Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Performance

Problem Solving Assessments

(Working Paper #15)

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INTRODUCTION

The purposes of the longitudinal/cross-sectional study of the impact of *Mathematics in Context* (MiC; National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998) on student performance were (a) to determine the mathematical knowledge, understanding, attitudes, and levels of student performance as a consequence of studying MiC for over three years; and (b) to compare student knowledge, understanding, attitudes, and levels of performance of students using MiC with those using conventional mathematics curricula. The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). For this study, information was gathered on 14 variables over a 3-year period for three groups of students (those in Grades 5, 6, and 7 in 1997). The variables were organized in five categories (prior, independent, intervening, outcome, and consequent). The Problem Solving Assessment System and the External Assessment System measure students' knowledge and understanding of mathematics and their application of mathematical ideas. (See Figure 1 for variables and hypothesized relationships.)

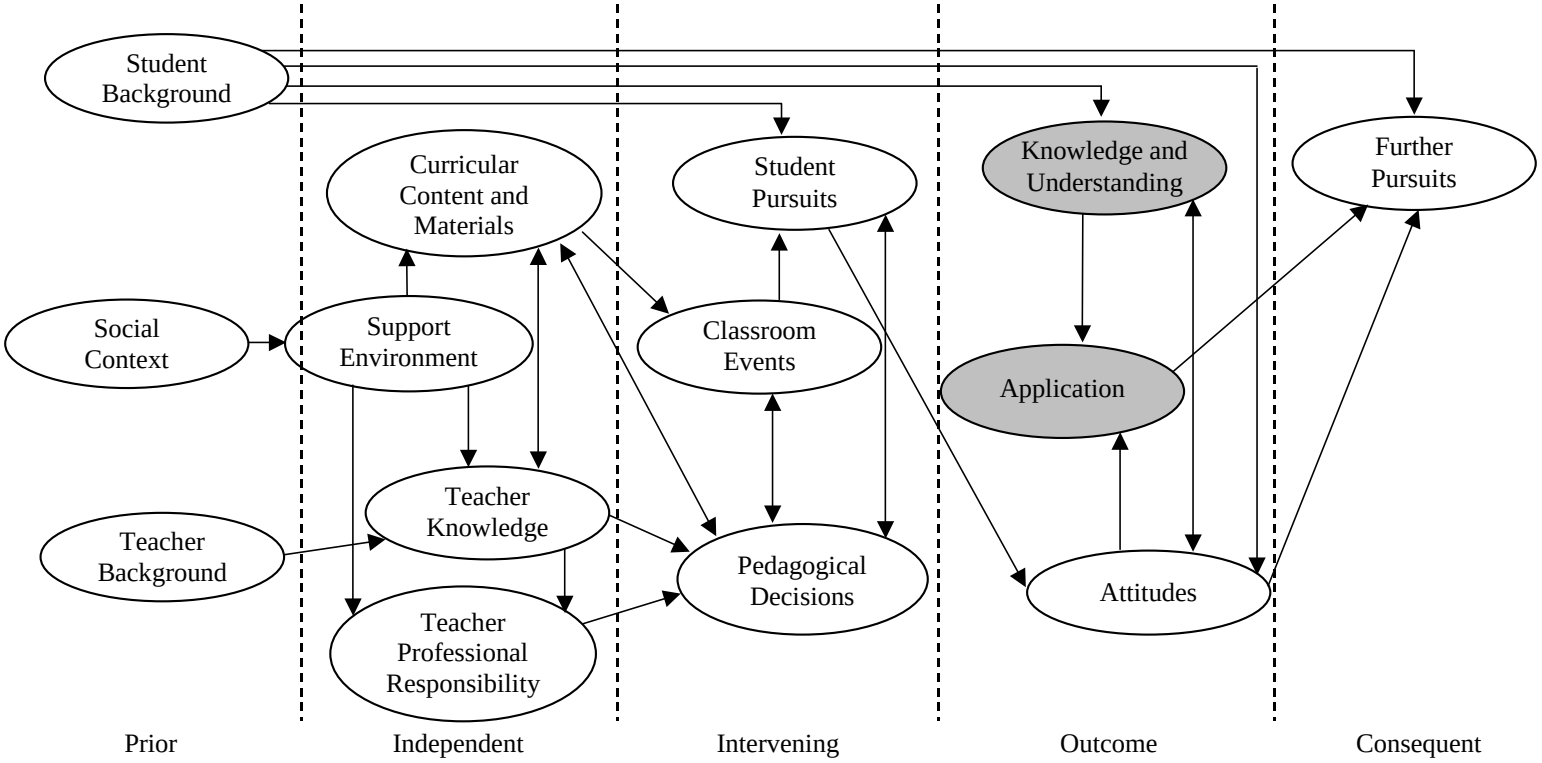


Figure 1. Revised model for the monitoring of school mathematics**.**

Overview

As part of the Longitudinal/Cross-Sectional Study of the impact of *Mathematics in Context* (MiC) on student performance, a set of problem solving assessments (PSAs) for grades 5–8 was designed to assess students' knowledge and understanding of mathematical content and to analyze students' ability to use progressively more abstract problem solving strategies as they advanced through middle school. The purpose of this paper is to describe the development of the PSAs and the accompanying rubrics used to score them.

The PSA, a set of grade-specific assessments composed of constructed-response items set in contexts, was designed to align with the goals of problem solving, communication, reasoning, and connections advocated in the NCTM *Curriculum and Evaluation Standards* (1989). The number of items in each context varied depending on the mathematical content, level of reasoning, and level of difficulty assessed. Partial-credit scoring rubrics were used to assign point values to student responses. Strategies students used in solving problems were also coded.

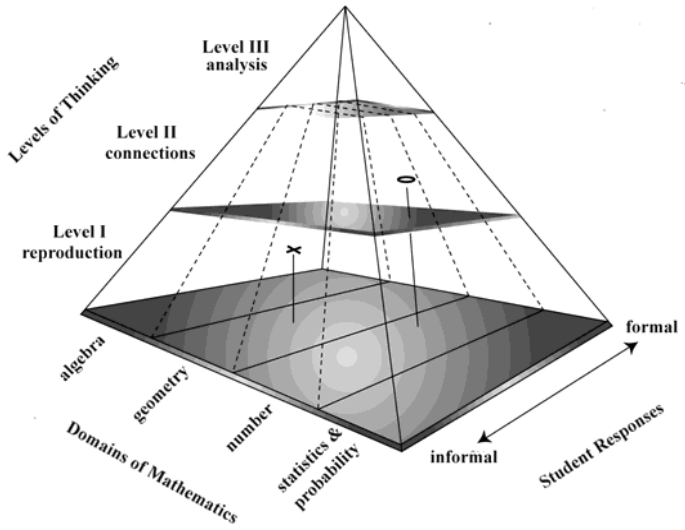
Description of the PSA

Although the problem solving assessments were designed as part of a study evaluating the impact of MiC, the assessments were not curriculum specific. The PSAs contained items that all students, regardless of the curriculum they studied, should be able to solve successfully. Moreover, items were designed for accessibility on a variety of levels so that students who rely on direct modeling or drawing strategies and students who use more sophisticated strategies could solve the same problems. To the extent that a paper-and-pencil assessment can be used as an indicator of student thinking, the PSAs were designed to elicit students' thinking processes in order to track their progress from informal methods to more formal symbolic notation over time. The general directions for the test and directions for individual items explicitly requested that students demonstrate their reasoning.

The first dimension involves domains of mathematics. For middle school curricula, four domains are common to most programs: algebra, geometry, number, and statistics and probability. The second dimension reflects the pedagogical notion of "progressive formalization," which assumes that student responses to tasks progress from informal, to pre-formal, to formal responses as the tasks progress from simple to complex. The third dimension reflects the level of reasoning. Level I consists of problems that require basic skills (i.e., conceptual and procedural knowledge). For example, students may perform specific calculations, reproduce memorized facts, or solve a particular equation (Shafer & Foster, 1997). Level I items primarily assess definitions, technical skills, and standard algorithms. Level I items may involve multiple steps, and they fall along a continuum of easy to difficult in complexity. Although Level I problems may be difficult, they can often be solved using routine skills or rote learning (Verhage & DeLange, 1997). Level II items require that students reason, communicate, integrate information, make connections between concepts or procedures, and solve problems in which no standard procedure is available. Problems on this level often invite a variety of strategies. Level III items challenge students to mathematize real or imaginable situations. That is, they must extract the mathematics embedded in a situation, make assumptions about the situation, develop models, generalize, critically analyze, and support their mathematical arguments. At this level, the distinction among mathematical content domains blurs, and the gap between easy and difficult items shrinks (Shafer & Foster, 1997). The distinction between the levels of reasoning is illustrated in Table 1 with tasks that might be used to assess students' understanding of

decimal operations at Grade 5. A complete assessment program utilizes items that, over time, "fill" the pyramid (Shafer & Foster, 1997). Thus, the PSAs were written to assess student thinking at all three levels, across all mathematical domains, and at increasing levels of difficulty.

The pyramid shown in Figure 2 illustrates the three dimensions considered in the creation of PSA items.



⊕ indicates the location of a task

Figure 2. Assessment pyramid. Source: Verhage, H. & deLange, J. (1997, April). Mathematics Education and Assessment. *Pythagoras*, 42, 14-20.

Table 1
Levels of Reasoning: Goals for Operations with Decimals, Grade 5

Level I Goals	Level II Goals	Level III Goals
Understand the relationship between benchmark fractions and their decimal representations Use decimals in a context such as money or measurement Estimate and compute with decimals	Understand place value and its use in ordering decimals Understand the metric system and its relationship to decimals Understand decimals as they relate to refinement in the measurement process	Use equivalent representations of fractions, decimals, and division notation Represent and use decimals in a variety of equivalent forms to solve problems in real-world and mathematical situations Choose an appropriate visual model or strategy to represent and solve problems involving decimals

These three levels correspond with the competency classes used in the Program for International Student Assessment (PISA) developed by the Organisation for Economic Co-Operation and Development (OECD, 1999): Class 1—reproduction, definitions, and computations; Class 2—connections and integration for problem solving; and Class 3—mathematical thinking, generalization and insight (OECD, 1999, p. 43). The classification of the items according to competency class is consistent with the use of the term "mathematization" in the PISA framework. Mathematization is defined as being able to "recognise and extract mathematics embedded in the situation and use mathematics to solve the problem; to analyse; to interpret; to develop their own models and strategies and to present mathematical arguments, including proofs and generalizations" (OECD 1999, p. 45).

The PISA definitions of the three competency classes were used to derive indicators of specific aspects of knowledge, skills and understandings that students must demonstrate in order to solve particular problems successfully. For Competency Class I (Level 1 in the pyramid model), the indicators are standard representation, computations, definition, routine procedures, and one solution method. For Competency Class 2 (Level 2) the indicators are modelling, problem solving, interpretation/reflection, and multiple well-defined solution methods. For Competency Class 3 (Level 3) the indicators are problem posing, reflection, original mathematical method, mathematical insight, multiple complex solution methods, and generalization. Each item in the MiC longitudinal/cross-sectional study was classified according to the indicators that applied. For most items, more than one indicator was often considered relevant. For each item, the most relevant or primary indicator was indicated by a 1 in the item classification matrix, and any secondary indicators were indicated by a 2 in the matrix. In general, items were assigned to competency class by primary indicator.

Development of the PSA

Researchers from the Freudenthal Institute in The Netherlands initially designed each PSA item, described the mathematical content assessed, listed point values and the level of reasoning addressed, and suggested codes for discriminating student strategies. The research staff at the Wisconsin Center for Education Research (WCER) "Americanized" the language, names, and contexts for use with American students. The mathematical intent of the items, however, was not altered. Items involving operations with fractions (often deemed a benchmark topic in middle

school instruction in the United States) were added to the sixth- and seventh-grade assessments. The WCER research staff assigned a difficulty rating to each item (easy, medium, or difficult) based on the perceived difficulty of the item for an eighth-grade student. Draft versions of the assessments were pilot-tested in classes using MiC and in classes using conventional texts and were administered over two class periods. Four-function calculators and other tools including rulers and protractors or compass cards were permitted during the assessments. The PSAs were revised based on the results of pilot-tests. Revisions included clarification of wording, additional prompts or diagrams, the addition, restructure or deletion of an item, and adjustment of the sequence of items. P-values (i.e., percentage of correct responses in the sample tested) from pilot-test results were then compared with the difficulty ratings previously assigned to each item. Inconsistent difficulty ratings were further discussed by the research staff until agreement was reached.

Item-specific scoring rubrics were created to assign point values to particular responses. Full or partial credit was awarded to responses based upon accuracy of response and thoroughness of explanation. In addition, student solution strategies were coded in order to monitor changes in strategies over time. On the basis of pilot-tests, scoring rubrics were enhanced with actual student work, and the number of points subtracted for particular errors was refined and consistently applied across all grade-specific assessments. Scores ranged from X (no response) or 0 (incorrect response) to 4, depending on the complexity of the problem. Correct responses for less complex items, for example, were worth one point; responses for the most complex items were worth as many as 4 points (see Table 2).

Table 2
Scoring Rubric for Problem Solving Assessment

Complexity	Scoring Scheme					
Less	X	0	1			
↓	X	0	1	2		
↓	X	0	1	2	3	
More	X	0	1	2	3	4

The original strategy codes proved cumbersome. As a result, strategy codes were categorized for consistency across all four grade-specific assessments. General strategy categories were identified for (a) computational strategies including tools such as charts, derived facts, and traditional algorithms; (b) explanations and descriptions; (c) use of patterns; (d) algebraic strategies; (e) geometric and measurement strategies; and (f) justifications. An example of specific codes identified for non-standard computation is shown in Table 3. Specific strategies within the general categories were identified using goals and student work for individual items.

Table 3
Strategy Codes: Alternative Methods of Computation

Code	Description
20	Converts rational numbers to compute (e.g., fractions to decimals to percents)
21	Uses diagrams or drawings to compute
22	Uses derived numbers (given numbers broken into more workable parts)
23	Uses ratio table
24	Uses the fraction strategy sum of partitions (e.g., each monkey gets ten 1/15 of a mango)
25	Combines quantities by category
26	Uses arrow language to compute
27	Uses the guess and check strategy
28	Uses chart or table
29	Uses other tool or model

Codes for solution strategies were also adjusted on the basis of pilot-test results. Strategy codes were added in order to investigate patterns in incorrect student responses such as responses that reflected the context but lacked mathematical basis and arbitrary calculation which randomly used numbers given in the problem. Strategies that rarely appeared were collapsed into an "Other" category. Codes for unscorable responses were also added to distinguish non-responses (blanks) from responses that were non-numerical or demonstrated confusion or emotion. On some items, strategy coding yielded little information because the point values adequately discriminated student responses. Strategy codes on such items were therefore eliminated.

Some scoring rubrics and strategy codes evolved during the scoring process. Two factors influenced this development. First, PSA items were pilot-tested with 75-100 students. Rubrics and strategy codes were created and revised based on those student samples. However, because during the study the PSA was administered to hundreds of students, additional types of student responses and solution strategies were detected. These newly discovered cases were integrated into existing scoring and coding schemes. Second, as a result of the pilot test, some items were rewritten and new items were included. Student work for changed and new items was unavailable prior to the administration of the assessments. As student responses were examined during the scoring process, rubrics and strategy codes were refined to better represent the variety of responses actually demonstrated by study students on specific items. When rubrics or strategies codes were changed, items scored prior to the changes were rescored.

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Grade 5 Problem Solving Assessment

The fifth grade PSA consisted of 22 constructed-response items with a total of 49 points. The assessment was organized around the theme of a class field trip to the zoo and included seven contexts (see Tables 4 and 5). The number of items per context varied from 1 to 5, and the number of points assigned to each item varied from 1 to 4. Most items required students to demonstrate their strategies for solving problems in order to earn full credit. The items examined students' application of mathematics and mathematical reasoning at three levels. Items designed to elicit reasoning at the second and third levels were more open-ended in nature and were more complex to score. Each item was also rated for difficulty (easy, medium, or difficult) based on the perceived difficulty of the item for an eighth-grade student. The item classification matrix which includes degree of formalization, competency class, and performance indicators is found in Appendix A.

Table 4

Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, Grade 5 Problem Solving Assessment, Part 1

Context	Item	Strand	Mathematical Content	Level of Reasoning	Level of Difficulty	Points
A Visit to the Zoo	1	Number	Multiplication of a whole number and a decimal number	I	Easy	2
	2	Number	Percent to determine a discount; operations with decimal numbers; rounding	I	Easy	3
	3	Number	Whole number division with remainder; interpretation of remainder	I	Easy	2
	4	Geometry	Interpretation of a map; identification of direction (cardinal or degree)	I	Easy	1
Monkeys	5	Number	Multiplication of whole numbers; multiplication of whole number and fraction	I	Easy	3
	6	Number	Rounding fractions with respect to context	I	Easy	2
	7	Number	Division of whole numbers with answer expressed as fraction	II	Easy	2
Buildings	8	Geometry	Interpretation of two-dimensional representations of three-dimensional situations	II	Medium	3
	9	Geometry	Reasoning using scale on a map and two-dimensional representations	II	Medium	2
Pools	10	Number/ Algebra	Reading decimal numbers from a number line	I	Easy	2
	11	Number	Operations with decimal numbers and integers	I	Easy	1
	12	Number	Placement of decimal numbers on a number line	I	Easy	2
Snail	13	Number/ Geometry	Use of ratio and scale; interpretation of two-dimensional representations of three-dimensional situations	II	Medium	3

Table 5

Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, Grade 5 Problem Solving Assessment, Part 2

Context	Item	Strand	Mathematical Content	Level of Reasoning	Level of Difficulty	Points
Playground	14	Number	Analysis of a pattern; counting	I	Easy	1
	15	Algebra/ Geometry	Determining geometrical and numerical patterns presented in a table	II	Easy	3
	16	Algebra/ Geometry	Determining geometrical and numerical patterns presented in a table	III	Easy	4
	17	Algebra/ Geometry/ Number	Knowledge of properties of odd and even numbers to draw conclusions about numerical and geometrical patterns	III	Medium	2
Questionnaire	18	Statistics	Analysis of statistical data; drawing conclusions	I	Easy	1
	19	Statistics	Critical analysis of statistical data; drawing conclusions	II	Easy	2
	20	Statistics	Appropriate representation of statistical data	II	Medium	2
	21	Statistics	Construction of bar graph to represent statistical data	I	Easy	4
	22	Statistics	Analysis of statistical data; drawing conclusions	II	Easy	2

Thirty-eight percent of the points on the fifth-grade PSA were related to students' understanding and application of number concepts, 28% of the points to geometric concepts and the remaining points were divided equally between algebra and statistics concepts. Forty-one percent of the points were designed to elicit Level I reasoning, 38% to elicit Level II reasoning, and 22% to elicit Level III reasoning (see Table 6).

Table 6

Point Distribution for Content Strands and Levels of Reasoning, Problem Solving Assessment, Grade 5

Level of Reasoning	Strand				
	Number	Geometry	Algebra	Statistics	Total
I	18 (28%)	1 (2%)	2 (3%)	5 (8%)	26 (41%)
II	5 (8%)	11 (17%)	3 (5%)	6 (9%)	25 (38%)
III	2 (3%)	6 (9%)	6 (9%)	0	14 (22%)
Total	25 (38%)	18 (28%)	11 (17%)	11 (17%)	65*

*The point total is greater than 49 because some assessment items fall into multiple categories.

A Visit to the Zoo

The first context of the fifth-grade assessment, A Visit to the Zoo, is composed of items associated with the cost of the trip and transportation from the school to the zoo. It contained four items for students to answer. The first item assesses students' understanding of multiplication with a whole number and a decimal number (see Figure 3). In order to solve the problem, students must calculate the admission fee for the class of 29 students. One solution strategy is to multiply 29 by $12.50 using the standard algorithm. An alternate strategy is to use derived numbers, such as for every 10 persons the admission is 10 x $12.50; add the price of admission for 10 persons three times; and subtract the admission for one person.

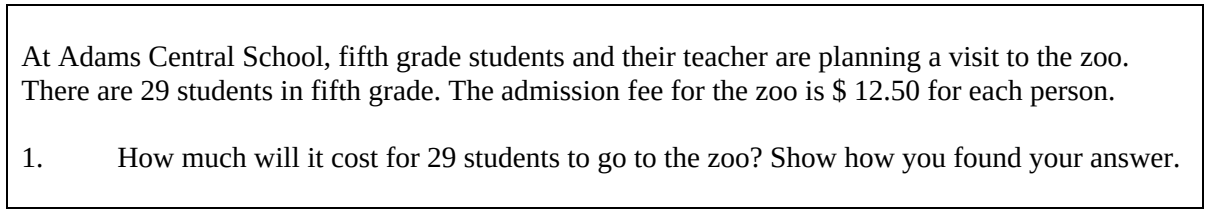


Figure 3. Grade 5 Problem Solving Assessment, Item 1.

The scoring rubric for this item contained four categories (see Table 7). For full credit, the student response must have had the correct answer with the work clearly shown. Partial credit was awarded for the correct solution without demonstrated work and responses with minor errors in calculation or inadvertent use of incorrect numbers. No credit was awarded for incorrect responses, including the incorrect placement of the decimal point. "X" was marked if the response was nonscorable or if no response was written.

Table 7
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 1

Points	Response
2	Correct answer: $362.50 *with* clear work shown
1	Correct answer only, no work shown *Or* Work shows multiplication strategy, however, incorrect answer results from one minor computation error (e.g., neglects to regroup or calculates one number fact incorrectly) *Or* Incorrect answer from error in writing numbers (27 x $12.50 instead of 29 x $12.50)
0	Incorrect response *Or* Incorrect placement of decimal (29 x $12.50 = $3625 or $36.25 or 29 x $125)
X	Nonscorable or no response

For this item, strategy codes were assigned to student responses (see Table 8). In this case, strategies varied from use of the standard algorithm to alternative strategies. Incorrect or partially incorrect responses were also coded. The use of random operations, for example, was assigned a particular strategy code. Codes 90-99 were used for all items in which strategies were coded. These special codes discriminated among types of nonscorable responses.

Table 8
Strategy Codes, Grade 5 Problem Solving Assessment, Item 1

Code	Description
11	Multiplication with a calculator
12	Answer only
13	Correct/incorrect use of standard algorithm
19C	Random operation that does not make sense
22	Derived number facts used to compute answer 30 x 12.50 = 375 – 12.50 = $362.50 or 29 x 12 = 348; 29 x .5 = 14.50; $348 + $14.50 = $362.50
19	Other computation strategy
90	Nonscorable response: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable response: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable response: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item 2 required the use of the response from Item 1. If a student used an incorrect solution from Item 1 to build an otherwise correct solution for Item 2, the response for Item 2 was awarded full credit. Item 2 assesses students’ understanding of calculating a discounted price (see Figure 4).

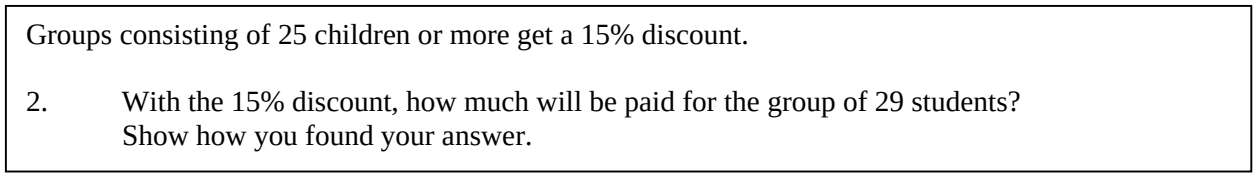


Figure 4. Grade 5 Problem Solving Assessment, Item 2.

In order to solve the problem, students must calculate a 15% discount for 25 or more children and find the price of admission for 29 students. One solution strategy is to multiply 29 by $12.50, multiply that product by 0.15, and find the difference. Another strategy is to find 10% of $12.50 and 5% of $12.50 and multiply the sum by 29.

The scoring rubric for this item contained 5 categories (see Table 9). For full credit, the student response must have had the correct answer with work clearly shown. Partial credit was awarded when the answer was incorrect due to errors in rounding, correct answer without explanation, minor computational error, and neglect to subtract the discount from the original price of tickets. No credit was awarded for incorrect responses. "X" was marked if the response was nonscorable or if no response was written. For this item, strategy codes were assigned to student responses (see Table 10).

Table 9
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 2

Points	Response
3	Correct answer: $308.12 OR $308.13 *with* Clear work shown For reference: 5% of $362.50 = $ 18.13 15% of $362.50 = $ 54.38
2	Incorrect answer due to rounding error or no rounding (e.g., $308.125)
1	Correct answer only, no work shown *or* Correct strategy to find percent and solution, however, incorrect answer due to minor error in computation, e.g., neglects subtracting 15% from total *or* minor regrouping error in addition or subtraction *or* calculates one number fact incorrectly in multiplying to find percent
0	Incorrect response
X	Nonscorable or no response

Table 10

Strategy Codes, Grade 5 Problem Solving Assessment, Item 2

Code	Description
11	Uses calculator: finds 15% and subtracts from $362.50 or Finds 85% of $362.50
12	Answer only
13	Uses standard algorithm to find percent and solution: finds 15% and subtracts from $362.50 or Finds 85% of $362.50
16	Incorrect operation used for calculating percent; student divides by 15 instead of multiplying by .15
19C	Random operation that does not make sense, e.g., student adds or subtracts
19D	Student's first step is 29 x 0.15 = 4.35
22	Student uses derived number facts to compute answer, e.g., finds 10% and 5% and adds them together
19	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 3 assesses students' ability to interpret the remainder to a division problem in the context of a problem (see Figure 5). In order to solve the problem, students must calculate the number of cars needed to take the children to the zoo; in interpreting the remainder students must realize that an additional car is needed. One solution strategy is to use division and state another car is needed to carry the additional child. Another strategy is to use multiplication to find a reasonable number of cars.

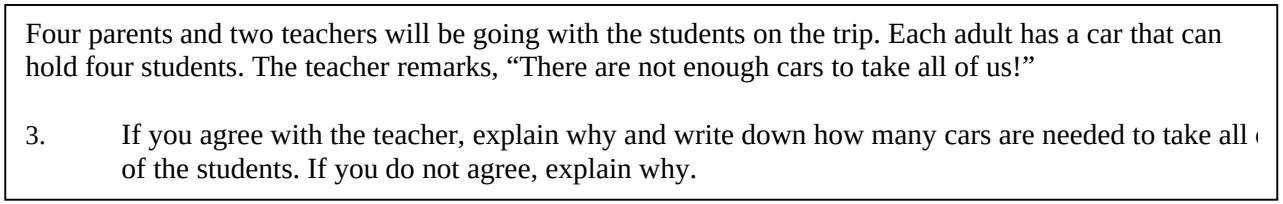


Figure 5. Grade 5 Problem Solving Assessment, Item 3.

The scoring rubric for this item contained 4 categories (see Table 11). For full credit, the student response must have had the correct answer with a conclusion clearly stated. Partial credit was awarded when the correct answer was given, but the conclusion was incorrect, missing, or incomplete. No credit was awarded for incorrect responses. "X" was marked if the response was nonscorable or if no response was written. For this item, strategy codes were assigned to student responses (see Table 12).

Table 11
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 3

Points	Response*
2	Correct conclusion: two more cars are needed or 8 cars are needed *with* Correct computation, 4 x 6 =24, 29 – 24 = 5, or correct explanation
1	Correct computation, 4 x 6 =24, 29 ÷ 4 = 7 R 1, but wrong, missing, or incomplete conclusion: e.g., 5 students are left or that's not enough *Or* Correct conclusion, 8 cars are needed, but unclear, incomplete, or missing explanation
0	Incorrect response (NOTE: this includes agreement with the teacher only)
X	Nonscorable or no response

*An indication of agreement or disagreement with the teacher is irrelevant in scoring this problem.

