Longitudinal/Cross-Sectional Study of the Impact of *Mathematics in Context* on Student Performance

Scale for Mapping Progress in Mathematical Competence

(Working Paper #49)

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Part 1: General Descriptions of Bands

Band vi

There are 8 full credit items in total for this band (6 PSA items; 2EA items). As there is only one item (EA, Town Populations, 3.97 full credit) with a logit value greater than 3.0, this item has been considered to be located in band vi. There are no partial credit items situated in band vi.

Degree of Mathematization

A very high level of mathematization is required in order to respond fully to items in this band, as students are typically required to translate highly contextualized real world problems (where comprehension of a detailed written stimulus is required) into mathematical terms, and then identify and use an appropriate mathematical strategy and use a range of tools to solve the problems.

In general, the demands required to receive full credit for these items are such that students typically need to identify the key elements of the problem; show an extensive working solution; and provide a summative statement in response to the problem. Students are typically required to identify, compare or combine elements of the problem, draw on assumptions based on real-world knowledge, and provide a fully justified conclusion supported by work, explanation, or reasoning.

Content specific demands at this level:

- Algebra problems typically require students to substitute values appropriately for several variables in expressions, combine like terms, and simplify.
- Number problems typically require students to interpret numerical data, use ratio and scale, and apply absolute (constant difference) or relative comparison (proportional change).
- Geometry problems typically require students to interpret spatial data to compare two-dimensional representations of three-dimensional objects involving different scales and perspectives.
- Statistics problems typically require students to identify an appropriate graph to represent statistical information and to explain why a particular graph is a suitable choice to support a conclusion.

Degree of Formalization

Items situated in this band are categorized as Preformal or Formal (65.5% and 37.5%, respectively) with Preformal items predominating. It should be noted however, that the number of items in the pool situated in this band is relatively small, so these figures may be misrepresentative of the band in this case. There are no Informal items situated in this band.

Competency Class/Subclass Composition

Items situated in this band are categorized as having a either a focus in Competency Class 1 or Class 2, with Class 2 items predominating (85%). This competency class is represented by the subcategories of *Problem Solving* (2 items) and *Interpretation/Reflection* (5 items).

Band v

There are 27 full credit items in total for this band (22 PSA items; 5 EA items).

Degree of Mathematization

A high level of mathematization is required in order to respond fully to items in this band, as students are typically required to translate contextualized real world problems into mathematical terms, and then identify and use an appropriate mathematical strategy and use a range of tools to solve the problems.

In general, the demands required to receive full credit for these items are such that students typically need to provide a correct answer accompanied by a complete explanation of the work needed to arrive at the solution that takes into

account the key points identified in the problem. The demands of the items in the band include the ability to interpret and to analyze, but the extent to which these skills are required is generally less compared to band vi.

Content specific demands at this level:

- Algebra problems typically require students to interpret a pattern and then generalize the pattern using variables to represent the pattern; interpret meaning of data points on a graph and use these to identify the slope, to construct an equation to represent a linear graph; or construct an accurate graph of exponential increase.
- Number problems typically require students to identify appropriate arithmetical calculations; divide decimal numbers; use both whole numbers and fractions; or compare calculated ratios.
- Geometry problems typically require students to interpret spatial data to reason about the relationship between angle measures and side lengths; or find missing dimensions using the properties of similar triangles.
- Statistics problems typically require students to analyze and compare two sets of statistical data in which the data sets are not presented in the same units (e.g., costs of x dollars per month or costs of x dollars per year).

Degree of Formalization

Items situated in this band are categorized as Informal (3.7%), Preformal or Formal, with Preformal and Formal co-dominating (44% and 52%, respectively).

Competency Class/Subclass Composition

Items situated in this band are categorized as having either a focus in Competency Class 1 or Class 2, with Class 2 items predominating (59%). This competency class is represented by the subcategories of *Modelling* (1 item), *Problem Solving* (10 items), and *Interpretation/Reflection* (5 items).

Band iv

There are 45 full credit items in total for this band (34 PSA items; 11 EA items).

Degree of Mathematization

A moderate level of mathematization is required in order to respond fully to items in this band, as students are typically required to translate either a contextualized or a non-contextualized, generally non-routine problem into mathematical terms. For contextualized problems, the solutions tend to depend on the application of a formula (.g., πr^2 for the area of a circle) or relationship (e.g., proportionality of corresponding side lengths) with which it is expected that the student would be familiar. Non-contextualized items also tend to depend on the application of specific knowledge (e.g., recognize an algebraic expression).

Content specific demands at this level:

- Algebra problems typically require students to recognize the equivalent algebraic form of an expression (m + m + m + m = 4m); identify a third point on a straight line given the coordinates of two points on the line.
- Number problems typically require students to use percent; use whole number division with remainder; add and subtract with integers.
- Geometry problems typically require students to find the area of an irregular shape; complete an accurate drawing of a two-dimensional representation of a three-dimensional object (e.g., a pyramid with a pentagonal base).
- Statistics problems typically require students to construct a bar graph to represent statistical data; describe the concept of the mean.

Degree of Formalization

Items situated in this band are categorized as as Informal (27%), Preformal or Formal (22%), with Preformal predominating (51%).

Competency Class/Subclass Composition

Items situated in this band are categorized as having either a focus in Competency Class 1 or Class 2, with Class 1 items predominating (62%). This competency class is represented by the subcategories of *Standard Representations* (5 items), *Computations* (15 items), *Definitions* (3 items), *Routine Procedures* (3 items), and *One Method* problems (5 items).

Band iii

There are 35 full credit items in total for this band (24 PSA items; 11 EA items).

Degree of Mathematization

Mathematization in this band is limited to the application of mathematical tools in order to solve predominantly routine problems and any contextualized problem-solving tasks that generally require only relatively simple computations for their solution.

Content specific demands at this level:

- Algebra problems typically require students to interpret the meaning of the y-intercept; read coordinates of a point from a graph.
- Number problems typically require students to place a decimal on a number line; use whole numbers to compare metric measures (e.g., convert mm to L).
- Geometry problems typically require students to visualise a three-dimensional object from its two-dimensional net (cube); calculate the area of a circle given its diameter.
- Statistics problems typically require students to apply a ratio to the whole to determine an unknown quantity.

Degree of Formalization

Items situated in this band are categorized as Informal, Preformal (40%) or Formal (6%), with Informal predominating (54%).

Competency Class/Subclass Composition

Items situated in this band are categorized as having either a focus in Competency Class 1 or Class 2, with Class 1 items predominating (63%). This competency class is represented by the subcategories of *Standard Representations* (7 items), *Computations* (10 items), *Definitions* (4 items), and *Routine Procedures* (1 item).

Band ii

There are 13 full credit items in total for this band (6 PSA items; 7 EA items).

Degree of Mathematization

Mathematization at this level is limited to the application of routine procedures to standard or familiar contextual representations of problems.

Content specific demands at this level:

- Algebra problems typically require students to interpret a simple algebraic expression; interpret a pattern presented in a series of diagrams.
- Number problems typically require students to read decimal numbers from a scale; use counting to analyze a pattern.
- Geometry problems typically require students to interpret a scale to estimate distance between two points on a map.
- Statistics problems typically require students to construct a sample space limited to the possible combinations of two selected items.

Degree of Formalization

Items situated in this band are categorized as Preformal or Informal (8%), with Preformal predominating (92%).

Competency Class/Subclass Composition

Items situated in this band are categorized as having either a focus in Competency Class 1 or Class 2, with Class 1 items predominating (92%). This competency class is represented by the subcategories of *Standard Representations* (3 items), *Computations* (4 items), and *Routine Procedures* (1 item).

Band i

As there is only one full credit item in this band (Stretch, PSA, 810), this item has been considered in conjunction with the partial credit items that are also located in band i for the purposes defining Degree of Mathematization below.

Degree of Mathematization

Mathematization at this level is limited to the application of routine procedures to standard or familiar contextual representations of problems. Partial credit items in this band tend to be of the type in which less credit is given when only some of the required criteria are met (e.g., for the construction of a bar graph, only one of following is met: labelled axes, consistent scales, correct bar lengths and widths, labelled bars), or when a criterion is met, but an explanation is lacking.

Degree of Formalization

The full credit item situated in this band is categorized as Preformal.

Competency Class/Subclass Composition

The full credit item situated in this band is classified as having a focus in Competency Class 1 and is subcategorized as addressing *Routine Procedures*.

General Trends

Degree of Formalization

Shifts from Informal at bands i and ii to Preformal (bands iv and v), with an overlap to more Formal items at bands v and then into vi.

Competency Class/Subclass

Shifts from a greater proportion of items with a focus in Competency Class 1 in the lower/middle bands [i, ii, iii, and iv (with iii and iv being approximately equivalent in both percentage of items in each Class and in subclasses addressed)] to a greater proportion of items with a focus in Competency Class 2 in the uppermost bands (v and vi).

Progress Map created by the Australian Council of Educational Research.

Part 2: PSA

Illustrative Items—PSA

Band vi

Logit — Full credit items

[3.0]

2.75

2.5

2.43 = most difficult full credit item in the set (Buildings 509)

Buildings 509

Full credit response: 2 points
This task requires students to compare two-dimensional representations of the same objects, drawn in different perspective views and to different scales, where only one view has a marked scale. Students need to apply a strategy to draw a conclusion (i.e., buildings are not on the same scale). Strategies employed by students include: use a visual estimate; refer to scale; and directly measure buildings on the map and in the pictures. Students are required to justify their conclusions by communicating an explanation of their reasoning.
The partial credit response (1 point) is situated in band iii.

2.43 Buildings (509) 2 pt

2.28 Key cards (807)

Key cards 807
This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. Students need to apply a strategy, taking into account constraints, to draw a conclusion about the validity of a mathematical statement and to provide mathematical evidence to support their conclusion. Strategies employed by students include: models and /or counting possible combinations and exponential models. Students are required to state clearly that the mathematical statement is supported by their conclusion.

2.25

2.0

Band v

[2.0]

1.82 Playground (517)

Playground 517
This task requires students to apply their understanding of odd and even numbers to draw a conclusion for a numerical pattern based on a geometrical representation. Students justify their conclusion with an appropriate explanation. Strategies employed by students include: a number-based strategy in which patterns of numbers are described (e.g., “To get the total number of tiles, you add the number of tiles in the ring to the total number of tiles in the previous playground. The ring is always even and the playground in always odd. An even + odd = odd”; a visually–based strategy in which the students refer to the diagrams (e.g., “The number of tiles in each ring is always even, and there is always one extra in the middle which makes it odd.” Students are required to

Pentagon 709
Full credit response: 3 points
This task requires students to define a shaded region to complete a drawing of the shadow cast by a two-dimensional representation of a three-dimensional object. Students need to define the shaded region taking into account: that the sun’s rays are perpendicular to the vertical axis of the building; the cast shadow width is constant in this scenario; and that the top edge of the shadow will be parallel to the bottom edge of the shadow as indicated at location ‘F’. Student drawing must fulfill all three criteria.
The partial credit responses (2 point and 1 point) are situated in bands iv and iii respectively.

1.75

1.56 Pentagon (709) 3 pt

1.5

1.36 Stretch (814)

Stretch 814
This task requires students to compare values in a graphical representation given in coordinate form. Students need to draw a conclusion by comparing the relative difference between the given data set and a second scenario in which the y-axis data point has to be used as the basis for comparison (given a specified x-value). Students are required to support their conclusion by communicating an explanation of their reasoning.

1.25

1.0

Band iv

[1.0]

0.75

0.55 Key Cards (806)

0.5

0.54 Pentagon (709) 2

Key cards 806
This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. The students determine all possible combinations in a sample space, given several constraints (zero, one, or two holes can be punched in each row). Students respond by providing either a numerical answer or by drawing the elements of the sample space on the diagrams provided to indicate a sample space of 20.

0.25

Band iii

[0]

-0.12 Stretch (813)

-0.13 Playground (516)

- 0.25

- 0.5

-0.6 Key cards (805)

- 0.75

- 1.0

Stretch 813
This task requires students to interpret a written description of a problem situation and link this to a graphical representation. Students need to draw a conclusion based on change in an experimental variable and link it to an understanding of the meaning of the y-intercept. Students are required to support their conclusion by communicating an explanation of their reasoning.

Playground 516
This task requires students to apply a strategy to extend a pattern beyond the values listed in a table (refer to 515). Strategies employed by students include: extending the table of values in tabular form; deriving the answer without extending the table (e.g., 9 X 8 = 72); generalizing the pattern (R X 8 = T). Students are required to provide the correct answer and communicate an explanation of their strategies. ***Note:*** The full credit response for this item is easier than the full credit response for 515. This problem requires the student to use the answer to 515, but if a student uses an *incorrect answer* to 515 to build *an otherwise correct* solution, the student receives full credit for item 516.

Key cards 805
This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. The students determine all possible combinations in a sample space given a constraint (one hole can be punched in each row only). Students respond by providing either a numerical answer or by drawing the elements of the sample space on the diagrams provided to indicate a sample space of eight.

