The Longitudinal/Cross-Sectional Study of the Impact of Teaching Mathematics using *Mathematics in Context* on Student Achievement

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Monograph 3

2005

Instruction, Opportunity to Learn with Understanding, and School Capacity

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The L/CSS Monograph Series

This is the third of eight monographs derived from the National Science Foundation-funded Longitudinal/Cross-Sectional Study of the impact of teaching mathematics using *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997-98) on student achievement.

In 1992, the National Science Foundation (NSF) funded several projects to develop new sets of instructional materials that reflected the reform vision of school mathematics espoused by the National Council of Teachers of Mathematics (NCTM, 1989). One of the funded projects was to the National Center for Research in Mathematical Sciences Education (NCRMSE) at the University of Wisconsin–Madison. The project was organized to develop a comprehensive mathematics curriculum for the Grades 5–8 (NSF Grant No. ESI-9054928). Assisted by the staff of the Freudenthal Institute (FI) at the University of Utrecht in The Netherlands, the *Mathematics in Context* (MiC) curriculum materials were created and field-tested prior to being published in 1997-98 by Encyclopaedia Britannica.

In 1996, as the development of the MiC materials was nearing completion, a proposal was submitted to the National Science Foundation to investigate how teachers were changing their instructional practices in schools whose staffs were using *Mathematics in Context* and how such changed practices affected student achievement. Two NSF grants were awarded to the University of Wisconsin–Madison: first, to conduct a three-year study of the impact of *Mathematics in Context* on student mathematical performance (NSF Grant No. REC-9553889); and second, to analyze the data gathered in that study (NSF Grant No. REC-0087511). This monograph series presents the methodologies used in and the results of scaling the instruction students experienced, their opportunity to learn comprehensive mathematics with understanding, and the capacity of their schools to support teaching and learning mathematics.

As students and teachers begin to use any of the new mathematics materials, district administrators, mathematics educators, teachers, parents, and funding agencies express cogent needs to demonstrate that the curricula have a positive impact on students' understanding of mathematics. They often want to know the bottom line—the results on measures of achievement that confirm improved student mathematical performance. However, while improved student performance is critical, we contend that just relying on outcome measures to judge the impact of a standards-based program is insufficient. In fact, it is not enough to consider student outcomes in the absence of the effects of the culture in which student learning is situated, the instruction students experience, and their opportunity to learn comprehensive mathematics content in depth and with understanding. The dynamic interplay of all these variables has an impact on student learning, and as such, these variables must be considered in making judgments about the impact of any standards-based curriculum.

This monograph series tells the complex story of the variations in how the MiC materials were used by teachers and students in classrooms that vary in location and ecological culture, and the impact of that variation on the achievement of their students. The story unfolds in eight monographs. This third monograph provides the results of scaling the instruction students experienced, their opportunity to learn mathematics with understanding, and the capacity of their schools to support teaching and learning mathematics.

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Introduction to Monograph 3

This monograph contains three chapters. The purpose of this monograph is to describe the instruction students experienced during the longitudinal/cross-sectional study, their opportunity to learn mathematics with understanding, and the capacity of their schools to support teaching and learning mathematics. During the field-testing of *Mathematics in Context* (MiC), teachers' teaching and classroom assessment practices varied from more traditional approaches to teaching mathematics for understanding, and some teachers supplemented MiC units with conventional worksheets and materials from external resources (Romberg, 1997). Thus, it is not enough to consider student performance as a consequence of studying MiC in the absence of the effects of the instructional setting in which student learning is situated and the students' opportunity to learn comprehensive mathematics content in depth and with understanding. The dynamic interplay of these variables has an impact on student learning.

The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). The model is composed of variables and their theoretical interrelationships (represented by arrows in the model). This model, illustrated in Figure 1, includes 14 variables in five categories (prior, independent, intervening, outcome, and consequent; see Monograph 1, Chapter 2 for descriptions of these categories).

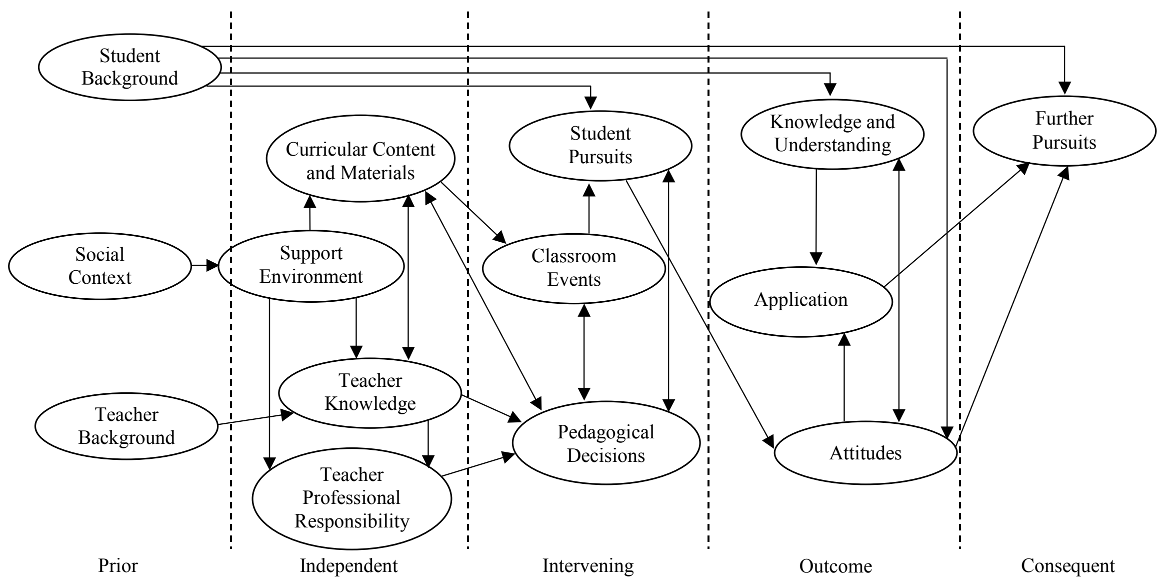

Figure 1. Revised model for the monitoring of school mathematics.

Because collinearity across indices posed a serious interpretation problem, a simplified model was designed during 1998-99 when Professor Romberg was a Fellow at the Center for Advanced Study in the Behavioral Sciences at Stanford. The statistics working group at the Center, headed by Lincoln Moses, examined the indices and the analysis plan for the study and suggested that four composite variables be created. This later became five composite variables during the examination of the information gathered in the study. The simplified model describes the relationship between variations in classroom achievement (CA), aggregated by strand, or total performance can be attributed to variations in prior achievement (PA), method of instruction (I), opportunity to learn with understanding (OTLu), and school context (SC). This relationship can be expressed as—

$$CA = PA + I + OTLu + SC$$

These composite indices, based on one or more sub-indices created for each variable in the original model, were then created. The sub-indices were intended to capture the variability across classes (and schools) in relation to each variable in the structural model. The details of how the composite indices for I, OTLu, and SC were created are discussed in this monograph.

In Chapter 1, the methodology used in characterizing the instruction students experienced is described, and levels on the composite index for instruction are illustrated. In Chapter 2, students' opportunity to learn comprehensive mathematics content in depth and with understanding is portrayed and the levels on the composite index for this variable are discussed. In Chapter 3, teachers' perceptions of capacity of their schools to support teaching and learning mathematics are described. In each chapter, the results show the extent of variation among teachers using MiC and teachers using conventional curricula, teachers at different grade levels, and teachers in different districts. Data from classroom observation reports and teacher journal entries, interviews, and questionnaires illustrate these differences.

CHAPTER 1: THE QUALITY OF INSTRUCTION

Mary C. Shafer

In the structural research model for the longitudinal/cross-sectional study, instruction is described through the intervening variables (see introduction to this monograph). In the model, three intervening variables were identified: pedagogical decisions, classroom events, and student pursuits. The variable *pedagogical decisions* represents the teacher's decisions in defining the actual curriculum. These decisions include deliberate advanced planning such as student grouping for instruction, time allotted to specific aspects of lessons, emphases given during instruction, and other modifications of the intended curriculum. Pedagogical decisions also include decisions made during instruction, for example, evaluating student solutions, resolving critical incidents, or assessing student knowledge. The variable *classroom events* represents the interactions among the teacher and students that promote learning mathematics with understanding. These events arise from a learning environment in which students explore mathematics and are encouraged to make sense of mathematics. The intervening variable *student pursuits* captures students' active involvement in learning. Thus, the intervening variables capture an array of complex teacher decisions that generate and maintain a learning environment in which students are actively engaged in making sense of mathematical ideas.

Koehler and Grouws (1992) argued that valid judgments about the quality of instruction arise from observing teaching as it occurs in classrooms through analytical and holistic methods. Analytical methods examine specific teaching actions, whereas holistic ways focus on teaching episodes or entire lessons. However, assessing the quality of instruction should consider the ways classroom events work together to shape meaningful learning experiences for students (Grouws, 1988; Koehler & Grouws, 1992). In the longitudinal/cross-sectional study, both analytical and holistic methodologies were used in examining the quality of instruction.