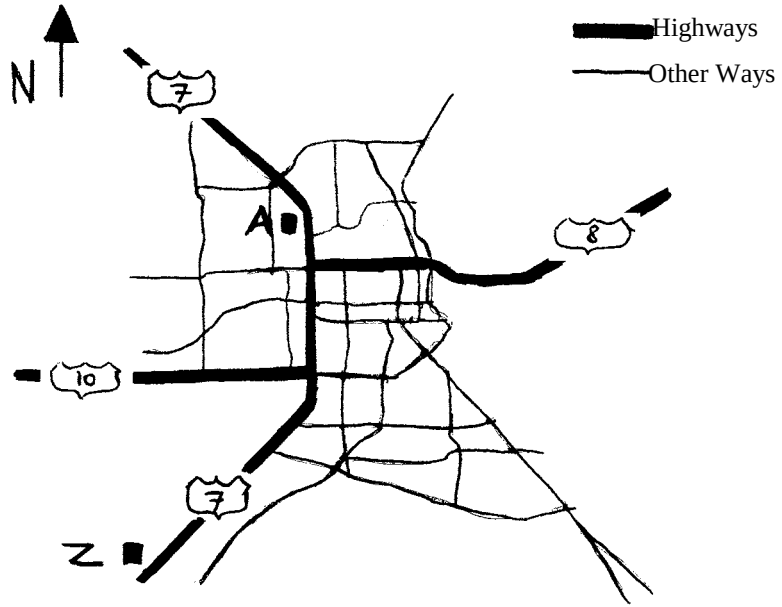
Table 12

Strategy Codes, Grade 5 Problem Solving Assessment, Item 3

Code	Description
12	Answer only
13	Uses division: 29 ÷ 4 = 7 R 1
14	Uses multiplication: 4 x 6 = 24 or 7 x 4 = 28, one more needed or 8 x 4 =32
19B	Descriptive answer based on problem context
21	Uses drawing or diagrams to model cars and occupants
19	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item4 assesses students' ability to interpret the remainder to a division problem in the context of a problem (see Figure 6). In order to solve the problem, students must calculate the number of cars needed to take the children to the zoo; in interpreting the remainder students must realize that an additional car is needed. One solution strategy is to use division and state another car is needed to carry the additional child. Another strategy is to use multiplication to find a reasonable number of cars. The scoring rubric is shown in Table 13. Strategy codes were not assigned for this item.

On the map below, Adams Central School is indicated with the letter A. The Zoo is indicated with the letter Z. The cars will follow Highway 7 to go from the school to the zoo. First, they drive south until there is a curve in Highway 7.



4. In which direction does Highway 7 continue from the curve to the zoo? Write down your answer using degrees or compass directions.

Figure 6. Grade 5 Problem Solving Assessment, Item 4.

Table 13
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 4

Points	Response
1	• 135 degrees (+ or - two degrees measured counter-clockwise from North) • 225 degrees (measured clockwise from North; + or – two degrees) • Make a turn of 45 degrees (+ or – two degrees) at the curve • Southwest
0	Incorrect response
X	Nonscorable or no response

The Monkeys

The second section of the assessment, The Monkeys, centers on feeding the monkeys at the zoo. It contains three questions for students to answer. The first item in this set, Item 5, assess students' abilities to identify appropriate arithmetical calculations; use whole number multiplication; and use multiplication of whole numbers and fractions (see Figure 7). The scoring rubric is shown in Table 14. Strategy codes were not assigned for this item.

At the monkey house, the zookeeper asks Alan to help her feed the monkeys.
There are 15 monkeys. Here is a list of some of the food they get:

food	*average per monkey*	*total for 15 monkeys*
bananas	1	____
cups of cereals	2	____
apples	$\frac{1}{2}$	____
carrots	$\frac{3}{4}$	____

5. Exactly how much of each food is needed for 15 monkeys? Fill in the missing numbers in the table above.

Figure 7. Grade 5 Problem Solving Assessment, Item 5.

Table 14
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 5

Points	Response
3	Four answers correct: 15 30 7 ½ (or 7.5) 11 ¼ (or 11.25 or $45/4$)
2	Three answers correct
1	Two answers correct
0	One or no answers correct
X	Nonscorable or no response

Item 6 uses the answer for problem 5. If a student used an incorrect answer from Item 5 to build an otherwise correct solution for Item 6, the student earned full credit. Item 6 assesses students' abilities to interpret mixed numerals in given context; and to provide correct answer and correct reasoning (see Figure 8). The scoring rubric is shown in Table 15. Strategy codes were not assigned for this item.

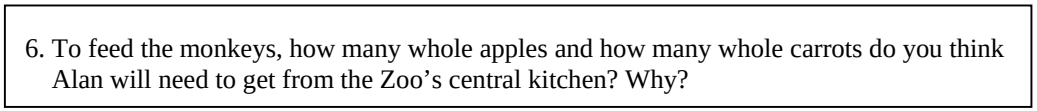


Figure 8. Grade 5 Problem Solving Assessment, Item 6.

Table 15
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 6

Points	Response
2	Correct answer: 8 apples, 12 carrots *With* Clear and correct reasoning: e.g., Alan must round up to the nearest whole number or he will not have enough food so that each monkey gets some.
1	Correct answer, 8 apples, 12 carrots, with incorrect or no explanation given *Or* Correct reasoning, e.g., Alan must round up to the nearest whole number or he will not have enough food so that each monkey gets some) with incorrect answer
0	Incorrect response
X	Nonscorable or no response

Item 7 assesses students' abilities to identify an appropriate method and to express the result as fraction, decimal, or ratio (see Figure 9). The scoring rubric is shown in Table 16, and strategy codes are shown in Table 17.

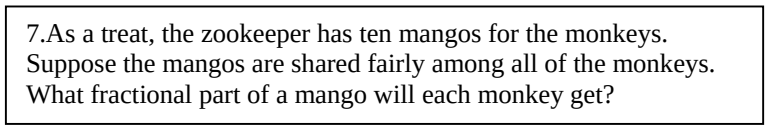


Figure 9. Grade 5 Problem Solving Assessment, Item 7.

Table 16

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 7

Points	Response
2	Correct answer: 2/3 or 10/15 or .666 or 10 parts of 1/15
1	Correct drawing but missing or incorrect numerical conclusion *Or* 10/15 simplified incorrectly
0	Incorrect response
X	Nonscorable or no response

Table 17

Strategy Codes, Grade 5 Problem Solving Assessment, Item 7

Code	Description
12	Answer only
14	Uses multiplication: each monkey gets 10 parts of $1/15^{th}$ of a mango
15	Uses division: 10 ÷ 15 = 10/15 or .667
21	Uses drawing or diagrams to model the problem
23	Uses ratio table
19	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Buildings

The third section of the assessment, Buildings, deals with the layout of the zoo. An overhead map of the zoo is provided which indicates the location of various buildings at the zoo (see Figure 10). On an adjacent page, side views of some of the zoo buildings are shown (see Figure 11). This section contains two questions for students to answer. Item 8 assesses students' abilities to interpret a two-dimensional representation of a three-dimensional situation; and match front and/or side views of buildings with top views of the same buildings on a map (see Figure 11). The scoring rubric is shown in Table 18. Strategy codes were not assigned for this item.

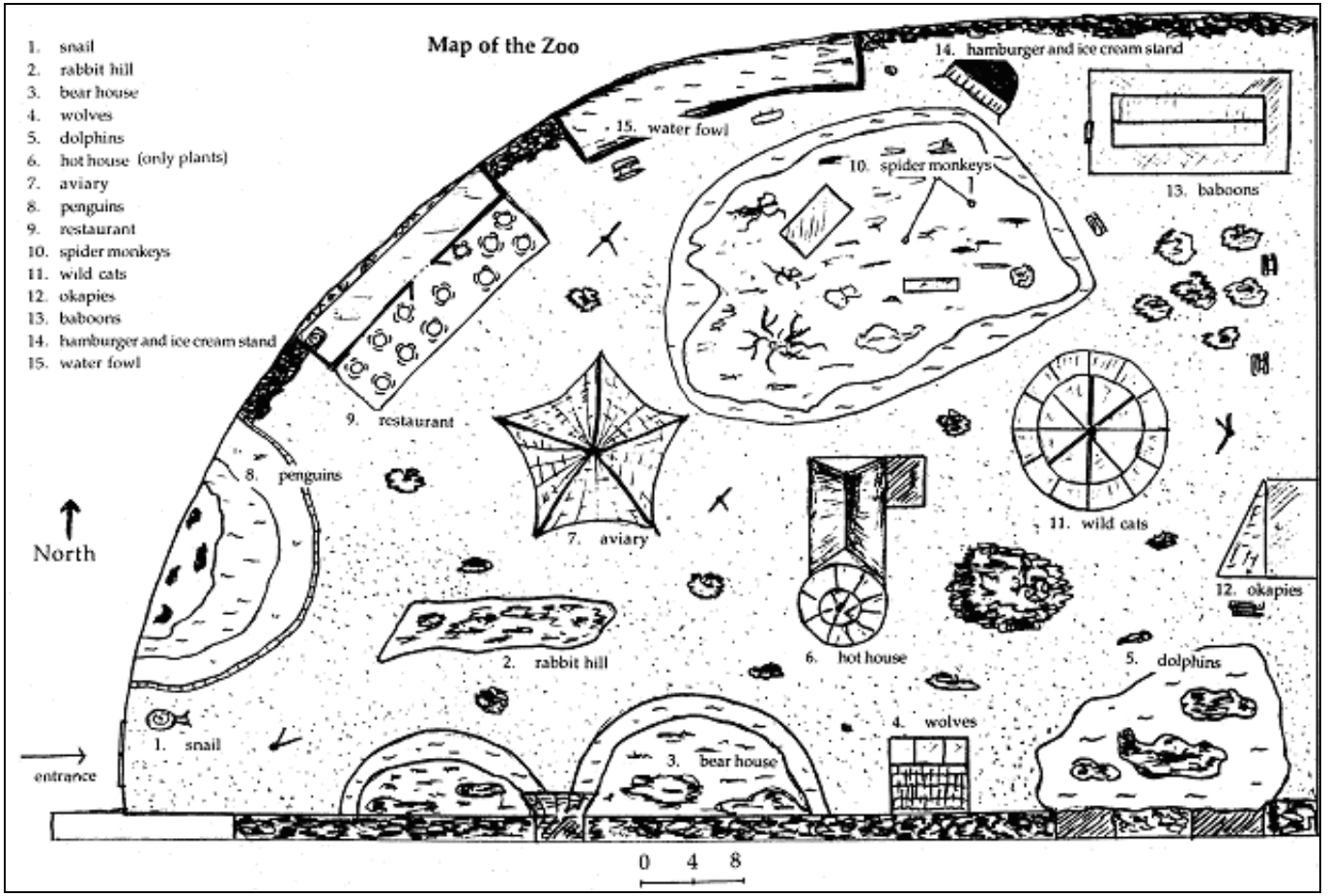
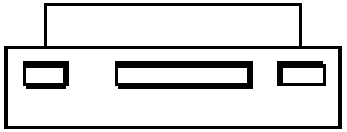
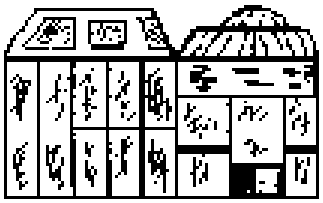


Figure 10. Grade 5 Problem Solving Assessment, map for Items 8 and 9.

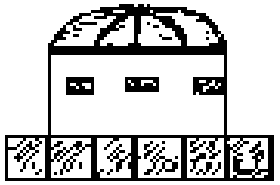
Below you see different side and front views of some of the buildings in the zoo.



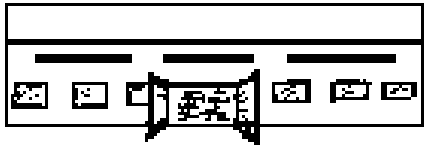
View I



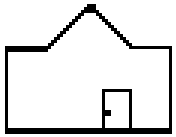
View II



View III



View IV



View V

8. Compare these side or front views with the top views on the map on page 5.
   On the lines below each building, write each building's correct name.

   NOTE: It is possible that the same building may be shown from more than one view.

Figure 11. Grade 5 Problem Solving Assessment, Items 8.

Table 18
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 8

Points	Response
3	Five answers correct: view I - baboons (#13) view II – hot house (#6) view III – wildcats (#11) view IV – restaurant (#9) view V – baboons (#13)
2	Four answers correct
1	Three answers correct
0	Two, one, or no answers correct
X	Nonscorable or no response

Item 9 assesses students' abilities to compare two-dimensional representations; use a visual estimate to determine that the representations are on different scales; and provide a correct answer and correct justification (see Figure 12). The scoring rubric is shown in Table 19, and strategy codes are shown in Table 20.

9. Lori declares, "The side and front views of the buildings shown above are drawn on the same scale as the buildings on the map." Explain why you agree or disagree with Lori's statement.

Figure 12. Grade 5 Problem Solving Assessment, Item 9.

Table 19

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 9

Points	Response
2	Correct answer: no, they are not on the same scale *With* Clear justification: e.g., the measurements of the buildings above are 1 ¼ times larger than those on the map.
1	Correct answer, no, they are not on the same scale, but no explanation given *Or* Correct answer but through visual estimate (e.g., I looked and could tell that the buildings are different sizes) *or* Incorrect conclusion, yes, they are on the same scale, or some are on the same scale and some are not, but with correct strategy to find solution, e.g., used a ruler to measure the buildings
0	Incorrect response
X	Nonscorable or no response

Table 20

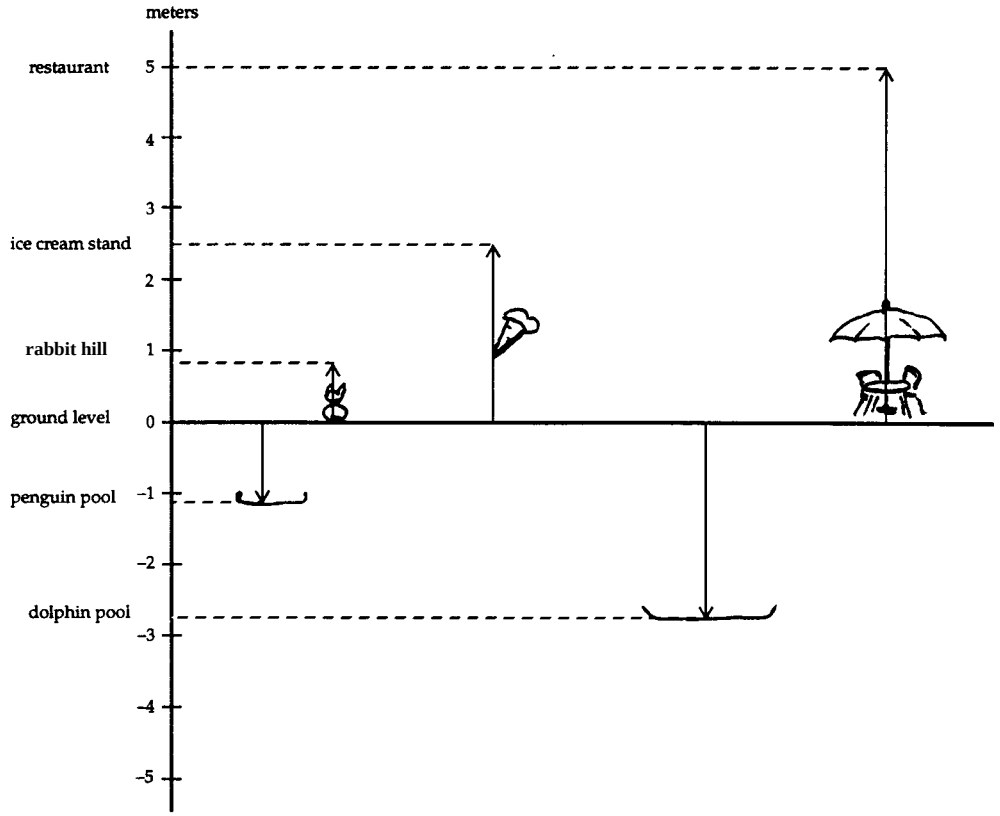
Strategy Codes, Grade 5 Problem Solving Assessment, Item 9

Code	Description
54	Visually compares the size of the buildings in the pictures to the size of the buildings on the map
56	Student incorrectly compares building views instead of scale
58	Answer only; no explanation
73	Measures the buildings on the map and in the pictures
89	Justification other: none of the above
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Pools

The fourth section of the assessment, Pools, focuses on a heights and depths of a variety of buildings and habitats at the zoo. It contains three questions for students to answer. Item 10 assesses students' abilities to read decimal numbers from a scale to determine the height of figure (see Figure 13. Scoring rubrics are shown in Table 21. Strategy codes were not assigned for this item.

Below is a scale, showing the heights and depths of several features on the Zoo map.



10. What is the height of the ice cream stand in meters?

Figure 13. Grade 5 Problem Solving Assessment, Item 10.

Table 21
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 10

Points	Response
2	Correct answer: 2.50 (meters), any answer between 2.45 (meters) and 2.55 (meters) deserves full credit, the word "meters" does not have to be included in the answer
1	Student estimated a measure between 2 and 3 meters that is outside the range of 2.45 to 2.55
0	Incorrect response
X	Nonscorable or no response

Item 11 assesses students' abilities to identify the appropriate arithmetic calculation (addition/subtraction); and use rational numbers (positive, negative decimals) or absolute value to solve a problem (see Figure 14). The scoring rubric is shown in Table 22, and the strategy codes are shown in Table 23.

The bottom of the dolphin pool is now at -2.75 meters. However, the dolphin pool is going to be changed. The new pool will be 1.50 meters deeper than the pool is now.

11. Which number should be used to mark the scale for the bottom of the new dolphin pool?

Figure 14. Grade 5 Problem Solving Assessment, Item 11.

Table 22
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 11

Points	Response
1	Correct answer: - 4.25 or 4.25 (meters) the word "meters" does not have to be included in the answer
0	Incorrect response
X	Nonscorable or no response

Table 23

Strategy Codes, Grade 5 Problem Solving Assessment, Item 11

Code	Description
12A	answer only: - 4.25, 4.25 m below ground
12B	answer only: 4.25
12	answer only: other
13	Direct computation using standard algorithm with an answer of –4.25 -2.75 + - 1.50 = - 4.25
13B	Direct computation using standard algorithm with an answer of 4.25 2.75 + 1.50 = 4.25
16	Student calculated -2.75 + 1.50 = -1.25
19	Other computation strategy
22	Student uses derived numbers 2 + 1 = 3; 0.75 + 0.50 = 1.25; 3 + 1.25 = 4.25
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item 12 assesses students' abilities to correctly place a decimal number on a number line. Item 12 uses the answer for Item 11. If a student used a n incorrect answer to build an otherwise correct solution, the student earned full credit. However, if a student answered 4.25 for Item 11 and correctly marked the depth at –4.25 in Item 12, the student received full credit for Item 12. The scoring rubric for Item 12 is shown in Table 24. Strategy codes were not assigned for this item.

12. On the scale above, mark the bottom level of the new dolphin pool.

Figure 15. Grade 5 Problem Solving Assessment, Item 12.

Table 24

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 12

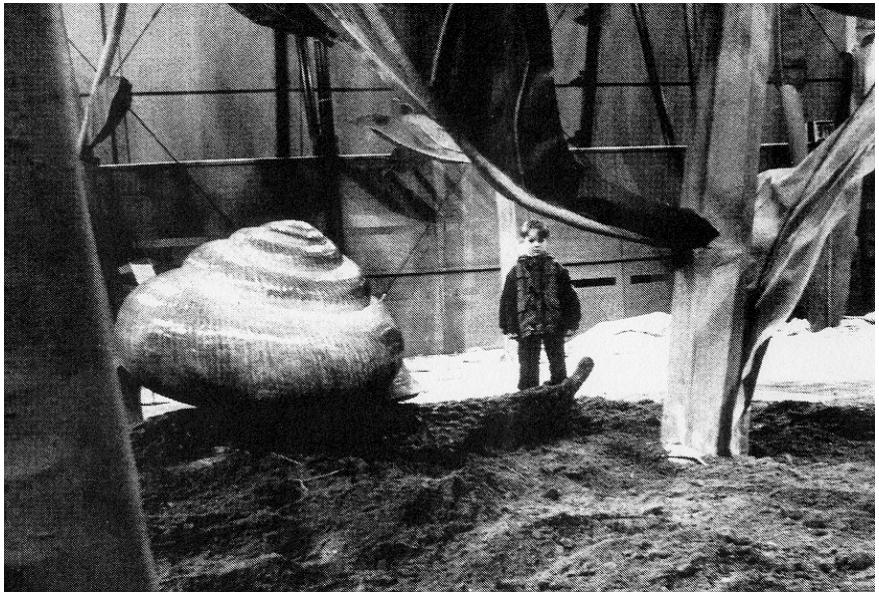
Points	Response*
2	Correct answer: -4.25; mark between -4.2 and -4.3 meters
1	Drawing at any measure between – 4.00 and – 4.50 meters outside of the range of – 4.2 and – 4.3 meters
0	Incorrect response
X	Nonscorable or no response

* Item 12 uses the answer for Item 11. If a student uses an incorrect answer to build an otherwise correct solution, the student deserves full credit. However, if a student answered 4.25 for Item 11 and correctly marks the depth at –4.25 in Item 12, the student receives full credit for Item 12.

Snail

The fifth section, Snail, includes only one task, Item 13, which involves a huge plastic snail that sits near entrance to the zoo. This task assesses students' abilities to interpret a two-dimensional representation of a three-dimensional object; draw a valid conclusion about a mathematical statement based on information gleaned from a photograph and assumptions based on real-world knowledge; and use ratio and scale (see Figure 16). The scoring rubric is shown in Table 25, and strategy codes are shown in Table 26.

Jack, who is about 10 years old, is standing next to a huge plastic snail near the entrance of the zoo. You can see him in the photograph below:



Jack exclaims, "This snail is nearly ten times as big as a real one!"

13. Use the information in the photograph and what you know about snails to explain why you agree or disagree with Jack.

Figure 16. Grade 5 Problem Solving Assessment, Item 13.

Table 25

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 13

Points	Response
3	*The following three components are necessary:* 1) Realistic information about the size of Jack or about snails given: "snail is approximately one inch tall" *or* "A real snail is smaller than Jack's hand." 2) Information from photograph used to make a comparison between Jack and the snail (Note: estimate of snail's height must be reasonably close to Jack's height.) "The plastic snail is as tall as a ten year old, about [53 to 65 inches]." *or* "The snail in the photograph is as tall as Jack." 3) Conclusion: connection between realistic measurements and information from photograph used to make a valid conclusion "So a plastic snail is about 30-70 times bigger than a real snail." *or* "Twenty hands could fit into Jack's height so Jack is wrong."
2	Only two out of the three components (realistic, photograph, & conclusion) are correct e.g., gives realistic and photograph measurements but does not connect the two
1	Only one correct assumption is made, (e.g., uses a ruler to conclude that Jack and the snail are about 3 centimeters in the photo), *or* Gives valid information that is trivial, anecdotal, or descriptive (e.g., snails are usually pretty small so Jack must be right).
0	Student makes no assumptions or comparisons *or* All three components are incorrect NOTE: If student only states that Jack is not correct, no points are given
X	Nonscorable or no response

Table 26
Strategy Codes, Grade 5 Problem Solving Assessment, Item 13

Code	Description
58	No justification (e.g., "I agree with Jack.")
71	Gives trivial or descriptive information w/ no comparison: "snails are pretty small...Jack must be right"
72	Gross comparisons: "Jack is like a thousand (anything ≥ 100) times bigger" *or* "a snail is smaller than my hand"
73	Specific numerical comparisons: Labeling the size of the snail or Jack's height with a specific numerical measure
89	Justification: other, none of the above
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care)
99	No response

Playground

The sixth section of the assessment, Playground, contains a set of four problems that focus on the construction of a playground at the zoo. In the Playground context, students are given a model of the plan for a playground that will be built at the zoo. The playground, whose dimensions remain undecided, will be in the shape of a square and will have a design made from white square tiles with a ring of shaded square tiles as shown in Figure 17. Item 14 assesses students' abilities to analyze a pattern and use counting to determine correct answer (see Figure 17). The scoring rubric is shown in Table 27. Strategy codes were not assigned on this item.

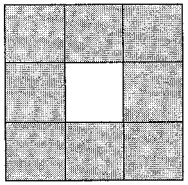


Diagram 1

The owners of the zoo are planning to make a playground in the zoo, between the okapis and the baboons. The playground will be paved with square tiles. The owners are not sure yet how large the playground will be. They start a design with one square tile in the middle and lay a ring of square tiles around it. In the picture above the first ring of tiles is shaded. It consists of 8 tiles.

A second ring of tiles is added. The tiles in the second ring are now shaded in the picture below.

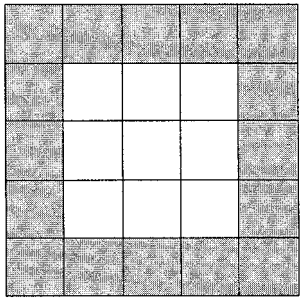


Diagram 2

14. How many tiles are in the shaded ring in Diagram 2?

Figure 17. Grade 5 Problem Solving Assessment, Item 14.

Table 27
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 14

Score	Response
1	Correct answer: 16 tiles no explanation required ("tiles" need not be given)
0	Incorrect response
X	Nonscorable or no response

Item 15 assesses students' abilities to discover and extend a numerical and geometrical pattern presented in a table (see Figure 18). The scoring rubric is shown in Table 28, and the strategy codes are shown in Table 29.

Suppose several more rings of tiles are added to make the playground larger.

15. Fill in the missing values in the table below.

Ring-number	Number of tiles in ring	Total number of tiles in playground
1	8	9
2	____	25
3	____	____
4	____	____

Figure 18. Grade 5 Problem Solving Assessment, Item 15.

Table 28

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 15

Points	Response
3	All numbers are correct ring # # tiles in ring total tiles 1 8 9 2 16 25 3 24 49 4 32 81
2	One wrong or missing number
1	Two wrong or missing numbers
0	Three or more wrong or missing numbers
X	Nonscorable or no response

Table 29

Strategy Codes, Grade 5 Problem Solving Assessment, Item 15

Code	Description
12	answer only; consistent pattern not evident
21	uses drawing
31	pattern: linear (e.g., 8, 16, 24, 32 or 8, 11, 14, 17)
32	pattern: doubling (x 2: 8, 16, 32, 64)
39	pattern: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 16 assesses students' abilities to extend a numerical and geometrical pattern presented in a table (see Figure 19). Item 16 uses the answer for Item 15. If a student used an incorrect answer to build an otherwise correct solution, the student earned full credit. The scoring rubric is shown in Table 30, and the strategy codes are shown in Table 31.

16. How many tiles will be in ring number 9? Explain how you found your answer.

Figure 19. Grade 5 Problem Solving Assessment, Item 16.