Birds of all sizes 621
Full credit response: 2 points
This task requires students to accurately read a graph to extract data points (x and y axes values) according to a given criterion. Students then need to substitute the values into a ratio (presented in fractional form) and to correctly calculate the ratio. Students are required to state the correct answer and to show the values positioned in the ratio correctly.
The partial credit response (1 point) is situated in band ii.

-0.17 Birds of all sizes (621) 2 pt

Playground 515
This task requires students to find a strategy for recognizing a pattern from data presented in tabular form (a numerical pattern based on a geometrical representation) and extending it. Strategies employed by students are the development of a numeric linear pattern; and the use of drawings. A full credit response required all five missing numbers to be correctly identified.

-0.72 Pentagon (709) 1 pt

-0.91 Buildings (509) 1 pt

Band ii

[-1.0]

- 1.25

-1.4 Playground (514)

Playground 514
This task requires students to interpret a diagrammatic representation of a problem. The students then need to count the number of tiles to satisfy a particular criterion (i.e., shaded tiles). The students need only to state the correct answer, without explanation.

-1.25 Birds of all sizes (621) 1 pt

- 1.5

-1.52 Playgrounds (722)

Playgrounds 722
This task requires students to interpret a pattern given in diagrammatic form and to extend the pattern by specified criteria. The students need only to state the correct answer without explanation.

-1.75

- 2.0

Band i

[- 2.0]

-2.25

- 2.5

-2.53 Stretch (810) = least difficult full credit item in set.

Stretch 810
This task requires students to understand and interpret a graphical representation (using a coordinate system) to identify the value of an unknown. Students need to interpolate to determine a value on the y-axis, given a data point on the x-axis. Students are required to define a value within reasonable limits, given the intervals marked on the y-axis. The context is a familiar one, commonly encountered in science and mathematics classrooms.

-2.69 = least difficult partial credit item in the set (Questionnaire

-2.75

[- 3.0]

Part 3: EA

Illustrative Items—EA

Band vi

Logit Full credit items (including representative items from each of N,G,S,A to indicate progression within content areas)

[3.0] **Note: There is one item (the most difficult full credit item in the set) located above band vi (Town Populations 3.97)**

2.75

2.5

2.43 Radio Station 815 — Geometry

Radio Station 815 Geometry
This task requires students to interpret a written description of a problem situation involving spatial data, to draw and label a diagram to indicate the distance between two loci on a straight line, and to then define an area of overlap (two-dimensional representation [concentric circles] of radio broadcast areas given specific constraints and conditions). Students need to indicate all of the specified features and apply a scale correctly. Students are required to communicate their working by providing a drawing that fully supports their answer.

2.25

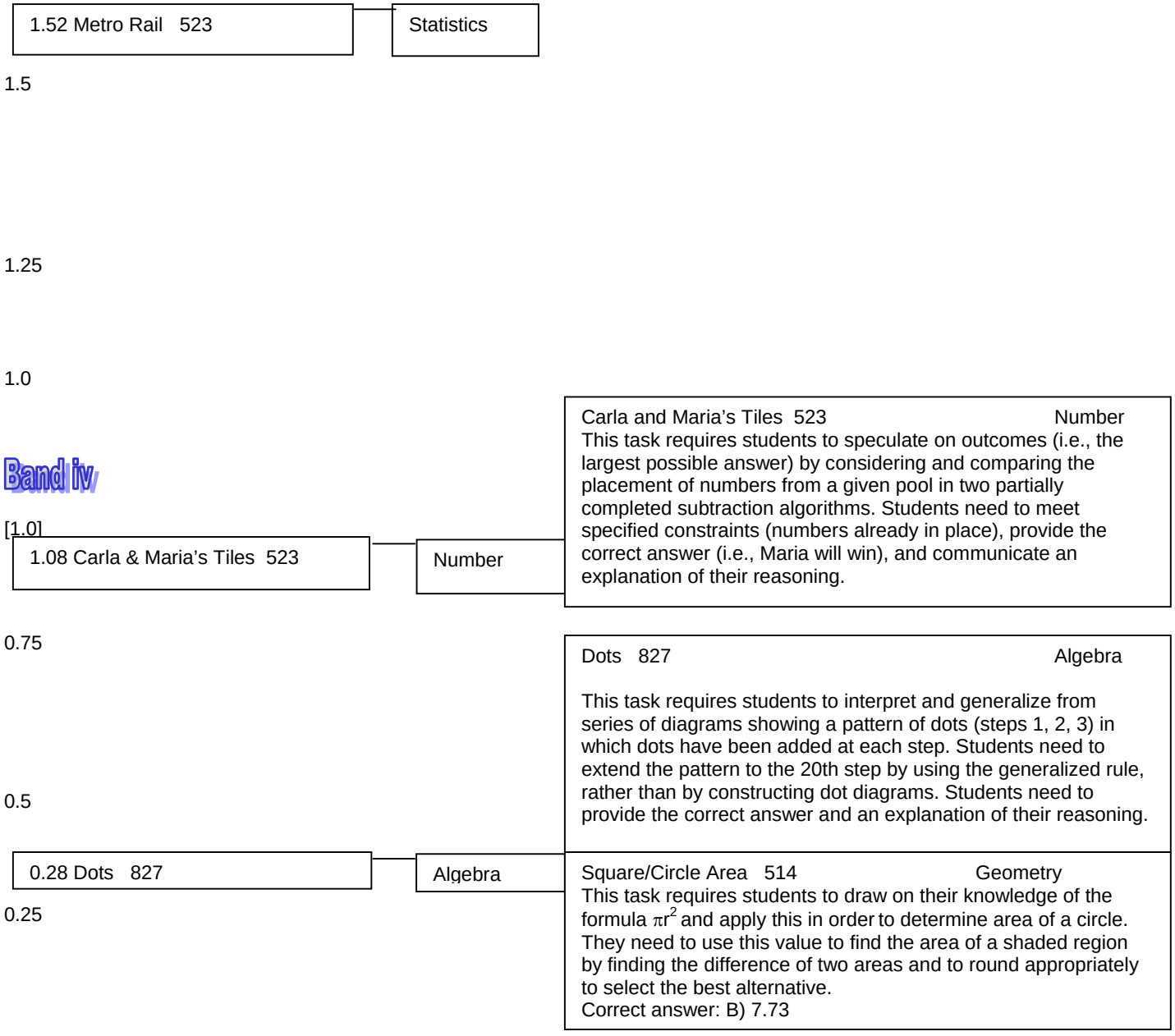
2.0

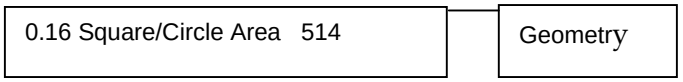
Band v

[2.0]

1.75

Metro Rail 523 Statistics
This task requires students to analyze two graphical representations of the same data set presented in tabular form to (i) chose the graph that emphasizes a relatively large increase in one parameter over a given range, and (ii) critically analyze this choice taking into account differences in scale. Students are required to justify their conclusion in part (ii) by communicating an explanation of their reasoning (e.g., one graph exaggerates a relatively small increase).


1.52 Metro Rail 523
Statistics
1.5
1.25
1.0
Band iv
[1.0]
1.08 Carla & Maria's Tiles 523
Number
Carla and Maria's Tiles 523 Number
This task requires students to speculate on outcomes (i.e., the largest possible answer) by considering and comparing the placement of numbers from a given pool in two partially completed subtraction algorithms. Students need to meet specified constraints (numbers already in place), provide the correct answer (i.e., Maria will win), and communicate an explanation of their reasoning.
0.75
Dots 827 Algebra
This task requires students to interpret and generalize from series of diagrams showing a pattern of dots (steps 1, 2, 3) in which dots have been added at each step. Students need to extend the pattern to the 20th step by using the generalized rule, rather than by constructing dot diagrams. Students need to provide the correct answer and an explanation of their reasoning.
0.5
0.28 Dots 827
Algebra
Square/Circle Area 514 Geometry
This task requires students to draw on their knowledge of the formula πr^2 and apply this in order to determine area of a circle. They need to use this value to find the area of a shaded region by finding the difference of two areas and to round appropriately to select the best alternative.
Correct answer: B) 7.73
0.25



0

Band iii

[0]

- 0.22 Red/Blue Cube 511 — Statistics

- 0.25

- 0.32 Tip Calculation 526 — Number

- 0.5

- 0.75

- 0.89 3(x+5)=30 525 — Algebra

- 1.0

Red/Blue Cube 511 — Statistics
This task requires students to apply a probability ratio (expressed as a fraction) to the six faces of a die in order to determine the number of faces of a specified color.
Correct answer: D) Four

Tip Calculation 526 — Number
This task requires students to round a decimal money amount (i.e., $24.99 rounded to $25) and to identify multiplication as the appropriate operation required in order to calculate a specified percent of that total (15% tip).
Correct answer: C) $3.75

3(x+5) = 30 525 — Algebra
This task requires students to solve an equation using the distributive law:
3(x +5) = 30
3x + 15 = 30
3x = 15
x = 5
Correct answer: B) 5

Flour for Cookies 502 — Number
This task requires students to identify an appropriate strategy (add or multiply) to solve a problem involving a mixed fraction and a whole number, where the context is quantity.
Correct answer: B) 4

Band ii

[-1.0]

- -1.03 Flour for Cookies 502 — Number
- -1.08 Oxford to Smithville 505 — Geometry

- 1.25

- -1.47 Nine Chips 520 — Statistics

Oxford to Smithville 505 — Geometry
This task requires students to interpret a scale (1 cm = 8 km) to estimate the distance between two reference points on a map.
Correct answer: C) 35 km

Nine Chips 520 — Statistics
This task requires students to calculate the probability of an event, given a constraint (consider even numbers only) and to select an answer expressed as a fraction (where simplification is not required). Correct answer: C) $\frac{4}{9}$

- 1.5

-1.72 k+6 508 — Algebra

k+6 508 Algebra
This task requires students to interpret an algebraic expression and to understand the term ‘infinitely’ in this context.
Correct answer: E) Infinitely many

-1.75 = least difficult full credit item in the EA set (Similar Triangles)

-1.97 = least difficult partial credit item in the EA set (Metro Rail, 1 pt response)

- 2.0

Band i **There are no full or partial credit EA items located in band i.**

[- 2.0]

-2.25

- 2.5

-2.75

[- 3.0]

Proposed Descriptive bands and illustrative items
EA DRAFT Dec 2001

Band descriptions

Band **vi**

Item thresholds

Equal to or greater than 2.0

EA	Illustrative items	Degree of mathematization	Degree of Formalization	Competency classes
There are 2 full credit items in this band. ▪ Interprets data two or more data sources, given in more than one form (numerical, graphical). ▪ Substantiates claim or determines solution given specific constraints and conditions.	▪ Town Populations.2 ▪ Radio Station.4	A very high level of mathematization is required in order to respond fully to items in this band. The information supplied is highly contextualized. The demands for full credit require that students identify the key elements of the problem; show an extensive working solution; and provide a summative statement in response to the problem. Calculations required are very complex in nature.	Preformal (1); Formal (1) No full credit items in this band are classified as Informal	Focus in: class 2 (2) Interpretation, Reflection (2)

EA	Illustrative items	Degree of mathematization	Degree of Formalization	Competency classes
There are 5 full credit items in this band.	▪ Metro			**Focus in class 2** (4)

• Critically analyse two graphical representations and fully justify conclusions • Converts rates and compares for specified time interval • Place numbers in algorithm to meet specified constraints	Rail.3 ▪ Zeds.3 [30] ▪ Carla and Maria's tiles.4	A high level of mathematization is required in order to respond fully to items in this band. A complete explanation is required that takes into account key points as noted in the problem situation; takes account of a comparative situation; and incorporates communication of a clear summative statement. Calculations are complex in nature.	Informal (1); Preformal (2); Formal (2)	▪ Interpretation, Reflection (1) ▪ Problem solving (3) **Focus in class 1** (1) ▪ Definition (1)

EA There are 11 full credit items in this band. • Interpret and extend or generalise from	Illustrative items ▪ Similar Triangles(b)/Dots.4 ▪ Dropped ball ▪ 4m/What is n?	Degree of mathematization A moderate level of mathematization is required in order to respond fully to	Degree of Formalization Preformal (6); Formal (5) No full credit items in this band are	Competency classes **Focus in class 2** (4) ▪ Multiple defined methods (1) ▪ Interpretation, Reflection (2)

a pattern of figures • Model a complex multistep problem • Demonstrate understanding of abstract variable and recognise equivalent algebraic form of an expression		items in this band. For contextualized problems, the solutions tend to depend on the application of a formula (eg area of a circle) or relationship (eg proportionality of corresponding side lengths) with which it is expected that the student would be familiar. Non-contextualized items also tend to depend on the application of specific knowledge (eg recognise algebraic expression).	classified as Informal	▪ Problem solving (1) **Focus in class 1** (7) ▪ Standard representation (3) ▪ Computations (1) ▪ Definition (1) ▪ Routine procedures (2)