Theoretical Framework

Three perspectives were used in framing this research on the quality of instruction: the NCTM *Standards*, the principles of Realistic Mathematics Education, and research on teaching and learning mathematics for understanding. Because MiC was developed to address the recommendations of the *Standards* (NCTM, 1989, 1991, 1995), these documents were used as an initial foundation for examining the quality of instruction. The predominant theme of the *Curriculum and Evaluation Standards* (1989) is that to develop students' mathematical abilities, all students should have opportunities to engage in problem solving, reasoning, communication, and making connections. Students investigate complex, nonroutine problems, conjecture and invent solutions, formulate conclusions and support their thinking with demonstrations, drawings, and calculations. Lessons are structured for students' active participation in discussing mathematical ideas, solution strategies, and arguments. As outlined in the *Professional Standards for Teaching Mathematics* (1991), the teachers' role is to facilitate classroom interaction. Teachers select or design tasks that involve important mathematics and allow for discussion of mathematical ideas. The tasks provide opportunities for students to extend and apply mathematical knowledge and practice important skills. Teachers nurture learning environments that provide the time necessary for students to explore mathematics and struggle with complex problems, and in which students are encouraged and expected to make sense of mathematics by working independently or collaboratively. As teachers strive to promote classroom discourse, students' roles in mathematics classes need to change in substantive ways. Student actions are characterized by verbs such as "explore, justify, represent, solve, construct, discuss, use, investigate, describe, develop, and predict" (NCTM, 1989, p. 17). Students are encouraged to initiate questions and raise problems; consider the validity of their own and others' representations, solutions, conjectures, and answers; and develop mathematical arguments. Throughout instruction, teachers make instructional decisions that affect their assessment of student learning and allow them to judge the effectiveness of instructional tasks. These roles were elaborated in the *Assessment Standards for School Mathematics* (1995) by emphasizing the integrated nature of classroom assessment practice and instruction. Teachers can take advantage of evidence of student learning through opportunities to listen to students as they work and through discussions centered on making sense of mathematics. In this way, assessment practice can move beyond merely paper-and-pencil methods. Teacher feedback to students can then be based on mathematical reasoning and solution strategies in addition to accuracy of procedures.

The second perspective used in characterizing the quality of instruction was the instructional approach of Realistic Mathematics Education (RME; Gravemeijer, 1994), which was one of the fundamental design elements for MiC (see Monograph 1, Chapter 1). In RME, mathematics is viewed as a dynamic set of interrelated ideas best learned by applying concepts and procedures in problem contexts and situations that make sense to students. Students are given the opportunity to reinvent significant mathematics under the guidance of their teachers, through interaction with their peers, and through the use of mathematical models introduced and developed during instruction. Such models act as bridges between concrete real-life problems and abstract formal mathematics. Through lessons that allow students to solve problems using a variety of strategies, teachers encourage students to discuss interpretations of the problem situation, express their thinking, and react to different levels and qualities of solution strategies shared in the group. Through instruction and discussion, more elaborate models and strategies are introduced. Students solve problems at different levels of abstraction, falling back to more concrete, less abstract strategies whenever they feel the need. As a result of exploration, reflection, and generalization, students theoretically progress from context-specific situations to more abstract mathematical reasoning.

The third perspective used in examining the quality of instruction in the MiC study is research on teaching and learning mathematics for understanding. This research is based on the principles that knowledge is constructed by the learner and is situated in the context of the learner's existing knowledge, skills, and beliefs; that the teacher's role is a guide for facilitating conceptual understanding; that mathematical tasks are nonroutine, accessible to all students, and engage students' thinking about important mathematics; that classrooms are communities of learners; and that mathematical tools are supports for learning (Cohen, McLaughlin, & Talbert, 1993; Fennema & Romberg, 1999; Hiebert et al., 1997). Research has documented important interrelated cognitive activities that support students' learning mathematics with understanding (Carpenter & Lehrer, 1999). Students need time and opportunity to develop relationships among mathematical ideas; extend and apply mathematical ideas in new situations; reflect on their own thinking; articulate mathematical ideas; and make mathematical knowledge their own. Understanding mathematics requires reflection, the “conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed” (Battista, 1999, p. 429). Through reflection, students can look for connections between new mathematical ideas and ones they already know. Teachers can specifically ask students to identify and articulate these relationships. Students can discuss how a procedure is linked to a particular notation or underlying mathematical concept. Cooperative groups can facilitate reflection on the part of individual students as they articulate “their assumptions, conjectures, and plans to one another” (Carpenter & Lehrer, 1999, p. 29). Student roles in learning mathematics with understanding are active and collaborative.

When teaching mathematics for understanding, lessons emphasize cognitive activities that are focused on mathematics. Lessons promote intellectual quality when they include attention to higher order thinking skills, depth of knowledge, and substantive conversation (Newmann, Secada, & Wehlage, 1995). Higher order thinking involves skills of prediction, interpretation, application of ideas, comparison, and raising questions. Tasks that promote depth of knowledge require that students “organize, interpret, evaluate, or synthesize complex information,” allow students to consider alternative solutions or perspectives, elaborate their thinking, or support their conclusions (Newmann, Secada, & Wehlage, 1995, p. 81, 83). Tasks, however, can be adjusted to either reduce or maintain cognitive demands. The difference resides in the actions of teachers who must “proactively and consistently support students’ cognitive activity without reducing the complexity and cognitive demands of the task” (Henningsen & Stein, 1997). To support high cognitive demand, teachers can, for example, scaffold questions, ask students to provide thorough explanations, and press students to look for meaningful connections among ideas. With appropriate guidance in large- and small-group settings, students discuss interpretations of problem situations, express their thinking, and react to the different levels and qualities of solution strategies shared in the group (Gravemeijer, 1994). Such conversation is characterized by reciprocity. Students listen carefully to each other’s ideas, build conversation on those ideas, and mutually construct their understanding (Newmann, Secada, & Wehlage, 1995). In contrast, student conversation with little or no substantive content involves reporting facts and procedures in ways that do not encourage further discussion of ideas and can ultimately result in students’ appropriating answers or solution strategies of a dominant peer.

The complementary nature of these perspectives provided linkages among the *Standards* documents, the RME instructional approach, and research on teaching for in-depth conceptual understanding. Collectively, these perspectives provided a substantial basis for examining instruction in the context of reform.

Methodology

In conducting research on the intervening variables, measurement of particular qualities of pedagogical decisions, classroom events, and student pursuits were designed to consider the complexity of these variable and variation encountered among study teachers and students. A set of categories was used to describe the intervening variables (see Table 1-1). An index was then created for each category to look for patterns of variation among teachers. The underlying single dimension was teaching mathematics for understanding. Therefore, the indices were preliminarily defined by describing subcategories of the intervening variables and identifying differences between conventional approaches and teaching for understanding that were measurable by the research team. For each category, three to six levels were outlined. Further distinctions in the levels for each category were identified through a review of literature that was specific to each category. Models for these indices were from previous research on authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993).

The levels in each index were positioned along a continuum from the least to the most reflective of teaching for understanding. For example, levels of *lesson presentation and development* ranged from no lesson presentation during instruction to lessons in which the teacher facilitated students' active participation in learning mathematics with understanding. The levels in each index were further refined (or levels were added) as data for fifth-grade study teachers in District 1 were reviewed. This process was based on Strauss' (1987) system of open, axial, and selective coding, which involved repeated coding of the data for interpretive codes. These codes included both external codes identified prior to reviewing the data (e.g., procedure or strategy demonstrated) and internal codes that were identified through the analysis (e.g., students were unable to use the presented procedure). Indices were further revised during review of data from sixth- and seventh-grade teachers in District 1 and all teachers in District 2. As a result, three to six levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts. Occasionally, a level was divided into subcategories to more accurately describe the variation found. For example, one level in the index for *lesson presentation and development* was subdivided further in order to more adequately describe the variation among lessons in which procedures or strategies were demonstrated to students. Sublevels categorized situations in which students were unable to complete exercises using the presented procedure and lessons during which students practiced the presented procedure in a rote fashion. When analyzing data from the second and third study years, similar adjustments were made in the levels as data from teachers were analyzed. When a code was added, the entire set of data (including data from the first study year) was re-read to see whether previous coding might change in light of the new codes. Data sources included teacher interviews, teaching journals and logs, evidence from classroom observation reports, and postobservation interviews. In this methodology, rich qualitative data were quantified according to the levels described for each subcategory. The qualitative data were then used to illustrate the assigned level for each teacher for each category.