Table 30

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 16

* This item uses the answer for problem 15. If a student uses a wrong answer to build an otherwise correct solution, the student deserves full credit.	
Points	Response
4	*Student must have both to receive four points:* Correct answer: 72 (tiles) *with* Clear explanation: 9 x 8 = 72 or "you multiply the ring number by 8" or demonstration of how student found answer (e.g., extends table to ring #9) NOTE: if a student uses the doubling strategy in question 15, the student should find 2048 tiles as the answer here.
3	Clear and complete demonstration of valid strategy, but one minor error in computation leads to incorrect answer, e.g., Completes drawings to ring #9 but makes minor mistake in counting (69-75) or Completes drawings to ring #9 but makes minor error in playground #9 (69-75 tiles in ring) or Extends table to ring #9 w/ one minor computational error *or* Student gives correct answer* for "the total number of tiles" (361) instead of the answer for "number of tiles in ring"
2	Clear and complete demonstration of valid strategy, but two minor errors in computation lead to incorrect answer
1	Correct answer only, no explanation or strategy evident *or* Valid strategy evident but incomplete (student finds the number of tiles in ring #6, #7 or #8 only)
0	Incorrect answer
X	Nonscorable or no response

Table 31

Strategy Codes, Grade 5 Problem Solving Assessment, Item 16

Code	Description
21	Uses diagrams or drawings
33	Extends table
34	Derives answer without extending table (but no general formula given) e.g., 9 • 8 = 72
38	Student generalizes beyond numerical pattern, e.g., R x 8 = T
39	Other strategy used
58	Answer only, no explanation given (e.g., 72 tiles)
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 17 assesses the students' abilities to use knowledge of even and odd numbers and to draw a conclusion based on numerical and geometric patterns (see Figure 20). The scoring rubric is shown in Table 32, and the strategy codes are shown in Table 33.

17. Will the total number of tiles always be even or always be odd? Explain your answer.

Figure 20. Grade 5 Problem Solving Assessment, Item 17.

Table 32

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 17

Points	Response
2	Correct conclusion: odd *with* Clear explanation, e.g., "The sides of the squares (playgrounds) have lengths of , 3, 5, 7, 9 .... tiles, the square of odd numbers is always odd." *or* "The number of tiles in each ring is always even, and there is always one extra in the middle which makes the total odd."
1	Correct conclusion but incomplete explanation or no explanation given *or* Correct explanation w/ wrong or missing conclusion NOTE: "They're all odd in table" is NOT a correct explanation
0	Incorrect response
X	Nonscorable or no response

Table 33
Strategy Codes, Grade 5 Problem Solving Assessment, Item 17

Code	Description
58	Answer only, no explanation given (e.g., “They are always even.”)
71	Justification: gives trivial or descriptive information only
76	Justification: uses evidence from table
77	Justification: number based; explanation using patterns of numbers e.g., “The sides of the playgrounds have lengths that are odd numbers; the square of odd numbers is always odd” or “To get the total number of tiles, you add the number of tiles in the ring to the total number of tiles in the previous playground. The ring is always even and the playground is always odd. An even + an odd = odd.”
78	Justification: visually based; student refers to diagrams e.g., “The number of tiles in each ring is always even, and there is always one extra in the middle which makes the total odd.”
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Questionnaire

The final section of the assessment, Questionnaire, contains five items for students to answer. The context pertains to a questionnaire students designed on their return from the zoo to determine the success of the field trip. The questionnaire and the results, represented with tally marks, are given. The tally sheet is shown in Figure 21. Graph paper is also provided for constructing the bar graph requested in Item 21.

Fifth Grade Zoo Questionnaire

Schooltrip to the zoo

1. Did you like going to the zoo?

yes	𝍸 𝍸 𝍸 𝍸 𝍸 /
no	///

2. Would you like to have a job in a zoo later?

yes	𝍸 ///
no	𝍸 𝍸 //
don't know	𝍸 ////

3. Which animals did you like best? You may name two animals.

rabbits	//
bears	𝍸
wolves	////
dolphins	𝍸 𝍸 ////
penguins	𝍸 ////
monkeys	𝍸 𝍸 𝍸 𝍸
wild cats	////

4. Give a mark for this schooltrip as a whole on a 1-10 scale.

mark 3	-
mark 4	-
mark 5	/
mark 6	///
mark 7	𝍸 ///
mark 8	𝍸 𝍸 ////
mark 9	//
mark 10	/

Figure 21. Grade 5 Problem Solving Assessment, questionnaire tally sheet for Item 18.

Item 18 assesses the students' abilities to analyze statistical data and interpret survey results (see Figure 22). The scoring rubric is shown in Table 34. Strategy codes were not assigned for this item.

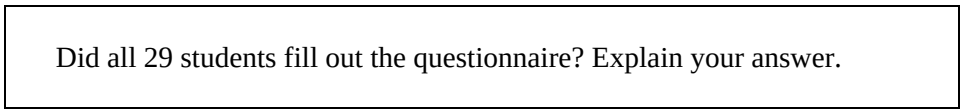


Figure 22. Grade 5 Problem Solving Assessment, Item 18.

Table 34
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 18

Points	Response
1	Correct answer: yes, *with* Clear explanation, e.g., the tallies on question #1 add up to 29
0	Incorrect response *or* "Yes," with incomplete, unclear, or no explanation
X	Nonscorable or no response

Item 19 assess students' abilities to analyze statistical data and draw conclusions based on survey results (see Figure 23). The scoring rubric is shown in Table 35. Strategy codes were not assigned for this item.

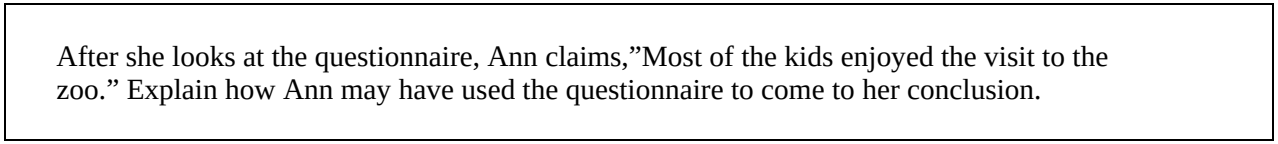


Figure 23. Grade 5 Problem Solving Assessment, Item 19.

Table 35
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 19

Points	Response
2	Correct explanation with reference to numerical results from the questionnaire "On question #1, 26 students out of 29 said they liked going to the zoo." *or* "25 out of 29 students rated the trip a 7 or higher on a scale of 1 to 10"
1	Explanation that alludes to pertinent information from the questionnaire but does not include specific numerical data, e.g., "Because most said yes," *or* "She looked at the question that asked if they liked the zoo."
0	Incomplete, incorrect, or irrelevant response, e.g., "She looked at the questionnaire," *or* "I think she's right because they participated in the votes and they liked a lot of the animals."
X	Nonscorable or no response

Item 20 assesses students' abilities to identify an appropriate graph to graphically represent statistical information and to explain why the graph is appropriate (see Figure 24). The scoring rubric is shown in Table 36, and strategy codes are shown in Table 37.

> Suppose you were asked to create a graph from the information in question number 2 on the questionnaire. Which graph would you choose, a line graph, a pie graph, or a bar graph? Explain why the graph you chose is the best one to display the information.

Figure 24. Grade 5 Problem Solving Assessment, Item 20.

Table 36

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 20

Points	Response
2	Correct answer: "I would use a pie graph" "I would use a bar graph" "I would use either a pie or bar graph" *with* Correct explanation: explanation should highlight the use of the graph in representing numbers For example, if student selects pie graph… "each piece of the pie graph represents a separate category of data" "pie graphs and bar graphs can compare categories of data"
1	Correct answer w/ wrong or missing explanation, e.g., "Bar graphs are easy to make."
0	Incorrect response
X	Nonscorable or no response

Table 37

Strategy Codes, Grade 5 Problem Solving Assessment, Item 20

Code	Description
58	No justification given (e.g., I would use a pie graph.)
71	justification: gives trivial or descriptive information, (e.g., "Bar graphs are easiest to make.")
74	justification: gives specific description of chosen graph's purpose
75	justification: extends Code 74 to compare purposes of two or more items (e.g., line graphs with bar graphs, pie graphs, etc.)
89	justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care)
99	No response

Item 21 assess students' abilities to construct a bar graph to represent statistical data and provide four correct criteria (labeled axes, consistent scales, correct bar lengths and consistent widths, and labeled bars; see Figure 25). Scoring rubrics are shown in Table 38.

Draw a bar graph to show the results of question number 3 of the questionnaire. Use the grid paper on page 13 to make your graph.

NOTE: The Fifth Grade Zoo Questionnaire is copied on page 12 to help you make your graph.

Figure 25. Grade 5 Problem Solving Assessment, Item 21.

Table 38

Scoring Rubric, Grade 5 Problem Solving Assessment, Item 21

Points	Response
4	Graph includes the following criteria: • both axes properly labeled • consistent scale given • lengths of bars correct with consistent width • bars properly labeled NOTE: bars are considered properly labeled if letters are used as labels, e.g., M (for monkey); *however, wolves and wildcats must be distinguished in some way, e.g., "Wo" and "Wi"*
3	Three criteria correct
2	Two criteria correct
1	One criterion correct only
0	Incorrect response
X	Nonscorable or no response

Item 22 assesses students' abilities to analyze statistical data and draw accurate conclusions (see Figure 26). The scoring rubric for this item is shown in Table 39.

Based on the bar graph you made, give at least two conclusions that you can make about the results of question number 3 on the questionnaire.

Figure 26. Grade 5 Problem Solving Assessment, Item 22.

Table 39
Scoring Rubric, Grade 5 Problem Solving Assessment, Item 22

Points	Response
2	Two correct conclusions *For reference*: In order from most liked to least liked animals you find: Monkeys, dolphins, penguins, bears, wolves/wildcats, rabbits,
	Other correct conclusions: The monkeys were liked best The dolphins were liked second best Wolves and wildcats are liked equally much The rabbits were liked least
1	One correct conclusion only
0	Incorrect response
X	Nonscorable or no response

Conclusion

Overall, the fifth-grade PSA was designed to include questions from all four content strands with a concentration on number concepts. This assessment contained approximately an equal number of points on Levels I and II and considerably fewer Level III items.

Grade 6 Problem Solving Assessment

The overarching theme of the sixth-grade PSA is a local park and two students who work as volunteers in the park's ranger station. The sixth-grade assessment contains six subsections, each providing specific contexts and problems for students to solve related to the park and to the student volunteers' responsibilities there.

The sixth grade assessment was designed to include questions from all strands of mathematics with a concentration on number concepts. The assessment contains an approximately equal distribution of points between Levels I and II and, reflecting the structure of de Lange's pyramid, considerably fewer Level III items (see Table 40).

Table 40
Points (and Percents) for Strands and Levels on the Sixth Grade Problem Solving Assessment*

LEVEL	STRAND				
	Number	Geometry	Algebra	Statistics	Total
I	17 (33.3%)	1 (2%)	9 (17.7%)	5 (9.8%)	32 (62.8%)
II	13 (25.5%)	7 (13.7%)	17 (33.3%)	2 (3.9%)	39 (76.5%)
III	2 (3.9%)	0 (0%)	2 (3.9%)	0 (0%)	4 (7.8%)
Total	32 (62.8%)	8 (15.7%)	28 (54.9%)	7 (13.7%)	

*Percents >100 because some questions fall into multiple categories.

The sixth-grade PSA consists of 24 constructed response items for a total of 51 points. The items on the assessment often require students to demonstrate their solution strategies in order to earn full credit. Scoring rubrics were created for each problem, allowing scorers to assign points for accuracy of answers and thoroughness of explanation. Moreover, many items have an accompanying strategy rubric which enables scorers to identify the strategies that students use to solve problems.

The Ranger Station

The first section of the assessment, The Ranger Station, revolves around the number of visitors who come to the park during a given week. This section contains six questions for students to answer. A partial item classification for items in this context is shown in Table 41. The entire item classification matrix for this assessment is found in Appendix B.

Table 41
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Ranger Station Context of the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
1	Statistics/Number	I	Easy	1	Computing the mean
2	Statistics	II	Medium	2	Reasoning about the meaning of mean
3	Number	I	Easy	3	Calculations (addition, subtraction, multiplication, division) with percentages and fractions
4	Number	I	Medium	2	Calculations (addition, subtraction, multiplication, division) with fractions
5	Number	I	Easy	2	Addition and subtraction of fractions with unlike denominators
6	Number	II	Medium	3	Addition and subtraction of integers Problem solving to find missing information on a chart

Item 1 assesses students' abilities to identify an appropriate arithmetic calculation to determine mean and use whole numbers in the calculation (see Figure 27). The scoring rubric is shown in Table 42. Strategy codes were not assigned for this item.

Beth and Gill work as volunteers at the ranger station in Wingra Park. They keep records of the number of visitors who come to the park. The park is open seven days a week. In the first week of April, 134 people visited the park.

1. What is the average number of people per day who visited the park this week in April?

Figure 27. Grade 6 Problem Solving Assessment, Item 1.

Table 42
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 1

Points	Response
1	20, 19.1, 19, $19\frac{1}{7}$ or equivalent value
0	Incorrect response
X	Nonscorable or no response

Item 2 assesses students' abilities to interpret a problem situation based on the mean; describe mean conceptually; and provide a correct answer and correct explanation (see Figure 28). The scoring rubric and list of strategy codes for this item are shown in Tables 43 and 44, respectively.

2. Is it possible that 40 visitors came to the park on Tuesday during the first week of April? Explain why or why not.

Figure 28. Grade 6 Problem Solving Assessment, Item 2.

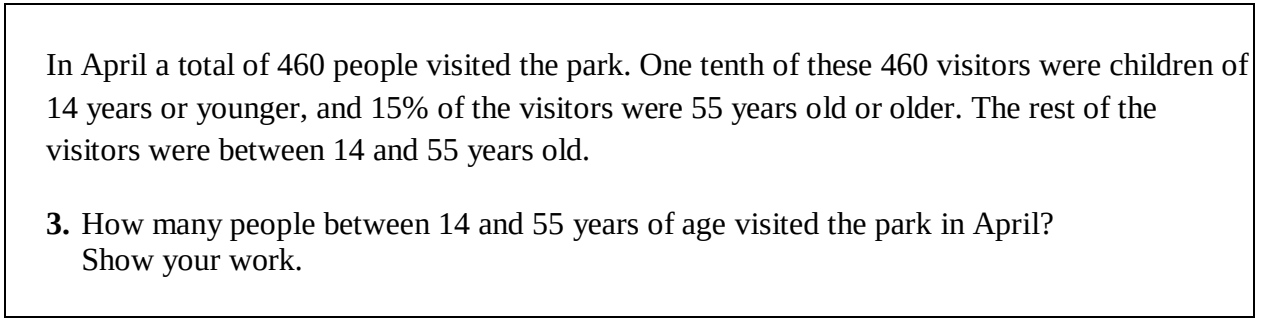
Table 43
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 2

Points	Response
2	One point for the correct answer: "Yes" One point for correct explanation: e.g., "Some days could have more and some days could have less"
1	Correct answer of yes w/ incomplete, incorrect, or no explanation, e.g., "Yes, but then the rest of the days would have to be lowered to about 16" *or* Correct explanation only with implied answer of yes: e.g., "It could be really busy on that day and then really slow for 3 or 4 days."
0	Incorrect response
X	Nonscorable or no response

Table 44
Strategy Codes, Grade 6 Problem Solving Assessment, Item 2

Code	Description
53	Explanation: with a diagram, drawing, or graph
55	Explanation: conceptual description of mean i.e., student notes that the numbers can vary about the mean; it is simply a one-number summary for a group of numbers. (e.g., "some days could have more than the average, some could have less" *or* "if less come the other days, it still works out")
58	Explanation: answer only, no explanation
77	Explanation: number based; (e.g., student gives specific numbers for each day, including 40)
19B	Contextual answer: cites real-world information unrelated to mathematics involved e.g., "It could have been a really sunny day and a lot of people wanted to go to the park."
59	Explanation: other, none of the above
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care)
99	No response

Item 3 assesses students' abilities to identify an appropriate series of arithmetic calculations; use whole numbers, fractions, and percent; and provide a correct answer and supporting work (see Figure 29). The scoring rubric and the list of strategy codes for this item are shown in Tables 45 and 46, respectively.


In April a total of 460 people visited the park. One tenth of these 460 visitors were children of 14 years or younger, and 15% of the visitors were 55 years old or older. The rest of the visitors were between 14 and 55 years old.

3. How many people between 14 and 55 years of age visited the park in April? Show your work.

Figure 29. Grade 6 Problem Solving Assessment, Item 3.

Table 45
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 3

Points	Response
3	Correct answer: 345 people *with* Appropriate work shown (e.g., 10% + 15% = 25%; 100%-25% = 75%; 0.75 x 460 = 345).
2	Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., .75 x 460 = 340 (instead of 345). *or* Answer of 75% with work shown.
1	Correct strategy used, but two minor errors in calculations are computed *or* One major error (e.g., neglects to subtract 115 from total) *or* Correct answer only (345 people) with no work shown
0	Incorrect response
X	Nonscorable or no response

Table 46

Strategy Codes, Grade 6 Problem Solving Assessment, Item 3

Code	Description (NOTE: Focus on strategy student uses for computation other than 10 percent)
11	Computes 15%, 25%, or 75% with calculator (e.g., 460 x .15 = 69)
12	Answer only (no other work shown)
13	Uses standard algorithm for multiplying by a decimal (e.g., partial products apparent in student work, regrouping, etc.)
16	Incorrect operation used: incorrectly tries to find percent (e.g., 460 ÷15)
20	Converts to fraction to compute 15%, 25%, or 75% (e.g., 75% = $\frac{3}{4}$)
21	Uses diagram or drawing to compute percent 15%, 25%, or 75%
22	Uses derived numbers to compute 15%, 25%, or 75%
23	Uses ratio table to compute 15%, 25%, or 75%
19C	Random operation that does not make sense, e.g., 55-14=41
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 4 assesses students' abilities to identify an appropriate series of arithmetic calculations; multiply a fraction by a fraction; and provide correct answer with clear supporting work (see Figure 30). The scoring rubric and the list of strategy codes for this item are shown in Tables 47 and 48, respectively.

To try to get more people to visit the park, Beth and Gill put ads on the radio and on TV. During April, they surveyed the visitors who came to the park to see if their ads had succeeded in getting new visitors to come to the park. Three-fifths of the 460 visitors said they had seen the ad on TV. Of the people that saw the ad on TV, one-fourth had also heard the ad on the radio.

4. How many visitors in April had seen the ad on TV *and* heard the ad on the radio? Show your work.

Figure 30. Grade 6 Problem Solving Assessment, Item 4.

Table 47

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 4

Points	Response
2	Correct answer: 69 people *with* Appropriate work shown
1	Answer only, 69 people, w/ no work shown *or* Answer of 3/20 *or* 69/460 with work shown *or* Incorrect answer resulting from one minor error (e.g., $\frac{2}{5}$ of 460 = 184 people, ¼ of 184 people = 46 people)
0	Incorrect response
X	Nonscorable or no response

Table 48

Strategy Codes, Grade 6 Problem Solving Assessment, Item 4

Code	Description *NOTE: If student does not compute fraction of a fraction, code as 16. Otherwise, use other strategy codes.
16*	Student misinterprets problem and adds fractions or products together e.g., $460 \times \frac{3}{5} = 276$ $460 \times ¼ = \underline{115}$ 391 people *or* $\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$ $\frac{17}{20} \times 460 = 391$ people
11	Computes with calculator
12	Answer only (no work shown)
13	Uses standard algorithm to compute with fractions – e.g., $\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{460}^{92}}{1} = 276$
20	Converts to rational numbers *and uses a standard algorithm* to compute
21	Uses diagrams or drawings to compute
22	Uses derived numbers (e.g., finds 1/5 of 460, then multiplies by 3)
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 5 assesses students' abilities to identify an appropriate series of arithmetic calculations; add and subtract fractions; rewrite fractions with common denominators to compute; and provide correct answer with clear supporting work shown (see Figure 31). The scoring rubric and list of strategy codes for this item are shown in Tables 49 and 50, respectively.

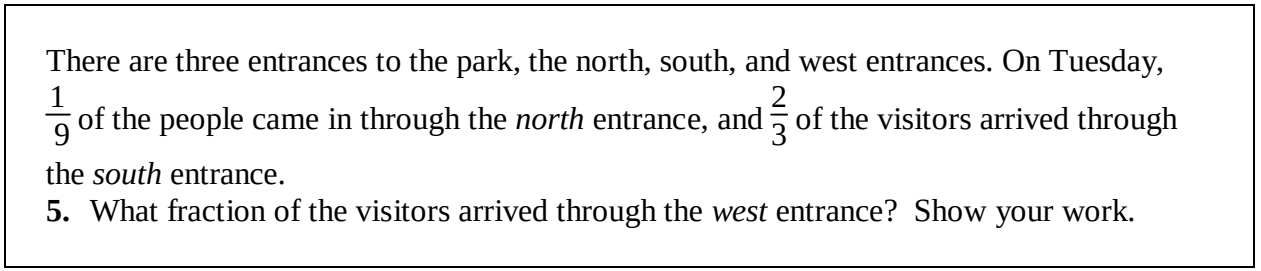


Figure 31. Grade 6 Problem Solving Assessment, Item 5.

Table 49

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 5

Points	Response
2	Correct answer: $\frac{2}{9}$ or .22 *with* Correct work shown (e.g., $\frac{1}{9}+\frac{6}{9}=\frac{7}{9}$; $\frac{9}{9}-\frac{7}{9}=\frac{2}{9}$). NOTE: Last step of subtracting $\frac{7}{9}$ need not be shown for full credit.
1	Answer only, $\frac{2}{9}$ or .22, with no work shown *or* One minor error in computation, e.g., $\frac{1}{9}+\frac{6}{9}=\frac{8}{9}$; $\frac{9}{9}-\frac{8}{9}=\frac{1}{9}$ *or* First step only (e.g., $\frac{1}{9}+\frac{6}{9}=\frac{7}{9}$) with appropriate work shown *or* Uses the 460 people (from problem 4) to get an answer of about 102 people with appropriate work shown (and no reference to $\frac{2}{9}$)
0	Incorrect response
X	Nonscorable or no response

Table 50

Strategy Codes, Grade 6 Problem Solving Assessment, Item 5

Code	Description
12	Answer only
13	Rewrites fractions with common denominators to add/subtract
16A	Student computes operation incorrectly (no conceptual basis), e.g., $\frac{2}{3} + \frac{1}{9} = \frac{3}{12}$
19A	Use numerical information not included in the problem to support computation (uses number of people rather than fractions in computation)
20	Converts rational numbers to compute: fractions to decimals to percents
21	Uses diagrams or drawings to compute
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 6 assesses students' abilities to interpret a problem situation; add and subtract integers; use information in chart; identify missing information; and provide correct answer with clear work shown (see Figure 32). The scoring rubric is shown in Table 51. Strategy codes were not assigned for this item.

Beth and Gill also keep records of the cash money that is at the ranger station. For each day they keep track of how many dollars came in or out. The records for one week are shown in the table below. On Wednesday Beth and Gill forgot to list the information on their record.

Day of Week	Total Dollars in the Morning	Dollars In/Out During the Day	Total Dollars at the End of the Day
Sunday	$130.00	$35.00 in	$165.00
Monday	$165.00	$40.00 out	
Tuesday		$30.00 in	
Wednesday			
Thursday		$55.00 out	
Friday		$20.00 in	
Saturday		$70.00 in	$155.00

6. Find the missing information and show how you found your answer.

Figure 32. Grade 6 Problem Solving Assessment, Item 6.

Table 51

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 6

Points	Response
3	Correct answer: $35 out ("out" must be included in answer) *with* Correct work shown, e.g., fills in chart or written explanation
2	Answer of "$35 in" or just "$35" with appropriate work shown *or* Correct totals at beginning ($155) and end ($120) of Wednesday with work shown, but no answer for dollars in/out *or* One minor error in computation (e.g., Friday $60 $20.00 in $85)
1	Correct answer, $35 out, only with no work *or* Total at the beginning of Wednesday, $155, with work shown *or* Total at the end of Wednesday, $120, with work shown
0	Incorrect response
X	Nonscorable or no response

A Patio

The second section of the assessment, A Patio, relates to the construction of a patio to be built in the park. This section contains three questions. These questions are part of a series of contexts used on all four grade level assessments to investigate growth over time of students' understanding of geometric and algebraic concepts and to trace students' use of strategies. Students are given models of the proposed patio and asked to reason about the algebraic and geometric patterns the patios show. The patios grow in the pattern shown in Figure 33. A partial item classification for items in this context is shown in Table 41. The entire item classification matrix for this assessment is found in Appendix B.

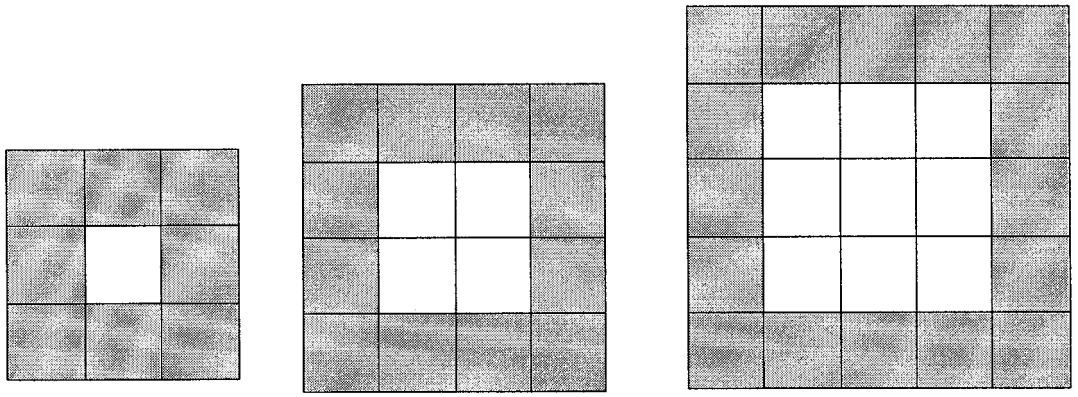


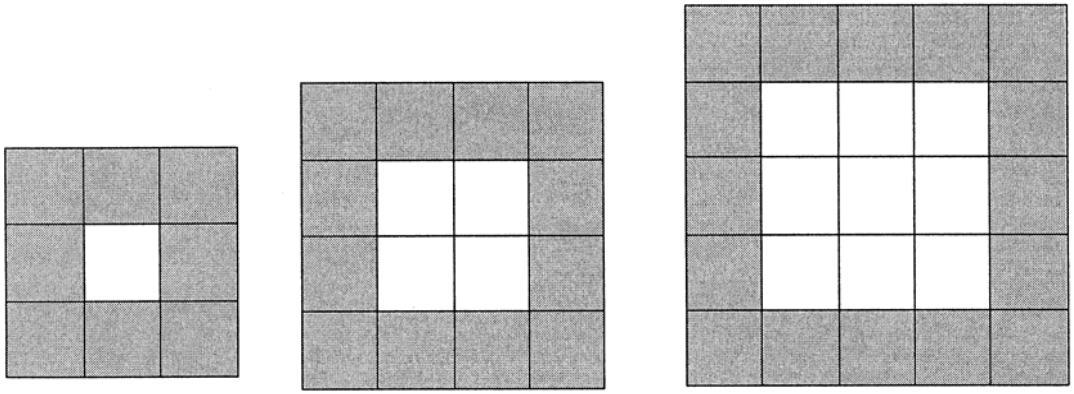
Figure 33. Patterns of Patio Tiles for Items 7, 8, and 9.

Table 52
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Patio Context on the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
7	Algebra/Geometry	I	Easy	1	Recognition and application of a pattern Analysis of a pattern
8	Algebra/Geometry	II	Easy	2	Recognition and application of a pattern Generalizing a pattern Or Extending/generalizing a pattern
9	Number/Algebra	II	Difficult	2	Using knowledge/properties of odd and even numbers to draw and support conclusions about geometrical and numerical patterns

Item 7 assesses students' abilities to interpret and extend a pattern (see Figure 34). The scoring rubric and the list of strategy codes for this item are shown in Tables 53 and 54, respectively.

The rangers are planning to build a patio in the park. The patio will be paved with white and gray square tiles arranged in a pattern. The pattern will be square with white tiles in the middle and gray tiles on the edge. The rangers have decided on the design, but not on the size. Three possible sizes are shown below.



size 1 size 2 size 3

7. What is the *total* number of tiles that will be in size 4?

Figure 34. Grade 6 Problem Solving Assessment, Item 7.