Band **iii**

Item thresholds

Greater than or equal to - 1.0; less than 0

EA	Illustrative items	Degree of mathematization	Degree of Formalization	Competency classes
There are 11 full credit items in this band. ▪ Interpret and apply ratio to determine unknown ▪ Recognize constant nature or ratio ▪ Identify appropriate operation to apply for percentage calculation ▪ Interpret and extrapolate to continue trend in graph ▪ Solve an equation using an appropriate strategy	▪ Red-Blue cube/ Radio Sales/Blue Pen ▪ Boys and Girls in class ▪ Tip Calculation ▪ Car speed – stopping distance ▪ 3(x+ 5) = 30	Mathematization at this level is limited to the application of concepts (such as ratio, extrapolation from a line graph, calculations involving fractional quantities) in order to respond fully to items in this band.	Informal (6); Preformal (4); Formal (1)	**Focus in class 2** (5) ▪ Interpretation, Reflection (4) ▪ Problem solving (1) **Focus in class 1** (6) ▪ Standard representation (1) ▪ Computations (2) ▪ Definition (3)

Band **ii**

Item thresholds

Greater than or equal to - 0.2; less than –0.1

EA	Illustrative items	Degree of mathematization	Degree of Formalization	Competency classes
There are 7 full credit items in this band. ▪ interprets simple algebraic expression ▪ Interpret scale to estimate distance ▪ Construct simple sample space ▪ Count number or congruent triangles for three figure sequence	▪ k + 6 ▪ Oxford to Smithville ▪ Blue and yellow balls ▪ Similar triangles	Mathematization at this level is limited to the application of routine procedures to standard representations of problems.	Informal (6); Preformal (1) No full credit items in this band are classified as Formal	**Focus in class 1** (7) ▪ Standard representation (2) ▪ Computations (2) ▪ Routine procedures (3)

EA	Illustrative items	Degree of mathematization	Degree of Formalization	Competency classes
▪ No full nor partial credit EA items in this band.	-			

Band **i**

Item thresholds

Less than – 0.2

EAS: Grade 5 – 8

Strand	Level	Test	Grade	FINAL ?s	Order	RME Level	Code	8th p-value	Prob Type	TYPE	5th Grade	Test # (5W)	6th Grade	Test # (6X)	7th Grade	Test # (7Y)	8th Grade	Test # (8Z)
A	A	T	1	X	1	I	**AAT1S1a**	75.0	PATT	CON	E	24A	E	24A				
A	A	N	1	X	2	I	**AAN1K1**	72.3	VARB	MC	E	8	E	5	E	4		
A	B	T	1	X	3	I	**ABT1O7**	72.0	EQUA	MC	A	25	A	22	A	20	A	17
A	B	T	1	X	4	I	**ABT1P10**	58.0	VARB	MC	A	10	A	9	A	6	A	2
A	B	T	1	X	5	I	**ABT1I8**	41.0	PLOT	MC	A	18	A	23	A	23	A	20
A	B	T	1	X	6	I	**ABT1L11**	34.0	PATT+	MC	A	15	A	14	A	11	A	7
A	B	T	1	X	7	II	**ABT1S1b**	26.0	PATT+	CON	A	24B	A	24B	A	22	A	16
A	C	T	I	X	8	I	**ACT1I1**	37.0	EQUA	MC					D	27	D	24
A	C	N	1	X	9	II	**ACN1K9**	10.9	MODL	CON							D	27
G	A	N	1	X	1	I	**GAN1C14**	54.6	VISI	MC	E	3	E	2				
G	A	T	1	X	2	I	**GAT1J17**	66.0	Scale	MC	E	5	E	3	E	2		
G	B	T	1	X	3	I	**GBT1L8**	60.0	Scale	MC	A	1	A	19	A	19	A	14
G	B	N	1	X	4	I	**GBN1O14**	32.3	PERI	MC	A	21	A	20	A	16	A	12
G	B	T	1	X	5	I	**GBT1U2a**	31.0	MEAS	CON	A	22A	A	25A	A	24A	A	22A
G	B	N	1	X	6	I	**GBN1M5**	29.2	AREA	MC	A	14	A	13	A	10	A	6
G	B	T	1	X	7	I	**GBT1U2b**	10.0	MEAS	CON	A	22B	A	25B	A	24B	A	22B
G	C	T	1	X	8	I	**GCT1P9**	38.0	SIMI	MC					D	17	D	25
G	C	N	1	X	9	II	**GCN1E13**	15.9	MODL	CON							D	15
N	A	T	1	X	1	I	**NAT1Q5**	65.0	Ratio	MC	E	4						
N	A	N	1	X	2	I	**NAN1E6**	63.8	FRAC	MC	E	2	E	1				
N	B	N	1	X	3	I	**NBN1E7**	58.7	COMP	CON	A	12	A	11	A	8	A	4
N	B	T	1	X	4	I	**NBT1I2**	58.0	FRAC	MC	A	16	A	15	A	12	A	8
N	B	P	1	X	5	I	**NBP1C5**	37.7	PERC	MC	A	26	A	6	A	1	A	19
N	B	N	1	X	6	I	**NBN1K4**	30.6	DECI	MC	A	19	A	18	A	15	A	11
N	B	P	1	X	7	II	**NBP1L5**	11.2	PERC	CON	A	9	A	17	A	25	A	23
N	C	P	1	X	8	II	**NCP1C13**	27.8	NUMB	CON			D	8	D	5	D	1
N	C	N	1	X	9	I	**NCN1M4**	21.9	OrdOp	MC					D	26	D	21
S	A	N	1	X	1	I	**SAN1E9**	59.1	Sampl	CON	E	6						
S	A	T	1	X	2	I	**SAT1N18**	56.0	PROB	MC	E	20	E	21				
S	B	T	1	X	3	I	**SBT1K7**	53.0	PROB	MC	A	7	A	4	A	3	A	13
S	B	T	1	X	4	I	**SBT1R8**	49.0	LINE	MC	A	13	A	12	A	9	A	5
S	B	T	1	X	5	I	**SBT1O5**	47.0	PROB	MC	A	11	A	10	A	7	A	3
S	B	T	1	X	6	I	**SBT1C18**	35.7	Sampl	MC	A	17	A	16	A	13	A	9
S	B	P	1	X	7	II	**SBP1L9**	25.5	STAT	CON	A	23	A	26	A	18	A	26
S	C	N	1	X	8	I	**SCN1M3**	22.8	Median	MC			D	7	D	14	D	10
S	C	T	1	X	9	II	**SCT1V2**	19.0	DATA	CON					D	21	D	18

EAS Grade 5

Strand	Level	Test	Grade	FINAL ?s	Order	RME Level	Code	8th p-value	Prob Type	TYPE	5th Grade	Test # (5W)
G	B	T	1	X	3	I	**GBT1L8**	60.0	Scale	MC	A	1
N	A	N	1	X	2	I	**NAN1E6**	63.8	FRAC	MC	E	2
G	A	N	1	X	1	I	**GAN1C14**	54.6	VISI	MC	E	3
N	A	T	1	X	1	I	**NAT1Q5**	65.0	Ratio	MC	E	4
G	A	T	1	X	2	I	**GAT1J17**	66.0	Scale	MC	E	5
S	A	N	1	X	1	I	**SAN1E9**	59.1	Sampl	CON	E	6
S	B	T	1	X	3	I	**SBT1K7**	53.0	PROB	MC	A	7
A	A	N	1	X	2	I	**AAN1K1**	72.3	VARB	MC	E	8
N	B	P	1	X	7	II	**NBP1L5**	11.2	PERC	CON	A	9
A	B	T	1	X	4	I	**ABT1P10**	58.0	VARB	MC	A	10
S	B	T	1	X	5	I	**SBT1O5**	47.0	PROB	MC	A	11
N	B	N	1	X	3	I	**NBN1E7**	58.7	COMP	CON	A	12
S	B	T	1	X	4	I	**SBT1R8**	49.0	LINE	MC	A	13
G	B	N	1	X	6	I	**GBN1M5**	29.2	AREA	MC	A	14
A	B	T	1	X	6	I	**ABT1L11**	34.0	PATT+	MC	A	15
N	B	T	1	X	4	I	**NBT1I2**	58.0	FRAC	MC	A	16
S	B	T	1	X	6	I	**SBT1C18**	35.7	Sampl	MC	A	17
A	B	T	1	X	5	I	**ABT1I8**	41.0	PLOT	MC	A	18
N	B	N	1	X	6	I	**NBN1K4**	30.6	DECI	MC	A	19
S	A	T	1	X	2	I	**SAT1N18**	56.0	PROB	MC	E	20
G	B	N	1	X	4	I	**GBN1O14**	32.3	PERI	MC	A	21
G	B	T	1	X	5	I	**GBT1U2a**	31.0	MEAS	CON	A	22.1
G	B	T	1	X	7	I	**GBT1U2b**	10.0	MEAS	CON	A	22.2
S	B	P	1	X	7	II	**SBP1L9**	25.5	STAT	CON	A	23
A	A	T	1	X	1	I	**AAT1S1a**	75.0	PATT	CON	E	24.1
A	B	T	1	X	7	II	**ABT1S1b**	26.0	PATT+	CON	A	24.2
A	B	T	1	X	3	I	**ABT1O7**	72.0	EQUA	MC	A	25
N	B	P	1	X	5	I	**NBP1C5**	37.7	PERC	MC	A	26

EAS Grade 6

Strand	Level	Test	Grade	FINAL ?s	Order	RME Level	Code	8th p-value	Prob Type	TYPE	6th Grade	Test # (6X)
N	A	N	1	X	2	I	**NAN1E6**	63.8	FRAC	MC	E	1
G	A	N	1	X	1	I	**GAN1C14**	54.6	VISI	MC	E	2
G	A	T	1	X	2	I	**GAT1J17**	66.0	Scale	MC	E	3
S	B	T	1	X	3	I	**SBT1K7**	53.0	PROB	MC	A	4
A	A	N	1	X	2	I	**AAN1K1**	72.3	VARB	MC	E	5
N	B	P	1	X	5	I	**NBP1C5**	37.7	PERC	MC	A	6
S	C	N	1	X	8	I	**SCN1M3**	22.8	Median	MC	D	7
N	C	P	1	X	8	II	**NCP1C13**	27.8	NUMB	CON	D	8
A	B	T	1	X	4	I	**ABT1P10**	58.0	VARB	MC	A	9
S	B	T	1	X	5	I	**SBT1O5**	47.0	PROB	MC	A	10
N	B	N	1	X	3	I	**NBN1E7**	58.7	COMP	CON	A	11
S	B	T	1	X	4	I	**SBT1R8**	49.0	LINE	MC	A	12
G	B	N	1	X	6	I	**GBN1M5**	29.2	AREA	MC	A	13
A	B	T	1	X	6	I	**ABT1L11**	34.0	PATT+	MC	A	14
N	B	T	1	X	4	I	**NBT1I2**	58.0	FRAC	MC	A	15
S	B	T	1	X	6	I	**SBT1C18**	35.7	Sampl	MC	A	16
N	B	P	1	X	7	II	**NBP1L5**	11.2	PERC	CON	A	17
N	B	N	1	X	6	I	**NBN1K4**	30.6	DECI	MC	A	18
G	B	T	1	X	3	I	**GBT1L8**	60.0	Scale	MC	A	19
G	B	N	1	X	4	I	**GBN1O14**	32.3	PERI	MC	A	20
S	A	T	1	X	2	I	**SAT1N18**	56.0	PROB	MC	E	21
A	B	T	1	X	3	I	**ABT1O7**	72.0	EQUA	MC	A	22
A	B	T	1	X	5	I	**ABT1I8**	41.0	PLOT	MC	A	23
A	A	T	1	X	1	I	**AAT1S1a**	75.0	PATT	CON	E	24.1
A	B	T	1	X	7	II	**ABT1S1b**	26.0	PATT+	CON	A	24.2
G	B	T	1	X	5	I	**GBT1U2a**	31.0	MEAS	CON	A	25.1
G	B	T	1	X	7	I	**GBT1U2b**	10.0	MEAS	CON	A	25.2
S	B	P	1	X	7	II	**SBP1L9**	25.5	STAT	CON	A	26