Table 1-1.
Characterization of the Intervening Variables

Pedagogical Decisions			**Classroom Events**	**Student Pursuits**
Instructional Planning		**Interactive Decisions**		
Unit Planning	**Lesson Planning**			
• Consideration of students' prior knowledge • Unit sequence • Pace of instruction	• Student performance in previous lesson • Purpose of the lesson • Forms of instruction that promote discourse for the purpose of the lesson • Student activities that promote discussion, problem solving, and reflection	• Explanation-oriented decisions • Task-oriented decisions • Shifts in pedagogical approach	• Lesson presentation and development • Development of conceptual understanding • Nature of student conjectures • Connections among mathematical ideas • Connections between mathematics and life experiences • Nature of student explanations • Elicitation of multiple strategies • Classroom assessment practice • Lesson closure, reflection, or summary	• Nature of student–student conversation • Nature of students' collaborative working relationships • Overall engagement during the lesson

When the decision was made to use the simplified research function (see introduction to this monograph), a composite variable was specified for instruction. This single composite was designed to account for all information gathered on the intervening variables. Categories of the intervening variables were reorganized into five major categories for the composite variable *Instruction*: unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice, and student pursuits during instruction (see Table 1-2). Unit planning, lesson planning, and classroom assessment were separated in the analysis to check their relative influence on the instruction composite variable. The subcategory *interactive decisions* was more appropriately classified as a subcategory under mathematical interaction during instruction. A composite index was created for *interactive decisions* to summarize the three aspects that were studied. Another change was made relative to four subcategories that previously described classroom events (development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and life experiences). These were summarized in a composite index called the *nature of inquiry*. The four subcategories that previously described classroom events in the structural research model (development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and life experiences) were used separately in characterizing students' opportunity to learn mathematics with understanding (see Chapter 2 of this monograph). Consequently, the research team decided to use a composite of this information as one subcategory of mathematical interaction. As a result, three to six subcategories of instruction were examined in each major category. In total, 19 subcategories of instruction were scaled in describing the quality of instruction that transpired in study classrooms. Although a single composite index was necessary for the research function, it could also be disaggregated to examine the relative influence of each subcategory on the instruction composite variable.

Table 1-2.
Characterization of the Instruction Composite Variable

Category	**Unit Planning**	**Lesson Planning**	**Mathematical Interaction during Instruction**	**Classroom Assessment Practice**	**Student Pursuits**
Subcategories	• Consideration of students' prior knowledge • Unit sequence • Pace of instruction	• Student performance in previous lesson • Purpose of the lesson • Forms of instruction that promote discourse for the purpose of the lesson • Student activities that promote discussion, problem solving, and reflection	• Lesson presentation and development • Nature of inquiry during lessons • Teachers' interactive pedagogical decisions • Nature of student explanations • Elicitation of multiple strategies • Lesson closure, reflection, or summary	• Evidence sought • Feedback coherence and purpose • Content of feedback	• Nature of student–student conversation • Nature of students' collaborative working relationships • Overall engagement during the lesson

Instructional Planning

Pedagogical decisions teachers make prior to and during instruction have a direct impact on student learning. Prior to instruction, teachers' decisions include deliberate advanced planning such as student grouping for instruction, time allotted to specific aspects of lessons, and emphases in instruction. In planning instruction, teachers draw on their knowledge of mathematics, including knowledge of concepts and procedures of mathematical domains, relationships among them, and processes involved in solving problems in the domains. Teachers also rely on their knowledge of pedagogical methods they have found to be effective. But pedagogical content knowledge is also important (Fennema & Franke, 1992; Shulman, 1987). Pedagogical content knowledge includes information about typical student learning trajectories in mathematical domains, the struggles students might encounter, and strategies for dealing with such difficulties. Shulman (1987) explained: "The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p. 15, original emphasis). This transformation of teacher knowledge into goals or expectations for student learning during the lesson is a distinguishing factor among the subcategories of unit planning and lesson planning in this study.

Standards-based curricula or curricula based on children's thinking (e.g., Cognitively Guided Instruction, Carpenter et al., 1999) present mathematics in ways that may be unfamiliar to teachers and require advanced planning to facilitate successful implementation (Bay, Reys, & Reys, 1999; Romberg, 1997). As teachers work through the problems themselves, they become aware of the differences in presentation of material from conventional texts, the context in which the lesson is set, and the types of problems students might encounter. In contrast, when the aim of the lesson is coverage of materials, the emphasis is often on pieces of information that are unconnected and on practice of repetitive procedures or heuristics that were determined by others (Battista, 1999). Such situations reduce the cognitive demands for students. Because lesson content in conventional textbooks tends to be the same for many years and lesson material conveniently parceled into 2-page spreads, lesson planning is often minimal.

Students' performance during a lesson may or may not affect teacher's future planning, even if difficulty in working with a particular concept or procedure is noted by the teacher. On some occasions, lessons might be extended to the next class period because the completion of a task is important to subsequent lessons. At other times, text or instructional problems might be disregarded because of time constraints for completing a particular unit, with or without regard for the benefit of exploring increasingly complex problems. The purpose of the lesson can be limited to learning particular mathematical content or can be expanded to include attention to higher order thinking skills. Information gathered through assessing students' understanding of the mathematical content of the lesson can lead to substantive changes in subsequent planning. For example, the teacher might allow for more in-depth exploration of the mathematical ideas, provide time for students to reflect on their thinking, introduce another approach for students who struggled with the initial one. The teacher might also use information from informal assessment to encourage thinking at more complex levels in the mathematical domain.

In teaching for understanding, less emphasis is expected on review of previous material and independent practice, lengthy warm-up activities focused on computation, and teacher presentation of material with the intent of students' receiving and working with isolated pieces of knowledge during the lesson. Rather, teachers must plan forms of instruction and tasks for active student engagement in lessons. Increased emphasis is given to engaging students in tasks with high cognitive demand and to whole-class discussion or small-group work focused on contributing mathematical ideas and building substantive conversation.

Unit Planning

Three subcategories were used to characterize the ways teachers planned to teach each unit or chapter: *consideration of students' prior knowledge during unit planning; unit sequence;* and *pace of instruction.* In this section, the highest rating for the index for each subcategory of unit planning is briefly described and illustrated with qualitative data. Indices are found in the appendix to this chapter. For descriptions of the level assigned for each teacher on each index, see Shafer, Marten, Webb, Folgert, & Davis (2003) and Shafer, Marten, Folgert, & Kwako (2003a, b).

Consideration of students' prior knowledge during unit planning. The index for *consideration of students' prior knowledge* measured the extent to which the teacher thought about and identified students' prior knowledge while planning to teach a unit. The highest rating was assigned when the teacher planned activities that were designed to (a) bridge the gap between students' prior knowledge and prerequisite skills for the unit or chapter; or (b) familiarize students with the contexts presented in the unit. For example, when planning to teach a MiC unit, Schroeder reviewed the unit, especially the section overviews, to become familiar with the content of the unit, unit goals, vocabulary, and the kinds of problems. For her special education students, she planned a unit introduction that included an overview and practice with problems from MiC ancillary materials (*Number Tools*) that were similar to those in the unit (Seventh grade, MiC, Schroeder, Interview 6/19/00). This introduction not only gave students an overall picture of what they would be learning in the unit, but provided students with some practice that would increase the potential for them to be successful in the unit.

Unit sequence. The index for *unit sequence* measured the extent to which the teacher considered the sequence of instructional units or chapters. The highest rating was assigned when the teacher sequenced the units to support the development of mathematical concepts. For example, Schroeder was aware of the recommended sequence of MiC units at her grade level, but modified the sequence to meet the needs of her students who were in special education classes: "I have to decide first of all what units [students] will do. They don't necessarily study units at the grade level the students are in. If they haven't had the previous units, we might go back and do a sixth-grade unit" (Seventh grade, MiC, Schroeder, Interview 6/19/00). Schroeder first considered the content she wanted students to learn. She then selected units designed for the grade level of the students. She also searched for related units in prior grade levels that provided prerequisite content that students would need to be successful in learning mathematics at their own grade level.

Pace of instruction. The index for *pace of instruction* measured the extent to which the teacher considered pacing when planning a unit. This information was deemed important because MiC units were designed to be completed in 3-4 weeks, although teachers often did not work within that time frame (see Monograph 1, Chapter 4). The highest rating was reserved for teachers who planned substantive supplemental activities for students who completed the lesson before most students in the class. None of the study teachers were given this rating. Several teachers, however, were assigned Level 3, indicating that they considered the learning styles and reasoning skills of their students when they planned the pace of instruction. For example, Teague used the amount of time she needed to work through the unit to estimate the pace of instruction. She also considered that many of her students had below average skills in mathematics and reading. She commented:

> If it takes me an hour to do an assignment, I know that it's going to take my students probably an hour and 40 minutes or two hours. Sometimes it's just too long to do this for the whole two-hour block. I look at the students. If they're getting restless, I know it's time to change the activity. But if they're into it, and they're working along, I'm certainly not going to stop. (Eighth grade, MiC, Teague, Interview 4/13/99)

Teague timed her own work on solving unit problems as a guide to estimate the pace of instruction. She also planned to modify the pacing according to student behaviors during individual lessons.

Lesson Planning

Four subcategories were used to characterize the ways teachers planned to teach individual lessons: *consideration of students' performance in the previous lesson; the purpose of the lesson; forms of instruction that promote discourse for the purpose of the lesson;* and *student activities that promote discussion, problem solving, and reflection on the content of the lesson*. The highest rating for the index for each subcategory of lesson planning is briefly described and illustrated with qualitative data. For descriptions of each index and the level assigned for each teacher on each index, see Shafer, Marten, Webb, Folgert, & Davis (2003) and Shafer, Marten, Folgert, & Kwako (2003a, b).