Table 53

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 7

Points	Response
1	Correct answer: 36 or 6^2 or "20 gray and 16 white". The word “tiles” is not needed for full credit. Student does not need to show work.
0	Incorrect response *Or* Drawing only
X	Nonscorable or no response

Table 54

Strategy Codes, Grade 6 Problem Solving Assessment, Item 7

Code	Description
12	Answer only (i.e., just 36 is written)
21	Uses diagrams or drawings to compute
19	Other computation strategy, e.g., 6^2
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 8 assesses students' abilities to interpret and extend a pattern and provide correct answer and correct explanation (see Figure 35). The scoring rubric and strategy codes are shown in Tables 44 and 56, respectively.

8. How many *white* tiles will be in size 9?
Show how you found your answer.

Figure 35. Grade 6 Problem Solving Assessment, Item 8.

Table 55
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 8

Points	Response
2	Correct answer: 81 or 9^2. The words "white tiles" are not needed for full credit. w*ith* Correct explanation (e.g., 9 x 9 = 81)
1	Correct answer w/ incomplete, incorrect, or no explanation o*r* Correct explanation without answer (e.g., "It's just the size times itself" or drawing only) *or* Correct drawing with a minor counting error
0	Incorrect response
X	Nonscorable or no response

Table 56

Strategy Codes, Grade 6 Problem Solving Assessment, Item 8

Code	Description
21	Uses diagrams or drawings to compute
33	Creates a table up to patio size #9
34	Derives answer without extending table (but no general formula given), e.g., 9 x 9 = 81 *or* 9^2 (answer does not need to be correct)
38	Student generalizes beyond numerical pattern (e.g., size x size)
39	Pattern: other
58	No explanation (e.g., 81 or other answer only)
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 9 assesses students' abilities to interpret and generalize a pattern; reason about odd and even numbers; and provide correct explanation (see Figure 36). The scoring rubric and strategy codes are shown in Tables 57 and 58, respectively.

9. Explain why the number of gray tiles is always even.

Figure 36. Grade 6 Problem Solving Assessment, Item 9.

Table 57
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 9

Points	Response
2	Correct explanation: e.g., "Every gray tile has a match on the opposite side of the square" *or* "There is an equal number on each side and any number times 2 is even"
1	Students provide correct but incomplete explanation, e.g., "I know that the number of gray tiles will always be even because it increases by four each time" (student did not include the first number) "The gray tiles are all even in sizes 1, 2, and 3, and this pattern will continue for the rest of the playgrounds."
0	Incorrect response
X	Nonscorable or no response

Table 58
Strategy Codes, Grade 6 Problem Solving Assessment, Item 9

Code	Description
71	Justification: Gives trivial or descriptive information only, e.g., "Yes, they are always even" *or* "The gray tiles surround the white tiles so they are always even" *or* "It is the perimeter."
76	Justification: Answer references evidence in student-created table
77	Justification: Number based, *explanation using computation or numbers* e.g., "4 x ? + 4 = even number" *or* "In size 3 it is 3 + 3 + 3 + 3 + the 4 corners = 16 (even)"
78	Justification: Visually based, *student refers to features of diagrams* e.g., "Every gray tile has a match on the opposite side of the square" *or* "It is a square so it must be even."
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care)
99	No response

Fly One Day

The third section of the assessment, Fly One Day, concerns birds' flying habits. This section contains four questions. A partial item classification matrix is shown in Table 59. The entire item classification matrix is found in Appendix B.

Table 59
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Fly One Day Context on the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
10	Number	I	Easy	1	Multiplication of a whole number and a decimal
11	Algebra/Number	I	Medium	2	Understanding a formula Order of operations Multiplication of a whole number and a decimal Subtraction of decimals
12	Algebra/Number	I	Medium	2	Understanding a formula Order of operations Division of a whole number by a decimal Subtraction of decimals
13	Algebra	II	Difficult	3	Interpretation of problem context Understanding and calculation of rate Multiplication of large whole numbers by fractions Division of large whole numbers by powers of ten Addition and subtraction of whole numbers

Item 10 assesses students' abilities to interpret a problem situation; identify an appropriate arithmetic calculation (multiplication); and use whole and decimal numbers to determine a correct answer (see Figure 37). The scoring rubric is shown in Table 60. Strategy codes were not assigned for this item.

Birds lose weight when they fly. For example, a swan loses about 0.1 kilogram of weight for each hour flying.

10. How much weight has a swan lost after 12 hours of flying?

Figure 37. Grade 6 Problem Solving Assessment, Item 10.

Table 60
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 10

Points	Response
1	Correct answer : 1.2 (kg); the word "kg" is not needed for full credit
0	Incorrect response
X	Nonscorable or no response

Item 11 assesses students' abilities to use a formula; calculate (multiply and subtract) with whole and decimal numbers; use order of operations; provide correct answer;r and show appropriate supporting work (see Figure 38). The scoring rubric is shown in Table 61. Strategy codes were not assigned for this item.

When you know the weight of a swan at the beginning of a flight—the starting weight in kilograms—you can compute the landing weight with the following formula:

landing weight = starting weight – N × 0.1

In this formula the landing weight is the weight in kilograms after a flight of **N** hours.

11. If a swan has a starting weight of 10.5 kilograms, how much will it weigh after flying 7 hours? Show your work.

Figure 38. Grade 6 Problem Solving Assessment, Item 11.

Table 61
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 11

Points	Response
2	Correct answer: 9.8 (kg.); the word "kg" is not needed for full credit *with* Clear work shown: (e.g., 7 x 0.1 = 0.7, 10.5 – 0.7 = 9.8)
1	Correct answer, 9.8, only or Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., 10.5 – 0.7 = 9.6 or Partial answer, 0.7 (kg), with appropriate work shown
0	Incorrect response
X	Nonscorable or no response

Item 12 assesses students' abilities to solve for the unknown in a given formula; calculate (subtract and divide) with whole and decimal numbers; and provide correct answer with appropriate supporting work (see Figure 39). The scoring rubric is shown in Table 62. Strategy codes were not assigned for this item.

A swan weighed 13 kilograms. After a flight its weight dropped to 11.9 kilograms.

12. How many hours has this swan been flying? Show your work.

Figure 39. Grade 6 Problem Solving Assessment, Item 12.

Table 62
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 12

Points	Response
2	Correct answer: 11 (hours); the word "hours" is not needed for full credit *with* Clear work shown (e.g., 13 – 11.9 = 1.1, 1.1 ÷ .1 = 11). NOTE: Last step of division by .1 need not be shown for full credit.
1	Answer, 11 (hours), only with no work shown *or* Partial answer, 1.1 (kg), with appropriate work shown *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., (13 – 11.9 =1.2 kg, 12 hours)
0	Incorrect response
X	Nonscorable or no response

Item 13 assesses students' abilities to interpret a problem situation; identify appropriate arithmetic calculations; use whole numbers and fractions; and determine correct answer with clear supporting work (see Figure 40). The scoring rubric is shown in Table 63. Strategy codes were not assigned for this item.

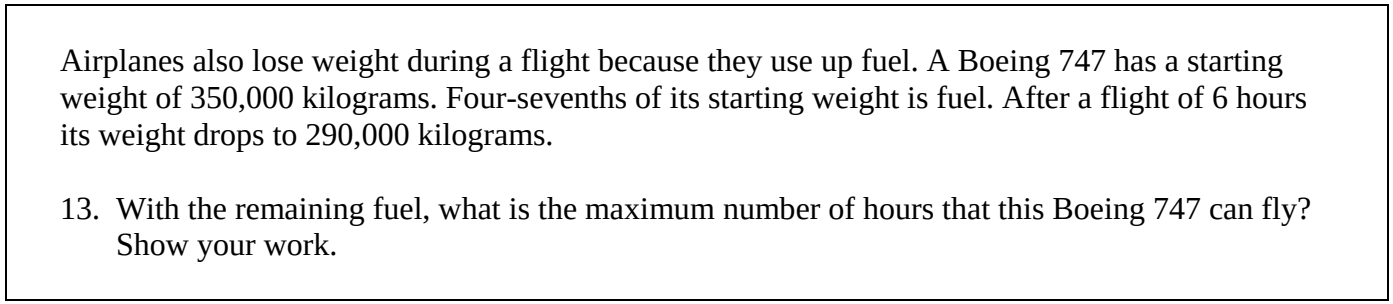


Figure 40. Grade 6 Problem Solving Assessment, Item 13.

Table 63

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 13

Points	Response
3	Correct answer: 14 (hours); the word "hours" is not needed for full credit *with* Clear work shown For full credit, students must compute or show evidence of using ✈ starting weight of fuel (200,000 kg.) ✈ weight of fuel used (60,000 kg.) ✈ fuel used per hour (10,000 kg/hr)
2	Work shows correct strategy, however, starting weight of fuel or weight of fuel used not computed *or* Work shows correct strategy, however, incorrect answer resulting from one missing step, e.g., the student neglects to divide 140,000 by 10,000 *or* One minor computation error, (e.g., student multiplies $\frac{4}{7}$ x 350,000 = 240,000), with work shown
1	Correct answer w/ no work shown *or* Work shows correct strategy, however, incorrect answer resulting from two minor computational errors (e.g., student finds weight of airplane to be 200,000 kg and finds kg of fuel/hr to be 6000)
0	Incorrect response
X	Nonscorable or no response

* 20 hours is also an acceptable answer *if* 14 + 6 = 20 is evident in the work shown

Bird Watchers Bulletin

The fourth section of the assessment, Bird Watchers' Bulletin, details the publication, advertising for, and cost of a magazine published by the workers at the ranger station. This section contains two questions. A partial item classification matrix is shown in Table 64. The entire item classification matrix is found in Appendix B.

Table 64
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the The Bird Watchers' Bulletin Context on the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
14	Number	I	Easy	2	Multiplication of a decimal by a whole number Subtraction of decimals
15	Number/Algebra (why is this A?)	II	Difficult	2	Problem solving to compare two situations Calculations with decimals

Item 14 assesses students' ability to interpret a problem situation; identify appropriate arithmetic calculations (multiplication and subtraction); and use whole and decimal numbers (see Figure 41). The scoring rubric is shown in Table 65. Strategy codes were not assigned for this item.

The ranger station publishes a magazine about birds called *Bird Watchers' Bulletin*. The price of one issue is $2.95. To increase the number of issues of *Bird Watchers' Bulletin* that are sold, the rangers start an advertising campaign. The following commercial is played daily on the local radio station.

> You heard it right! For only $21.95 a year (that's 12 fabulous issues), you can have *Bird Watchers' Bulletin* delivered to your home every month, rain or shine! And being a subscriber means you'll receive each issue before it becomes available at the newsstand!

14. How much money do you save buying a one-year subscription instead of buying a single copy each month for one year worth of *Bird Watchers' Bulletin* at a newsstand? Show your work.

Figure 41. Grade 6 Problem Solving Assessment, Item 14.

Table 65

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 14

Points	Response
2	Correct answer: $13.45 (or $13.44) w*ith* Work clearly shown
1	Correct answer only *or* Work shows correct strategy, however, incorrect answer resulting from one minor mistake in computation (e.g., $35.40 -$21.95 = $13.40) *or* Student only calculates newsstand price per year, $35.40 *or* Student computes price per issue for subscription at $1.83 and subtracts from $2.95 to get $1.12 without multiplying by 12.
0	Incorrect answer
X	Nonscorable or no response

Item 15 assesses students' abilities to interpret a problem situation; identify appropriate arithmetic calculations (multiplication, division, and subtraction); use whole and decimal numbers; compare decimals; find unit price as part of solution; and provide correct answer with clear supporting work (see Figure 42). The scoring rubric and strategy codes are shown in Tables 66 and 67, respectively.

Gill does not buy *Bird Watchers' Bulletin* every month. He wonders if he will save money by buying a one year subscription.

15. After how many newsstand issues does it become cheaper to buy a subscription? Explain your answer.

Figure 42. Grade 6 Problem Solving Assessment, Item 15.

Table 66

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 15

Points	Response
2	Correct answer: “7” or "after seven” or “from eight issues on" or “after 8” *with* Appropriate computational work, written explanation, or correct estimation including supporting work, e.g., “7 x 2.95 = 20.65” and/or “8 x 2.95 = 23.60” *or* “If you add $2.95 eight times it goes over $21.95”, *or* “22 ÷ 3 is about seven so after seven issues”
1	Correct answer only without computation or written explanation *or* Answer: “between 7 and 8” or “after 8 issues” with appropriate work shown *or* Student makes one computational error (e.g., 7 issues = $23.60 and answers, “After six issues.”)
0	Incorrect response
X	Nonscorable or no response

Table 67
Strategy Codes, Grade 6 Problem Solving Assessment, Item 15

Code	Description
12	Answer only
15	Student finds price per issue as part of solution (finds $21.95 ÷12 = $1.83)
17	Computes with estimation strategy, e.g., "22 ÷ 3 is about seven so after seven issues"
23	Uses ratio table
27	Guess and check *or* systematic guess and check, e.g., "I added $2.95 eight times until it was more than $21.95."
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care)
99	No response

Selling Tickets

The fifth section of the assessment, Selling Tickets, centers on the cost of admission to the park. This section contains one question. A partial item classification matrix is shown in Table 68. The entire matrix is available in Appendix B.

Table 68
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for The Selling Tickets Context of the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
16	Algebra	II	Difficult	4	Solving for two unknowns in simultaneous equations

Item 16 assesses students' abilities to interpret a problem situation; solve two equations with two unknown variables; calculate with decimals; use and provide correct answer with clear supporting work (see Figure 43). The scoring rubric and strategy codes are shown in Tables 69 and 70, respectively.

Today Gill sells the tickets at the main entrance of Wingra Park. The woman in front of the row says, "Two adults and five children." Gill answers, "That makes $6.50, please." The next visitor wants tickets for three adults and three children. He has to pay $6.15. Then a child buys a ticket.

16. What is the price of the child's ticket? Show how you found your answer.

Figure 43. Grade 6 Problem Solving Assessment, Item 16.

Table 69

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 16

Points	Response
4	Correct answer, 1 child = $0.80, with clear work shown
3	Work clearly shown and complete with one minor computational error (e.g., 7.20 ÷ 9 = .90)
2	Student derives 1 A + 1 C = $2.05 from 3A + 3C = $6.15 (÷ all terms by 3) *or* Correct strategy evident but student does not follow through to a solution *or* Correct strategy evident but student makes two minor computational errors that lead to an incorrect solution
1	Incorrect strategy is used to derive a solution (e.g., 3A + 3C = 6.15; 6.15 ÷ 6 = 1.03; 2A + 5C = 6.50; 6.50 ÷ 7 = .93; 1 child = .93) *or* Correct answer only with no work shown
0	Incorrect response
X	Nonscorable or no response

Table 70

Strategy Codes, Grade 6 Problem Solving Assessment, Item16

Code	Description
12	Answer only
27	Guess and check (no fair exchange)
40	Fair exchange model (notebook strategy -- table without variables) *may include drawings*
41	Formulas or equations (w/o other strategies) are used to solve problem
42	Coordinate graphs used to solve problem
43	Uses both: 1) coordinate graphs and 2) linear equations to solve problem
44	Combination chart used to solve problem
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Birds of All Sizes

The final section of the assessment, Birds of All Sizes, focuses on the weights, wingspreads, and surface areas of several birds. Several visual models of birds as well as a scatter plot of birds' wingspreads and weights are included. This section contains eight questions. A partial item classification matrix is shown in Table 71. The entire matrix is available in Appendix B.

Table 71
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for The Birds of All Sizes Context of the Grade 6 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
17	Geometry	II	Medium	3	Drawing a triangle with a given area
18	Geometry	II	Medium	2	Using an appropriate strategy to estimate the area of an irregular shape
19	Algebra/Statistics	I	Easy	2	Reading a point from a graph
20	Algebra/Statistics	I	Easy	2	Plotting a point on a graph
21	Number	I	Medium	2	Calculating and interpreting ratio
22	Number/Algebra	II	Difficult	2	Finding an unknown in equivalent ratios
23	Number/Algebra	III	Difficult	2	Using a graph or table to justify agreement or disagreement with a statement
24	Number/Algebra	II	Difficult	2	Using a graph or table to justify agreement or disagreement with a statement

Item 17 assesses students' abilities to draw a triangle with a given area and provide accurate drawing with clear and correct explanation (see Figure 44). The scoring rubrics and strategy codes are shown in Tables 72 and 73, respectively.

Small birds usually have small wings. A hummingbird has about the smallest wings of all birds.

The surface area of one wing of a hummingbird is only 12 square centimeters.

17. Draw a *triangle* that has an area of 12 square centimeters. Show how you know the triangle has an area of 12 square centimeters.

Figure 44. Grade 6 Problem Solving Assessment, Item 17.

Table 72

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 17

Points	Response
3	Correct drawing (*NOTE: a. For a triangle drawn to scale, the correctness of a drawing is based only on the lengths of the base and height of the triangle. The correct area of a triangle drawn to scale must be between 11-13 cm.*^{2} *b. If the triangle is not drawn to scale, the drawing must include labels)* with Correct written explanation, e.g., the student calculates the area using the height and base from the drawing using the formula ½ x b x h *or* the student includes accurate squares in the triangle that add up to 12 *or* the student draws a quadrilateral with an area of 24 cm^2 and partitions it accurately.
2	Correct drawing but with incomplete, incorrect, or no explanation
1	Incorrect or no drawing with correct explanation
0	Incorrect response
X	Nonscorable or no response

*If the base and height of the drawing are not labeled, it is necessary to measure the dimensions with a ruler and compute the area of the student's triangle.

Table 73

Strategy Codes, Grade 6 Problem Solving Assessment, Item 17

Code	Description
41	Formulas or equations used to solve problem, e.g., A = ½ x b x h
60	Drawing of triangle only
60A	Drawing OTHER THAN TRIANGLE only
61	Computes area by partitioning a quadrilateral into triangles 
62	Computes area by drawing square units inside a triangle 
64	Student draws a triangle with a PERIMETER of 12 cm. 
64B	Student draws a triangle with 3 sides of 12 cm. Each (i.e., perimeter=36 cm.) 
69	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 18 assesses students' abilities to find the area of an irregular shape (see Figure 45). The scoring rubric and strategy codes are shown in Tables 74 and 75 respectively.

An eagle has much larger wings than a hummingbird. A top view of an eagle is shown below. The top view is drawn to scale.

17. Estimate the surface area in square decimeters of one wing of the eagle in the drawing above. Write down how you found your answer.

Figure 45. Grade 6 Problem Solving Assessment, Item 18.

Table 74

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 18

Points	Response
2	Correct answer: between 43 and 57 (dm^2) *with* Correct description or explanation (e.g., "I found the length to be 11 and the width to be 4 so that is 44 plus there is about 3 in the feathers so that is 47 dm^2 or 11 x 4 + 3 = 47 or strategy obvious from picture)
1	Correct answer only without explanation *or* Appropriate strategy evident with answer of 38-42 dm^2 or 58-62 dm^2 (e.g., student draws boxes on figure that are approximately the same size as the box given) *or* Found area of both wings (and tail): between 100 and 120 dm^2 (Note: this must be evident from figure, i.e., blocks drawn on both wings and tail or from written explanation)
0	Incorrect response
X	Nonscorable or no response

NOTE: The student deserves full credit if she/he finds the surface area (*both sides) of one wing*: between 86 and 114 (dm^2). However, this must be evident from the figure or explanation.

Table 75

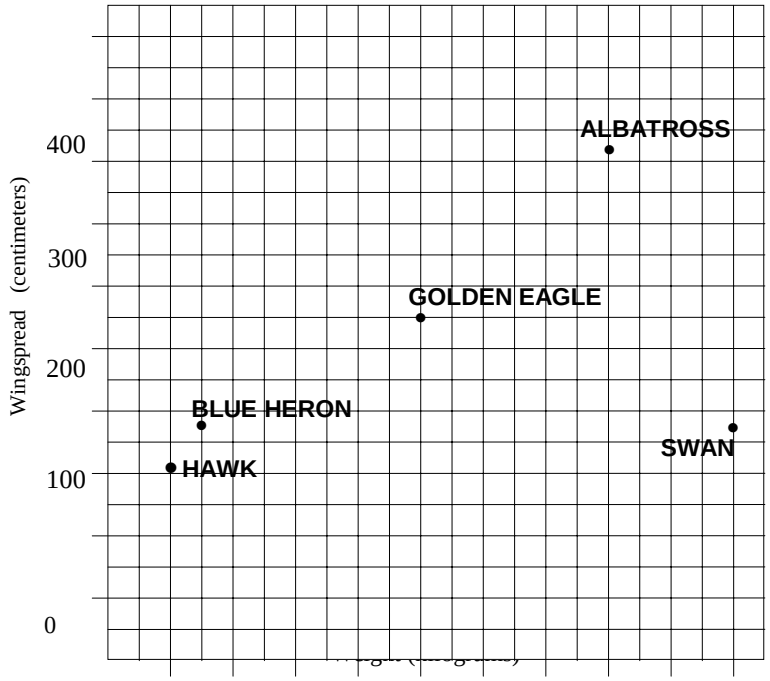
Strategy Codes, Grade 6 Problem Solving Assessment, Item 18

Code	Description
12	Answer only
41	Computes area with a formula or equation
62	Computes area by drawing square units inside the wing
69	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 19 assesses students' abilities to read coordinates of a point from a graph (see Figure 46). The scoring rubric is shown in Table 76. Strategy codes were not assigned for this item.

To compare small and large birds you can look at the weight in relation to the wingspread. The wingspread of a bird is measured from wing tip to wing tip:

In the graph below, you see the values for wingspread and weight plotted for several birds.



19. What is the weight of the hawk? What is the wingspread of the hawk?

Figure 46. Grade 6 Problem Solving Assessment, Item 19.

Table 76
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 19

Points	Response
2	Correct answers: 1 (kg); the word "kg" is not needed for full credit 120 - 125 (cm); the word "cm" is not needed for full credit
1	One answer correct only
0	Incorrect response
X	Nonscorable or no response

Item 20 assesses students' abilities to plot and label a point on a graph from given coordinates (see Figure 47). The scoring rubric is shown in Table 77. Strategy codes were not assigned for this item.

20. A white tailed eagle weighs 6.5 kilograms and its wingspread is 250 cm. Using these values, plot and label the point for the white tailed eagle on the graph above.

Figure 47. Grade 6 Problem Solving Assessment, Item 20.

Table 77
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 20

Points	Response
2	Correct answers: Point plotted correctly at (6.5, 250) Point labeled
1	Point is plotted correctly but not labeled *Or* Only one value, either 6.5 or 250, is correctly plotted and point labeled
0	Incorrect response
X	Nonscorable or no response

Item 21 assesses students' abilities to identify point on a graph; provide a correct ratio based on coordinates of plotted point; and simplify the ratio correctly (see Figure 48). The scoring rubric is shown in Table 78. Strategy codes were not assigned for this item.

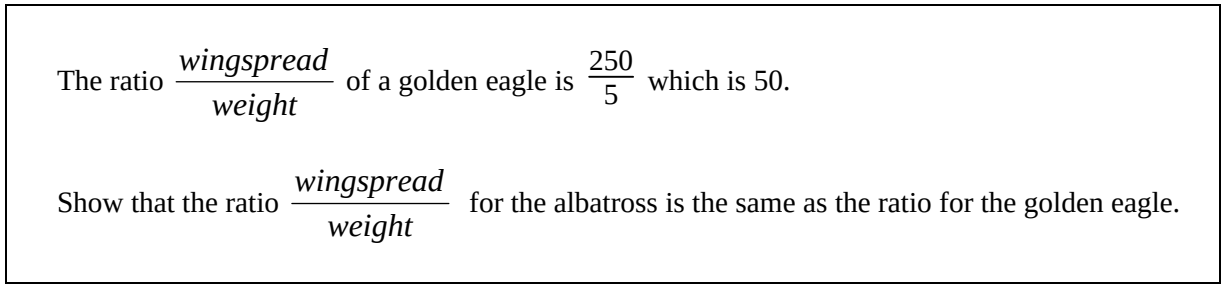


Figure 48. Grade 6 Problem Solving Assessment, Item 21.

Table 78
Scoring Rubric, Grade 6 Problem Solving Assessment, Item 21

Points	Response
2	One point for correct calculation of 50 *and* Shows appropriate ratio $\left(e.g.\frac{400}{8}\right)$
1	Shows appropriate ratio only $\left(\text{e.g. } \frac{400}{8}\right)$ *or* Shows appropriate ratio $\left(\text{e.g. } \frac{400}{8}\right)$ but makes calculation error in dividing *or* Computes a ratio $\left(\mathbf{e.g.}\frac{350}{\mathbf{8}}\right)$ based on an inaccurate reading of the graph *or* Answer only, 50
0	Incorrect or illegible answer
X	Nonscorable or no response

Item 22 assesses students' abilities to identify the relationship between a simplified ratio and the corresponding terms in a ratio; calculate the unknown in ratio given one term and simplified ratio; and provide correct answer and clear work (see Figure 49). The scoring rubric is shown in Table 79. Strategy codes were not assigned for this item.

The table below gives the wingspread, weight, and the $\frac{wingspread}{weight}$ ratios for three birds.

(NOTE: The ratios have been rounded.)

Bird	wingspread (in cm)	weight (in kilograms)	$\frac{wingspread}{weight}$
golden eagle	250	5	50
condor	275	6.6	42
buzzard	150		42

22. Use the information to find the weight of the buzzard (in kilograms). Show how you found your answer.

Figure 49. Grade 6 Problem Solving Assessment, Item 22.

Table 79

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 22

Points	Response
2	Correct answer: 3.5, 3.6, $3\frac{4}{7}$ or equivalent (kg not necessary) *with* Work clearly shown
1	Answer only (3.5, 3.6, $3\frac{4}{7}$) *or* One minor computational mistake (e.g., student divides 42 into 150 and gets 3 with a remainder of 24 and writes 3.24) *or* Reasonable estimate with supporting work (e.g., "between three and four" or "about four" or any number between 3 and 4 with supporting work)
0	Incorrect response
X	Nonscorable or no response

Item 23 assesses students' abilities to Use a graph or table to calculate ratios; reference specific points on a graph to justify disagreement with mathematical statement about ratios; provide correct conclusion; and provide correct explanation with supporting work (see Figure 50). The scoring rubric and strategy codes are shown in Tables 80 and 81, respectively.

Barbara says, "I see in the table that the ratio $\frac{wingspread}{weight}$ is always a number between 40 and 50. I think that this is true for all birds."

23. Use the graph, the table, or both to explain if you agree or disagree with Barbara's statement.

Figure 50. Grade 6 Problem Solving Assessment, Item 23.

Table 80

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 23

Points	Response
2	Correct mathematical explanation with supporting work that indicates that Barbara is not always right, such as a counterexample, e.g., the swan has a ratio of 15.
1	Student answers "I disagree" only with incomplete or incorrect explanation *or* One minor mistake (e.g., the albatross is $\frac{350}{9}$ which is 38.9, so I disagree)
0	Incorrect response
X	Nonscorable or no response

Table 81
Strategy Codes, Grade 6 Problem Solving Assessment, Item 23

Code	Description
58	Answer only (e.g., “Barbara is not always right.”)
71	Justification: gives trivial or descriptive information
76	Justification: references specific birds on the graph but does not provide numerical data
77	Justification: number based (e.g., student uses a counterexample such as the coordinates of the swan)
78	Justification: visually based (e.g., refers to the graph in a global manner; *does not* reference specific points)
89	Justification: other, none of the above
19B	Contextual answer: cites real-world information unrelated to mathematics involved
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 24 assesses students' abilities to use a graph or table to explain a statement about ratio; reference specific points on the graph; and provide correct explanation to support disagreement with mathematical statement (see Figure 51). The scoring rubric and strategy codes are shown in Tables 82 and 83, respectively.

