)	2	1	1	4	4					2.40
Newberry-Stark (5)	1	3	1	2	1					1.60

Level of Conceptual Understanding: This measures the extent to which the lesson fostered the development of conceptual understanding.

4 The continual focus of the lesson was on building connections or linking procedural knowledge with conceptual knowledge.

3 Some lesson questions fostered students' conceptual development of mathematical ideas, or some aspects of the lesson focused on conceptual understanding, but the main focus of the lesson was on building students' procedural understanding without meaning.

2 Few questions fostered students' conceptual development of mathematical ideas or conceptual understanding was a small part of lesson design.

1 The lesson as presented did not promote conceptual understanding.

Table I11

Conjectures, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Conjectures Level
— MiC —										
Guggenheim-Keeton (6)	2	4	4	4	1	4				3.17
Guggenheim-Teague (6)	1	1	2	3	2	2				1.83
HirschMetro-Draski (3)	1	4	1							2.00
— Conventional —										
Newberry-Cunningham (5)	3	1	1	2	3					2.00
Newberry-Stark (5)	1	2	1	1	1					1.20

Level of Conjectures: This index measures the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas.

4 Students made generalizations about mathematical ideas.

3 Observed conjectures consisted mainly of student investigations about the truthfulness of particular statements.

2 Observed conjectures consisted mainly of making connections between a new problem and problems previously seen.

1 No conjectures of any type were observed in the lesson. Students were not encouraged to make connections.

Table I12

Mathematical Connections, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Mathematical Connections Level
— MiC —										
Guggenheim-Keeton (6)	3	2	2	2	1	3				2.17
Guggenheim-Teague (6)	2	2	4	3	3	3				2.83
HirschMetro-Draski (3)	2	2	2							2.00
HirschMetro-McFadden (3)	2	2	2							2.00
— Conventional —										
Newberry-Cunningham (5)	2	1	1	2	2					1.60
Newberry-Stark (5)	1	2	2	1	1					1.40

Level of Connections Within Mathematics: This index measured the extent to which connections within mathematics were explored in the lesson.

4 The mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics.

3 Connections among mathematical topics were discussed by teacher and students or connections were clearly explained by the teacher.

2 The teacher or students might have briefly mentioned that the topic was related to others, but these connections were not discussed in detail.

1 The mathematical topic was presented in isolation of other topics, and teacher and students did not talk about connections between the topic of the lesson and other mathematical topics.

Table I13

Connections to Life Experiences, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Observation 1	2	3	4	5	6	7	8	9	Connections to Life Level
— MiC —										
Guggenheim-Keeton (6)	3	3	3	3	3	3				3.00
Guggenheim-Teague (6)	1	3	3	2	2	2				2.17
HirschMetro-Draski (3)	1	1	2							1.33
HirschMetro-McFadden (3)	1	3	1							1.67
— Conventional —										
Newberry-Cunningham (5)	3	1	2	3	1					1.80
Newberry-Stark (5)	1	1	1	1	1					1.00

Level of connections between mathematics and students' daily lives: This index measured the extent to which connections between mathematics and mathematics and students' daily lives were apparent in the lesson.

3 Connections between mathematics and students' daily lives were clearly apparent in the lesson.

2 Connections between mathematics and students' daily lives were not apparent to the students, but would be reasonably clear if explained by the teacher.

1 Connections between mathematics and students' daily lives were not apparent in the lesson.

Opportunity to Learn with Understanding Composite

A single index, a composite of multiscaled information from each category and dimension of opportunity to learn with understanding, represents opportunity to learn with understanding in the simplified research function. The following table summarizes the ratings for each category and dimension for each teacher and indicates the composite level for each teacher.

Table I14

Opportunity to Learn with Understanding, Grade 7 Teachers in District 2, 1997-1998

School-Teacher (No. of Observations)	Curricular Content	Modification of Curricular Materials	Conceptual Understanding	Conjectures	Connections within Mathematics	Connections between Mathematics and Students' Daily Lives	Total	Composite*
— MiC —								
Guggenheim-Keeton (6)	5	4	3.50	3.17	2.17	3.00	20.83	**4**
Guggenheim-Teague (6)	3B	4	3.50	1.83	2.83	2.17	17.33	**3**
Hirsch Metro-Draski (3)	2	2B	2.67	2.00	2.00	1.33	12.00	**2**
Hirsch Metro-McFadden (3)	2	2B	2.00	1.67	2.00	1.67	11.33	**2**
— Conventional —								
Newberry-Cunningham (5)	4	5A	2.40	2.00	1.60	1.80	16.80	**3**
Newberry-Stark (5)	1	5A	1.60	1.20	1.40	1.00	11.20	**2**

* 1 < 9.12, 2 = 9.12 to 14.00, 3 = 14.01 to 19.12, 4 > 19.12; see next page for key.

Level of Opportunity to Learn with Understanding: This index includes six major dimensions: curricular content, modification of curricular materials, the development of conceptual understanding, the nature of student conjectures, discussion of connections among mathematical ideas, and discussion of connections between mathematics and students' life experiences.

4 High Level of Opportunity to Learn with Understanding: Teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

3 Moderate Level of Opportunity to Learn with Understanding: Teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

2 Limited Opportunity to Learn with Understanding: Teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both experimental and control teachers, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

1 Low Level of Opportunity to Learn with Understanding: Teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. Lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.