Students' performance in the previous lesson. The index for *students' performance in the previous lesson* measures the extent to which a particular teacher considered students' performance on the previous lesson when planning to teach the subsequent lesson. The highest rating was reserved for teachers who planned to vary problem structure or setting to encourage thinking at higher levels or to emphasize connections with related concepts. None of the study teachers were given this rating. Several teachers, however, were assigned Level 3, indicating that they used the information gathered to allow a more in-depth exploration of the mathematical content or introduce another approach to encourage students' understanding. For example, Piccolo used information gathered during informal classroom assessment to change instructional approaches in subsequent lessons. To illustrate this, Piccolo reported one day that students in her class did not fully understand the compensation strategy for finding the mean. She noted that during the next class period she would demonstrate the compensation strategy by using manipulatives on the overhead projector (Fifth grade, MiC, Piccolo, Teaching log 3/16/98). Piccolo used the information she gathered about her students to select another approach for assisting students to develop an understanding of this concept. Piccolo made changes in subsequent instruction that moved students toward deeper understanding of mathematics.

Purpose of the lesson. The index for the *purpose of the lesson* measured the extent to which the teacher thought about and identified the purpose of the lesson prior to instruction. The highest rating was reserved for teachers who planned questions to engage students in interpreting a solution in terms of the problem context, exploring connections among equivalent representations of numbers, or summarizing the mathematics in a series of lessons. None of the study teachers were given this rating. Several teachers, however, were assigned Level 3, indicating that they went beyond just becoming familiar with the content of the lesson, the presentation of the mathematics, or the context in which the lesson was situated. These teachers considered potential student questions, possible misunderstandings, various solution strategies, or conceptual development in the unit. For example, when planning to teach an individual lesson, Mitchell solved some of the problems in the student edition herself or at least reviewed them to get "a general idea of how they should be solved" (Fifth grade, MiC, Mitchell, Interview 4/9/98). She also read the comments in the teacher guide for each problem. This process enabled her to anticipate difficulties the students might have with the content and to decide whether modifications were warranted. For instance, on one occasion Ms. Mitchell used a formal assessment that included symmetric patterns. She reported that the students were unable to do the work individually, but she had anticipated the trouble spots and was prepared with specific ways to help the students (MiC, fifth grade, Mitchell, Observation 10/20/97). Mitchell's planning went beyond becoming familiar with curricular content. She made decisions for students based on potential student questions or misunderstanding.

Forms of instruction that promote classroom discourse. The index for *instructional forms that promote classroom discourse* measured the extent to which the teacher planned various forms of instruction for a lesson that promoted classroom discourse. The highest level was assigned when the teacher planned forms of instruction that promoted substantive conversation. For example, Downer physically organized her classroom so that students sat in groups of five or six. If a lesson had several problems, Downer assigned different problems to different groups. Each group member was responsible for reporting at least a portion of the group's findings to the whole class. When there were differences in group's findings, the whole class looked for reasons and justifications. Downer thought small-group work was generally a good form of instruction because it allowed the opportunity for each student to participate and succeed (Eighth grade, MiC, Downer, Interview 6/13/00). The highest rating was assigned for Downer because she planned instruction to promote discourse. Students were expected to contribute to both small-group and whole-class discussion by talking about their ideas, presenting conclusions to others, and responding to others' questions. To do this, students had to actively listen to their peers, compare their ideas with their own, and build conversation.

Student activities that promote discussion, problem solving, and reflection. The index for *student activities that promote discussion* measured the extent to which the teacher included various student activities that promote discussion, problem solving, and reflection in lesson plans. The highest rating was assigned when investigation of problems, discussion of answers and solution strategies, reflection on or summarization of the lesson were dominant in lesson plans. Information from Nash's teaching logs illustrate this emphasis. Investigation of problems was an important element in Nash's instruction: It was noted on nearly every reported day, generally for at least half of the class period. Students discussed answers and solution strategies on two-thirds of the reported days for approximately one-third of the class period. Several student activities were reported frequently and were usually given less than 10 minutes of class time: listening to the teacher or taking notes (on 81% of the reported days), reflecting on or summarizing lesson concepts (on 74% of the reported days), and participating in whole-class discussions (on 50% of the reported days; fifth grade, MiC, Nash, Teacher Log 1997-98). Nash's teaching logs indicated the strong emphasis she placed on planning for students to investigate problems, discuss solutions, and reflect on lesson concepts. This emphasis was also noted in classroom observations.

Mathematical Interaction during Instruction

Six subcategories characterized the nature of mathematical interaction that transpired in study classrooms: *lesson presentation and development; nature of mathematical inquiry during instruction; teachers' interactive decisions; nature of students' explanations; elicitation of multiple strategies;* and *lesson closure, reflection, or summary*. In this section, the highest rating for the index for each subcategory of mathematical interaction during instruction is briefly described and illustrated with qualitative data. For descriptions of each index and the level assigned for each teacher on each index, see Shafer, Marten, Webb, Folgert, & Davis (2003) and Shafer, Marten, Folgert, & Kwako (2003a, b).

Lesson Presentation and Development

In every lesson students construct mathematical knowledge. Instruction is enhanced when students are allowed to do the mathematical work that is the substance of the lesson. In other words, when students are given opportunities to explore mathematical concepts and generate solutions with the teacher's support, instruction is enhanced. On the other hand, instruction in which the teacher consistently demonstrates how to find a solution and students practice the teacher's method, few opportunities exist for students to do mathematical work. In such instruction, the work was done by their teacher. The ways in which lessons are presented and developed also vary in the attention to conceptual understanding, coherence among lesson activities, levels of reasoning, and mathematical connections (Stigler & Hiebert, 1999).

The index for *lesson presentation and development* measured the extent to which lesson content was presented in ways that encouraged learning mathematics with understanding. The highest level of this index (Level 6) was assigned when the lesson emphasized conceptual understanding and teachers supported active student participation. The lesson presentation set the stage for students to explore the mathematical content of the lesson on their own. Student solutions and generalizations were later presented and compared during discussions orchestrated by the teacher.

To illustrate *lesson presentation and development* at Level 6, a lesson presented LaSalle (Observation, 5/19/98) from a class using the MiC fifth-grade number unit *Measure for Measure* (Gravemeijer, Boswinkel, Meyer, & Shew, 1997), is used. In the lesson (pp. 4–6), students were asked to determine the amount of castor oil on each of several measuring sticks and to express the amount using both a common fraction and Egyptian symbols. At the beginning of the lesson, LaSalle read a definition of castor oil and discussed its uses. She related oil from castor beans to oil from coffee beans. When she introduced measuring sticks, LaSalle talked about how measuring sticks are used in measuring the oil level in cars. She then used two soda cans to demonstrate the use of measuring sticks. Students were asked to measure the part of each can that was filled with soda. One was about ¼ full while the other was about 2/3 full. When the soda was poured into one can, one student estimated that the amount was 11/12 because it was almost full. LaSalle then distributed whiteboards to each group. A student read 5, and groups were given five minutes to work on the problem. Each group wrote their solution on their whiteboard and presented their solution to the class. LaSalle encouraged students to develop their own strategies to solve problems. For example, in their solutions to 5, the observer noted:

> One group added the fractions by getting common denominators and found they had a sum of 63/64 so they were still missing 1/64. Another group used an area model with rectangles and shaded each fraction. 1/64 of the rectangle was unshaded.

The classed worked on 7 on the whiteboards and discussed the problem as a large group. Students worked with fractions, estimating, and representing quantities. The observer noted that LaSalle worked with students to build discussion: “Classroom discussion is a strength for Ms. LaSalle. Students discuss issues with other students, and she asks meaningful questions to keep the discussion on track.”

On this occasion, the demonstration LaSalle used promoted understanding of measurement, estimation with fractions, and solving problems with quantities represented as fractions. LaSalle presented the lesson in a way that supported students’ mathematical reasoning. She did not present a procedure for students to use in solving problems in this context. Rather, she set the stage for students to work on their own. Student groups were given the responsibility to develop solutions for one problem at a time. Students were expected to carry out the mathematical work through developing their own solution strategies, writing them on whiteboards, and presenting them to the class. The presentations permitted the groups to compare various solutions to the same problem.

Nature of Inquiry

The index for the *nature of inquiry* is a composite index of subcategories that previously described classroom events in the structural research model: development of conceptual understanding, nature of student conjectures, connections among mathematical ideas, and connections between mathematics and life experiences. For each teacher, the sum of the ratings from each classroom observation report for each of the four subcategories was calculated, and cluster analysis was completed. Cluster analysis, which tests the proximity between two objects, permitted the classification of teachers. Cluster analysis was beneficial for two reasons. First, because it considers grand total means, cluster analysis lessened whatever distortion that might have been caused by relying solely on researchers' intuition in grouping teachers with similar ratings. Second, because the means of each subcategory of instruction are considered in the cluster analysis, the level descriptions became more acute. This process permitted the classification of teachers into four groups. For each group of teachers, common characteristics from the ratings on the separate indices were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each separate index. These descriptions became the levels of the composite index for the nature of inquiry, which captured the variation among study teachers. The highest level of the composite index for the *nature of inquiry* (Level 4) was assigned when mathematical content was explored in enough detail for students to think about relationships among mathematical ideas or link procedural and conceptual knowledge. Students were encouraged to make generalizations about mathematical ideas. Connections between mathematics and students’ life experiences were discussed.