EAS Grade 7

Strand	Level	Test	Grade	FINAL ?s	Order	RME Level	Code	8th p-value	Prob Type	TYPE	7th Grade	Test # (7Y)
N	B	P	1	X	5	I	**NBP1C5**	37.7	PERC	MC	A	1
G	A	T	1	X	2	I	**GAT1J17**	66.0	Scale	MC	E	2
S	B	T	1	X	3	I	**SBT1K7**	53.0	PROB	MC	A	3
A	A	N	1	X	2	I	**AAN1K1**	72.3	VARB	MC	E	4
N	C	P	1	X	8	II	**NCP1C13**	27.8	NUMB	CON	D	5
A	B	T	1	X	4	I	**ABT1P10**	58.0	VARB	MC	A	6
S	B	T	1	X	5	I	**SBT1O5**	47.0	PROB	MC	A	7
N	B	N	1	X	3	I	**NBN1E7**	58.7	COMP	CON	A	8
S	B	T	1	X	4	I	**SBT1R8**	49.0	LINE	MC	A	9
G	B	N	1	X	6	I	**GBN1M5**	29.2	AREA	MC	A	10
A	B	T	1	X	6	I	**ABT1L11**	34.0	PATT+	MC	A	11
N	B	T	1	X	4	I	**NBT1I2**	58.0	FRAC	MC	A	12
S	B	T	1	X	6	I	**SBT1C18**	35.7	Sampl	MC	A	13
S	C	N	1	X	8	I	**SCN1M3**	22.8	Median	MC	D	14
N	B	N	1	X	6	I	**NBN1K4**	30.6	DECI	MC	A	15
G	B	N	1	X	4	I	**GBN1O14**	32.3	PERI	MC	A	16
G	C	T	1	X	8	I	**GCT1P9**	38.0	SIMI	MC	D	17
S	B	P	1	X	7	II	**SBP1L9**	25.5	STAT	CON	A	18
G	B	T	1	X	3	I	**GBT1L8**	60.0	Scale	MC	A	19
A	B	T	1	X	3	I	**ABT1O7**	72.0	EQUA	MC	A	20
S	C	T	1	X	9	II	**SCT1V2**	19.0	DATA	CON	D	21
A	B	T	1	X	7	II	**ABT1S1b**	26.0	PATT+	CON	A	22
A	B	T	1	X	5	I	**ABT1I8**	41.0	PLOT	MC	A	23
G	B	T	1	X	5	I	**GBT1U2a**	31.0	MEAS	CON	A	24.1
G	B	T	1	X	7	I	**GBT1U2b**	10.0	MEAS	CON	A	24.2
N	B	P	1	X	7	II	**NBP1L5**	11.2	PERC	CON	A	25
N	C	N	1	X	9	I	**NCN1M4**	21.9	OrdOp	MC	D	26
A	C	T	I	X	8	I	**ACT1I1**	37.0	EQUA	MC	D	27

EAS Grade 8

Strand	Level	Test	Grade	FINAL ?s	Order	RME Level	Code	8th p-value	Prob Type	TYPE	8th Grade	Test # (8Z)
N	C	P	1	X	8	II	**NCP1C13**	27.8	NUMB	CON	D	1
A	B	T	1	X	4	I	**ABT1P10**	58.0	VARB	MC	A	2
S	B	T	1	X	5	I	**SBT1O5**	47.0	PROB	MC	A	3
N	B	N	1	X	3	I	**NBN1E7**	58.7	COMP	CON	A	4
S	B	T	1	X	4	I	**SBT1R8**	49.0	LINE	MC	A	5
G	B	N	1	X	6	I	**GBN1M5**	29.2	AREA	MC	A	6
A	B	T	1	X	6	I	**ABT1L11**	34.0	PATT+	MC	A	7
N	B	T	1	X	4	I	**NBT1I2**	58.0	FRAC	MC	A	8
S	B	T	1	X	6	I	**SBT1C18**	35.7	Sampl	MC	A	9
S	C	N	1	X	8	I	**SCN1M3**	22.8	Median	MC	D	10
N	B	N	1	X	6	I	**NBN1K4**	30.6	DECI	MC	A	11
G	B	N	1	X	4	I	**GBN1O14**	32.3	PERI	MC	A	12
S	B	T	1	X	3	I	**SBT1K7**	53.0	PROB	MC	A	13
G	B	T	1	X	3	I	**GBT1L8**	60.0	Scale	MC	A	14
G	C	N	1	X	9	II	**GCN1E13**	15.9	MODL	CON	D	15
A	B	T	1	X	7	II	**ABT1S1b**	26.0	PATT+	CON	A	16
A	B	T	1	X	3	I	**ABT1O7**	72.0	EQUA	MC	A	17
S	C	T	1	X	9	II	**SCT1V2**	19.0	DATA	CON	D	18
N	B	P	1	X	5	I	**NBP1C5**	37.7	PERC	MC	A	19
A	B	T	1	X	5	I	**ABT1I8**	41.0	PLOT	MC	A	20
N	C	N	1	X	9	I	**NCN1M4**	21.9	OrdOp	MC	D	21
G	B	T	1	X	5	I	**GBT1U2a**	31.0	MEAS	CON	A	22.1
G	B	T	1	X	7	I	**GBT1U2b**	10.0	MEAS	CON	A	22.2
N	B	P	1	X	7	II	**NBP1L5**	11.2	PERC	CON	A	23
A	C	T	I	X	8	I	**ACT1I1**	37.0	EQUA	MC	D	24
G	C	T	1	X	8	I	**GCT1P9**	38.0	SIMI	MC	D	25
S	B	P	1	X	7	II	**SBP1L9**	25.5	STAT	CON	A	26
A	C	N	1	X	9	II	**ACN1K9**	10.9	MODL	CON	D	27

PSA Grade 5

Problem #	Strand	Difficulty	RME Level	Problem Context	Max Pts
1	N	E	I	Zoo	2
2	N	E	I	Zoo	3
3	N	E	I	Zoo	2
4	G	E	I	Zoo	1
5	N	E	I	Monkeys	3
6	N	E	I	Monkeys	2
7	N	E	II	Monkeys	2
8	G	A	II	Buildings	3
9	G	A	II	Buildings	2
10	N/A	E	I	Pools	2
11	N	E	I	Pools	1
12	N	E	I	Pools	2
13	N/G	A	II	Snail	3
14	N	E	I	Playground	1
15	A/G	E	II	Playground	3
16	A/G	E	III	Playground	4
17	A/N/G	A	III	Playground	2
18	S	E	I	Questionnaire	1
19	S	E	II	Questionnaire	2
20	S	A	II	Questionnaire	2
21	S	E	I	Questionnaire	4
22	S	E	II	Questionnaire	2

PSA Grade 6

Problem #	Strand	Difficulty	RME Level	Problem Context	Max Pts
1	S/N	E	I	Ranger Station	1
2	S/N	A	II	Ranger Station	2
3	N	E	I	Ranger Station	3
4	N	A	I	Ranger Station	2
5	N	E	I	Ranger Station	2
6	N	A	II	Ranger Station	3
7	A/G	E	I	Patio	1
8	A/G	E	II	Patio	2
9	N/A	D	II	Patio	2
10	N	E	I	Fly One Day	1
11	A/N	A	I	Fly One Day	2
12	A/N	A	I	Fly One Day	2
13	A	D	II	Fly One Day	3
14	N	E	I	Bird Watcher Bulletin	2
15	N/A	D	II	Bird Watcher Bulletin	2
16	A	D	II	Selling Tickets	4
17	G	A	II	Birds of All Sizes	3
18	G	A	II	Birds of All Sizes	2
19	A/S	E	I	Birds of All Sizes	2
20	A/S	E	I	Birds of All Sizes	2
21	N	A	I	Birds of All Sizes	2
22	N/A	D	II	Birds of All Sizes	2
23	A/N	D	III	Birds of All Sizes	2
24	A/N	D	II	Birds of All Sizes	2

PSA Grade 7

Problem #	Strand	Difficulty	RME Level	Problem Context	Max Pts
1	N/A	E	I	Baby Feeding	1
2	N/A	E	I	Baby Feeding	2
3	A/S	A	I	Baby Feeding	2
4	N/A/G	D	II	Baby Feeding	2
5	N	E	I	Baby Feeding	2
6	G	D	II	Baby Feeding	3
7	G	A	I	Baby Feeding	2
8	G	D	I	Pentagon	2
9	G	A	II	Pentagon	3
10	G	D	III	Pentagon	3
11	N	D	III	Airships	2
12	A/S	A	II	Airships	3
13	A/S	A	I	Airships	1
14	N	A	II	Airships	2
15	G	E	I	Pyramids	2
16	G	E	I	Pyramids	2
17	A/G	A	I	Pyramids	2
18	A	A	II	Pyramids	2
19	A	D	II	Pyramids	1
20	A	D	I	Pyramids	1
21	A	D	II	Pyramids	2
22	A	E	I	Playgrounds	1
23	A/N	E	I	Playgrounds	2
24	A	D	III	Playgrounds	3
25	A	D	II	Playgrounds	2
26	G/N	A	I	Playgrounds	2

PSA Grade 8

Problem #	Strand	Difficulty	RME Level	Problem Context	Max Pts
1	S	A	II	Club Members	2
2	G	D	II	Lopsided	2
3	A/N	D	II	Lopsided	2
4	A/G	D	II	Lopsided	3
5	S/N	A	II	Keys	2
6	S/N	D	II	Keys	2
7	S/N	D	III	Keys	2
8	G	D	II	Seesaw	2
9	G	D	III	Seesaw	2
10	A/N	E	I	Stretch	1
11	A/N	D	I	Stretch	1
12	A	D	II	Stretch	2
13	A/G	A	II	Stretch	1
14	A/G	A	III	Stretch	2
15	G	E	I	Parking	1
16	N	A	II	Parking	2
17	G	D	III	Parking	3
18	A	A	I	Cubes	2
19	A	A	I	Cubes	2
20	A/G	D	II	Cubes	2
21	A	D	III	Cubes	3

Part 4: Items sorted by threshold

Item title (**bold** for full credit response)	Item ID ('p' prefaces PSA items)	Map code	Threshold (logits)	Domain
Town Populations .2	509	C2F	3.97	N
Radio Station.4	815	C2P	2.43	G
BuildingsQ2.2	p509	C2P	2.43	G
PyramidsQ7.2	p721	C1F	2.34	A
QuestionnaireQ3.2	p520	C2P	2.32	S
KeysQ3.2	p807	C2F	2.28	S
BuildingsQ1.3	p508	C2P	2.26	G
Snail.3	p513	C2P	2.05	N
PlaygroundQ4.2	p517	C2F	1.82	N
PentagonQ3.3	p710	C1F	1.79	G
Baby Feeding Q4.2	p704	C1P	1.76	N
PentagonQ3.2	p710	C1F	1.75	G
KeysQ3.1	p807	C2F	1.72	S
CubesQ4.3	p821	C2F	1.71	A
PatioQ3.2	p609	C2F	1.61	N
PentagonQ2.3	p709	C2P	1.56	G
PyramidsQ7.1	p721	C1F	1.56	A
LopsidedQ2.2	p803	C1P	1.53	N
Metro Rail.3	523	C2F	1.52	S
Fly One DayQ4.3	p613	C2P	1.52	A
AirshipsQ1.2	p711	C2F	1.5	N
PentagonQ1.2	p708	C2P	1.48	G
AirshipsQ4.2	p714	C1P	1.47	N
AirshipsQ1.1	p711	C2F	1.44	N
Rectangle Ratio (area).2	522b	C1F	1.37	G
StretchQ5.2	p814	C2F	1.36	A
Snail.2	p513	C2P	1.35	N
AirshipsQ2.3	p712	C1P	1.35	A
StretchQ3.2	p812	C1F	1.32	A
StretchQ2.2	p811	C1F	1.3	A
ParkingQ3.3	p817	C2F	1.27	G
CubesQ4.2	p821	C2F	1.18	A
StretchQ3.1	p812	C1F	1.16	A
Zeds.3	721	C2P	1.11	S
ParkingQ2.2	p816	C1P	1.1	N
LopsidedQ3.3	p804	C1P	1.09	N
Carla's& Maria's tiles.4	608	C2P	1.08	N
Ranger StationQ4.2	p604	C1P	1.07	N
Fly One DayQ4.2	p613	C2P	1.06	A
SeesawQ2.2	p809	C2F	1.06	G
Draw Rectangle.2	522a	C2I	1.03	G

Birds of All SizesQ7.2	p623	C2F	1.03	N
Selling Tickets.4	p616	C2F	1.02	A
Birds of All SizesQ1.3	p617	C1P	0.99	G
CubesQ1.2	p818	C2I	0.97	A
Selling Tickets.3	p616	C2F	0.96	A
AirshipsQ3	p713	C1I	0.96	A
Birds of All SizesQ2.2	p618	C1P	0.95	G
SeesawQ2.1	p809	C2F	0.94	G
PlaygroundsQ3.3	p724	C2F	0.93	A
AirshipsQ4.1	p714	C1P	0.92	N
Order of Operations	726	C1F	0.9	N
Visit ZooQ2.3	p502	C1P	0.88	N
Ranger StationQ4.1	p604	C1P	0.88	N
Fly One DayQ4.1	p613	C2P	0.88	A
Baby FeedingQ6 .3	p706	C2F	0.88	G
Similar Triangles (b)	524a	C2P	0.86	A
QuestionnaireQ4.4	p521	C1I	0.84	S
Ranger StationQ3.3	p603	C1I	0.84	N
PentagonQ3.1	p710	C1F	0.84	G
CubesQ4.1	p821	C2F	0.84	A
PlaygroundsQ4.2	p725	C1F	0.83	A
Selling Tickets.2	p616	C2F	0.82	A
PyramidsQ1.2	p715	C2I	0.81	G
LopsidedQ2.1	p803	C1P	0.81	N
Rectangle Ratio (area).1	522b	C1F	0.78	G
Visit ZooQ2.2	p502	C1P	0.78	N
PoolsQ2	p511	C1P	0.76	N
Birds of All SizesQ1.2	p617	C1P	0.76	G
Baby Feeding Q6.2	p706	C2F	0.74	G
MonkeysQ2.2	p506	C1I	0.69	N
PlaygroundsQ4.1	p725	C1F	0.69	A
Visit ZooQ3.2	p503	C1I	0.66	N
Radio Station.3	815	C2P	0.64	G
Draw Rectangle.1	522a	C2I	0.63	G
Visit ZooQ2.1	p502	C1P	0.63	N
Zeds.2	721	C2P	0.62	S
MonkeysQ1.3	p505	C1I	0.62	N
MonkeysQ3.2	p507	C1P	0.62	N
Baby FeedingQ6 .1	p706	C2F	0.61	G
Fly One DayQ3.2	p612	C1P	0.6	N
Birds of All SizesQ8.2	p624	C2F	0.6	N
PatioQ3.1	p609	C2F	0.59	N
MonkeysQ3.1	p507	C1P	0.56	N
Ranger StationQ2.2	p602	C2P	0.56	S
PyramidsQ5	p719	C1P	0.55	A
KeysQ2.2	p806	C2F	0.55	S
PentagonQ2.2	p709	C2P	0.54	G