Tonio says, "After looking at all these different birds, I see that the heavier a bird is the larger its wingspread is."

24. Use the graph, the table, or both to explain if you agree or disagree with Tonio.

Figure 51. Grade 6 Problem Solving Assessment, Item 24.

Table 82

Scoring Rubric, Grade 6 Problem Solving Assessment, Item 24

Points	Response
2	Correct mathematical explanation that indicates no e.g., "If this were true all birds would be approximately on a line with a positive slope and the swan is a not with the other birds"
1	Student answers "I disagree" with incomplete or incorrect explanation *or* One minor mistake in reading the graph or calculating ratio *or* Incomplete explanation (e.g., "In general it is true except for the swan.")
0	Incorrect response
X	Nonscorable or no response

Table 83
Strategy Codes, Grade 6 Problem Solving Assessment, Item 24

Code	Description
58	Answer only without justification (e.g., “I don’t agree with Tonio.”)
71	Justification: gives trivial or descriptive information
76	Justification: references specific birds on the graph but does not provide numerical data (e.g., gives an example of a bird to support agreement or disagreement with Tonio)
77	Justification: number based (e.g., “the swan has a weight of 10kg but its wingspread is only 150cm”)
78	Justification: visually based (e.g., student uses graph but does not mention specific numbers or data points, but looks at graph globally)
89	Justification: other, none of the above
19B	Contextual answer: cites real-world information unrelated to mathematics involved
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Grade 7 Problem Solving Assessment

The seventh grade assessment was designed to include questions from all strands of mathematics with an increasing concentration on concepts other than number. This assessment contains a similar distribution of points on Levels I and II, and, reflecting the structure of de Lange's pyramid, considerably fewer Level III items (see Table 84).

Table 84

Point Distribution for Content Strands and Levels of Reasoning, Problem Solving Assessment, Grade 7

LEVEL	STRAND				
	Number	Geometry	Algebra	Statistics	Total
I	9 (17.3%)	10 (19.2%)	14 (26.9%)	3 (5.8%)	36 (69.2%)
II	4 (7.7%)	8 (15.4%)	12 (23.1%)	3 (5.8%)	27 (51.9%)
III	2 (3.9%)	3 (5.8%)	3 (5.8%)	0 (0%)	8 (15.4%)
Total	15 (28.9%)	21 (40.4%)	29 (55.8)	6 (11.4%)	

*Percentages >100 because some questions fall into multiple categories.

The seventh-grade PSA consists of 26 constructed response worth a total of 52 points. The items on the assessment often require students to demonstrate their solution strategies in order to earn full credit. Scoring rubrics were created for each problem, allowing scorers to assign points for accuracy of answers and thoroughness of explanation. Moreover, many items have an accompanying strategy rubric which enables scorers to identify the strategies that students use to solve problems.

Baby Feeding

The seventh grade PSA is organized around five distinct contexts. The first context on this assessment, Baby Feeding, investigates the amount of formula to prepare for newborns and the packaging of a specific brand of formula. A table indicating how much formula a baby needs per kilogram of weight is included in this section as well as a line graph that shows the weight gain of a newborn over the first six months of life. This section contains seven questions. A partial item classification matrix is shown in Table 85. The entire item classification matrix is found in Appendix C.

Table 85

Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Baby Feeding Context on the Grade 7 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
1	Number/Algebra	I	Easy	1	Reading information from a table
2	Number	I	Easy	2	Multiplication of whole numbers Comparison of metric units (milliliters to liters)
3	Algebra/Statistics	I	Medium	2	Reading a graph Reading a chart Multiplication of whole numbers
4	Number/Algebra/Geometry	II	Difficult	2	Calculate using a rule of thumb Multiplication and division of whole numbers Recognizing the need to make and state assumptions
5	Number	I	Easy	2	Division of a fraction by a whole number
6	Geometry	II	Difficult	3	Reasoning about the relation between length and volume
7	Geometry	I	Medium	2	Calculating volume

Item 1 assesses students' abilities to read information correctly from a chart (see Figure 52). The scoring rubric is shown in Table 86. Strategy codes were not assigned for this item.

Until they are about 6 months old, the only food babies eat is milk. One type of milk is made from a special kind of milk powder. This powder is called Nutri. Each box of Nutri contains a table that indicates how much Nutri a baby needs. This table is shown below.

weight of baby in kilograms	water in milliliters	+ spoonfuls of Nutri	is about milliliters of milk (each feeding)	number of feedings each day
< 3	60	2	65	7
3 - 3.5	90	3	100	6
3.5 - 4	120	4	135	5
4 - 5	150	5	165	5
5 - 6	180	6	200	5
> 6	180	6	200	4

For example: To prepare one feeding for a baby who weighs 2 kilograms you take 60 milliliters of water
and add two spoonfuls of Nutri. This results in 65 milliliters of milk.

Note: 24 hours = 1 day
1000 milliliters = 1 liter

Chris weighs 4.5 kilograms.

1. Use the table to find how many milliliters of milk Chris needs for **each feeding**.

Figure 52. Grade 7 Problem Solving Assessment, Item 1.

Table 86
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 1

Points	Response
1	Correct answer: 165 ml (“ml” does not need to be in answer)
0	Incorrect response
X	Nonscorable or no response

Item 2 assesses students’ abilities to identify appropriate arithmetic calculation (multiplication); use whole numbers; compare metric measures; determine correct answer; and provide clear supporting work (see Figure 53). This item uses the answer for Item 1. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the rubric. The scoring rubric is shown in Table 87. Strategy codes were not assigned for this item.

2. Does Chris receive more or less than one liter of milk **each day**? Show your work.

Figure 53. Grade 7 Problem Solving Assessment, Item 2.

Table 87

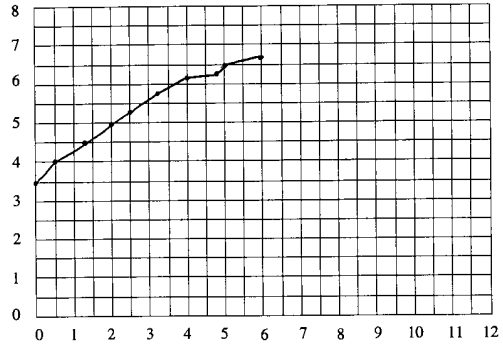
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 2

Points	Response
2	Correct conclusion: "less" or "less than one liter" or "no" *with* Correct supporting computation: 5 x 165 (ml) = 825 ml or 5 x 0.165 (L) = .825 L
1	Correct conclusion only, "less than one liter," with incomplete or no work shown *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error, use of incorrect conversion factor, or misreading of the table, e.g., "5 x 165 = 1025, so more than one liter" or "100 ml = 1 L so 825 ml = 8.25 L so more than one liter" "200 ml x 5 feedings = 1000 ml/day so equal to one L" *or* Correct computation only but no conclusion given
0	Incorrect response
X	Nonscorable or no response

This item uses the answer for Item 1. If a student used an incorrect answer to build an otherwise correct solution, the student receives full or partial credit in accordance with the criteria given in the rubric.

Item 3 assesses students' abilities to read a graph and a table; connect information from the graph with information in the table; identify appropriate arithmetic calculation (multiplication); and use information from the graph and table to calculate a correct answer (see Figure 54). The scoring rubric is shown in Table 88. Strategy codes were not assigned for this item.

In the graph below the line shows how much Chris weighed during the first 6 months of his life.



3. Use the graph above and the table on page 1 to find how many milliliters of milk Chris needed **per day** when he was 4 months old. Show your work.

Figure 54. Grade 7 Problem Solving Assessment, Item 3.

Table 88
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 3

Points	Response
2	Correct answer: 800 ml ("ml" does not need to be in answer); *with* Correct computation: e.g., 200 x 4
1	Correct conclusion only, "800 (ml)," with no work shown *or* Partial answer, "200 (ml)" with supporting work shown, e.g., Chris is > 6 kg. *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error or misreading of the table, e.g., "200 ml/day x 4 = 600 ml/day" or "200 ml/day x 5 = 1000 ml/day"
0	Incorrect response
X	Nonscorable or no response

Item 4 assesses students' abilities to use a formula and convert from one metric unit to another (see Figure 55). The scoring rubric is shown in Table 89. Strategy codes were not assigned for this task.

Another way of determining how much milk a baby needs is stated in the general rule below:

***Each day**, a baby needs 150 milliliters of milk per kilogram of its body weight.*

4. Kathleen, Chris' sister, is 13 years old. She wonders how much milk she would need if milk were her only food. Kathleen weighs about 40 kilograms. Using the rule above, calculate how many liters of milk Kathleen would need for **each meal**. Show your work.

Figure 55. Grade 7 Problem Solving Assessment, Item 4.

Table 89

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 4

Points	Response
2	Correct answer: (depends on assumption) e.g., "1.5 L" (for 4 meals) *or* "2 L" (for 3 meals), etc. *with the following three criteria:* • Correct computation shown: 40 x 150 ml = 6000 ml • Correct conversion to liters: 6 L (NOTE: correct conversion to liters can be implied from answer given in liters) • Assumption for number of meals given: $2 \leq$ # of meals/day ≤ 6 e.g., "she eats three meals per day" or assumption clear from computation (e.g., "6 L ÷ 3")
1	Correct answer only, "1.5 L" (for 4 meals) *or* "2 L" (for 3 meals), etc., with no work shown *or* Partial answer meeting only two criteria (above), e.g., 40 x 150 ml = 6000 ml = 6 L or 40 x 150 ml = > 6000 ml ÷ 3 meals/day => 2000 ml *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., 40 x 150 ml => 5000 ml => 5 L ÷ 3 => $1\frac{2}{3}$
0	Incorrect response
X	Nonscorable or no response

Item 5 assesses students' abilities to identify appropriate arithmetic calculation (division); convert from metric unit to another; use whole numbers; and provide correct answer with clear supporting work (see Figure 56). The scoring rubric and strategy codes are shown in Tables 90 and 91, respectively.

5. Mrs. Simons has a set of quadruplets who are now 1 year old. Each morning she divides 1½ liters of milk into 4 bottles, one bottle for each baby. How much milk is in each bottle? Show how you found your answer.

Figure 56. Grade 7 Problem Solving Assessment, Item 5.

Table 90
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 5

Points	Response
2	Correct answer: 3/8 0.375 (l) 375 (ml) (label not needed) *with* Clear work shown: e.g., $1.5 \div 4$ $\frac{3}{2} \div 4$
1	Correct answer only, 3/8, 0.375 (l), 375 (ml), with no work shown *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., $1500 \div 4 = 345$ ml
0	Incorrect response
X	Nonscorable or no response

Table 91

Strategy Codes, Grade 7 Problem Solving Assessment, Item 5

Code	Strategy description
12	Answer only, no strategy evident
13	Divides 1500 ml by 4 = 375 ml.
15	Computation with standard algorithm for division of fractions (e.g., 3/2 ÷ 4 = 3/8 → 3/2 x 1/4 = 3/8 → etc.)
16	Inverts divisor and dividend (e.g., 4 ÷ 1 1/2 = 2 2/3)
20	Divides 1.5 ÷ 4 = 0.375
21	Models situation using a diagram (fraction bars, draws bottles)
22	Uses derived numbers to compute (e.g., computes 1 ÷ 4 and ½ ÷ 4 separately)
23	Uses ratio table
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item 6 assess students' abilities to reason about the relationship between the dimensions of a rectangular prism and its volume when all dimensions are halved; compare volumes; and provide a clear explanation (see Figure 57). The scoring rubric and strategy codes are shown in Tables 92 and 93, respectively.

Nutri is available in refill boxes of 900 grams.
The dimensions of these boxes are: height: 12 cm
width: 18 cm
depth : 11 cm

Because young babies need very little Nutri per day, the company wants to develop a new box which contains half as much Nutri as the box they are now using.

6. Sheila suggests, "To make this new box we should divide all dimensions of the regular box by two." Explain why you agree or disagree with Sheila.

Figure 57. Grade 7 Problem Solving Assessment, Item 6.

Table 92
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 6

Points	Response
3	Valid strategy with supporting explanation given: e.g., "old box: 12 x 18 x 11 = 2376 cm^3, Sheila's box: 6 x 9 x 5.5 = 297 cm^3. 297 x 2 < 2376, so she is not right." *or* Valid explanation only (no strategy): e.g., "I do not agree with Sheila because her box is 8 times as small" or "If you cut only one dimension in half and kept the other two the same then the box would become half as large."
2	Valid strategy but incomplete or no conclusion drawn, e.g., "old box: 12 x 18 x 11 = 2376 cm^3, Sheila's box: 6 x 9 x 5.5 = 297 cm^3 so Sheila is wrong." *or* Valid strategy evident but incorrect answer due to one minor computation error *or* Partially correct explanation, e.g., "If you split each one in half it would be one fourth the size. If you only cut the width in half it would be okay."
1	Volume of old box is calculated only: "12 x 18 x 11 = 2376 cm^3" *or* Volume of Sheila's box is calculated only: "6 x 9 x 5.5 = 297 cm^3" *or* Incomplete explanation that indicates that Sheila's box is less than ½ the volume: e.g., "I don't agree because it's only ¼ the size" *or* "If you cut all the dimensions in half then there wouldn't be enough."
0	Incorrect response
X	Nonscorable or no response

Table 93

Strategy Codes, Grade 7 Problem Solving Assessment, Item 6

Code	Strategy description
58	explanation: answer only, "I disagree," no explanation
71	Justification: trivial or descriptive (no reference to computation) "Yes, because that would be half" *or* "Half of each dimensions is half of the box" *or* "If you split all the dimensions in ½ it would not fit ½ as many grams."
77	Justification: computation based or specific numerical example provided "Want ½ of 900 = 450; 6 x 9 x 5.5 is less than 450" *or* "12 x 18 x 11 = 2376; 6 x 9 x 5.5 = 297; 2376 ÷ 297 = 8, so 2376 is 8 times as much as 297"
78	Justification: visually based e.g., draws one or more diagrams to explain conclusion
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 7 assesses students' abilities to calculate the height, width, and depth for a rectangular prism with a given volume (see Figure 58). This item uses the answer fro Item 6. If a student correctly uses an incorrect strategy from Item 6 to derive an answer for Item 7, the student received full credit in accordance with the criteria given in the rubric. The scoring rubric is shown in Table 94. Strategy codes were not assigned for this item.

7. Find the height, width, and depth for the new box.

Figure 58. Grade 7 Problem Solving Assessment, Item 7.

Table 94

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 7

Points	Response
2	Full credit is given for three correct dimensions that give a volume of 1188 cm^3 (*NOTE: Many correct sets of dimensions are possible*) Example: 6 cm, 18 cm, 11 cm ("cm" does not need to be in answer)
0	Incorrect response
X	Nonscorable or no response

* This item uses the answer fro Item 6. If a student correctly uses an incorrect strategy from Item 6 to derive an answer for Item 7, the student received full credit in accordance with the criteria given in the rubric.

The Pentagon

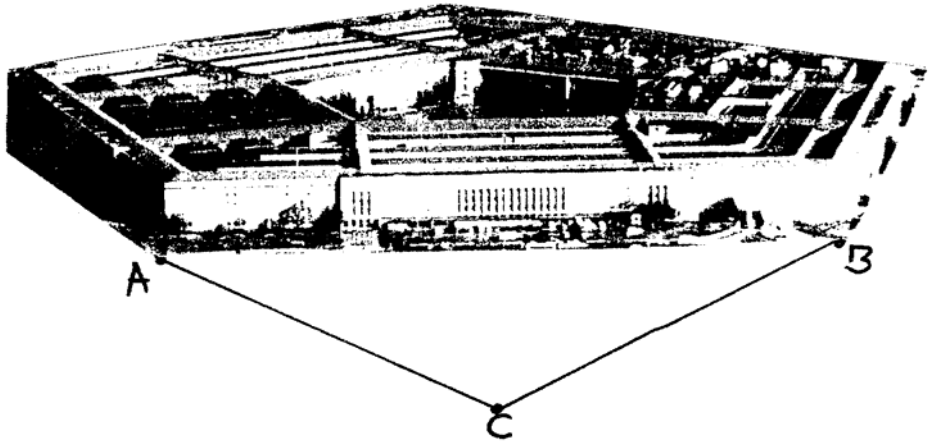
The second section of the assessment, The Pentagon, requires students to answer questions related to geometric properties of the Pentagon in Washington, D. C. A picture of the Pentagon is provided as well as a drawing of the outline of the Pentagon. This section contains three questions. A partial item classification matrix is shown in Table 95. The entire matrix can be found in Appendix C.

Table 95
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Pentagon Context on the Grade 7 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
8	Geometry	I	Difficult	2	Reasoning about the length of sides relative to angles in a triangle
9	Geometry	II	Medium	2	Completing a drawing of a shadow on a building caused by the sun
10	Geometry	III	Difficult	3	Calculating the interior angle of a pentagon

Item 8 assesses students' abilities to reason about the relationship between angle measures in a triangle and length of sides in a triangle and provide the correct answer with clear and complete reasoning (see Figure 59). The scoring rubric is shown in Table 96. Strategy codes were not assigned for this item.

The following picture represents the Pentagon, a famous building in Washington D.C.
This picture is not drawn to scale.



In front of the building is a huge parking lot. A car is parked at point C.
The vision lines AC and BC are drawn in the picture.

The angle BAC is 75º and the angle ABC is 50º.

8. Is the car closer to A or to B? How do you know?

Figure 59. Grade 7 Problem Solving Assessment, Item 8.

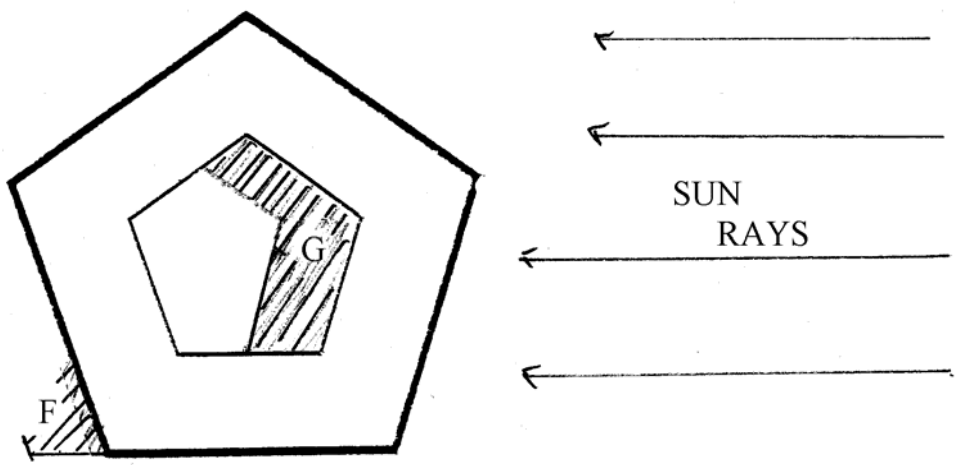
Table 96

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 8

Points	Response
2	Correct conclusion: "A" or "car is closer to A" *with* Correct/complete reasoning, e.g., "in a triangle the side opposite the smaller angle is shorter, so AC is shorter" *or* "closer to A because angle BAC is bigger than angle ABC"
1	Correct/complete reasoning with incorrect or no conclusion *or* Correct answer with incomplete reasoning that references angles or sides
0	Incorrect response
X	Nonscorable or no response

Item 9 assesses students' abilities to analyze the effect the sun's rays have on the shadow of a building and complete an accurate drawing of the shadow (see Figure 60). The scoring rubric is shown in Table 97. Strategy codes were not assigned for this item.

A top view of the Pentagon is shown below. The garden is in the center. The shaded part G shows the shadow of the building caused by the sun. A small part of the shadow on the left of the building is drawn (see F).



9. In the drawing above, finish the shadow of the building.

Figure 60. Grade 7 Problem Solving Assessment, Item 9.

Table 97
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 9

Points	Response
3	*Drawing of shadow must include the following criteria:* • shadow is *only* drawn on both the *entire* upper and *entire* lower left sides of figure • *where drawn*, the shadow is equally wide, horizontally, AND the horizontal (not perpendicular) distance from building is always 1 cm • the top edge of the shadow is parallel to the sun rays
2	Two criteria correct
1	One criterion correct
0	No criteria correct
X	Nonscorable or no response

Item 10 assesses students' abilities to identify the properties of angles in a two-dimensional shape; compute the interior angle of a regular pentagon; and provide correct answer with clear work shown (see Figure 61). The scoring rubric and strategy codes are shown in Tables 98 and 99, respectively.

A drawing of a **regular** pentagon is shown on the right.

10. Compute the angle indicated in this drawing without using a compass card or a protractor. Show how you found your answer.

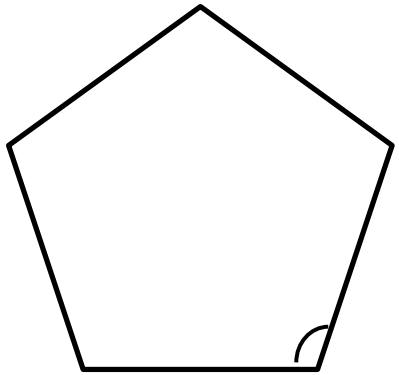


Figure 61. Grade 7 Problem Solving Assessment, Item 10.

Table 98

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 10

Points	Response
3	Correct answer: 108 (degrees) (“degrees” does not need to be in answer) *with* Clear work shown, e.g., 360 ÷ 5 = 72 .... 180 - 72 = 108 or Student makes a drawing showing turns (i.e., explains process visually)
2	Work shows correct strategy, however, incorrect answer resulting from one minor computation error
1	Correct answer only *or* Correct first step only (e.g., 360 ÷ 5 = 72) with appropriate work shown *or* Student makes a drawing showing turns (i.e., explains process visually), but does not mark turns with degree measurements *or* Correct answer found by measuring w/ protractor or compass card (within two degrees, 106° – 110°) *or* Correct answer found through visual estimation with supporting explanation or drawing (within two degrees, 106° – 110°)
0	Incorrect response
X	Nonscorable or no response

Table 99
Strategy Codes, Grade 7 Problem Solving Assessment, Item 10

Code	Strategy description
12	Answer only; no strategy given
18	Correctly computes first step only: "360 ÷ 5 = 72"
66	Uses protractor or compass card (or uses straightedge to extend sides to measure with protractor or compass card)
71	Justification: descriptive information with no additional mathematical explanation "the angles are all the same"
77	Justification: number based e.g., "360 ÷ 5 = 72 .... 180 - 72 = 108" *or* "the angles are all the same so I divided 360 by 5 then…"
78	Justification: visually based e.g., student makes a drawing showing turns *or* estimate made on the basis that the angles are more than 90 degrees
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Airships

The third section of this assessment, Airships, focuses on dirigibles and the amount of hydrogen they require to stay aloft. Graph paper with two labeled axes is provided for the students. This section contains four questions. A partial item classification matrix is shown in Table 100. The entire matrix can be found in Appendix C.

Table 100
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Airships Context on the Grade 7 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
11	Number	III	Difficult	2	Reasoning about the relative nature of percent decrease
12	Algebra	II	Medium	3	Constructing a graph of exponential decrease
13	Algebra	I	Medium	1	Reading values from a graph
14	Number	II	Medium	2	Using a fraction to represent a relationship

Item 11 assesses students' abilities to interpret a situation involving percent; reason about the relative nature of percent; and provide complete and correct reasoning (see Figure 62). The scoring rubric and strategy codes are shown in Tables 101 and 102, respectively.

Airships were very popular in the 1930s.

Airships used hydrogen gas to stay aloft. Part of the hydrogen leaked away through the skin of the airship. The first airships lost 50% of the hydrogen in the ship during each 10 day period.

11. Jan claims, "After 20 days such an airship will have lost 100% of its hydrogen." Explain why you agree or disagree with Jan's statement.

Figure 62. Grade 7 Problem Solving Assessment, Item 11.

Table 101
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 11

Points	Response
2	Clear and complete explanation including reasoning why less than 100% is lost, e.g., "I disagree. Because after 10 days the amount of hydrogen decreases by 50% of what it was before." *or* "50% of 50% is not 100%, it's 25%" *or* "50% of 180,000 is 90,000 and 50% of 90,000 is 45,000 not 0"
1	Student provides partially correct response that indicates that 100% of the hydrogen is not gone, e.g., "50% of 50% is not 100%" *or* "Jan is wrong because it will be cut in half each 10 day period" NOTE: "I do not agree with Jan" is not considered a partially correct response
0	Incorrect response
X	Nonscorable or no response

Table 102
Strategy Codes, Grade 7 Problem Solving Assessment, Item 11

Code	Strategy description
31	Uses linear model (e.g., "50% + 50% = 100%")
32	Uses exponential model with specific numerical information (e.g., "180,000 → 90,000 → 45,000 → 22,500 etc.)
38	Uses general exponential model (e.g., "½ of ½ is ¼" or "50% of 50% is 25%")
58	Justification: no explanation (e.g., Jan is not correct.)
78	Justification: visual model (e.g., fraction bars)
19B	Contextual response (e.g., "the gas leaks out more in the beginning because of pressure")
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 12 assesses students' abilities to construct an accurate graph of exponential decrease; include a complete consistent scale on both axes; and reflect an exponential decrease using at least three points (see Figure 63). If the plotted points were based upon an incorrect answer to Item 11, the student received credit for plotting. However, the graph must have a scale and a curve/line drawn to receive any credit. The scoring rubric is shown in Table 103. Note: If a student drew an increasing graph in Item 12 and answered Item 13 correctly, then the student received full credit in Item 12. Strategy codes were not assigned for this item.

Airships were very popular in the 1930s. Airships used hydrogen gas to stay aloft.

A large airship could hold as much as 180,000 cubic meters of hydrogen. Fifty percent of the hydrogen gas in the ship leaked away every 10 days.

12. On the grid provided on the next page, draw a graph of the amount of hydrogen in this airship per day for at least 20 days.

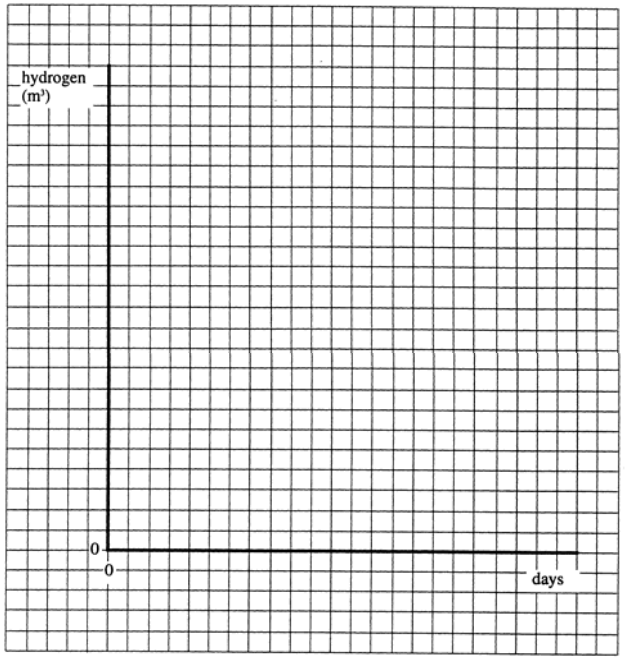


Figure 63. Grade 7 Problem Solving Assessment, Item 12.

Table 103
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 12

Points	Response
3	*For full credit the following three criteria must be satisfied:* • Each axis contains complete consistent scale (0 to 180,000 m^3 or 0 to 100% *and* 0 to 20 days) • Graph must show an exponential or linear* decrease • At least three points (including 0 and 20 days and one point in between) are correctly placed on the graph and a curved line is drawn through the points* (* or straight line could be drawn if reasoning in #11 is linear or if the scale on the y-axis is logarithmic)
2	Two criteria are correct
1	Only one criterion is correct *or* An accurate bar graph is drawn including a correct, consistent scale and *all* points plotted accurately
0	No criteria are correct
X	Nonscorable or no response

*If the plotted points were based upon an incorrect answer to Item 11, the student received credit for plotting. However, the graph must have a scale and a curve/line drawn to receive any credit.