A lesson presented by Fiske (Observation, 1/23/98) using the MiC number unit *Some of the Parts* (van Galen, Wijers, Burrill, & Spence, 1997) is used to illustrate this rating. In this lesson, the ratio table was used to support students' reasoning about fractional amounts, specifically in finding the amounts of ingredients if the number of servings in a recipe decreases. For their homework assignment, students were asked to bring their favorite recipes from home. Fiske began the class with looking at a recipe for Potato Chip Sandwiches, which she found in a cookbook. The class doubled the recipe. Fiske read the recipe in *Some of the Parts* and led the class in halving the recipe. The lesson promoted conceptual understanding of the mathematics: Fiske continually drew pictures that depicted the fractions and shaded half of the represented fraction. Students expressed generalizations they made, as noted by the observer:

> At one point, after many drawings, students were asked what was half of 1/4. Some students were able to respond without the help of a drawing. They had generalized the situation mentally. By the time they got to half of 1/6, all hands were waving to answer without a drawing to guide them. With respect to taking half of a fraction, one student said, "You can keep multiplying the denominator by 2." She was so excited to share her generalization that she could hardly be restrained from yelling out her discovery. This contribution was done without Ms. Fiske asking for a generalization. (MiC, fifth grade, Fiske, Observation 1/23/98)

Many students reported that they regularly cooked at home. The recipes used in the unit and the recipes students brought in for their homework assignments connected the mathematics to their lives. In this lesson, Fiske encouraged students to think about mathematical relationships (representing halves of fractional quantities) and linking procedural and conceptual knowledge (using diagrams to understand the resulting amounts). She also encouraged students to state generalizations. Connections between mathematics and their life experiences were emphasized as students worked with their own recipes.

Teachers' Interactive Decisions

Planning shapes the possibilities of events that might occur during instruction. Once instruction begins, however, teachers' interactive decisions become more important. Research on the content of teachers' interactive thoughts suggests that teachers most frequently reported concern for the learner and referred to their students' understanding, attention, behavior, and involvement in lessons. Studies suggest that teachers make decisions frequently, on the average of every two minutes (Clark & Peterson, 1986). For experienced teachers who used conventional pedagogy, many of these decisions were automatic: Teachers developed routines for handling incidents that frequently occurred during classroom interaction. These routines minimized teachers' conscious decision-making (Borko & Shavelson, 1990). The interactive decisions of teachers using prepublication versions of MiC became more complex as students assumed more active roles in their own learning. For example, Clarke (1995) investigated the nature of critical incidents that developed as two teachers used MiC for the first time. In their efforts to understand students' solution strategies at various levels of sophistication, these teachers found it difficult both to structure lessons that allowed for some guidance without limiting opportunities for student thinking and to determine the length of time to let students struggle with a problem before intervening. They also faced challenges of "valuing all genuine attempts at problems, while seeking to move students toward increasingly mathematically elegant methods" (p. 156), and they found that it was "very difficult to be interacting, to be listening in an attentive way, trying to understand students' solutions, and thinking of an appropriate response, many times during a lesson" (p. 163). Thus, the nature of teachers' interactive decision making changes dramatically as teachers use MiC.

Teaching for understanding involves the flexible use of various pedagogical methods. As such, teachers frequently make on-the-spot decisions, on the basis of students' mathematical understanding, their instructional goals, and the dynamic interaction occurring in the classroom. In this study, three types of interactive decisions were examined: *explanation-oriented decisions* (e.g., direct method introduced, procedures and correct answers emphasized, connections promoted, mathematical processes emphasized), *task-oriented decisions* (e.g., attention focused on pertinent elements of the task or reasonableness of solutions), and *shifts in pedagogical approach* (e.g., limited changes made, another context or review added, process modeled, modification made based on student inquiry). The index measured the extent to which a *teacher's interactive decisions* supported teaching mathematics for understanding Shafer, 2002). The decisions in each category were classified as least aligned with teaching for understanding, reflective of traditional pedagogy, or most aligned with teaching for understanding. Five levels of interactive decisions were identified based on profiles of interactive decisions for individual teachers. The highest level of this index was assigned when teachers' interactive decisions were predominantly aligned with teaching for understanding and were least likely to be coded as least aligned with teaching for understanding (around 10%). Evidence of teachers' interactive decisions was culled from journal entries, classroom observations, and postobservation interviews. For example, the profile for Fulton, a fifth-grade teacher who used a conventional curriculum, 77% of the coded decisions were more aligned with teaching for understanding. Explanation-oriented decisions coded for Fulton promoted connections among mathematical ideas or between mathematics and students' lives (11% of the evidence), and questioning focused on mathematical processes (22% of the evidence). One-third of the interactive decisions coded for Fulton were task-oriented decisions that focused students' attention on the reasonableness of their solutions. Eleven percent of the decisions involved shifts in pedagogical approach by modifying a lesson based on student statements or inquires. Fulton's interactive decisions demonstrated a significant alignment with teaching for understanding.

Nature of Students' Explanations

The index for the *nature of student explanation*s measured the extent to which students elaborated on their solutions orally or in written form by justifying their approaches to a problem, explaining their thinking, or supporting their results, rather than simply stating answers. The highest level was assigned when student explanations were focused on such mathematical processes. For example:

> Students were continually asked to justify and explain their answers and conjectures. They were asked questions about the lesson's logic problem. Students made statements about who was taking or not taking [particular] language courses. They had

> to give the reasoning for their statements . . . and the rest of the class either agreed or disagreed. (Seventh grade, conventional curriculum, Cunningham, Observation 3/16/98)

In this lesson, students were expected to move beyond description of procedures toward the more complex processes of validating conjectures and supporting conclusions.

Elicitation of Multiple Strategies

The index for the *elicitation of multiple strategies* measured the extent to which students were asked to consider different perspectives in approaching the solution to a problem. In a classroom in which multiple strategies are encouraged and valued, this discourse is an important element of instruction. When students discuss advantages and efficiency of various solution strategies for the same task, opportunities to develop conceptual understanding are opened (Hiebert et al, 1997). Multiple strategies might be elicited by the teacher during whole-class or small-group discussion in which students explicitly share their strategies. The task itself might clearly involve students in solving the problem in different ways (e.g., find the discount in another way), or the task may require students to consider alternative approaches for successful completion (e.g., list as many ways as you can to calculate 15 x $1.98). The highest rating was assigned when the discussion of alternative strategies was frequent, substantive in nature, and an important element of classroom instruction. For example:

> Ms. Kipling encouraged the use of a variety of strategies to solve the problems. [One student said,] "I chose [the first one] because 1/2 is 2/4 and 2/4 + 3/4 is 5/4 or $1.25. So all together I have $4.25." Other [students] only used hundredths since they were dealing with money. (Fifth grade, MiC, Kipling, Observation 5/19/98).

In this example, the discussion of alternative strategies was an important element of classroom instruction.

Lesson Closure, Reflection, or Summary

The index for *lesson closure, reflection, or summary* measured the extent to which students were given opportunities to reflect on or summarize mathematics they learned. The highest rating was assigned when students were frequently provided with opportunities to reflect on the mathematics in a lesson or in a series of lessons or for students to summarize what they had learned in a lesson. These opportunities included items embedded in curricular materials, teacher-designed writing prompts, or questions posed during whole-class discussions. For example, on 74% of the days in which her teaching log was completed, Nash reported that students reflected on or summarized lesson concepts (Fifth grade, MiC, Nash, Teacher log 1997-98).

Classroom Assessment Practice

Teachers seek evidence to predict and confirm their intuition regarding students' skills, disposition, knowledge and understanding. They gather evidence, for example, to evaluate whether students correctly interpret the parameters of the task, possess the prerequisite knowledge, and are ready to engage in tasks on their own. Opportunities to gather evidence are opened when student solutions are supported by thorough explanations. When criteria are established, both students and teacher can work toward high quality verbal and written responses. Developing a classroom environment conducive to discourse-based assessment is not a trivial endeavor. The assessment evidence sought by a teacher depends on variables such as the tasks a teacher selects and the rapport between teacher and students. In order for valid evidence to be gathered, the learning environment must be non-threatening and conducive to sharing, attentive listening, and valuing others' perspectives (Graue & Smith, 1996; Lampert, 1990; Turner, Meyer, & Cox, 1998). Effective classroom assessment practice requires a wide range of evidence that attends to both process (reasoning and communication) and product (solutions). When teachers give attention to mathematical procedures without regard to how students have made sense of those procedures, opportunities to counteract misconceptions are limited. On the other hand, teachers who exhibit exemplary classroom practice create and seek opportunities to assess student learning beyond paper-and-pencil assessment, recognizing that multiple sources of evidence are embedded in instruction (NCTM, 1995; Shafer, 1996; Shafer & Romberg, 1999; de Lange, 1999). Informal options (such as class work and homework) as well as instruction-based (or interactive) options are used regularly.

Feedback, whether oral or written, given to students has a direct impact on their learning. The purpose of feedback is to develop students' ability to assess their own work and recognize it as a satisfactory (or exemplary) representation of mathematics. One of the underlying objectives of feedback is to allow multiple opportunities for students to compare their work to teacher-presented exemplars and performance criteria. According to Wiggins (1998), feedback should "provide students with training in how to evaluate (and score) the work they must eventually produce" (p. 329). Substantive feedback provides students with information to improve their responses to mathematical tasks and their articulation of mathematical principles. Feedback is most effective when it is "highly specific, directly revealing or highly descriptive of what actually resulted, clear to the performer, and available or offered through specific targets and standards" (Wiggins, 1998, p. 46). When performance criteria and exemplars are provided, students have a greater opportunity to internalize standards of quality for responding to mathematical tasks. Students should be given the opportunity to share and critique explanations, arguments, strategies and responses.