AirshipsQ2.2	p712	C1P	0.54	A
Birds of All SizesQ1.1	p617	C1P	0.53	G
Sit-ups scatterplot	607	C1P	0.52	S
PlaygroundQ2.3	p515	C2P	0.52	A
Ranger StationQ5.2	p605	C1I	0.47	N
Ranger StationQ3.2	p603	C1I	0.46	N
PatioQ2.2	p608	C2I	0.46	A
Ranger StationQ6.3	p606	C1P	0.45	N
Bird WatchersQ2.2	p615	C2P	0.45	N
Selling Tickets.1	p616	C2F	0.44	A
LopsidedQ1.2	p802	C1P	0.44	G
LopsidedQ3.2	p804	C1P	0.44	N
CubesQ3.2	p820	C2P	0.42	A
QuestionnaireQ2.2	p519	C2I	0.41	S
Carla's& Maria's tiles.3	608	C2P	0.4	N
PyramidsQ4.2	p718	C2P	0.4	A
PlaygroundsQ3.2	p724	C2F	0.38	A
StretchQ2.1	p811	C1F	0.38	A
ParkingQ3.2	p817	C2F	0.38	G
SeesawQ1.2	p808	C1P	0.37	G
4m	510	C1P	0.33	A
Birds of All SizesQ6.2	p622	C1P	0.33	N
PlaygroundQ2.2	p515	C2P	0.32	A
PentagonQ1.1	p708	C2P	0.31	G
PlaygroundsQ5.2	p726	C1P	0.31	N
Dots.4	827	C2P	0.28	A
MonkeysQ2.1	p506	C1I	0.28	N
Ratio Similar Triangles	717	C1F	0.25	G
PatioQ2.1	p608	C2I	0.25	A
ParkingQ2.1	p816	C1P	0.25	N
Birds of All SizesQ2.1	p618	C1P	0.22	G
Dropped Ball	515	C2P	0.2	A
Ranger StationQ3.1	p603	C1I	0.2	N
PyramidsQ2.2	p716	C1I	0.18	G
Square/Circle Area	514	C1F	0.16	G
SeesawQ1.1	p808	C1P	0.16	G
Town Populations.1	509	C2F	0.13	N
Ranger StationQ6.2	p606	C1P	0.13	N
Baby Feeding Q4.1	p704	C1P	0.13	N
Radio Station.2	815	C2P	0.12	G
Points on Line	518	C1F	0.11	A
Dots.3	827	C2P	0.08	A
PlaygroundsQ5.1	p726	C1P	0.06	N
What is "n"	727	C1F	0.04	A
Batteries	517	C2P	0.03	S
LopsidedQ1.1	p802	C1P	0.03	G
CubesQ3.1	p820	C2P	0.02	A

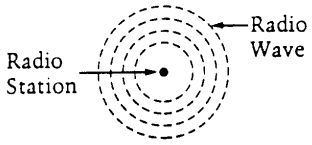
PlaygroundsQ3.1	p724	C2F	0	A
Fly One DayQ3.1	p612	C1P	-0.03	N
Perimeter Shapes	521	C1P	-0.05	G
LopsidedQ3.1	p804	C1P	-0.05	N
Ranger StationQ5.1	p605	C1I	-0.06	N
Birds of All SizesQ6.1	p622	C1P	-0.06	N
AirshipsQ2.1	p712	C1P	-0.06	A
Baby FeedingQ3 .2	p703	C2P	-0.09	S
PyramidsQ6	p720	C1P	-0.1	A
StretchQ4	p813	C2P	-0.12	A
CubesQ2.2	p819	C1I	-0.12	A
PlaygroundQ3.4	p516	C2P	-0.13	A
Fly One DayQ2.2	p611	C1I	-0.16	A
Bird Watchers'Q2.1	p615	C2P	-0.16	N
Birds of All SizesQ3.2	p619	C1I	-0.16	S
Birds of All SizesQ5.2	p621	C1P	-0.17	N
Dots.2	827	C2P	-0.18	A
PlaygroundQ3.3	p516	C2P	-0.21	A
Red/Blue Cube	511	C1I	-0.22	S
PatioQ1	p607	C2I	-0.23	A
Carla's& Maria's tiles.2	608	C2P	-0.25	N
ParkingQ1.2	p815	C1I	-0.26	G
CubesQ2.1	p819	C1I	-0.27	A
PlaygroundQ3.2	p516	C2P	-0.28	A
PlaygroundQ4.1	p517	C2F	-0.28	N
ParkingQ3.1	p817	C2F	-0.28	G
Metro Rail.2	523	C2F	-0.29	S
Baby FeedingQ2 .2	p702	C1I	-0.29	N
Radio Sales	519	C2F	-0.31	N
Tip Calc.	526	C1I	-0.32	N
PyramidsQ3.2	p717	C2P	-0.32	G
Ranger StationQ6.1	p606	C1P	-0.33	N
Bird Watchers'Q1.2	p614	C1I	-0.33	N
Birds of All SizesQ8.1	p624	C2F	-0.34	N
PoolsQ3.2	p512	C1I	-0.35	N
Dots.1	827	C2P	-0.36	A
Baby FeedingQ5.2	p705	C1I	-0.37	N
PlaygroundQ3.1	p516	C2P	-0.38	A
Birds of All SizesQ4.2	p620	C1I	-0.39	S
PyramidsQ3.1	p717	C2P	-0.41	G
ParkingQ1.1	p815	C1I	-0.41	G
PlaygroundQ2.1	p515	C2P	-0.44	A
PlaygroundsQ2.2	p723	C2I	-0.44	N
Fly One DayQ2.1	p611	C1I	-0.45	A
Blue Pen	507	C1I	-0.51	S
PyramidsQ4.1	p718	C2P	-0.53	A
CubesQ1.1	p818	C2I	-0.53	A

PoolsQ3.1	p512	C1I	-0.56	N
PyramidsQ1.1	p715	C2I	-0.56	G
Club Members.2	p801	C2F	-0.56	S
Flattened Cube	503	C2I	-0.57	G
BuildingsQ1.2	p508	C2P	-0.6	G
KeysQ1.2	p805	C1P	-0.6	S
Birds of All SizesQ7.1	p623	C2F	-0.63	N
Baby FeedingQ5.1	p705	C1I	-0.63	N
PlaygroundsQ2.1	p723	C2I	-0.63	N
KeysQ2.1	p806	C2F	-0.63	S
StretchQ5.1	p814	C2F	-0.63	A
Ranger StationQ2.1	p602	C2P	-0.66	S
Fly One DayQ1	p610	C1P	-0.67	N
Jose's Tree	501	C2I	-0.68	G
Boys and Girls in Class	504	C2P	-0.69	N
Snail.1	p513	C2P	-0.69	N
QuestionnaireQ4.3	p521	C1I	-0.69	S
Baby FeedingQ3 .1	p703	C2P	-0.69	S
PentagonQ2.1	p709	C2P	-0.72	G
Radio Station.1	815	C2P	-0.75	G
Tourists on Bus	516	C1P	-0.77	N
Visit ZooQ3.1	p503	C1I	-0.78	N
Baby Feeding Q7	p707	C1P	-0.82	G
Zeds.1	721	C2P	-0.84	S
Bird Watchers'Q1.1	p614	C1I	-0.84	N
QuestionnaireQ5.2	p522	C2I	-0.85	S
MonkeysQ1.2	p505	C1I	-0.87	N
Car Speed/Stopping Dist.	513	C2I	-0.89	S
3(x+5)=30	525	C1P	-0.89	A
BuildingsQ2.1	p509	C2P	-0.91	G
Visit ZooQ4	p504	C1I	-0.95	G
QuestionnaireQ1	p518	C1I	-0.98	S
QuestionnaireQ2.1	p519	C2I	-1	S
Flour for Cookies	502	C1I	-1.03	N
Visit ZooQ1.2	p501	C1I	-1.03	N
Oxford to Smithville	505	C1I	-1.08	G
KeysQ1.1	p805	C1P	-1.08	S
Blue and Yellow Balls	506	C1I	-1.09	S
Birds of All SizesQ5.1	p621	C1P	-1.25	N
Baby Feeding Q2.1	p702	C1I	-1.25	N
Baby Feeding Q1	p701	C1I	-1.27	S
PyramidsQ2.1	p716	C1I	-1.31	G
PoolsQ1.2	p510	C1I	-1.32	N
BuildingsQ1.1	p508	C2P	-1.33	G
QuestionnaireQ5.1	p522	C2I	-1.33	S
Ranger StationQ1	p601	C1I	-1.33	S
PoolsQ1.1	p510	C1I	-1.39	N

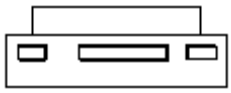
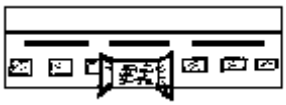
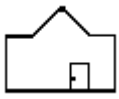
PlaygroundQ1	p514	C1I	-1.4	N
Visit ZooQ1.1	p501	C1I	-1.44	N
Nine Chips	520	C1I	-1.47	S
Jill's Trip	512	C1I	-1.5	N
PlaygroundsQ1	p722	C2I	-1.52	A
Club Members.1	p801	C2F	-1.53	S
Birds of All SizesQ4.1	p620	C1I	-1.59	S
k+6	508	C1P	-1.72	A
Similar Triangles (a)	524b	C1I	-1.75	A
QuestionnaireQ4.2	p521	C1I	-1.75	S
Birds of All SizesQ3.1	p619	C1I	-1.91	S
Carla's& Maria's tiles.1	608	C2P	-1.94	N
Metro Rail.1	523	C2F	-1.97	S
MonkeysQ1.1	p505	C1I	-2.19	N
StretchQ1	p810	C1I	-2.53	A
QuestionnaireQ4.1	p521	CII	-2.63	S
QuestionnaireQ3.1	p520	C2P	-2.69	S

Part 5: Illustrative items and annotations

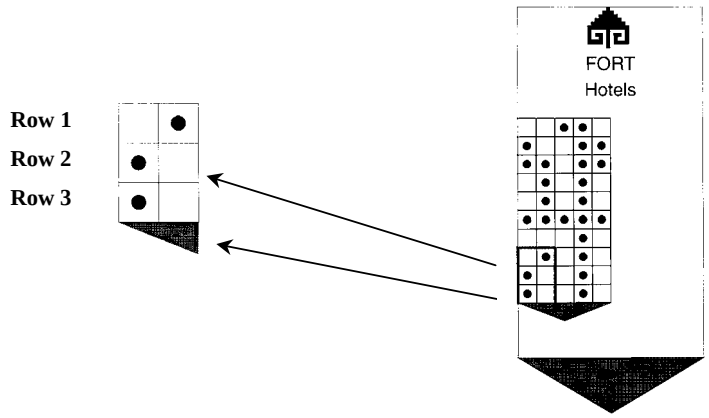
Band vi

RADIO STATION.4 [2.43LOGITS]	ANNOTATION
This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answers should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work. Radio station KMAT in Math City is 200 miles from radio station KGEO in Geometry City. Highway 7, a straight road, connects the two cities. KMAT broadcasts can be received up to 150 miles in all directions from the station and KGEO broadcasts can be received up to 125 miles in all direction. Radio waves travel from each radio station through the air, as represented below.  On the next page, draw a diagram that shows the following. • Highway 7 • The location of the two radio stations • The part of Highway 7 where both radio stations can be received Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.	Illustrative of band vi. This task requires students to interpret a written description of a problem situation involving spatial data to draw and label a diagram to indicate the distance between two loci on a straight line, and to then define an area of overlap (two dimensional representation - concentric circles) of radio broadcast areas given specific constraints and conditions. The students need to indicate all of the specified features and apply a scale correctly. Students are required to communicate their working by providing a drawing that fully supports their answer.

SCORING RUBRIC	Points		
	4 [2.43 logits]	Correct answer (75 miles must be stated)	
	3 [0.64 logits]	Map with cities or stations and 200 miles labelled (or a clear and correct application of scale) and identifies common broadcast area on Highway 7 but omits length of common area.	
	2 [0.12 logits]	Map with cities or stations and 200 miles labelled (or some attempt at using a scale); the highway should be shown as straight, and identifies incorrect common broadcast area (e.g., not on Highway 7) or insufficiently identifies an area. (Insufficiently means that there is not enough information labelled to determine the length of the common broadcast area). Bounds of common area may or may not be labelled.	
	1 [- 0.75 logits]	Map with cities or stations and 200 miles labelled (or some attempt to use a scale). Highway should be shown as straight. There is no indication of how student determined common broadcast area. (It may, for example, be represented as a single point or not at all.) *or* Map uses some but not all of given information with no indication of how common broadcast area was determined.	
	0	Work is completely incorrect, irrelevant, or off task.	