Item 13 assesses students' abilities to read a value from a graph (see Figure 64). The scoring is based on the graph constructed in Item 12. If a student drew an increasing graph in Item 12 and answered Item 13 correctly, then the student received full credit in Item 12. The scoring rubric is shown in Table 104. Strategy codes were not assigned for this item.

13. When only 30,000 cubic meters of hydrogen remained in the airship, it was forced to land. After how many days was the airship forced to land?

Figure 64. Grade 7 Problem Solving Assessment, Item 13.

Table 104

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 13

Points	Response
1	Student finds the correct number of days for 30,000 m^3, according to the graph constructed in #12 for reference: curvilinear graph, day 26-27 for linear graph, day16-17
0	Incorrect response
X	Nonscorable or no response

* Scoring is based on the graph constructed in Item 12. If a student drew an increasing graph in Item 12 and answered Item 13 correctly, then the student received full credit in Item 12.

Item 14 assesses students' abilities to interpret meaning of a point on a graph; use a fraction to represent a relationship; and provide correct answer with clear supporting work (see Figure 65). The scoring rubric is shown in Table 105.

14. When the airship is forced to land, what fractional part of the hydrogen has been lost? Show your work.

Figure 65. Grade 7 Problem Solving Assessment, Item 14.

Table 105
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 14

Points	Response
2	Correct answer: $\frac{5}{6}$ or equivalent such as 0.83, ≈83%, $\frac{150000}{180000}$, $\frac{15}{18}$ *with* Clear work shown ($\frac{150000}{180000}$ is considered clear work)
1	Correct answer only, no work shown *or* Correct answer, 0. 83, $\frac{5}{6}$ or equivalent, with unclear or incomplete work *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error, e.g., "1 – 1/6 means that 4/6 is lost" or incomplete response, e.g., "1/6 left" *or* "≈17%," or minor rounding error *or* Student estimates. e.g., "180000 * $\frac{1}{10}$ = 18000; 180000 * $\frac{2}{10}$ = 36000. So somewhere between $\frac{1}{10}$ and $\frac{2}{10}$, with clear supporting work.
0	Incorrect response
X	Nonscorable or no response

Pyramids

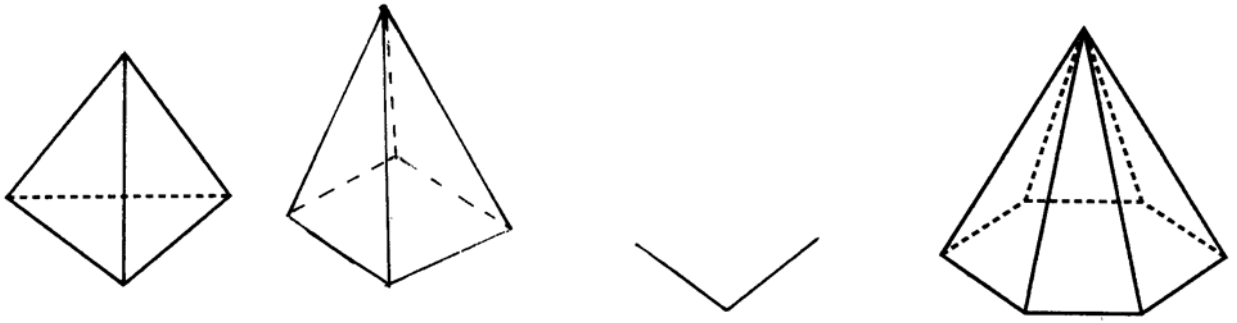
The fourth section of this assessment, Pyramids, students use several models of polyhedra as well as a table partially filled in with the number of faces, vertices, and edges for given polyhedra. Euler's formula, which describes the relationship between the number of faces, vertices, and edges of polyhedra, is also provided. This section requires that students discover relationships and patterns between the number of faces, vertices, and edges of polyhedra. This section contains seven questions. A partial item classification matrix is shown in Table 106. The entire matrix can be found in Appendix C.

Table 106
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Ptramids Context on the Grade 7 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
15	Geometry	I	Easy	2	Completing the drawing of a three-dimensional object
16	Geometry	I	Easy	2	Naming two dimensional shapes
17	Algebra/Geometry	I	Medium	2	Analyzing geometric and numerical patterns found in drawings and a table
18	Algebra	II	Medium	2	Extending/generalizing a numerical pattern
19	Algebra	II	Difficult	1	Using variables to express a general algebraic pattern
20	Algebra	I	Difficult	1	Substituting numbers in an equation or expression
21	Algebra	II	Difficult	2	Substituting expressions in a formula Combining like terms to simplify an expression

Item 15 assesses students' abilities to complete an accurate drawing of a pyramid with a pentagonal base (see Figure 66). The scoring rubric for this item is shown in Table 107. Strategy codes were not assigned for this item.

Some pyramids are shown below. Regular polygons (which are sometimes called *n*-gons) are used as the bases of these pyramids. The pyramids with the 3-gon, the 4-gon, and the 6-gon as bases are shown below.



n=3 *n*=4 *n*=5 *n*=6

n is the number of sides of the polygon that forms the base of the pyramid.

15. Finish the drawing for the pyramid for $n = 5$. Use a ruler or a straightedge.

Figure 66. Grade 7 Problem Solving Assessment, Item 15.

Table 107
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 15

Points	Response
2	*For full credit the following three criteria must be satisfied:* • Correct base (regular pentagon) • Correct sides • Straightedge used
1	Two criteria correct
0	Only one or no criteria correct
X	Nonscorable or no response

Item 16 assesses students' abilities to correctly name the shape of the base and the faces of a pentagonal pyramid (see Figure 67). The scoring rubric is shown in Table 108. Strategies codes were not assigned for this item.

11. Name the shapes of **all** the faces of the pyramid for $n = 5$ including the shape of the base.

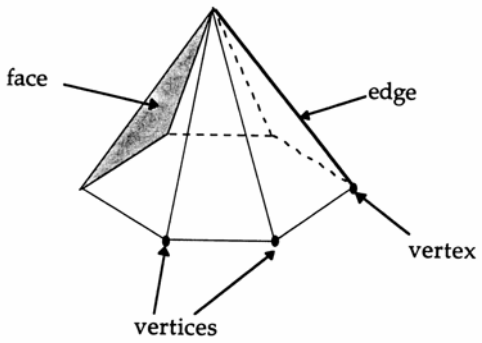
Figure 67. Grade 7 Problem Solving Assessment, Item 16.

Table 108
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 16

Points	Response
2	Correct base: pentagon, 5-gon, or five-sided polygon *and* Correct faces: triangles or isosceles triangles
1	Student correctly names the shape of the base or faces, but not both
0	Incorrect response
X	Nonscorable or no response

Item 17 assesses students' abilities to identify the number of faces, vertices, and edges of given pyramids; complete table of values for given pyramids; recognize patterns; and provide correct answers (see Figure 68). The scoring rubric is shown in Table 109. Strategy codes were not assigned for this item.

Each of the pyramids has vertices (the corners), edges (the lines) and faces (the sides). These are labeled in the picture below.



17. The number of faces, vertices, and edges for pyramids is given in the table below. Fill in the number of vertices and edges for the 4-gon, the 5-gon, the 6-gon, and the 7-gon.

Pyramid base n-gon	Number of faces	Number of vertices	Number of edges
3-gon	4	4	6
4-gon	5		
5-gon	6		
6-gon	7		
7-gon	8		
12-gon			
n-gon	*n* + 1		

Figure 68. Grade 7 Problem Solving Assessment, Item 17.

Table 109

Scoring Rubric, Grade 7 Problem Solving Assessment, Item 17

Points	**Response**
2	Correct "Number of vertices" column Correct "Number of edges" column Number of faces / Number of vertices / Number of edges 4 / 4 / 6 5 / 5 / 8 6 / 6 / 10 7 / 7 / 12 8 / 8 / 14
1	One error in table
0	Two or more errors in table
X	Nonscorable or no response

Item 18 assesses students' abilities to determine a numerical pattern in a table; extend the pattern for each intermediate step; and provide correct answers and complete explanations (see Figure 69). If the pattern from Item 17 was incorrect, full credit was awarded for correctly extending the incorrect pattern. The scoring rubric and strategy codes are shown in Tables 110 and 111, respectively.

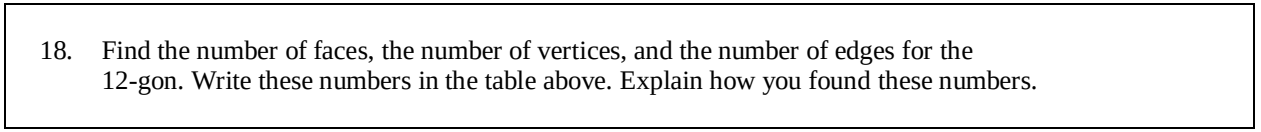


Figure 69. Grade 7 Problem Solving Assessment, Item 18.

Table 110
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 18

Points	Response
2	Correct answer: F = 13, V = 13, and E = 24 *with* Clear explanation: pattern or formula must be explained, e.g., "The number of faces and vertices is one more than the number of sides on the base of the n-gon, and the number of edges is twice the numbers of sides on the base of the n-gon."
1	Correct answer with missing, incorrect, or incomplete explanation e.g., "I used the pattern"
0	Incorrect response
X	Nonscorable or no response

* If the pattern from Item 17 was incorrect, full credit was awarded for correctly extending the incorrect pattern.

Table 111
Strategy Codes, Grade 7 Problem Solving Assessment, Item 18

Code	Strategy description
58	Answer only, no explanation
33	Justifies answer by extending table (finds 8-gon, 9-gon, etc.)
38	Generalizes pattern descriptively or with variables e.g., “the number of faces and vertices are 1 more than the polygon #. The number of edges is twice the number of the polygon”
71	Justification: trivial (e.g., “I used a pattern”)
77	Justification: number based (e.g., “12+1 base, 12+1 faces; 13 x 2 edges”)
78	Justification: emphasizes visual cues, tried to draw picture
89	Justification: other
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care)
99	No response

Item 19 assesses students' abilities to determine a numerical pattern found in a chart; generalize the pattern; and use variables to express the pattern (see Figure 70). Full credit was given for answers that are based on an incorrect pattern used in Item 17. The scoring rubric is shown in Table 112. Strategy codes were not assigned for this item.

19. Fill in the last line of the table for the *n*-gon. Fill in **expressions that use *n***.

Figure 70. Grade 7 Problem Solving Assessment, Item 19.

Table 112
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 19

Points	**Response**
1	Correct answer: Vertices: V = n + 1; Edges: E = n + n *or* E = 2 x n *or* E = 2n *or* E = n x 2
0	Incorrect response
X	Nonscorable or no response

* Full credit was given for answers that are based on an incorrect pattern used in Item 17.

Item 20 assesses students' abilities to use a given formula; substitute numbers found in a table for variables in a formula; determine if equation is valid; and provide a correct explanation (see Figure 71). A student might have used incorrect numbers from Item 17 to show that Euler's formula does or does not work. The scoring rubric is shown in Table 113. Strategy codes were not assigned for this item.

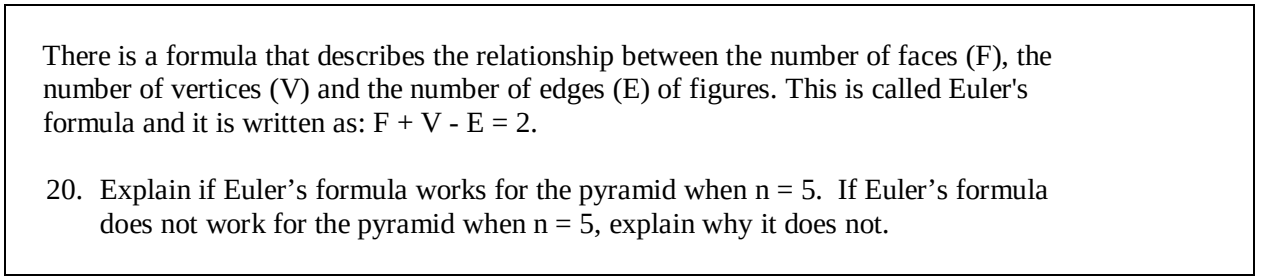


Figure 71. Grade 7 Problem Solving Assessment, Item 20.

Table 113
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 20

Points	Response
1	Correct answer: 6 + 6 –10 = 2 [Note: an equation is considered an explanation here.]
0	Incorrect response (e.g., "Yes, it works on everything.")
X	Nonscorable or no response

* A student might have used incorrect numbers from Item 17 to show that Euler's formula does or does not work.

Item 21 assesses students' abilities to substitute expressions for variables in a formula; combine like terms; simplify to show equality; and provide correct answer and correct explanation (see Figure 21). A student might have used expressions based upon a faulty pattern from Item 17. The scoring rubric is shown in Table 114. Strategy codes were not assigned for this item.

21. Show that Euler's formula works for the expressions in the last line of the table.

Figure 72. Grade 7 Problem Solving Assessment, Item 21.

Table 114
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 21

Points	Response
2	Correct equation simplified to clearly suggest a result of 2: e.g., "n + 1 + n + 1 – 2n = 2n – 2n + 2 = 2" *or* Clear and correct explanation in words or with variables: e.g., "If you add F and V you have 2n + 2 and E is 2n so 2 is left"
1	Student uses variables from chart but work has one minor computation error so that equation does not result in the proper conclusion of 2 *or* Student uses variables from chart to write an equation that = 2 but does not simplify it so that a result of 2 is clearly suggested
0	Student uses numbers to show instead of variables or work *or* Incorrect response
X	Nonscorable or no response

* A student might have used expressions based upon a faulty pattern from Item 17.

Playgrounds

The final context in this assessment, Playgrounds, is an extension of the Playground and Patio contexts found on the fifth- and sixth- grade assessments and is used to assess growth in algebraic reasoning over time. This context begins to formalize some of the ideas found earlier on the fifth and sixth grade assessments. This context focuses on the construction of a large square playground that is to be built in a park. The playground, whose dimensions are undecided, will be paved with gray and white square tiles in the pattern illustrated in Figure 73. Students are asked to reason about patterns in the playground as the length increases. This section contains five questions. A partial item classification is shown in Table 115. The entire matrix can be found in Appendix C.

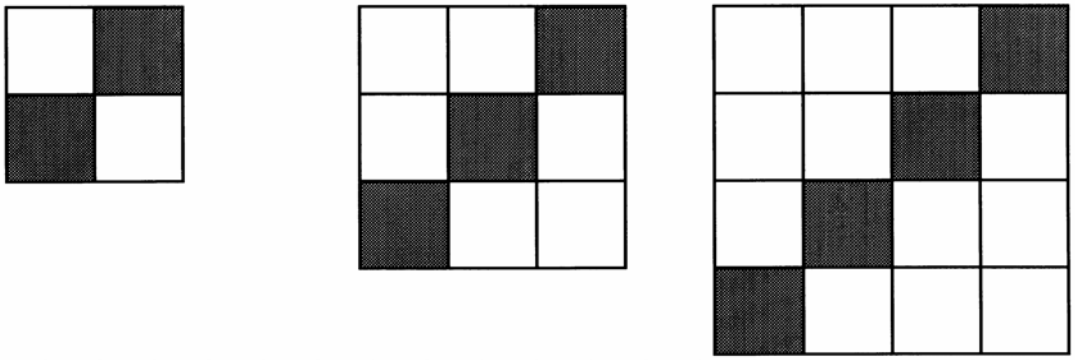
Table 115
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Playgrounds Context on the Grade 7 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
22	Algebra	I	Easy	1	Extending a pattern
23	Algebra/Number	I	Easy	2	Extending a pattern
24	Algebra	III	Difficult	3	Recognizing a pattern Generalizing a pattern and expressing it with variables
25	Algebra	II	Difficult	2	Problem solving Reasoning about number patterns
26	Geometry/Number	I	Medium	2	Finding area Calculation with fractions or decimals

Item 22 assesses students' abilities to interpret a pattern demonstrated in diagrams and the extend pattern (see Figure 73). The scoring rubric is shown in Table 116. Strategy codes were not assigned for this item.

A large **square** playground will be built in a park. It will be paved with gray and white the following square tiles in pattern: one diagonal will be paved with gray tiles, the rest of the playground will be paved with white tiles. The dimensions of the playground have not been decided.

Some playgrounds of different sizes are shown below. The shaded tiles are the gray tiles.



LENGTH = 2 TILES LENGTH = 3 TILES LENGTH = 4 TILES

22. If the playground has a length of 10 tiles, how many gray tiles will be in the playground?

Figure 73. Grade 7 Problem Solving Assessment, Item 22.

Table 116
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 22

Points	Response
1	Correct answer: 10
0	Incorrect response
X	Nonscorable or no response

Item 23 assesses students' abilities to interpret a pattern demonstrated in diagrams; extend the pattern; and provide a correct answer and clear explanation (see Figure 23). The scoring rubric and strategy codes are shown in Tables 117 and 118, respectively.

23. If a playground has a length of 10 tiles, how many white tiles will there be?
Show how you found your answer.

Figure 74. Grade 7 Problem Solving Assessment, Item 23.

Table 117
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 23

Points	Response
2	Correct answer: 90 *with* Clear explanation: e.g., descriptive: "I multiplied the length by itself and then subtracted the length" or calculations given: "10 x 10 – 10 = 90" or "100 – 10"
1	Correct answer only *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error
0	Incorrect response
X	Nonscorable or no response

Table 118
Strategy Codes, Grade 7 Problem Solving Assessment, Item 23

Code	Strategy description
12	Answer only, no strategy evident
21	Uses diagrams or drawings to compute (if diagram is included with calculation, code as strategy 21)
37	Uses pattern (e.g., “10 x 10 – 10 = 90”)
19	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item 24 assesses students' abilities to interpret a pattern demonstrated in diagrams; generalize the pattern; use variables to represent pattern; and provide a correct direct formula (see Figure 75). The scoring rubric and strategy codes are shown in Tables 119 and 120, respectively.

24. Write a formula you can use to find the number of white tiles in a playground if you know its length.

Figure 75. Grade 7 Problem Solving Assessment, Item 24.

Table 119
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 24

Points	Response
3	Correct formula: white = length x length – length *or* white = length x (length – 1) *or* white = $L^2 - L$ *or* $L^2 - L = ?$
2	Minor mistake in formula (e.g., student writes white = length x length – 1 or L x L – gray tiles = answer) *or* Recursive formula (e.g., next white = current white + 2 x current length) *or* "Two-part" formula (e.g., length x length = total – length = # white, also includes arrow language) *or* Expression without equal sign (e.g., $L^2 - L$)
1	Student makes an attempt to describe the formula in words but it is incomplete or lacks several parts, *or* Two minor errors appear in the given formula, (e.g., length – gray = white)
0	Incorrect response
X	Nonscorable or no response

Table 120
Strategy Codes, Grade 7 Problem Solving Assessment, Item 24

Code	Strategy description
21	Recursive formula
26	Arrow language
35	Uses number expression (e.g., "10 x 10 – 10 = 90")
38	Direct formula (e.g., "white = length x length – length")
48	Incomplete algebraic response (e.g., "L = white tiles … L = ??") *or* "Two-part" formula (e.g., length x length = total – length = # white) *or* Expression
49	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 25 assesses students' abilities to identify a pattern demonstrated in diagrams and determine correct answer with clear work shown (see Figure 76). The scoring rubric and strategy codes are shown in Tables 121 and 122, respectively.

25. The contractor building the playground has 1600 white tiles. Using the pattern given on the previous page, what is the length of the largest playground the contractor can make? Show how you found your answer.

Figure 76. Grade 7 Problem Solving Assessment, Item 25.

Table 121
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 25

Points	Response
2	Correct answer: 40 with Clear work shown
1	Correct answer only, no work shown or Work shows correct strategy, however, incorrect answer resulting from one minor computation error or Valid strategy but incomplete or no conclusion drawn
0	Incorrect response
X	Nonscorable or no response

Table 122

Strategy Codes, Grade 7 Problem Solving Assessment, Item 25

Code	Strategy description
12	Answer only, no strategy evident
16	Divides 1600 by 2
21	Uses drawings or diagrams to compute
27	Guess and check
41	Other formulas or equations used (e.g., “1600 - 40 = 1560”),$\sqrt{1600}$
71	Justification: descriptive (e.g., “No playground can be built that has 1600 white tiles.”)
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Item 26 assesses students' abilities to identify appropriate the arithmetic calculation (multiplication) to determine the area of a rectangle; compute with fractions or decimal numbers; and determine correct answer with clear work shown (see Figure 26). The scoring rubric and strategy codes are shown in Tables 123 and 124, respectively.

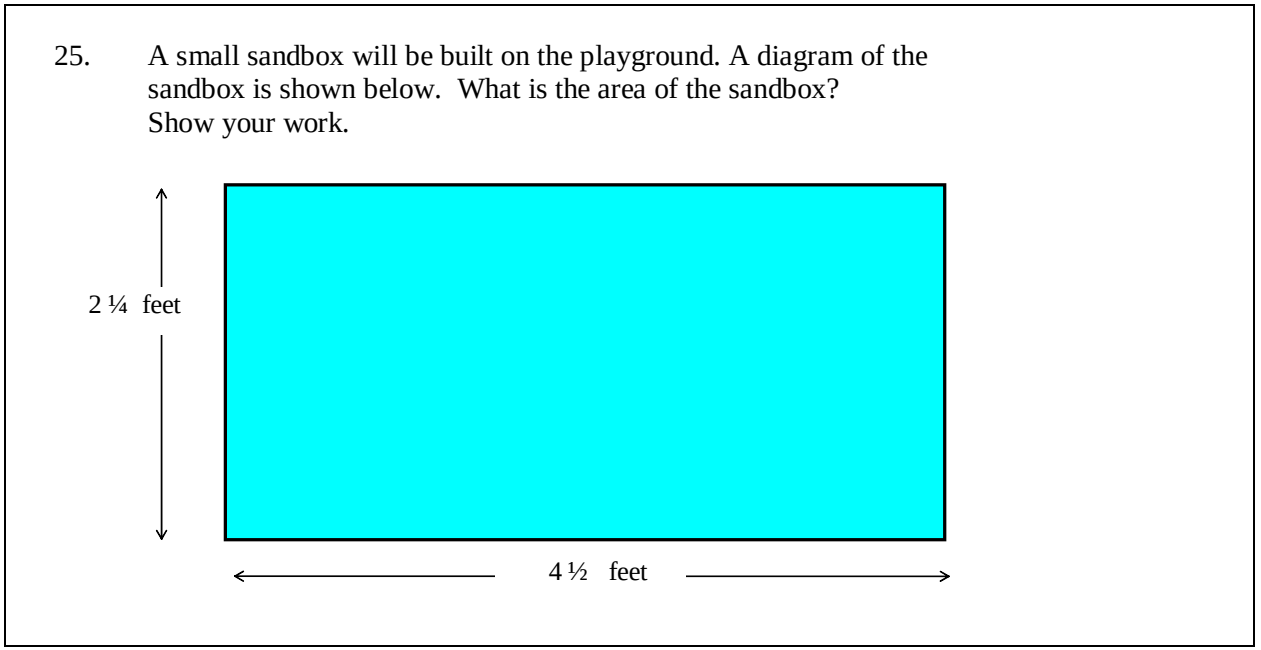


Figure 77. Grade 7 Problem Solving Assessment, Item 26.

Table 123
Scoring Rubric, Grade 7 Problem Solving Assessment, Item 26

Points	**Response**
2	Correct answer: 10.125 (sq. ft.) *or* $10\frac{1}{8}$ (sq. ft.) *or* $\frac{81}{8}$ (sq. ft) (label not needed for full credit) *with* Clear work shown
1	Correct answer only, no work shown *or* Incorrect answer resulting from one minor error in partitioning of drawing *or* Work shows correct strategy, however, incorrect answer resulting from one minor computation error (e.g., 2.25 x 4.5 = 10.105) (NOTE: incorrect placement of decimal point is NOT a minor error)
0	Incorrect response
X	Nonscorable or no response

Table 124

Strategy Codes, Grade 7 Problem Solving Assessment, Item 26

Code	Strategy description
11	Calculator with supporting work
12	Answer only
13	Uses standard algorithm for multiplication of decimals
14	Computes by multiplying fractions (e.g., $2\frac{1}{4}$ x $4\frac{1}{2}$ = $10\frac{1}{8}$)
16	Incorrect operation used
16A	Uses operation incorrectly
61	Partitions diagram horizontally and vertically into parts and wholes
64	Student finds perimeter
19	Other computation strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Grade 8 Problem Solving Assessment

The eighth-grade problem solving assessment requires students to engage with more formalized mathematical ideas than are presented on earlier assessments. Moreover, the problems on this assessment can generally be categorized as difficult even if they are problems that are identified as those on Levels I and II. This assessment is organized around seven distinct contexts.

The eighth-grade assessment was designed to include questions from all strands of mathematics with a decreased emphasis on number concepts as compared with earlier years. This assessment contains a majority of points on Levels I and II, and, reflecting the increasing sophistication of the mathematics involved in the contexts, a considerable number of points on Level III (see Table 125).

Table 125
Point Distribution for Content Strands and Levels of Reasoning, Problem Solving Assessment, Grade 8

LEVEL	STRAND				
	Number	Geometry	Algebra	Statistics	Total
I	2 (4.9%*)	6 (14.6%)	1 (2.4%)	0 (0%)	9 (22.8%)
II	8 (19.5%)	10 (24.4%)	10 (24.4%)	6 (14.6%)	34 (82.9%)
III	2 (4.9%)	5 (12.2%)	7 (17.1%)	2 (4.9%)	16 (39%)
Total	12 (29.3%)	21 (51.2%)	18 (43.9%)	8 (19.5%)	

*Percentages >100 because some questions fall into multiple categories.

The eighth grade PSA consists of 21 constructed response worth a total of 41 points. The items on the assessment often require that students must demonstrate their strategies for solving problems in order to earn full credit. Scoring rubrics were created for each problem, allowing scorers to assign points for accuracy of answers and thoroughness of explanation. Moreover, many items have an accompanying strategy rubric which enables scorers to identify the strategies that students used to solve problems.

Four problem solving assessments were created to evaluate students' problem solving fluency as well as to characterize their choice and use of strategies to solve problems at Grades 5, 6, 7, and 8. The assessment questions were written to encompass problems from four mathematical strands, at three levels of reasoning, and at three levels of difficulty. Although the problems are accessible for students using a variety of strategies, the assessments become progressively more difficult as the grade levels increase and the mathematics becomes more formalized. The item classification matrix for this assessment can be found in Appendix D.

Club Members

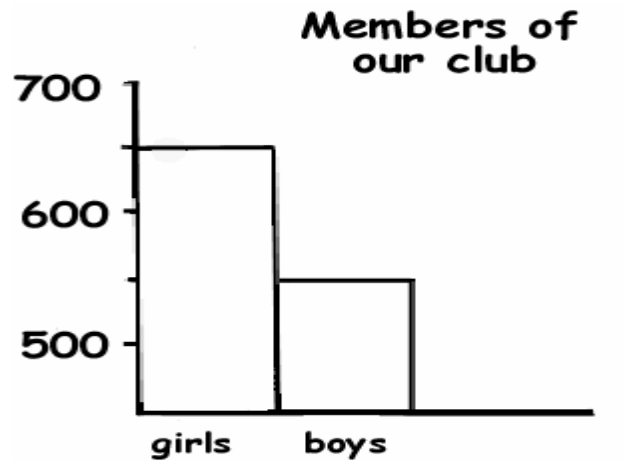
The first context on this assessment, Club Members, presents students with a graph of club membership comparing the number of boys and girls enrolled in a swim club. This section contains one question. A partial item classification matrix is shown in Table 126. The entire matrix can be found in Appendix D.