Teachers who have limited understanding of mathematics may struggle with providing feedback of mathematical substance. Although teachers can learn through student interpretations of problems, feedback needs to be provided at some point to further develop, connect, and extend student responses. If feedback and guidance for particular responses lack mathematical substance, students may focus on the superficial features of their responses rather than on the mathematics. On these occasions, teachers offer limited feedback such as whether an answer is correct or incorrect or their guidance is directed toward organization and neatness. On the other hand, teachers exhibiting exemplary classroom practice include references to mathematical principles and concepts in their feedback to students. Exemplary classroom practice provides an assortment of feedback to allow students to think about potential connections and extensions among shared explanations, models used to support thinking, and solution strategies (NCTM, 1995, Bransford, Brown, & Cocking, 1999, Carpenter & Lehrer, 1999, de Lange, 1999).

In this study, three subcategories of teachers' *classroom assessment practice* were characterized: *evidence sought during classroom assessment; purpose and coherence of feedback given in response to classroom assessment;* and *content of feedback* provided in response to classroom assessment. The highest rating for the index for each subcategory of *classroom assessment practice* is briefly described and illustrated with qualitative data. Indices are found in the appendix to this chapter. For descriptions of the level assigned for each teacher on each index, see Shafer, Marten, Webb, Folgert, & Davis (2003) and Shafer, Marten, Folgert, & Kwako (2003a, b).

Evidence Sought

The ways in which teachers sought evidence for student learning with understanding were coded according to the type of *evidence sought* (process and explanation vs. products and answers) and the opportunities utilized to gather evidence (whether through homework, classwork, or discourse). The highest level was assigned when the teacher viewed student explanations as evidence of learning. Although the teacher sought both process and product as evidence, answers, solutions, and procedures were recognized as inadequate forms of demonstrating understanding. Rather, the teacher actively sought demonstration of student learning through verbal or written communication of process. Interaction during instruction was used as an opportunity to gather evidence of student learning. Teachers at this level regularly elicited student ideas and used misconceptions to guide instruction. For example, Murphy used questioning techniques to gather evidence about student learning:

> I question a lot. I walk around the room questioning. I at least try to encourage a response from all of them, especially the ones who cannot write about it. They can talk to me and tell me. That's how I know that they know. (Fifth grade, MiC, Murphy, Interview 3/24/98)

Murphy created opportunities during instruction to elicit ideas about student understanding of the mathematics. She focused her assessment practice on verbal and written communication of process.

Feedback Coherence and Purpose

This index described the method and goal orientation of feedback that was available to students. The highest level was assigned when the process and criteria used by the teacher to evaluate mathematical work was revealed to students, and students were invited to assess their own work and work of others. The teacher provided opportunities for students to participate in the creation or modification of performance standards and revise more complex assignments that require elaborated communication such as projects, reports, and written explanations. For example, in LaSalle's class, students worked toward making sense of the mathematics, and they often provided feedback to each other. LaSalle explained:

> They check with each other. When they work in small groups, I witness that. "Well, what did you get?" Sometimes they will explain their answers to each other. They'll say, "How did you get that? Well, I did this and then this." So they do some counseling together. (Fifth grade, MiC, LaSalle, Interview 4/21/98)

In this class, students frequently provided feedback to each other as they worked on assignments. This feedback was not limited to answers. Strategies were also shared and considered by others.

Content of Feedback

The index for *content of feedback* described the content of feedback provided to students from teachers and students. The content of feedback was coded on the basis of the following questions: Did the feedback and guidance offered to students address mathematics, context, or general work habits? Did the mathematics content address skill and procedures or concepts? Were metacognitive strategies offered to assist students? Were exemplars of expected performance given? The highest level was assigned when feedback was directed toward conceptual understanding. Student misconceptions were addressed through probing questions, counterexamples, or alternative representations. Interactive verbal discourse was characterized by substantive discussions of mathematics. Feedback related to procedures and skills was used to prompt students to consider sense making over recall. For example, on difficult tasks Murphy worked with her students toward conceptual understanding, as noted in a journal entry:

> On February 23, we discussed the graph on p. 18. Students felt the graph was slightly misleading. They could easily see that the graph was divided into four parts. They could also see that the section News and Information was the largest. But it was difficult to see that Movies and Entertainment [represented] 1/6 of the 100 people surveyed. So we made the graph together, and I was able to emphasize how the graph was an estimate, not an exact picture. We were able to discuss degrees of the graph [measures of the central angle in each sector] and compare an exact picture with an estimate. (Fifth grade, MiC, Murphy, Journal entry 2/28/98)

In this situation, Murphy's feedback was directed toward conceptual understanding of the mathematics. She assessed student difficulties and made the decisions to emphasize the difference between an estimate and an exact picture of the data. Murphy's feedback was focused on making sense of this graphical representation of survey data.

Student Pursuits

In this study, three subcategories of *student pursuits* were characterized: *the nature of student–student conversation; the nature of student collaborative working relationships;* and *the overall level of student engagement*. The highest rating for the index for each subcategory of student pursuits is briefly described and illustrated with qualitative data. Indices are found in the appendix to this chapter. For descriptions of the level assigned for each teacher on each index, see Shafer, Marten, Webb, Folgert, & Davis (2003) and Shafer, Marten, Folgert, & Kwako (2003a, b).

Nature of Student–Student Conversation

The index for *student–student conversation* measured the extent to which student exchanges with peers reflected substantive conversation of mathematical ideas. The highest rating was assigned when conversation among students was substantive and characterized by reciprocal interaction that involved careful listening to others' ideas in order to understand those ideas, build conversation around them, or extend them to a new level. For example, "Group members were working well when deciding how to tally data, display the data, and what conclusions to present to the class" (Sixth-grade, conventional curriculum, Krittendon, Observation 10/23/97). During this class period, students actively participated in discussion about conducting a survey on a particular topic, how to display the data in a graphical representation, and the conclusions they would share in a class presentation. They needed to listen to one another, consider different perspectives, and formulate a group plan in each phase of conducting and presenting survey results.

Students' Collaborative Working Relationships

The collaborative nature of the classroom can be thought of as students working together, exchanging ideas, and finding solutions to the same problem. This includes providing assistance to one another, making sure that everyone understands and is working on the same problem, exchanging ideas, and seeking help from each other when it is needed. Student collaboration can occur in a small-group or large-group setting. The highest rating was assigned when most students were involved with their classmates in solving problems and made sure that other group members were caught up and understood the problems before moving on to the next problem. For example, in Redling's class, the observer noted: "All groups worked cooperatively to solve the problem and to display their solution for their presentation. In all presentations, all members of the group stood up together to present their solutions" (Sixth grade, MiC, Redling, Observation 3/23/99). On this occasion, students worked in small groups on a given assignment, which involved a presentation of group solutions to the whole class. Students in each group worked collaboratively on the task, and all group members were available for the presentation of the solution.

Overall Level of Student Engagement during Lessons

Ideas for the index for *level of student engagement during lessons* were drawn from the index created by Secada & Byrd (1993). The index measured the extent to which students remained on task during the lesson. Engagement was exemplified by behaviors in which students were attentive, completed assigned work, participated by raising questions, contributed to both large-group and small-group discussions, and helped their peers. The highest rating was assigned when engagement in the lesson was widespread. Most students were on task pursuing the substance of the lesson most of the time. Most seemed to take the work seriously and put forth much effort. For example, the observer noted: "Students were on task. They readily volunteered to put up solutions on the chalkboard and read out answers" (Sixth grade, MiC, Davenport, Observation 1/16/98). In another class, the observer noted: "All students were taking notes, copying down what the teacher wrote on the overhead projector. They did occasionally asked questions. They were attentive and on task" (Sixth grade, conventional curriculum, Friedman, Observation 2/24/99). During these classes, the level of student engagement was high. In the Davenport's MiC class, students were actively engaged in volunteering solutions and reporting answers. In Friedman's class using a conventional curriculum, students listened to their teacher and actively took down the notes she presented. In both cases, students were engaged in activities related to the mathematics content, and they took their work seriously.

The Composite Index Instruction

Although teachers in all four research sites completed interviews, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index for instruction, therefore, was created only for teachers in Districts 1 and 2 for whom there was a complete set of ratings on all 19 indices. Thirty-four teachers were involved in the analysis for the first year of data collection, 32 teachers in the second year, and 17 teachers in the third year. Some teachers were in the study multiple years. For example, Broughton, Dillard, and St. James participated in the study during all three years of data collection.

The composite index *Instruction* was created in a multiple-step process. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the Instruction Total.[1] Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subcategories and the Instruction Total (see Table 1-3).

[1] The sum of the weighted results was taken as a measure of the quality of instruction. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of instruction.

Table 1-3.