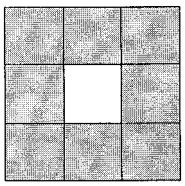
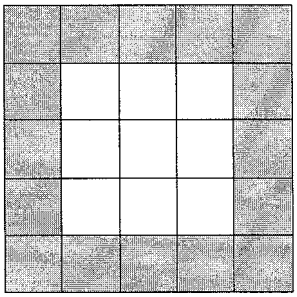
BUILDINGS Q2.2 [2.43 LOGITS]	ANNOTATION
Below you see different side and front views of some of the buildings in the zoo.  View I _____  View II _____  View III _____  View IV _____  View V _____ Are the drawings of the buildings shown above drawn on the same scale as the buildings on the map? Explain why or why not.	Illustrative of band vi. A full credit response requires students to compare two-dimensional representations of the same objects, drawn in different perspective views and to different scales, where only one view has a marked scale. Students need to apply a strategy to draw a conclusion (i.e. buildings are not on the same scale). Strategies employed by students include: refer to scale; and directly measure buildings on the map and in the pictures. Students are required to justify their conclusion by communicating an explanation of their reasoning. The partial credit response (Buildings Q2.1) is situated in band iii.

SCORING RUBRIC	Points		
	2 [2.43 logits]	**Correct answer:** no, they are not on the same scale *with* **Clear justification**: e.g., the measurements of the buildings above are 1 ¼ times larger than those on the map.	
	1 [- 0.91 logits]	**Correct answer**: no, they are not on the same scale, **but no explanation given** *or* Correct answer but through visual estimate (e.g. I looked and could tell that the buildings are different sizes) *or* **Incorrect conclusion**, yes they are on the same scale, or some are on the same scale and some are not**, but with correct strategy to find solution**, e.g., used ruler to measure the buildings	
	0	Incorrect response	

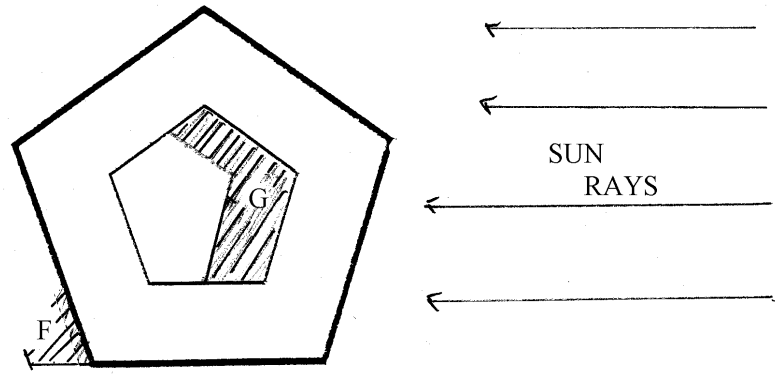
KEYS Q3.2 [2.28 LOGITS]	ANNOTATION
 Look at the enlarged section of the key card pictured on the left again. The manager of the hotel complains, "For our 50 room hotel, punching only three holes in that section does not give us enough codes. I suggest that each card can have from *zero to six holes* punched in that section. Then over 50 different codes could be made, and we would have enough for all of the rooms." Is the manager correct? Use mathematical evidence to support your conclusion.	Illustrative of band vi. This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. Students need to apply a strategy – taking into account constraints – to draw a conclusion about the validity of a mathematical statement, and to provide mathematical evidence to support their conclusion. Strategies employed by students include: drawing models and/or counting possible combinations; and exponential models, Students are required to state clearly that the mathematical statement is supported by their conclusion.

SCORING RUBRIC	Points		
	2 [2.28 logits]	**Correct conclusion:** yes, the manager is correct *with* Correct explanation or strategy that indicates the manager is correct e.g., 'For each place there are 2 possibilities. For 6 places there are 2 X 2 X 2 X 2 X 2 X 2 = 2^6 = 64 possibilities. So the manager is right.' *or* Student finds the number of possibilities for 0 through 6 holes **Holes possibilities** Zero = 1 One = 5 Two = 15 Three = 20 Four = 15 Five = 6 Six = 1 64 possibilities total so the manager has 14 more than needed.'	
	1 [1.72 logits]	Correct conclusion with partially correct explanation To receive credit for a counting strategy students must show evidence of drawings for counting the number of different possibilities for each hole *or* **Correct strategy with 'minor computation' error**	
	0	**Correct conclusion with no or trivial/non-mathematical explanation** **Incorrect conclusion**	

Band v

PLAYGROUND 4.2 [1.82 LOGITS]	**ANNOTATION**
 **Diagram 1** The owners of the zoo are planning to make a playground in the zoo, between the okapis and the baboons. The playground will be paved with square tiles. The owners are not sure yet how large the playground will be. They start a design with one square tile in the middle and lay a ring of square tiles around it. In the picture above the first ring of tiles is shaded. It consists of 8 tiles. A second ring of tiles is added. The tiles in the second ring are now shaded in the picture below.  **Diagram 2** Suppose several more rings of tiles are added to make the playground larger. Will the total number of tiles always be even or always be odd? Explain your answer.	Illustrative of band v. This task requires students to apply their understanding of odd and even numbers to draw a conclusion for a numerical pattern based on a geometrical representation. Students justify their conclusion with an appropriate explanation. Strategies employed by students include: a number-based strategy in which patterns of numbers are described (e.g. 'To get the total number of tiles, you add the number of tiles in the ring to the total number of tiles in the previous playground. The ring is always even and the playground in always odd. An even + odd = odd'; a visually – based strategy in which the students refer to the diagrams (e.g. 'The number of tiles in each ring is always even, and there is always one extra in the middle which makes it odd.' Students are required to support their conclusion by communicating such an explanation of their reasoning.

SCORING RUBRIC	Points		
	2 [1.82 logits]	**Correct conclusion:** odd *with* **Clear explanation,** e.g., 'The sides of the squares (playgrounds) have lengths of 3, 5, 7, 9 ….. tiles, the square of odd numbers is always odd.' *or* 'The number of tiles in each ring is always even, and there is always one extra in the middle which makes the total odd.'	
	1 [- 0.28 logits]	Correct conclusion but incomplete explanation or no explanation given *or* Correct explanation with wrong or missing conclusion NOTE: 'They're all odd in table' is **NOT** a correct explanation.	
	0	Incorrect response.	

<table>
<tr><td colspan="3">PENTAGON Q2.3 [1.56 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">A top view of the Pentagon is shown below. The garden is in the center. The shaded part G shows the shadow of the building caused by the sun. A small part of the shadow on the left of the building is drawn (see F).
 

In the drawing above, finish the shadow of the building.</td><td rowspan="6">Illustrative of band v.

For a full credit response (Pentagon Q2.3), students are required to define a shaded region to complete a drawing of the shadow cast by a two-dimensional representation of a three-dimensional object. Students need to define the shaded region taking into account: that the sun’s rays are perpendicular to the vertical axis of the building; the cast shadow width is constant in this scenario; and that the top edge of the shadow will be parallel to the bottom edge of the shadow as indicated at location ‘F’. The students’ drawing must fulfil all three criteria.
The partial credit responses Pentagon Q2.2 and Pentagon Q2.1) are situated in bands iv and iii respectively.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td rowspan="4"></td><td>3

[1.56 logits]</td><td>Drawing of shadow must include the following criteria:
▪ Shadow is only drawn on both the entire upper and entire lower left sides of figure
▪ Where drawn, the shadow is equally wide, horizontally, AND the horizontal (not perpendicular) distance from building is always 1 cm
▪ The top edge of the shadow is parallel to the sun rays</td></tr>
<tr><td>2

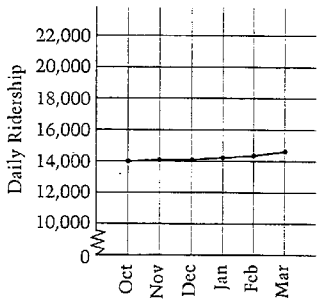
[0.54 logits]</td><td>Two criteria correct</td></tr>
<tr><td>1

[- 0.72 logits]</td><td>One criteria correct</td></tr>
<tr><td>0</td><td>No criteria correct</td></tr>
</table>

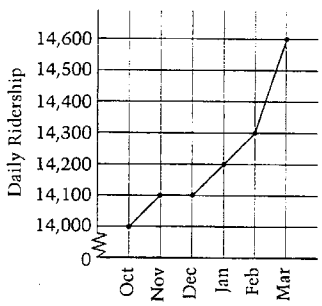
METRO RAIL.3 [1.52 LOGITS]	ANNOTATION
This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work. METRO RAIL COMPANY The data in the table above has been correctly represented by both graphs shown below. Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October? Explain your reason for making this selection Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?	Illustrative of band v. This task requires students to analyze two graphical representations of the same data set presented in tabular form to (i) chose the graph that emphasizes a relatively large increase in one parameter over a given range and (ii) critically analyze this choice taking into account differences in scale. Students are required to justify their conclusion in part (ii) by communicating an explanation of their reasoning (e.g. one graph exaggerates a relatively small increase).

Month	Daily Ridership
October	14,000
November	14,100
December	14,100
January	14,200
February	14,300
March	14,600

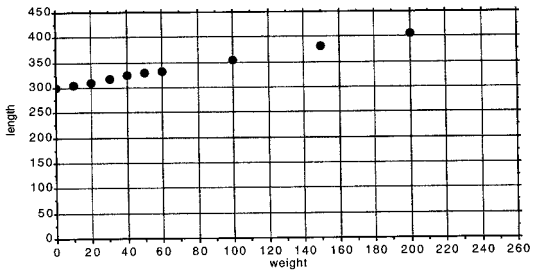
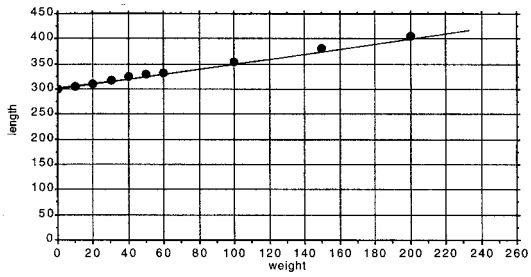
Graph A



Graph B

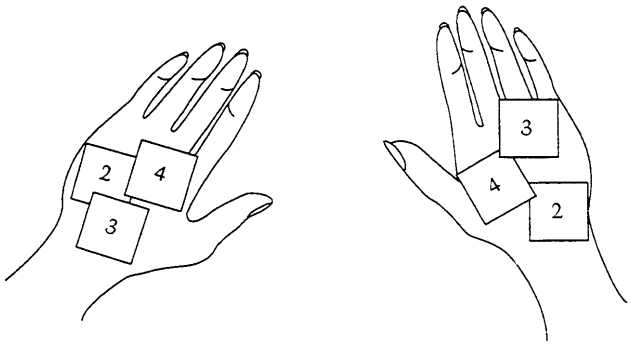


SCORING RUBRIC	Points		
	3 [1.52 logits]	Correct response – B with a complete explanation that addresses each of the following two points in the total response (i.e., both parts are taken together) ▪ B because it (appears to) show(s) a large increase in ridership. ▪ Misleading because B exaggerates a relatively small increase (i.e., misuse of scale)	
	2 [- 0.29 logits]	B with an incomplete (but partially correct) explanation for the first part or the second part or both parts.	
	1 [- 1.97 logits]	B with a missing or incorrect explanation.	
	0	Incorrect response – does not select graph B.	

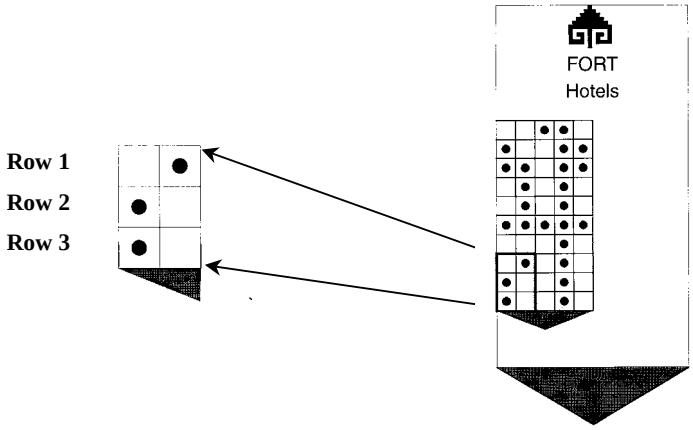
STRETCH Q5.2 [1.36 LOGITS]	ANNOTATION
 In science class, Nicola conducted an experiment involving a spring and different weights. The spring was stretched by hanging weights on a hook at the bottom of the spring. Nicola recorded the length of the spring after hanging each weight on the hook. The results of the experiment are shown in the table below. Nicola used a computer to plot the results of her experiment on a coordinate system. This graph is shown below.  In the graph below Nicola drew a straight line that best fits the data points.  When Paul hung a weight of 180 grams on a hook at the bottom of his spring, the spring stretched to 450 mm. Did the spring Paul use stretch more or less than Nicola's spring? Explain how you found your answer.	Illustrative of band v. This task requires students to compare values in a graphical representation given in coordinate form. Students need to draw a conclusion by comparing the relative difference between the given data set and a second scenario, where the y-axis data point has to be used as the basis for comparison (given a specified x-value). Students are required to support their conclusion by communicating an explanation of their reasoning.