Table 126
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Club Members Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
1	Statistics	II	Medium	2	Analyzing misleading statistical data presented on a bar graph

Item 1 assesses students' abilities to critically analyze a graphical representation of data (bar graph); recognize misleading scale of y-axis; and draw a correct conclusion (see Figure 78). The scoring rubric is shown in Table 127. Strategy codes were not assigned for this item.

Anika is a member of a swimming club. She made the diagram you see below.



7. Use the graph to explain why you agree or disagree with Anika.

Figure 78. Grade 8 Problem Solving Assessment, Item 1.

Table 127

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 1

Points	Response
2	Correct explanation that indicates Anika is wrong: *A correct explanation should indicate that student has:* *1) read the graph correctly, and* *2) used information to show why Anika's statement is incorrect* e.g., "If there were twice as many girls there would have to be 1100 girls because there are 550 boys" *or* "650 is not 2 x 550" *or* "I disagree. There are 650 girls and 550 boys and that's not twice as many" *or* "I don't agree with her because there are 650 girls and 550 boys and that's only 100 more girls than boys" NOTE: For full credit, students who explain why Anika's statement might be perceived as correct must also explain why Anika did not correctly read the graph.
1	Student reads bar graph correctly but explanation of why Anika is incorrect is incomplete, inaccurate, or missing e.g., "I disagree. There are 650 girls and 550 boys" *or* Correct conclusion with incomplete explanation e.g., "I don't agree with her because there are only 100 more girls than boys"
0	Conclusion with no explanation Other incorrect response
X	Nonscorable or no response

Lopsided

The second section on the eighth grade assessment, Lopsided, revolves around three structures that have begun to lean: a tower, a house, and the Tower of Pisa. This section contains three questions. A partial item classification matrix is shown in Table 128. The entire matrix can be found in Appendix D.

Table 128
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Lopsided Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
2	Geometry	II	Difficult	2	Using scale and ratio
3	Number	I	Difficult	2	Using a given formula Calculating and rounding with decimal numbers
4	Geometry	II	Difficult	3	Problem solving using ratio Comparing decimals

Item 2 assesses students' abilities to determine the scale of a model given scaled and actual building measures; and solve for an unknown in a proportional situation (see Figure 79). The scoring rubric and strategy codes are shown in Tables 129 and 130, respectively.

Here is a drawing of a bell tower called the Marti Tower. The height of the Marti Tower is
96 meters. Joan wants to make a scale drawing of the Marti Tower. In the drawing, the height of the Marti Tower will be 9.6 centimeters.
(1 meter = 100 centimeters)

2. How many meters in the actual tower does each centimeter in Joan's drawing represent?
   Show how you determined the scale of Joan's drawing.

Figure 79. Grade 8 Problem Solving Assessment, Item 2.

Table 129

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 2

Points	Response
2	Correct answer: 1 cm = 10 m *or* 1 cm :1000 cm *With* Clear supporting work
1	Correct answer only *Or* Answer or explanation has reversed scale (e.g., 10 to 1; 1 = 10) with clear supporting work *or* Answer or explanation does not include units* *Or* Response includes correct unsimplified ratio (e.g., writes 9.6 : 96, or equivalent, as answer or as part of explanation)
0	Incorrect response
X	Nonscorable or no response

* If a student gave an answer of 1:1000, then units are not necessary and the student received full credit

Table 130

Strategy Codes, Grade 8 Problem Solving Assessment, Item 2

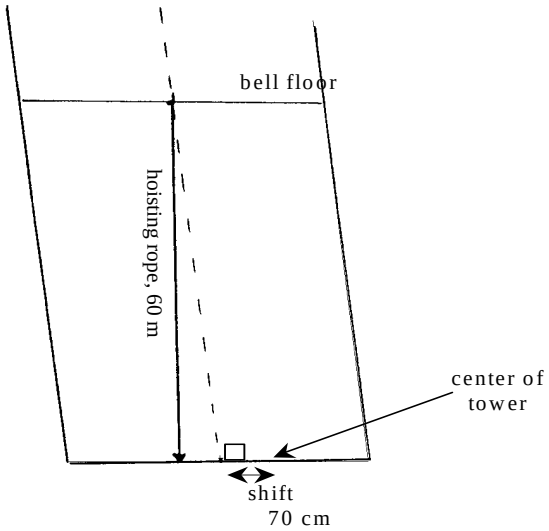
Code	Description
12	Answer only, no strategy evident
12C	Answer only: 1 m = 100 cm (i.e., "it's given above")
13	Standard algorithm for proportions e.g., $\frac{96}{9.6} = \frac{x}{1}$
21	Uses diagrams or drawings (incl. number line model)
23	Ratio table e.g., Actual | 96 m | 9600 | 1000 cm Drawing | 9.6 cm | 9.6 | 1 cm
41	Formulas or equations used to solve problems (e.g., 96 m ÷ 9.6 cm = 10 m)
19	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 3 assesses students' abilities to identify variable values; use values in a given formula; convert values to same unit; divide decimal numbers; round answer to three decimal places; and provide correct answer with computation shown (see Figure 80). The scoring rubric is shown in Table 131. Strategy codes were not assigned for this item.

PRESS RELEASE:
The Marti Tower is leaning! Yesterday, when the new bells were hoisted up, the construction crew discovered that the tower had shifted from its original position by 70 centimeters!

In order to hoist the new bells to their positions in the tower, the construction crew used a 60-meter rope. The rope hung perpendicular to the ground. The shift of the building was measured from the bottom of the rope to the center of the tower.

Joan drew a sketch to show the length of the rope and the shift of the tower. The sketch was not drawn to scale.



The length of rope and the shift of a building can be used to determine the lean of the tower. This is shown in the following formula:

$$lean = \frac{shift}{length\ of\ rope}$$

1. Compute the lean of the Marti Tower. Round your answer to three decimal places. (1 meter = 100 centimeters)

Figure 80. Grade 8 Problem Solving Assessment, Item 3.

Table 131
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 3

Points	Response
2	Correct answer: 0.012 *With* Computation shown: equivalent ratios, etc
1	Correct answer with no work shown O*r* Minor computation error *Or* Rounding error (e.g., 1.2 – student needs to "round to three decimal places") *Or* Incorrect use of units (e.g., 70 ÷ 60 = 1.167)
0	Incorrect response *Or* Lean and length of rope are reversed when calculating (e.g., 60 ÷ 70 = .857)
X	Nonscorable or no response

Item 4 assesses students' abilities to use a given formula to calculate ratios of three buildings; divide using decimal numbers; compare calculated ratios; and draw a correct conclusion (see Figure 81). This item uses the answer for Item 3. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the rubric. The scoring rubric is shown in Table 132. Strategy codes were not assigned for this item.

Three buildings are shown below.

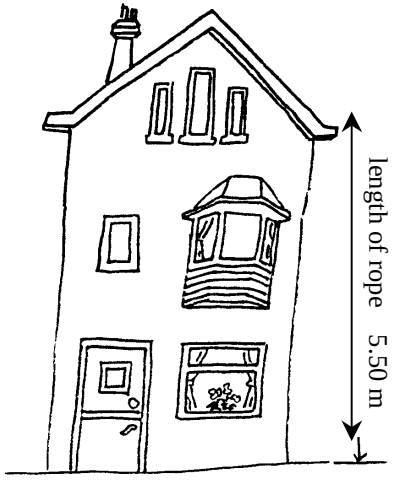
The first picture shows the Marti Tower.

The second picture is a sketch of a lopsided house. A rope 5.50 meters long was dropped from the edge of the roof to the ground. The shift of the house is 25 centimeters.

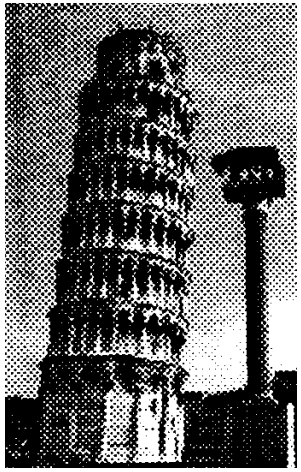
The third picture shows the leaning Tower of Pisa. A 55.9-meter rope was dropped from the tower. The shift of the tower is 5.2 meters (520 centimeters).

(NOTE: The pictures are NOT to scale)

Marti Tower



Lopsided House



Tower of Pisa

1. Which building leans the most: the Marti Tower, the lopsided house, or the Tower of Pisa? Show how you found your answer.

Figure 81. Grade 8 Problem Solving Assessment, Item 4.

Table 132

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 4

Points	Response
3	For full credit the student must have: • Correct ratio for House: 0.045, • correct ratio for Tower of Pisa: 0.093 NOTE: a minor conversion error made in calculating both ratios should only be deducted once • a valid conclusion BASED ON CALCULATED RATIOS: the Tower of Pisa leans most
2	Two criteria met
1	Only one criterion met
0	No ratios (or proportional reasoning) used (i.e., student compares the "shift" not the "lean") *or* Proper conclusion with no evidence or computation *Or* Incorrect conclusion
X	Nonscorable or no response

* This item uses the answer for Item 3. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the rubric.

Key Cards

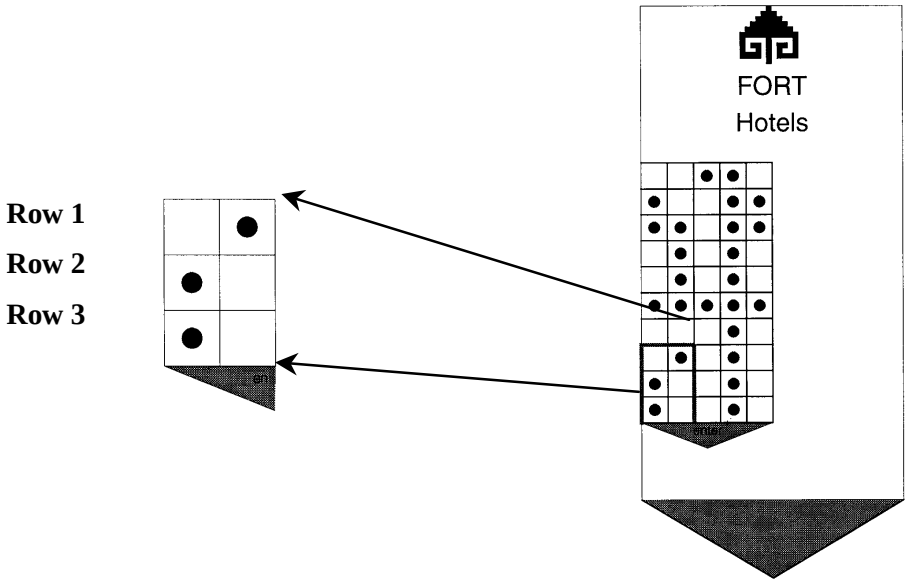
The third section on the eighth grade assessment, Keys, involves cards used as hotel keys and the codes encrypted on them. Drawings of the keys are provided for the students. This section contains three questions. A partial item classification matrix is shown in Table 133. The entire matrix can be found in Appendix D.

Table 133
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Key Card Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
5	Statistics/Number	II	Medium	2	Recognizing and extending patterns while adhering to given restrictions Smart counting
6	Statistics/Number	II	Difficult	2	Recognizing and extending patterns while adhering to given restrictions Smart counting
7	Statistics/Number	III	Difficult	2	Recognizing and extending patterns while adhering to given restrictions Smart counting

Item 5 assesses students' abilities to interpret a problem situation; determine all possible combinations in sample space given constraints; and provide correct answer and/or accurate drawings (see Figures 82and 83). The scoring rubric is shown in Table 134. Strategy codes were not assigned for this item.

The rooms in some hotels have keys in the shape of a card that has holes in it. A picture of a key card is shown on the right.



A section of the key card is shown on the left. The section consists of six places that determine the code for a room. A code consists of three holes, one in each row.

1. How many different codes can be made in this section of the key card if *exactly one hole* is punched in each row? You may use the diagrams on the next page to try different possibilities.

Figure 82. Grade 8 Problem Solving Assessment, Item 5.

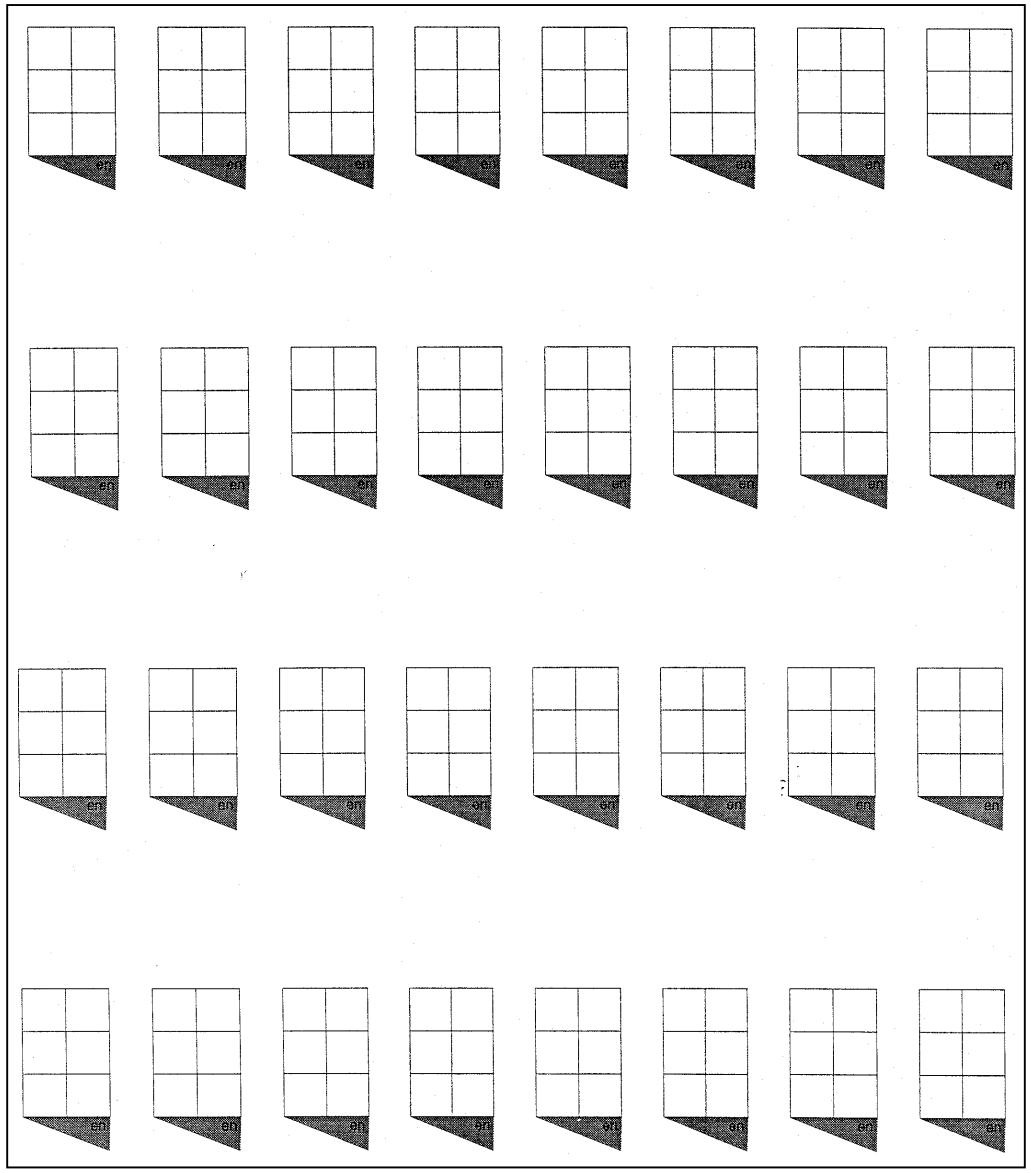
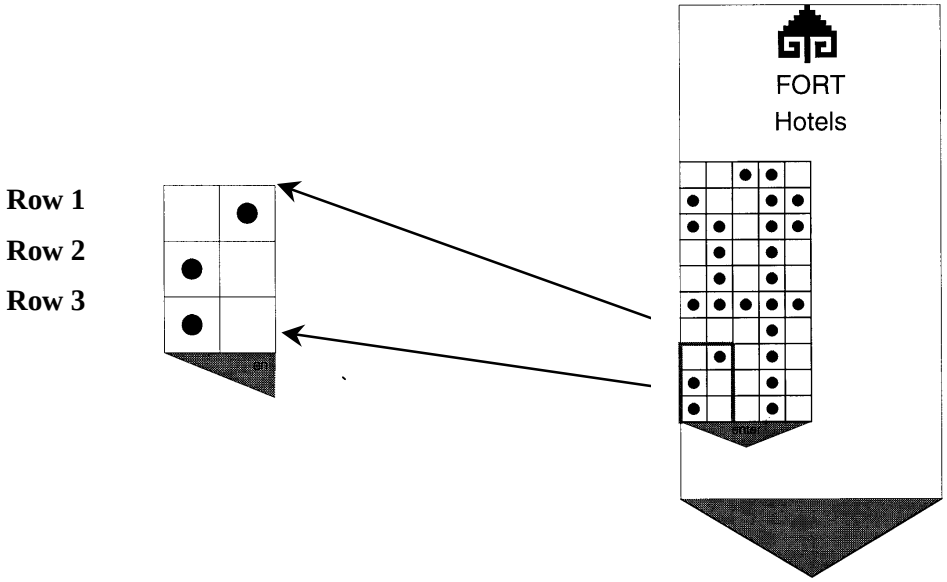


Figure 83. Grade 8 Problem Solving Assessment, Accompanying Diagram to Item 5.

Table 134
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 5

Points	Response NOTE: Student should receive full credit for drawing the correct number of different keys without providing numerical response.
2	Correct answer: 8
1	Answer given: 7 or 9
0	Answer given: ≤ 6 or ≥ 10
X	Nonscorable or no response

Item 6 assesses students' abilities to interpret a problem situation; determine all possible combinations in sample space given constraints; and provide correct answer and/or accurate drawings (see Figures 84 and 85). The scoring rubric is shown in Table 135. Strategy codes were not assigned for this item.



1. Look at the enlarged section of the key card pictured on the left again. Suppose only three holes can be punched in this section of the key card, but there can be *zero, one, or two holes* in each row. If a code consists of exactly three holes, how many different codes can be made in this section of the key card? You may use the diagrams on the next page to try different possibilities.

Figure 84. Grade 8 Problem Solving Assessment, Item 6.

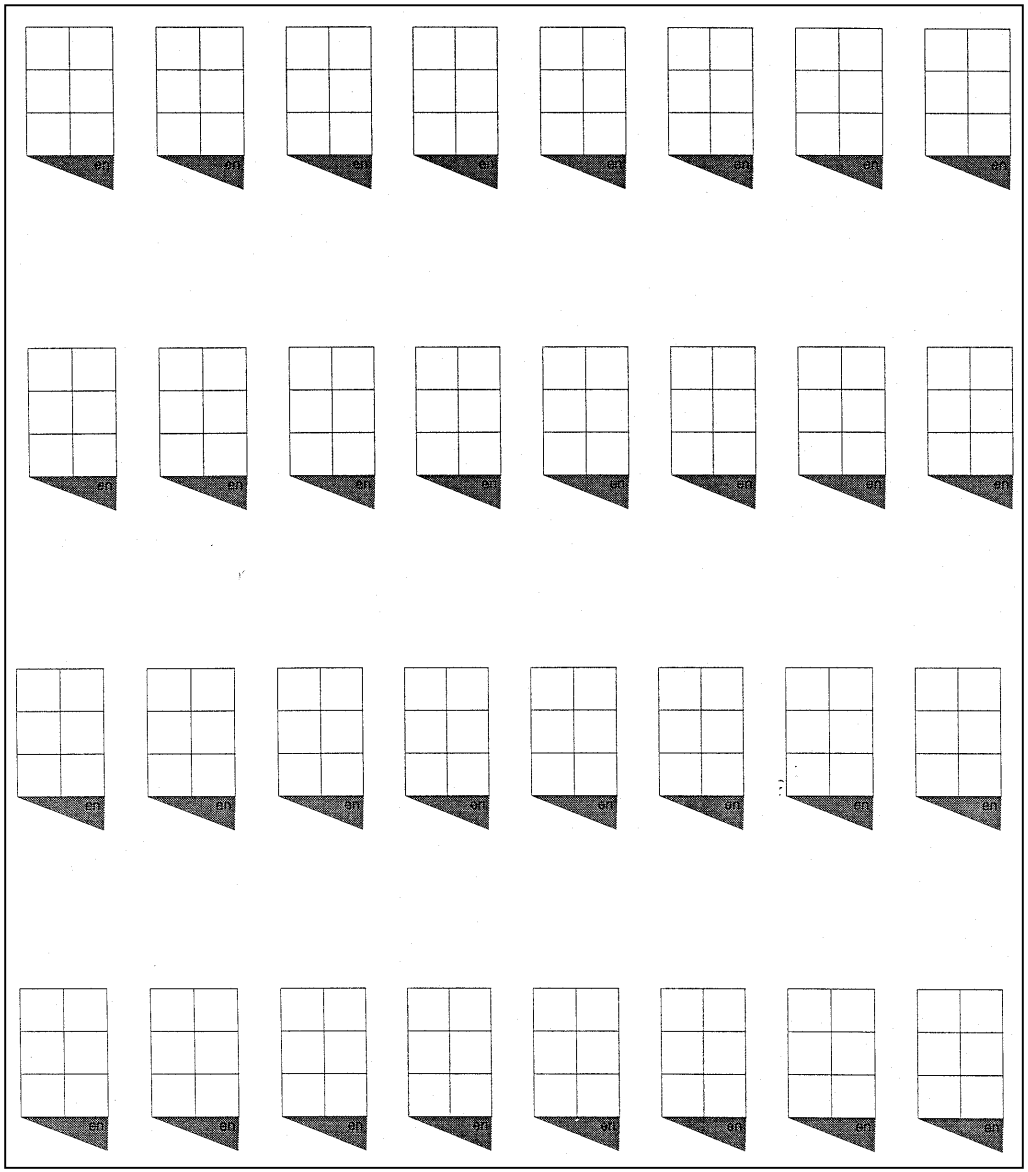


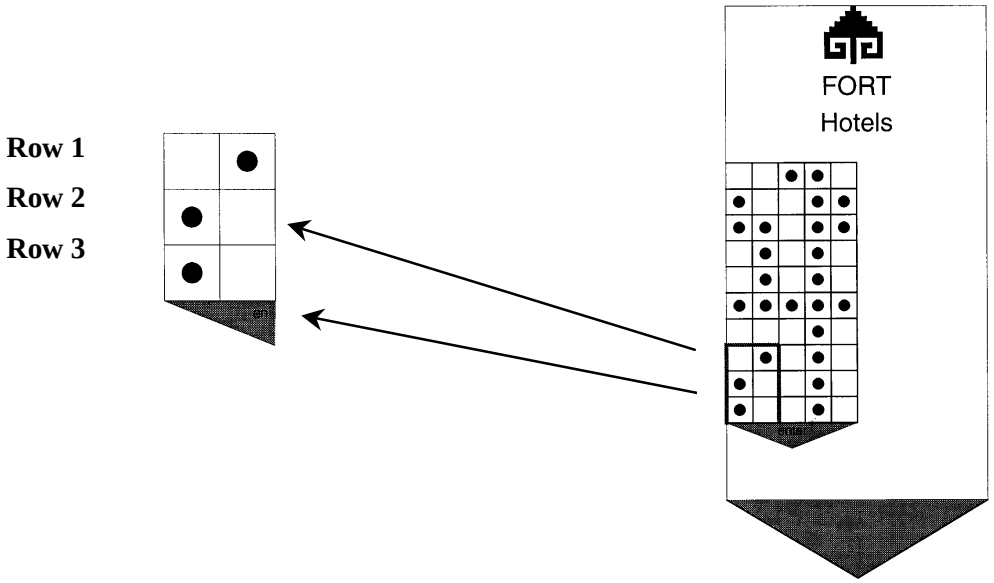
Figure 85. Grade 8 Problem Solving Assessment, Accompanying Diagram to Item 6.

Table 135
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 6

Points	Response
2	Correct answer: 20
1	Answer is in the following range: 16 – 19 *or* 21 – 24
0	Other incorrect response: ≤ 15 or ≥ 25
X	Nonscorable or no response

* The student received full credit for drawing the correct number of different keys without providing a numerical response.

Item 7 assesses students' abilities to interpret a problem situation; determine possible combinations in sample space given constraints; draw a conclusion about the validity of a mathematical statement; and provide correct conclusion with correct explanation (see Figure 86). The scoring rubric and strategy codes are shown in Tables 136 and 137, respectively.



Look at the enlarged section of the key card pictured on the left again.

The manager of the hotel complains, “For our 50 room hotel, punching only three holes in that section does not give us enough codes. I suggest that each card can have from *zero to six holes* punched in that section. Then over 50 different codes could be made, and we would have enough for all of the rooms."

1. Is the manager correct? Use mathematical evidence to support your conclusion.

Figure 86. Grade 8 Problem Solving Assessment, Item 7.

Table 136

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 7

Points	Response
2	Correct conclusion: yes, the manager is correct *With* Correct explanation or strategy that indicates the manager is correct e.g., "For each place there are 2 possibilities. For 6 places there are 2 x 2 x 2 x 2 x 2 x 2 = 2^6 = 64 possibilities. So the manager is right." *Or* Student finds the number of possibilities for 0 through 6 holes holes possibilities zero = 1 one = 6 two = 15 three = 20 four = 15 five = 6 six = 1 64 possibilities total so the manager has 14 more than needed.
1	Correct conclusion with partially correct explanation To receive credit for a counting strategy students must show evidence of drawings or counting the number of different possibilities for each hole
0	Correct conclusion with no or trivial/non-mathematical explanation Incorrect conclusion
X	Nonscorable or no response

Table 137
Strategy Codes, Grade 8 Problem Solving Assessment, Item 7

Code	Description
58	Answer only
71	Trivial explanation (with no restrictions there has to be more than 50 codes)
81	Counting strategy
82	Exponential strategy (2 x 2 x 2 x 2 x 2 x 2 = 64; $2^6 = 64$)
89	Other strategy or explanation
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Seesaw

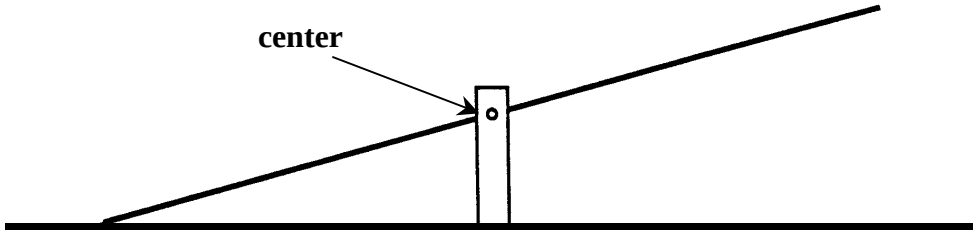
The fourth section of the eighth grade assessment, Seesaw, is structured around a seesaw. A diagram of the seesaw with the length of the seesaw and its height above the ground at its center is given. This section contains two questions. A partial item classification is shown in Table 138. The entire matrix can be found in Appendix D.

Table 138
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Seesaw Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
8	Geometry	II	Difficult	2	Problem solving using similar triangles
9	Geometry	III	Difficult	2	Problem solving using similar triangles Interpreting results

Item 8 assesses students' abilities to interpret a problem situation to determine the missing dimension of a triangle; use properties of similar triangles; and provide correct answer with clear work or correct explanation (see Figure 87). The scoring rubric and strategy codes are shown in Tables 139 and 140, respectively.