Correlation between the Instruction Total and the Subcategories of Instruction

Subcategory	Unit Planning			Lesson Planning				Mathematical Interaction						Classroom Assessment			Student Pursuits		
	SPK	US	PI	SPPL	PL	FIPD	SAPD	LPD	NI	ID	SE	MS	LCS	ES	FCP	FC	SC	SWR	OSE
SPK																			
US	0.092																		
PI	0.321**	0.017																	
SPPL	0.164	0.295**	0.051																
PL	0.136	-0.026	0.147	0.409***															
FIFD	0.038	-0.077	0.229*	0.177	0.326**														
SAPD	0.016	-0.181	0.134	0.118	0.474***	0.640***													
LPD	-0.023	0.104	0.176	0.157	0.312**	0.537***	0.546***												
NI	-0.026	-0.116	0.208	0.217*	0.506***	0.665***	0.549***	0.647***											
ID	-0.033	-0.015	0.285**	0.099	0.309**	0.568***	0.527***	0.676***	0.702***										
SE	-0.008	-0.072	0.141	0.175	0.260*	0.497***	0.468***	0.559***	0.692***	0.660***									
MS	-0.088	-0.060	0.048	0.025	0.227*	0.523***	0.426***	0.552***	0.635***	0.619***	0.788***								
LCS	-0.032	0.000	0.084	0.172	0.291**	0.476***	0.534***	0.556***	0.476***	0.495***	0.445***	0.471***							
ES	0.005	0.000	0.015	0.152	0.257*	0.408***	0.457***	0.650***	0.573***	0.602***	0.635***	0.599***	0.537***						
FCP	-0.036	0.055	0.108	0.079	0.266*	0.442***	0.423***	0.570***	0.520***	0.532***	0.561***	0.523***	0.480***	0.791***					
FC	-0.136	0.030	0.142	0.159	0.228*	0.398***	0.401***	0.624***	0.547***	0.613***	0.551***	0.469***	0.464***	0.695***	0.753***				
SC	0.015	-0.002	0.101	0.189	0.375**	0.493***	0.425***	0.436***	0.624***	0.532***	0.506***	0.475***	0.409***	0.483***	0.503***	0.578***			
SWR	0.073	0.008	0.063	0.109	0.336**	0.437***	0.422***	0.369***	0.532***	0.487***	0.521***	0.480***	0.385***	0.450***	0.486***	0.492***	0.845***		
OSE	-0.049	0.015	0.082	0.220	0.272*	0.322**	0.336**	0.525***	0.562***	0.512***	0.623***	0.572***	0.364***	0.548***	0.489***	0.518***	0.592***	0.593***	
Instr. Total	0.089	0.081	0.272*	0.309**	0.501**	0.697***	0.665***	0.801***	0.833***	0.803***	0.781***	0.721***	0.654***	0.787***	0.748***	0.751***	0.723***	0.667***	0.688***

*p<.05
**p<.01
***p<.001

Key

SPK--Consideration of Students' Prior Knowledge
US--Unit Sequence
PI--Pace of Instruction
SPPL--Students' Performance in the Previous Lesson
PL--Purpose of the Lesson
FIFD--Forms of Instruction That Promote Classroom Discourse for the Purpose of the Lesson
SAPD--Student Activities That Promote Discussion, Problem Solving, and Reflection on the Content of the Lesson
LPD--Lesson Presentation and Development
NI--Nature of Inquiry
ID--Teachers' Interactive Decisions
SE--Nature of Student Explanations
MS--Elicitation of Multiple Strategies
LCS--Lesson Closure, Reflection, or Summary
ES--Evidence Sought
FCP--Feedback Coherence and Purpose
FC--Content of Feedback
SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction
Instr. Total--Instruction Total

Five subcategories were not well correlated with the Instruction Total and other subcategories: *consideration of students' prior knowledge; unit sequence; pace of instruction; students' performance in the previous lesson;* and *the purpose of the lesson*. To verify these results, a principle component factor analysis was completed using SAS. Factors 1 and 2 accounted for a significant amount of the variance among the subcategories. Fourteen subcategories were included in Factors 1 and 2 (see Table 1-4). The five subcategories that had weak correlations to the Instruction Total were not influential in Factors 1 and 2 and were important only in the composition of other factors. Consequently, these subcategories were excluded from the analysis. The Instruction Total for each teacher was then recalculated.

Table 1-4.
Contribution of Subcategories to Principle Component Factors

Subcategory	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
SPK	-18	11	16	72*	9
US	4	-5	7	18	83*
PI	21	-5	-4	85*	7
SPPL	5	9	75*	0	46*
PL	19	21	78*	11	-12
FIFD	60*	15	33	23	-32
SAPD	59*	9	44*	9	-41*
LPD	83*	10	17	7	6
NI	65*	39	33	12	-23
ID	76*	28	8	19	-11
SE	68*	45*	5	5	-10
MS	68*	39	-2	-3	-17
LCS	68*	6	29	-4	-5
ES	79*	30	4	-10	12
FCP	75*	332	-1	-4	15
FC	74*	35	0	-8	17
SC	36	80*	22	6	-6
SWR	29	85*	15	8	-8
OSE	46*	65*	8	-5	9

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

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SC--Nature of Student–Student Conversation
SWR--Students' Collaborative Working Relationships
OSE--Overall Student Engagement during Instruction

Using the revised Instruction Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into six groups. For each group of teachers, common characteristics from the subcategories of instruction were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each subcategory of instruction. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. Similar to the index for each subcategory, the underlying single dimension of the composite index was teaching mathematics for understanding. The levels of the composite index *Instruction* were on a continuum from least to most reflective of teaching mathematics for understanding. The six levels are summarized in Table 1-5.

Level 6: Most reflective of teaching for understanding
Level 5: Reflective of teaching for understanding
Level 4: Attempted to teach mathematics for understanding
Level 3: Limited attention to conceptual understanding
Level 2: Focus on procedures
Level 1: Underdeveloped lessons.

Table 1-5.
Summary of the Levels of the Composite Index for Instruction

Level 6: Most Reflective of Teaching for Understanding	Level 5: Reflective of Teaching for Understanding	Level 4: Attempt to Teach for Conceptual Understanding
Mathematical Interaction Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students with teacher support • Discussion of solutions, generalizations, connections Interactive decisions • Predominantly aligned with understanding • Frequent questions on articulation of thinking, understanding mathematics, or reasonable solutions *Classroom Assessment Practice* • Attention to mathematical processes • Ongoing, purposeful feedback from teacher, students • Feedback: making sense of mathematics, solutions • Student assessment of own work and others' work *Student Pursuits* • Occasional substantive conversation • Student-student conversation about procedures *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Emphasis on conceptual understanding • Active participation by students and teacher • Discussion of solutions Interactive decisions • Attentive to teaching for understanding • Teacher explanations promote connections *Classroom Assessment Practice* • Student explanations as evidence of mathematical processes *or* procedural understanding • Feedback consistent with Level 6 *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Student discussion, problem solving, reflection planned	*Mathematical Interaction* Inquiry and lesson presentation • Attempt for conceptual understanding, but focus on procedural understanding • General acceptance of teacher's procedures Interactive decisions • More attentive to good standard pedagogy • Additional exercises, mini-lessons, contexts, review *Classroom Assessment Practice* • Evidence from student explanations • Focus on procedural understanding • Teacher feedback related to concepts, contexts • Student-student feedback: answers, procedures *Student Pursuits* • Engagement mildly enthusiastic, teacher encouraged *Lesson Planning* • Student discussion, problem solving, planned

Table 1-5 (continued).
Summary of the Levels of the Composite Index for Instruction

Level 3: Limited Attention to Conceptual Understanding	**Level 2: Focus on Procedures**	**Level 1: Underdeveloped Lessons**
Mathematical Interaction Inquiry and lesson presentation • Students use invented or demonstrated strategies • Student explanations focused on procedures Interactive decisions • More reflective of good standard pedagogy • Some attention to articulation of thinking, reasonable solutions • Occasional addition of different context or review *Classroom Assessment Practice* • Evidence from homework, classwork, occasionally student explanations • Teacher feedback: concepts, contexts, *or* procedures, answer format • Student-student feedback: answers *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion of vocabulary, steps in procedures planned, not elaboration of thinking	*Mathematical Interaction* Inquiry and lesson presentation • Predominantly lower order thinking • Students expected to use demonstrated procedures Interactive decisions • Predominantly least aligned with understanding • Limited changes in response to student difficulties, misunderstanding *Classroom Assessment Practice* • Evidence from homework, classwork • Emphasis on procedures, format of answers • Teacher feedback indirectly responsive to students, inattentive to student misconceptions • Student-student feedback: minimal *Student Pursuits* • Student-student conversation limited, answers shared *Lesson Planning* • Discussion anticipated but not planned	*Mathematical Interaction* Inquiry and lesson presentation • No formal lesson presentation • Procedures demonstrated to individual students • Student dependence on teacher for mathematical work • Frequent confusion or misunderstanding Interactive decisions • Least likely to support teaching for understanding • Teacher explanations preferred, no changes to address student needs *Classroom Assessment Practice* • Teacher feedback inattentive to student misconceptions, misleading, lacked mathematical substance • Student-student feedback: nonexistent *Student Pursuits*: Conversation not encouraged *Lesson Planning* • Student discussion, problem solving not considered

Each of the six levels of the composite index *Instruction* is described below and is illustrated with information from classroom observations, postobservation interviews, and teacher journal entries.