SCORING RUBRIC	Points		
	2 [1.36 logits]	**Correct conclusion with clear explanation:** **e.g.,** 'Stretched more. Paul's went from 200 to 450 and Nicola's went from 300 to only 406 with even more weight on it.' *or* 'Paul's stretched more. His spring went to 450 cm and hers stretched only to 400 mm. *or* 'Paul's stretched more. If you draw his line on the graph, it is steeper than Nicola's.'	
	1 [-0.63 logits]	**Correct conclusion, 'more', with partially complete explanation** **e.g.,** 'Paul's stretched more. There are bigger differences in the lengths' *or* 'More, because it went up to 450 and hers only when to 380 at the same weight' *or* 'More because Paul's went to 450 and Nicola's only went to 406 altogether.' *or* **Clear explanation that implies an answer of more.**	
	0	**Correct conclusion with incorrect or no explanation** **e.g.,** 'More, Paul's spring is longer' *or* 'More. Length measurements are bigger.' *or* Other incorrect response.	

Band iv

CARLA AND MARIA'S TILES.4 [1.08 LOGITS] – BORDERLINE BAND IV/V	**ANNOTATION**
In a game, Carla and Maria are making subtraction problems using tiles numbered 1 to 5. The player whose subtraction problem gives the largest answer wins the game. Look at where each girl placed her two tiles. Carla 1 _ _ – 5 _ Maria _ _ 5 – _ 1  Who will win the game?_____ Explain how you know this person will win.	Illustrative of band iv. This task requires students to speculate on outcomes (i.e. the largest possible answer) by considering and comparing the placement of numbers from a given pool in two partially completed subtraction algorithms. Students need to meet specified constraints (numbers already in place) and are required to provide the correct answer (i.e. Maria will win) and communicate an explanation of their reasoning.

SCORING RUBRIC	**Points**	Maria will win the game. The following reasons may be given: a) The largest possible difference for Carla is less but Maria will get several larger differences. b) Carla will only get a difference of 91 or less but Maria will get several larger differences. c) Carla can have only up to 143 as her top number but Maria will get several larger differences. d) Carla has only 1 hundred but Maria can have 2, 3 or 4 hundreds. e) Maria can never take away as much as Carla. f) 143 – 52; 345 – 21 Any combination of problems to show that Maria's difference is greater.	
	4 [1.08 logits]	Student answers Maria and give explanation such as a) or b), or an appropriate combination of the other explanations.	
	3 [0.4 logits]	Student answers Maria and gives explanation such as c), d), or e).	
	2 [- 0.25 logits]	Student answers Maria with partially correct, or incomplete but relevant, explanation.	
	1 [- 1.94 logits]	Student answers Maria and give sample such as f) but no explanation. *or* Maria with an incorrect explanation.	
	0	Incorrect response.	

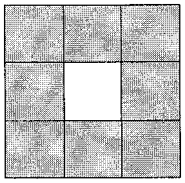
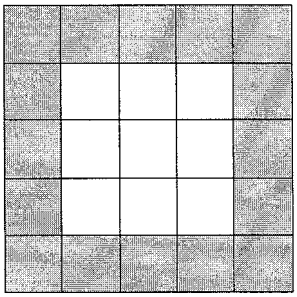
<table>
<tr><th colspan="3">KEYS Q2.2 [0.55 LOGITS]</th><th>ANNOTATION</th></tr>
<tr><td colspan="3"> 

Look at the enlarged section of the key card pictured on the left again. Suppose only three holes can be punched in this section of the key card, but there can be *zero, one, or two holes* in each row. If a code consists of exactly three holes, how many different codes can be made in this section of the key card? You may use the diagrams on the next page to try different possibilities.</td><td rowspan="5">Illustrative of band iv.

This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. The students determine all possible combinations in a sample space, given several constraints (zero, one, or two holes can be punched in each row). Students respond by providing either a numerical answer, or by drawing the elements of the sample space on the diagrams provided, to indicate a sample space of 20.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td>Note: Students should receive full credit for drawing the correct number of different keys without providing numerical response.</td></tr>
<tr><td rowspan="3"></td><td>2

[0.55 logits]</td><td>Correct answer: 20</td></tr>
<tr><td>1

[- 0.63 logits]</td><td>Answer is in the following range:
16 – 19 or 21- 24</td></tr>
<tr><td>0</td><td>Other incorrect response:
≤ 15 or ≥ 25</td></tr>
</table>

PLAYGROUND Q2.3 [0.52 LOGITS]	ANNOTATION
 **Diagram 1** The owners of the zoo are planning to make a playground in the zoo, between the okapis and the baboons. The playground will be paved with square tiles. The owners are not sure yet how large the playground will be. They start a design with one square tile in the middle and lay a ring of square tiles around it. In the picture above the first ring of tiles is shaded. It consists of 8 tiles. A second ring of tiles is added. The tiles in the second ring are now shaded in the picture below.  **Diagram 2** Suppose several more rings of tiles are added to make the playground larger. Fill in the missing values in the table below. Ring-number / Number of tiles in ring / Total number of tiles in playground 1 / 8 / 9 2 / ____ / 25 3 / ____ / ____ 4 / ____ / ____	Illustrative of band iv. This task requires students to find a strategy for recognising a pattern from data presented in tabular form (a numerical pattern based on a geometrical representation); and extending it. Strategies employed by students are: the development of a numeric linear pattern; and the use of drawings. A full credit response required all five missing numbers to be correctly identified.

<table>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td><td></td></tr>
<tr><td rowspan="4"></td><td>3

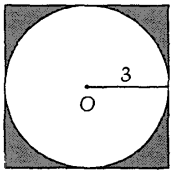
[0.52 logits]</td><td>All numbers are correct
<table>
<tr><th>ring #</th><th># tiles in ring</th><th>total tiles</th></tr>
<tr><td>1</td><td>8</td><td>9</td></tr>
<tr><td>2</td><td>16</td><td>25</td></tr>
<tr><td>3</td><td>24</td><td>49</td></tr>
<tr><td>4</td><td>32</td><td>81</td></tr>
</table></td><td rowspan="4"></td></tr>
<tr><td>2

[0.32 logits]</td><td>One wrong or missing number</td></tr>
<tr><td>1

[-0.44 logits]</td><td>Two wrong or missing numbers</td></tr>
<tr><td>0</td><td>Three or more wrong or missing numbers</td></tr>
</table>

DOTS.4 [0.28 LOGITS]	**ANNOTATION**
A pattern of dots is shown below. At each step, more dots are added to the pattern. The number of dots added at each step is more than the number added in the previous step. The pattern continues infinitely. (1st step) (2nd step) (3rd step)  2 Dots 6 Dots 12 Dots Marcy has to determine the number of dots in the 20th step, but she does not want to draw all 20 pictures and then count the dots. Explain or show how she could do this and give the answer that Marcy should get for the number of dots.	Illustrative of band iv. This task requires students to interpret and generalise from series of diagrams showing a pattern of dots (steps 1, 2, 3), in which each step dots have been added. Students need to extend the pattern to the 20th step by using the generalised rule, rather than by constructing dot diagrams. Students need to provide the correct answer and an explanation of their reasoning.

SCORING RUBRIC	**Points**	Explanation should include one of the following ideas with no false statements. a) For each successive step, the number of rows and the number of columns is increasing by 1, forming a pattern. For example, the first step forms 1 by 2 rows and columns, the next step 2 by 3, the third step 3 by 4 and so on. Continuing this pattern would mean that the 20^{th} step has 20 by 21 rows or 420 dots. b) Look at successive differences between consecutive steps. The differences 4, 6, 8, 10, … form a pattern. There are 19 differences forming the pattern 4, 6, 8, 10, ….., 30, 40 and this sum is (8 X 44) + 22 or 418. However, 2 must be added for the 1^{st} step, yielding a response of 420.	
	4 [0.28 logits]	Correct answer. (Must state 420; must tie step 20 back to beginning of pattern in some specific form of generalization)	
	3 [0.08 logits]	Correct explanation of pattern but does not include or omits the correct number of dots (420).	
	2 [- 0.18 logits]	A partial (incomplete) correct explanation (i.e., does not tie together well).	
	1 [- 0.36] logits]	An attempt to generalize *or* draw all 20 pictures in the pattern (with a clear understanding of the pattern).	
	0	The work is completely incorrect, irrelevant, or off task. (A response of 420 is a score of 0).	

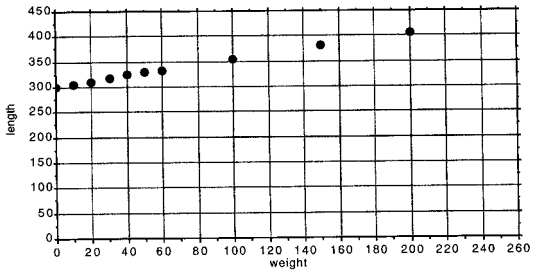
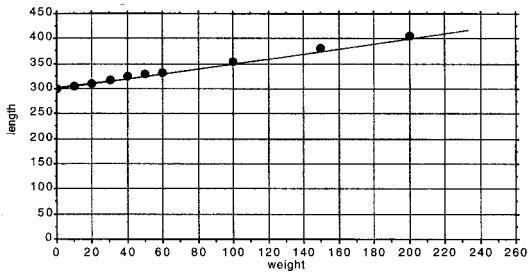
<table>
<tr><td colspan="3">SQUARE/CIRCLE [0.16 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3"> 

In the figure above, a circle with center O and radius of length 3 is inscribed in a square. What is the area of the shaded region?

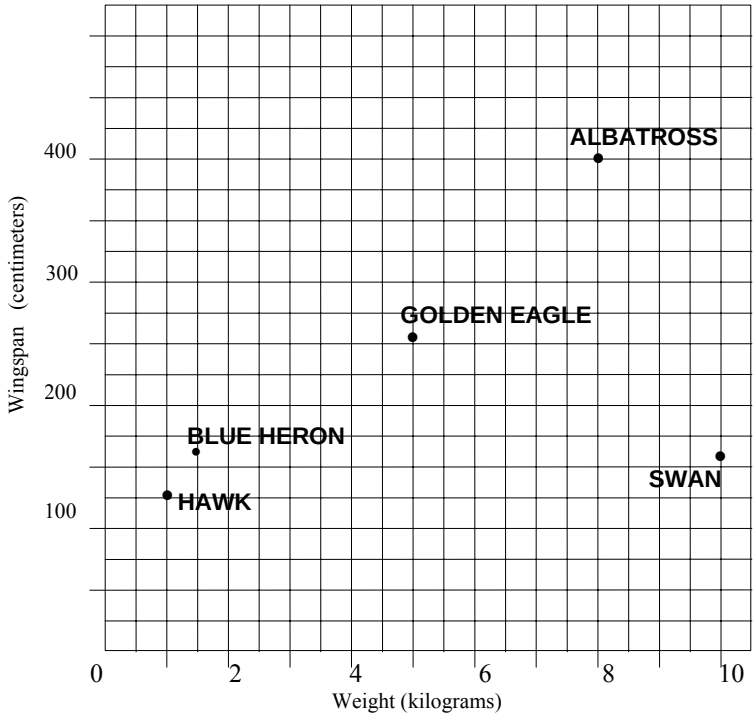
A) 3.86 B) 7.73 C) 28.27 D) 32.86 E) 36.00</td><td rowspan="3">Illustrative of band iv.

This task requires students to draw on their knowledge of the formula πr^2 and apply this in order to determine area of a circle. They need to use this value to find the area of a shaded region by finding the difference of two areas, and to round appropriately to select the best alternative.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: B) 7.73</td></tr>
</table>

Band iii

STRETCH Q4 [-0.12 LOGITS]	ANNOTATION
 In science class, Nicola conducted an experiment involving a spring and different weights. The spring was stretched by hanging weights on a hook at the bottom of the spring. Nicola recorded the length of the spring after hanging each weight on the hook. The results of the experiment are shown in the table below. Nicola used a computer to plot the results of her experiment on a coordinate system. This graph is shown below.  In the graph below Nicola drew a straight line that best fits the data points.  Paul performed an experiment similar to the one Nicola conducted. The straight line he drew started at (0, 200). Was the spring Paul used in his experiment longer or shorter than Nicola's spring? Explain how you found your answer.	Illustrative of band iii. This task requires students to interpret a written description of a problem situation and link this to a graphical representation. Students need to draw a conclusion based on change in an experimental variable, and link it to an understanding of the meaning of the y-intercept. Students are required to support their conclusions by communicating an explanation of their reasoning.