A drawing of a seesaw is shown below (NOTE: this drawing is NOT to scale).



The center of the seesaw is 20 inches above the ground. The length of the seesaw board is 16 feet (192 inches).

1. The end of one side of the seesaw is at its highest point above the ground. What is its height? Show how you found your answer.

Figure 87. Grade 8 Problem Solving Assessment, Item 8.

Table 139
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 8

Points	Response
2	Correct answer: 40 in And Correct work shown or explanation given
1	Correct answer with no work shown or no explanation given *Or* Correct work shown with minor calculation error
0	Incorrect response
X	Nonscorable or no response

Table 140
Strategy Codes, Grade 8 Problem Solving Assessment, Item 8

Code	Description
12	Answer only
65*	Uses similar triangles to justify solution
68A*	Uses Pythagorean theorem
71	Trivial or descriptive justification
77	Number based justification (e.g., 20 x 2 = 40)
78	Visual or descriptive model that represents the exchange of heights
89	Other justification
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

*If both Pythagorean Theorem and similar triangles are used to justify solution, code as 65.

Item 9 assesses students' abilities to interpret a problem situation to determine the missing dimension of triangle; use properties of similar triangles; and provide correct answer with clear work or correct explanation (see Figure 88). The scoring rubric and strategy codes are shown in Tables 141 and 142, respectively.

Sean wanted the highest point of the seesaw to be higher than it is in the drawing so he doubled the length of the board.
Remember, the board must still be centered on the beam.

9. At what height is the highest point of the seesaw now? Show how you found your answer.

Figure 88. Grade 8 Problem Solving Assessment, Item 9.

Table 141

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 9

Points	Response
2	Correct answer: 40 in And Correct work shown or explanation given
1	Correct answer with no work shown or no explanation given *Or* Correct work shown with minor calculation error
0	Incorrect response

Table 142

Strategy Codes, Grade 8 Problem Solving Assessment, Item 9

Code	Description
12	Answer only
16	Doubles answer to problem 8
65*	Uses similar triangles to justify solution
68A*	Uses Pythagorean theorem
71	Trivial or descriptive justification
77	Number based justification (e.g., 20 x 2 = 40)
78	Visual or descriptive model that represents the exchange of heights
89	Other justification
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

*If both Pythagorean Theorem and similar triangles are used to justify solution, code as 65.

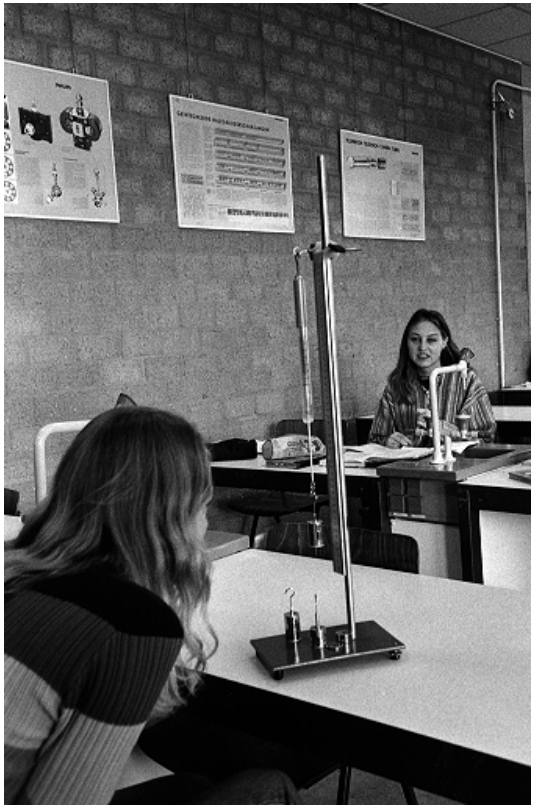
Stretch

The fifth section of the eighth grade assessment, Stretch, focuses on an experiment performed in an eighth-grade science class. The experiment involved hanging different weights on a spring and recording the length of the spring after each weight was hanged on it. The results of the experiment are presented in both a table and on a line graph. This section contains five questions. A partial item classification is shown in Table 143. The entire matrix can be found in Appendix D.

Table 143
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Stretch Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
10	Algebra	I	Easy	1	Recognizing patterns presented in a graph and table
11	Algebra	I	Difficult	1	Calculating slope of a given line
12	Algebra	II	Difficult	2	Using information on a graph to derive a linear formula
13	Algebra	II	Medium	1	Analyzing a situation Interpreting information from a graph Understanding the meaning of coordinates
14	Algebra	III	Medium	2	Analyzing a situation Interpreting information from a graph Understanding the meaning of coordinates

Item 10 assesses students' abilities to identify pattern based on data in table or plotted in graph; use a pattern to determine y-value for data point given x coordinate; and provide correct answer within reasonable range (see Figures 89 and 90). The scoring rubric is shown in Table 144. Strategy codes were not assigned for this item.

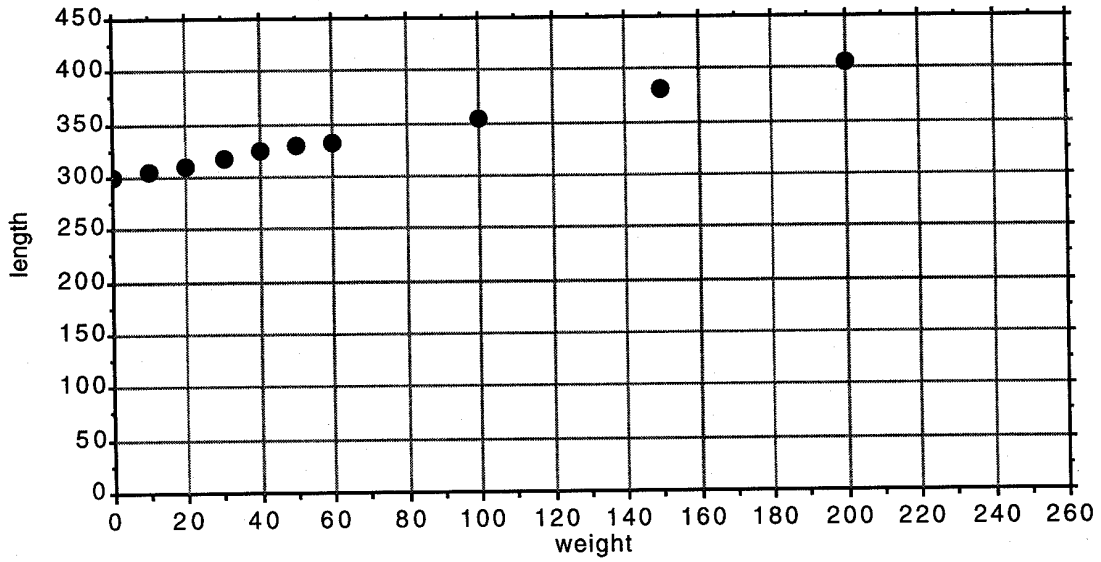


In science class, Nicola conducted an experiment involving a spring and different weights. The spring was stretched by hanging weights on a hook at the bottom of the spring. Nicola recorded the length of the spring after hanging each weight on the hook. The results of the experiment are shown in the table below.

weight (g)	0	10	20	30	40	50	60	70	100	150	200
length (mm)	300	305	310	316	325	329	332	…	354	380	406

Nicola used a computer to plot the results of her experiment on a coordinate system. This graph is shown below.

Figure 89. Grade 8 Problem Solving Assessment, Item 10, Part 1.



10. Nicola did not record the length the spring stretched when a weight of 70 grams was hung on the hook. Using the graph, what do you expect this length to be?

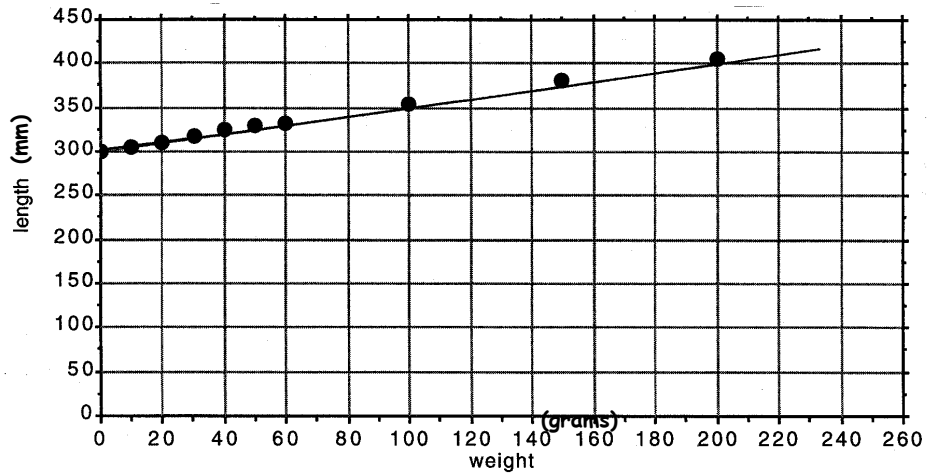
Figure 90. Grade 8 Problem Solving Assessment, Item 10, Part 2.

Table 144
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 10

Points	Response
1	Any answer between 325 and 351 answer receives credit if point is correctly placed on graph
0	Answer: < 325 or > 351
X	Nonscorable or no response

Item 11 assesses students' abilities to interpret the meaning of data points on a graph and calculate the slope of a given line based on values read from graph; provide correct answer (see Figure 91). The scoring rubric is shown in Table 145. Strategy codes were not assigned for this item.

In the graph below Nicola drew a straight line that best fits the data points.



Nicola remarked that the line she drew had a small slope. Calculate the slope of the line that Nicola drew.

Figure 91. Grade 8 Problem Solving Assessment, Item 11.

Table 145
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 11

Points	Response
2	Answer: 0.5 *or* equivalent value
1	Student selects a change in y and a corresponding change in x in the ratio of 1:2 but calculates the slope as 2 or 1/5 *or* Student correctly selects a change in y and a corresponding change in x with a relationship of 1:2 or 2:1, but does not simplify the slope e.g., 400, 200 *or* Minor calculation eror in simplifying accurate slope
0	Incorrect response
X	Nonscorable or no response

Item 12 assesses students' abilities to interpret the meaning of data points on a graph; identify the y-intercept on graph of plotted line; write a formula for graphed line (see Figure 92). This item uses the answer from Item 11. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the scoring rubric. The scoring rubric is shown in Table 146. Strategy codes were not assigned for this item.

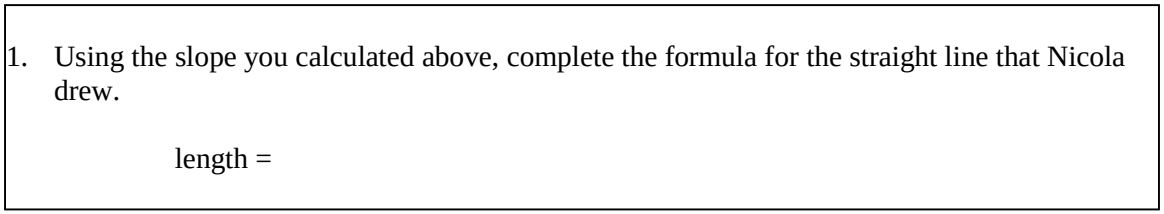


Figure 92. Grade 8 Problem Solving Assessment, Item 12.

Table 146
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 12

Points	Response
2	Correct slope and y-intercept are given in a formula Length = 300 + 0.5 x *weight* *Or* Length = 0.5*w* + 300
1	Error in formula 300 or 0.5 are given in the formula, but variable for weight is missing or misplaced (e.g., Length = 300 + 0.5 or Length = 300 x weight + 0.5)
0	Incorrect response
X	Nonscorable or no response

Item 13 assesses students' abilities to interpret the meaning of y-intercept; use a graph to draw conclusion; and provide correct conclusion with clear supporting explanation (see Figure 93). The scoring rubric is shown in Table 147. Strategy codes were not assigned for this item.

Paul performed an experiment similar to the one Nicola conducted. The straight line he drew started at (0, 200).

13. Was the spring Paul used in his experiment longer or shorter than Nicola's spring? Explain how you found your answer.

Figure 93. Grade 8 Problem Solving Assessment, Item 13.

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Table 147
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 13

Points	Response
1	Correct conclusion with explanation that uses mathematical evidence or references specific points on the graph: e.g., “The spring is shorter because at weight 0, the length is only 200”
0	Correct conclusion with incomplete, incorrect, or no explanation *Or* Other incorrect response
X	Nonscorable or no response

Item 14 assesses students’ abilities to use coordinates of points on a graph to make a comparison and to provide correct conclusion with clear supporting explanation (see Figure 94). The scoring rubric is shown in Table 148. Strategy codes were not assigned for this item.

1. When Paul hung a weight of 180 grams on a hook at the bottom of his spring, the spring stretched to 450 mm. Did the spring Paul use stretch more or less than Nicola's spring? Explain how you found your answer.

Figure 94. Grade 8 Problem Solving Assessment, Item 1

Table 148
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 14

Points	Response
2	Correct conclusion with clear explanation: e.g., "Stretched more. Paul's went from 200 to 450 and Nicola's went from 300 to only 406 with even more weight on it." *or* "Paul's stretched more. His spring went to 450 cm and hers stretched only to 400 mm." *or* "Paul's stretched more. If you draw his line on the graph, it is steeper than Nicola's."
1	Correct conclusion, "more," with partially complete explanation e.g., "Paul's stretched more. There are bigger differences in the lengths." *or* *"More, because it went up to 450 and hers only went u[to 380 at the same weight" or* *"More because Paul's went to 450 and Nicola's only went to 406 altogether."* *or* Clear explanation that implies an answer of more
0	Correct conclusion with incorrect or no explanation e.g., "More, Paul's spring is longer" or "More. Length measurements are bigger." *or* Other incorrect response
X	Nonscorable or no response

The sixth section on the eighth grade assessment, Parking, is organized around a circular stone-paved parking lot. It contains three questions. A partial item classification matrix is shown in Table 149. The entire matrix can be found in Appendix D.

Parking

Table 149

Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Parking Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
15	Geometry	I	Easy	1	Computing the area of a circle using a given formula
16	Number	II	Medium	2	Calculating with percentages Rounding
17	Geometry	III	Difficult	3	Problem solving using symmetry or measuring angles Interpreting results

Item 15 assesses students' abilities to calculate the area of circle given the diameter and multiply with decimals (see Figure 95). The scoring rubric is shown in Table 150. Strategy codes were not assigned for this item.

In the photograph shown below, the parking lot next to the house is circular. Only a part of the parking lot is shown in the photograph.

The inner circle of this parking lot has a diameter of 15 meters.

1. Compute the area of the inner circle in square meters.
   (NOTE: area of a circle = π x radius x radius)

Figure 95. Grade 8 Problem Solving Assessment, Item 15.

Table 150
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 15

Points	Response
2	Answer should be in the range 176 – 177 m^2 *or* 56.25π (Area = 7.5 x 7.5 x π = 176.625 m^2) *No work needs to be shown for full credit* *The word "meters" not needed for full credit*
1	7.5 x 7.5 x π is given with a minor computation error
0	15 x 15 x π (which is approx. 706.5) *or* Other incorrect answer
X	Nonscorable or no response

Item 16 assesses students' abilities to identify appropriate series of arithmetic calculations (multiplication, addition) and use whole numbers, decimals, and percent (see Figure 96). This item used the answer from Item 16. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the scoring rubric. The scoring rubric is shown in Table 151. Strategy codes were not assigned for this item.

If you did not find an answer to question 16, you may use 190 m^2 as the area of the circle for question 17.

To pave the inner circle, 70 stones per square meter plus an extra 5% were needed.

16. How many stones were needed to pave the inner circle?

Figure 96. Grade 8 Problem Solving Assessment, Item 16.

Table 151

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 16

Points	Response
2	There are two possible answers depending on number of m^2 students use: For students who use 177 m^2 (or other answer from problem 16) Answer: 13,010 stones Work: (177) x 70 = 12,390; 12,390 + 5% = 13,010 (or 12,390 x 1.05) For students who use 190 m^2 Answer: 13,965 stones Work: 190 x 70 = 13,300; 13,300 + 5% = 13,965 (*or* 13,300 x 1.05 = 13,965) NOTE: for students who use any other m^2, the answer will vary 1 pt each for: 1) calculation of stones per m^2 ; 2) adding 5%
1	Correct calculation of stones per m^2 only For students who use 177 m^2, 177 x 70 = 12,390 For students who use 190 m^2 , 190 x 70 = 13,300 For students who use any other m^2, answer varies
0	Incorrect response
X	Nonscorable or no response

* This item used the answer from Item 16. If a student used an incorrect answer to build an otherwise correct solution, the student received full or partial credit in accordance with the criteria given in the scoring rubric.

Item 17 assesses students' abilities to interpret a problem situation in a geometric context (see Figures 97 and 98). The scoring rubric and strategy codes are shown in Tables 152 and 153, respectively.

In the parking lot, one parking space is used as an entrance. A scale drawing of one parking space is shown on the next page.

17. Not including the entrance, how many parking spaces are there in the parking lot? Show how you found your answer.

Figure 97. Grade 8 Problem Solving Assessment, Item 17, Part 1.

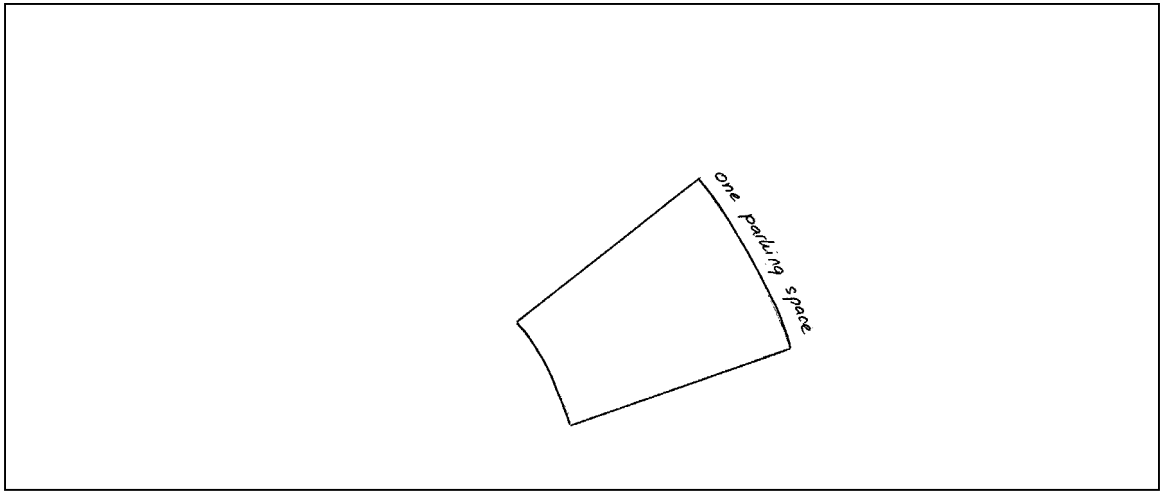


Figure 98. Grade 8 Problem Solving Assessment, Item 17, Part 2.

Table 152

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 17

Points	Response
3	Answer: 14 – 20 spaces 19° = 18 spaces; 18° = 19 spaces; 17° = 20 spaces; *For full credit the following three criteria must be satisfied:* • Valid strategy (e.g., student finds central angle by extending sides of parking space *or* student calculates circumference and divides by estimated arc length *or* Student draws approximately equal sized parking spaces that fit around a circle) • Accuracy of student work • Entrance is subtracted
2	Student uses a valid strategy and shows accurate work but neglects to subtract entrance *or* Student uses a valid strategy and subtracts the entrance but makes a minor computation error *or* Student makes drawings of parking spaces that are inconsistent with the size of the original parking space and subtracts the entrance (10-13 or 21-24)
1	Student uses a valid strategy makes a minor computation error and neglects to subtract the entrance *or* Student makes drawings of parking spaces that are inconsistent with the size of the original parking space and neglects to subtract the entrance (10-13 or 21-24) Answer only 14-20
0	Incorrect response
X	Nonscorable or no response

Table 153
Strategy Codes, Grade 8 Problem Solving Assessment, Item 17

Code	Description
12	Answer only
53	Repeats drawing of piece to count
66	Uses angle measure with direct computation (w/o drawing repeated pieces)
67	Uses estimated arc length and circumference to compute
69	Other strategy
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., “I don’t understand,” “This is confusing”)
92	Nonscorable: emotional (e.g., expletive or “This is stupid,” “I don’t care”)
99	No response

Cubes

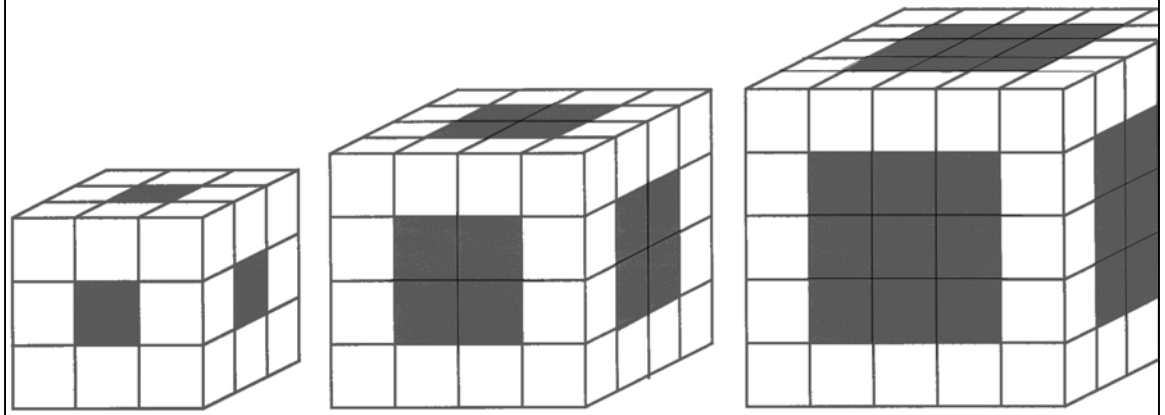
The final section on the eighth grade assessment, Cubes, extends the Patio and Playground contexts found on earlier assessments. Whereas on the fifth, sixth, and seventh grade assessments students were asked to reason about two-dimensional patterns, on the eighth grade assessment, these patterns are three-dimensional. The first three cubes of the sequence are shown in Figure 99. This section contains four questions. A partial item classification matrix is shown in Table 154. The entire matrix can be found in Appendix D.

Table 154
Mathematical Content, Levels of Reasoning and Difficulty, and Point Scores, for the Cubes Context on the Grade 8 Problem Solving Assessment

Problem	Strand	Level	Difficulty	Points	Mathematical Goal
18	Algebra	I	Medium	2	Recognizing a pattern or using a formula to compute a number
19	Algebra	I	Medium	2	Using a formula to compute a number
20	Algebra	II	Difficult	2	Analyzing three-dimensional shapes Recognizing patterns
21	Algebra	III	Difficult	3	Recognizing a pattern Generalizing a pattern and expressing it with variables

Item 18 assesses students' abilities to interpret a pattern demonstrated in diagrams and provide correct answer and explanation (see Figure 99). The scoring rubric and strategy codes are shown in Tables 155 and 156, respectively.

The first three cubes of a larger sequence are shown below.



The first cube is a 3 x 3 x 3 cube. The second cube is a 4 x 4 x 4 cube.

Small white cubes form the edges of the large cubes. The other cubes on the outside are black. **Ignore the cubes on the inside**.

18. How many white cubes are needed for the 4 x 4 x 4 cube?
    Show how you found your answer.

Figure 99. Grade 8 Problem Solving Assessment, Item 18.

Table 155
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 18

Points	Response
2	Correct answer: 32 *with* Clear explanation
1	Correct answer with incomplete or no explanation (including an explanation of "I counted them" without additional description or supporting work) *or* Correct strategy with minor computation error that leads to an incorrect answer *or* Student identifies the correct number of white tiles (12) on one face of the cube
0	Incorrect response
X	Nonscorable or no response

*An explanation of "I counted them" without additional description or supporting work is not sufficient for full credit.

Table 156
Strategy Codes, Grade 8 Problem Solving Assessment, Item 18

Code	Description
12	Answer only, no strategy evident
21	Uses diagrams or drawings to compute (if diagram is included with calculation, code as strategy 21)
37	Creates a table or pattern
19	Other strategy (includes "I counted" without evidence)
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response

Item 19 assesses students' abilities to use a formula to compute and calculate with whole numbers (see Figure 100). The scoring rubric is shown in Table 157. Strategy codes were not assigned for this item.

A formula to compute the total number of white cubes on the surface of an $n \times n \times n$ cube is:

$$number\ of\ white\ cubes = 12 \times n - 16$$

19. Using the formula, compute the number of white cubes in a 25 x 25 x 25 cube.

Figure 100. Grade 8 Problem Solving Assessment, Item 19.

Table 157
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 19

Points	Response
2	Correct answer: 284
1	Minor calculation error that leads to incorrect answer
0	Incorrect response
X	Nonscorable or no response

Item 20 assesses students' abilities to analyze a pattern in the construction of a cube; extend the pattern; and provide a correct answer and explanation (see Figure 101). The scoring rubric is shown in Table 158. Strategy codes were not assigned for this item.

20. How many black cubes are needed to build a 7 x 7 x 7 cube? Show how you found your answer. **Remember to ignore the cubes on the inside**.

Figure 101. Grade 8 Problem Solving Assessment, Item 20.

Table 158
Scoring Rubric, Grade 8 Problem Solving Assessment, Item 20

Points	Response
2	Answer: 150 *With* Clear supporting work: e.g., 5 x 5 x 6
1	Student identifies the number of black cubes on one side, 25, but neglects to multiply by 6 *or* Student makes a minor error in calculating cubes on one side and then multiplies by 6 sides
0	Incorrect response
X	Nonscorable or no response

Item 21 assesses students' abilities to interpret a pattern demonstrated in diagrams; generalize the pattern; use variables to represent pattern; and provide a correct formula (see Figure 102). The scoring rubric and strategy codes are shown in Tables 159 and 160, respectively.

21. Write a formula or equation to compute the total number of black cubes needed for an n x n x n cube. **Remember to ignore the cubes on the inside.**

Figure 102. Grade 8 Problem Solving Assessment, Item 21.

Table 159

Scoring Rubric, Grade 8 Problem Solving Assessment, Item 21

Points	Response
3	Correct formula: black = (n – 2) x (n – 2) x 6 or black = $(n – 2)^2$ x 6
2	Minor mistake in formula, e.g., *n* – 2 x n - 2 x 6 = black *or* Recursive formula *or* "Two-part" formula e.g., $(n – 2)^2$ = the number of black cubes on a side x 6 = total number of black cubes, also includes arrow language *or* Expression without equal sign e.g., $(n – 2)^2$ x 6 *Or* Student describes the formula in words
1	Student makes an attempt to describe the formula in words but it is incomplete or lacks several parts *or* Two minor errors appear in the given formula, (e.g., white cubes^2 x 6 = black)
0	Incorrect response
X	Nonscorable or no response

Table 160

Strategy Codes, Grade 8 Problem Solving Assessment, Item 21

Code	Description
21	Recursive formula
26	Arrow language
35	Uses number expression (e.g., "$(3\text{-}2)^2$ x 6 = 6")
38	Direct formula (e.g., "black = $(n - 2)^2$ x 6") related to the picture
47	Descriptive explanation
48	Incomplete algebraic response *or* "Two-part" formula *or* Expression
49	Other strategy (including direct formulas unrelated to the cubes)
90	Nonscorable: non-numerical, irrelevant doodles, unclear, illegible
91	Nonscorable: confused (e.g., "I don't understand," "This is confusing")
92	Nonscorable: emotional (e.g., expletive or "This is stupid," "I don't care")
99	No response