SCORING RUBRIC	Points		
	1	**Correct conclusion with explanation that uses mathematical evidence or references specific points on the graph:** e.g., 'The spring is shorter because at weight 0, the length is only 200.'	
	0	**Correct conclusion with incomplete, incorrect, or no explanation** *or* **Other incorrect response.**	

BIRDS OF ALL SIZES Q5.2 [-0.17 LOGITS]	ANNOTATION
To compare small and large birds you can look at the weight in relation to the wingspread. The wingspread of a bird is measured from wing tip to wing tip:  In the graph below, you see the values for wingspread and weight plotted for several birds.  The ratio $\frac{wingspread}{weight}$ of a golden eagle is $\frac{250}{5}$ which is 50. Show that the ratio $\frac{wingspread}{weight}$ for the albatross is the same as the ratio for the golden eagle.	Illustrative of band iii. For a full credit response, this task (Birds of all sizes Q5.2) requires students to accurately read a graph to extract data points (x and y axes values) according to a given criterion. Students then need to substitute the values into a ratio (presented in fractional form) and to correctly calculate the ratio. Students are required to state the correct answer and to show the values positioned in the ratio correctly. The partial credit response (Birds of all sizes Q5.1) is situated in band ii.

SCORING RUBRIC	Points		
	2 [- 0.17 logits]	One point for correct calculation of 50 *and* **Shows appropriate ratio** [e.g. $\frac{400}{8}$]	
	1 [- 1.25 logits]	**Shows appropriate ratio only** [e.g. $\frac{400}{8}$] *or* **Shows appropriate ratio** [e.g. $\frac{400}{8}$] but makes calculation error in dividing *or* **Computes a ratio** [e.g. $\frac{350}{8}$] **based on an inaccurate reading of the graph** *or* Answer only, 50	
	0	**Incorrect or illegible answer**	

<table>
<tr><td colspan="3">RED/BLUE CUBE [-0.22 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">Each of the six faces of a certain cube is painted either red or blue. When the cube is tossed, the probability of the cube landing with a red face up is $\frac{2}{3}$. How many faces are red?

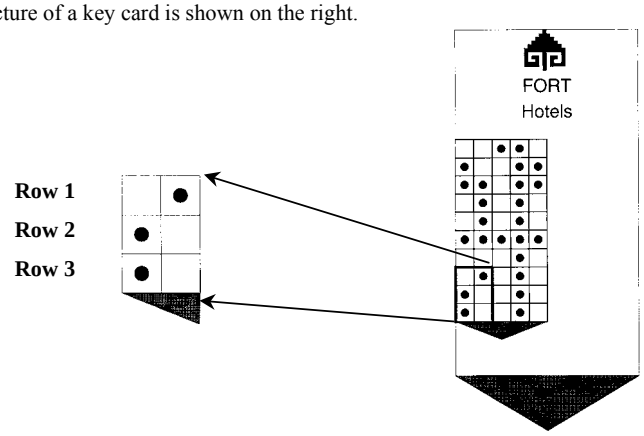
A) One
B) Two
C) Three
D) Four
E) Five</td><td rowspan="3">Illustrative of band iii.

This task requires students to apply a probability ratio (expressed as a fraction) to the six faces of a die in order to determine the number of faces of a specified color.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: D) Four</td></tr>
</table>

<table>
<tr><td colspan="3">TIP CALCULATION [-0.32 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?

A) $2.50
B) $3.00
C) $3.75
D) $4.50
E) $5.00</td><td rowspan="3">Illustrative of band iii.

This task requires students to round a decimal money amount (i.e. $24.99 rounded to $25) and to identify multiplication as the appropriate operation required in order to calculate a specified percentage of that total (15% tip).</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: C) $3.75</td></tr>
</table>

<table>
<tr><th colspan="3">KEYS Q1.2 [-0.6 LOGITS]</th><th>ANNOTATION</th></tr>
<tr><td colspan="3">The rooms in some hotels have keys in the shape of a card that has holes in it. A picture of a key card is shown on the right.
 

A section of the key card is shown on the left. The section consists of six places that determine the code for a room. A code consists of three holes, one in each row.

How many different codes can be made in this section of the key card if exactly one hole is punched in each row? You may use the diagrams on the next page to try different possibilities.</td><td rowspan="5">Illustrative of band iii.

This task requires students to interpret a written description of a problem situation and link this to a diagrammatic representation. The students determine all possible combinations in a sample space, given a constraint (one hole can be punched in each row only). Students respond by providing either a numerical answer, or by drawing the elements of the sample space on the diagrams provided, to indicate a sample space of eight.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td>Note: Students should receive full credit for drawing the correct number of different keys without providing numerical response.</td></tr>
<tr><td rowspan="3"></td><td>2

[- 0.6 logits]</td><td>Correct answer: 8</td></tr>
<tr><td>1

[- 1.08 logits]</td><td>Answer given: 7 or 9</td></tr>
<tr><td>0</td><td>Answer given: ≤ 6 or ≥ 10</td></tr>
</table>

<table>
<tr><td colspan="3">3(X+5)=30 [-0.89 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">If 3(x + 5) = 30, then x =

A) 2
B) 5
C) 10
D) 95</td><td rowspan="3">Illustrative of band iii.

This task requires students to solve an equation using the distributive law:

3(x +5) = 30

3x + 15 = 30

3x = 15

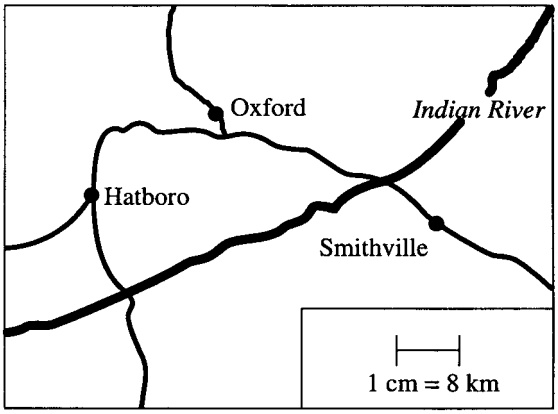
x = 5</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: B) 5</td></tr>
</table>

Band ii

<table>
<tr><td colspan="3">FLOUR FOR COOKIES [-1.03 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">If $1\frac{1}{3}$ cups of flour are needed for a batch of cookies, how many cups of flour will be needed for 3 batches?

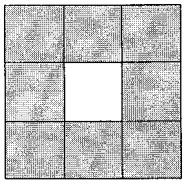
A) $4\frac{1}{3}$
B) 4
C) 3
D) $2\frac{2}{3}$</td><td rowspan="3">Illustrative of band ii.

This task requires students to identify an appropriate strategy (add or multiply) to solve a problem involving a mixed fraction and a whole number, where the context is quantity.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: B) 4</td></tr>
</table>

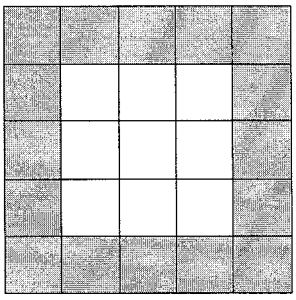
<table>
<tr><th>OXFORD TO SMITHVILLE [-1.08 LOGITS]</th><th>ANNOTATION</th></tr>
<tr><td>


About how far apart are Oxford and Smithville on the land?
A) 4 km
B) 16 km
C) 35 km
D) 50 km
<table>
<tr><th>SCORING RUBRIC</th><th>Points</th><th></th></tr>
<tr><td></td><td>1</td><td>Correct answer: C) 35 km</td></tr>
</table>
</td><td>Illustrative of band ii.

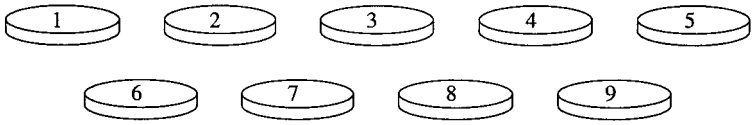
This task requires students to interpret a scale (1cm = 8 km) to estimate the distance between two reference points on a map.</td></tr>
</table>

<table>
<tr><td colspan="3">PLAYGROUND Q1 [-1.4 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3"> 
Diagram 1
The owners of the zoo are planning to make a playground in the zoo, between the okapis and the baboons. The playground will be paved with square tiles. The owners are not sure yet how large the playground will be. They start a design with one square tile in the middle and lay a ring of square tiles around it. In the picture above the first ring of tiles is shaded. It consists of 8 tiles.

A second ring of tiles is added. The tiles in the second ring are now shaded in the picture below.

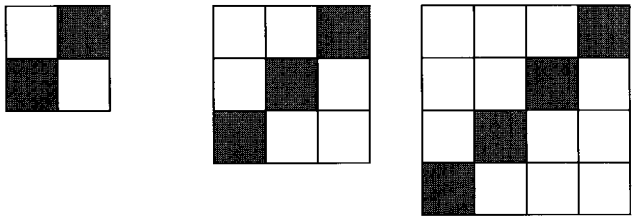
 
Diagram 2
How many tiles are in the shaded ring in Diagram 2?</td><td rowspan="4">Illustrative of band ii.

This task requires students to interpret a diagrammatic representation of a problem. The students then need to count the number of tiles to satisfy a particular criterion (i.e. shaded tiles). The students need only to state the correct answer, without explanation.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td rowspan="2"></td><td>1</td><td>Correct answer: 16 tiles no explanation required (‘tiles’ need not be given).</td></tr>
<tr><td>0</td><td>Incorrect response.</td></tr>
</table>

<table>
<tr><td colspan="3">NINE CHIPS [-1.47 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">The nine chips shown are placed in a jar and mixed.
 

Madeleine draws one chip from the jar. What is the probability that Madeleine draws a chip with an even number?
A) $\frac{1}{9}$ B) $\frac{2}{9}$ C) $\frac{4}{9}$ D) $\frac{1}{2}$</td><td rowspan="3">Illustrative of band ii.

This task requires students to calculate the probability of an event, given a constraint (consider even numbers only) and to select an answer expressed as a fraction (where simplification is not required).</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: C) $\frac{4}{9}$</td></tr>
</table>

PLAYGROUNDS Q1 [-1.52 LOGITS]			ANNOTATION
A large **square** playground will be built in a park. It will be paved with gray and white square tiles in the following pattern: one diagonal will be paved with gray tiles, the rest of the playground will be paved with white tiles. The dimensions of the playground have not been decided. Some playgrounds of different sizes are shown below. The shaded tiles are the gray tiles.  LENGTH = 2 TILES LENGTH = 3 TILES LENGTH = 4 TILES If the playground has a length of 10 tiles, how many gray tiles will be in the playground?			Illustrative of band ii. This task requires students to interpret a pattern given in diagrammatic form and to extend the pattern according to specified criteria. The students need only to state the correct answer, without explanation.
SCORING RUBRIC	**Points**		
	1	**Correct answer: 90**	
	0	**Incorrect response.**	

<table>
<tr><td colspan="3">K+6 [-1.72 LOGITS]</td><td>ANNOTATION</td></tr>
<tr><td colspan="3">If k can be replaced by any number, how many different values can the expression $k + 6$ have?

A) None
B) One
C) Six
D) Seven
E) Infinitely many</td><td rowspan="3">Illustrative of band ii.

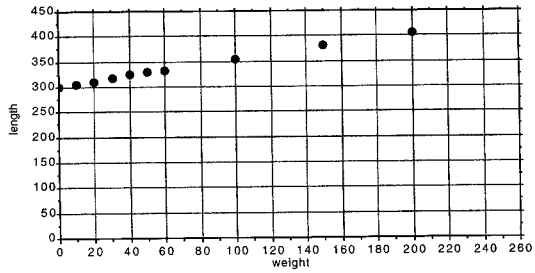
This task requires students to interpret an algebraic expression and to demonstrate an understanding of the term ‘infinitely’ in this context.</td></tr>
<tr><td>SCORING RUBRIC</td><td>Points</td><td></td></tr>
<tr><td></td><td>1</td><td>Correct answer: E) Infinitely many</td></tr>
</table>

Band i

STRETCH Q1 [-2.53 LOGITS]	**ANNOTATION**
 In science class, Nicola conducted an experiment involving a spring and different weights. The spring was stretched by hanging weights on a hook at the bottom of the spring. Nicola recorded the length of the spring after hanging each weight on the hook. The results of the experiment are shown in the table below.	Illustrative of band i. This task requires students to understand and interpret a graphical representation (using a coordinate system) to identify the value of an unknown. Students need to interpolate to determine a value on the y-axis, given a data point on the x-axis. Students are required to define a value within reasonable limits, given the intervals marked on the y-axis. The context is a familiar one, commonly encountered in science and mathematics classrooms.

weight (g)	0	10	20	30	40	50	60	70	100	150	200
length (mm)	300	305	310	316	325	329	332	…	354	380	406

Nicola used a computer to plot the results of her experiment on a coordinate system. This graph is shown below.



Nicola did not record the length the spring stretched when a weight of 70 grams was hung on the hook.

Using the graph, what do you expect this length to be?

SCORING RUBRIC	**Points**	
	1	**Any answer between** 325 and 351 any answer receives credit if point is correctly placed on graph
	0	**Answer:** ≤ 325 OR ≥ 351

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