Level 6: Most Reflective of Teaching for Understanding

At Level 6, lessons emphasized conceptual understanding, and students actively participated in lessons with the support of their teacher. Lesson presentations set the stage for students to explore the mathematical content on their own. Students explored the mathematical content in enough detail to think about relationships among mathematical ideas or to link procedural and conceptual knowledge. Students were encouraged to make generalizations about mathematical ideas, and they presented and compared solutions and generalizations during whole-class discussions. Teachers' interactive decisions were predominantly aligned with teaching for understanding. Teachers frequently asked students to articulate their thinking, discuss differences in solutions and reasonableness of answers, and reflect on the mathematics in a particular lesson and across lessons. Teachers sought evidence of student learning through mathematical processes (e.g., reasoning expressed) and procedural competence. They used students' misconceptions to guide instruction by using probing questions, counterexamples, and alternative representations. Feedback promoted making sense of mathematics and mathematical conventions.

The description of LaSalle's class, which was used earlier in this chapter to illustrate the highest level for lesson presentation and development, also exemplifies teaching at Level 6. LaSalle introduced activities but allowed students to solve the problems on their own. She used a demonstration with soda cans to provide meaning for the context and allowed students to pursue a solution to each problem. In this way, LaSalle did not reduce the mathematical work for the students. They had the opportunity to develop their own solutions, record them, and present them to the class. LaSalle provided visual clues (measuring and combining liquid in soda cans), but she did not interject comments that might reduce students' cognitive activity on problems done by student groups. She worked toward students' developing conceptual understanding of the mathematics by allowing students to explore the activities in detail and talk about the relationships they found. The lesson promoted connections among mathematical ideas and representations (measuring sticks to identify quantities of oil, representations of the quantities using equivalent fractions or rectangular models), and connections between mathematics and students' life experiences (combining amounts of soda from two cans) were discussed. LaSalle viewed student explanations as evidence of their learning. She sought both verbal and written communication of process. When solving the problems, students elaborated on their thinking in both oral and written forms. Students worked toward sense-making and often provided feedback to each other. They assessed their own work and the work of others. Multiple solution strategies were encouraged and valued. Generally, student exchanges with peers reflected substantive conversation, as described by the observer: "This group is very strong when working with their peers. They discuss the mathematics and work together to arrive at a solution." With respect to students' collaborative working relationships, the observer noted: "Some groups have dominant members that at times roll over the others. But for the most part, they discuss strategies and proceed together towards a solution." Students were on task pursuing the substance of the lesson. They took the work seriously and put forth much effort: "Students were very involved in the lesson, and they were motivated to perform well."

Level 5: Reflective of Teaching Mathematics for Understanding

At Level 5, lessons emphasized conceptual understanding of the mathematical content, and students and their teachers actively participated in lessons. Student conjectures were characterized by investigating the veracity of particular statements. Teachers and students discussed connections among mathematical ideas, or teachers clearly explained such connections. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Teachers' interactive decisions were characterized by less attention to teaching for understanding than at Level 6. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools such as fraction bars and focused students' attention on pertinent elements of tasks. Their questioning techniques focused on articulation of student thinking, understanding mathematics, or reasonableness of solutions. They added a different context, a review lesson, or explanations that promoted connections among mathematical ideas or between mathematics and students' lives. Teachers were more likely to modify a lesson based on a student's statement or inquiry than at Level 4.

Teachers' classroom assessment practices were characterized by accepting students' explanations as evidence of their learning. Some teachers orchestrated discussions around students' solutions to learn about their procedural competence. Other teachers sought evidence of student learning through students' verbal or written communication of mathematical processes. These teachers used students' misconceptions to guide instruction through the use of probing questions, counterexamples, and alternative representations. Feedback was an ongoing, purposeful, and shared responsibility of both teachers and students. Feedback promoted making sense of mathematical tasks, students' solutions, and mathematical conventions.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Occasionally, students discussed procedures or asked each other for clarification of procedures demonstrated by their teachers. Although seated in groups, students generally worked at different paces on the assigned work. Most students were on task, seriously pursuing the substance of the lesson.

A lesson presented by Nash (Observation, 3/25/98) from a class using the MiC fifth-grade statistics unit *Picturing Numbers* (Boswinkel et al., 1997) is used to illustrate Level 5 of the composite index for instruction. In the lesson (pp. 31–33), students interpreted and created bar graphs for categorical data and line graphs for changes over time using information situated in realistic contexts. Content of previous lessons in the unit included interpretation and construction of bar graphs, stacked bar graphs, pie charts, pictographs, and number line plots; conclusions about data sets; and arguments using data and graphs.

The lesson began with a review of the homework assignment, a weather pictograph for the month of April from a supplementary resource. Students exchanged papers for grading, and they volunteered answers. When offering responses to the question, "On how many days were there clouds?," many students had not included the days designated with thunderstorms as days with clouds. At that point, Nash led a discussion on reading data *and* using inferences from the data. Her guidance encouraged students to critically analyze the provided information, which nurtured the development of a critical attitude toward investigation of data sets. Nash used student inquiries as a guide to shape the mathematical content of the lesson. For example, when checking homework, a student asked if an answer was wrong if the fraction was not simplified correctly (something like 12/30 = 3/5). Nash used this question to explain that, although fractions are usually simplified, 12/30 was really a better answer because April has 30 days and the weather chart showed 12 cloudy days. So the student decided to give credit for the response. In this interaction, Nash's explanation encouraged students to think about numbers in relation to the context. The focus changed from performing a rote procedure after writing an expression with fractions to clearer communication of the situation being analyzed.

Nash introduced the next section of the unit by asking students a question about the context, "What does Bicentennial Celebration mean to you?" Students volunteered to read the introductory paragraphs about a concert being planned for the celebration. Nash then asked students questions about the bar graph of favorite performers of students in New England, which was displayed on p. 31 (e.g., labels on the vertical and horizontal axes, title). Question 1, which had two parts, was discussed orally in class. When students compared the most popular performer with other performers, Nash used questioning techniques that promoted thinking. She helped students refine their responses and kept probing until more types of comparisons were offered by students. In this way, Nash continually supported higher order thinking by expecting meaningful and varied explanations. She pressed students to effectively communicate their thinking and compare various answers. Nash brought to the attention of the class that in the past students had answered the first part of the question, but not the second part. She reiterated her expectation for students to write answers to both parts of the question. Data collected by students prior to the lesson were distributed for question 4 (favorite musical performers of classmates) and question 7 (hourly temperatures outside the school building recorded during the previous school day). Nash assigned questions 1–12 for group work.

Inquiry during this class emphasized conceptual understanding of the mathematical content. Nash continuously required students to make sense of the data. Students' conjectures consisted mainly of making connections between a new problem and problems previously seen. Students applied their knowledge about bar graphs in order to answer the questions. Connections between data tables and bar graphs and between bar graphs and verbal interpretations of data were very evident during class. Ms. Nash clearly explained these connections or elicited explanations from students. Connections between mathematics and students' daily lives were clearly apparent in the lesson. Students related to the context of favorite musical performers and to the temperature data they collected. The use of multiple strategies was not a primary goal of instruction, although student graphs did vary. The observer noted:

> [Ms. Nash] pointed out that in question 4, students were not restricted to making a bar graph. They could select any type of graph. I noticed that most students used bar graphs, but some used pie graphs and pictographs.

Student conversation in small groups focused on the collected data, and interactions among the students reflected collaborative working relationships. Students seemed to take the work seriously and were trying hard. The observer noted: "Students were motivated. They were knowledgeable about making graphs, and they didn't have to be prodded to begin. They knew what to do."

This lesson is reflective of instruction at Level 5. The lesson was not teacher centered. Nash took an active role in participating in the mathematical activity with her students, and she seized opportunities to enhance students' understanding of mathematics. For example, when she realized students had not included stormy days as cloudy days, Nash pointed out the importance of reading and making inferences from data. In this way, she promoted students' development of a critical attitude in statistical analysis. Her questions and comments supported higher order thinking by encouraging active listening, reflection, and comparison.

Level 4: Attempt to Teach Mathematics for Understanding.

At Level 4, teachers attempted to teach for conceptual understanding, but the primary focus of lessons was on building students' procedural understanding. On some occasions, students were asked if different strategies were used in solving particular problems, but this was not a primary goal of instruction. Although students were allowed to find their own solution strategies, they generally used a procedure or strategy presented by the teacher. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Interactive decisions were characterized by greater attention to standard pedagogy and teaching for understanding than at Level 3. Teachers emphasized understanding of mathematical vocabulary or correct use of mathematical tools, included additional exercises based on student interest or need for practice, or focused students' attention on pertinent elements of tasks. They added minilessons on algorithms or procedures, a different context, or a review lesson. Teachers also asked students about the reasonableness of their solutions. At times, interactive decisions involved the introduction of alternative strategies by teachers or students.

Teachers' classroom assessment practices were characterized by gathering evidence from student explanations as evidence of their skills and procedures. Teachers elicited students' solution strategies and orchestrated whole-class discussions around them, but the primary goal of their assessment practice remained demonstration of procedural competence. Student misconceptions were rarely used as opportunities for instruction. Feedback students received from other students rarely went beyond sharing answers and procedures. Teachers' feedback included attention to mathematical concepts or the contexts in which problems were situated.

Student–student conversation occurred on a limited basis and usually consisted of sharing answers. Although seated in groups, students worked at different paces on the assigned work. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Teachers' planning for teaching instructional units or chapters was consistent with Level 5.

A lesson presented by Heath (Observation, 10/20/97) from a class using the MiC seventh-grade algebra unit *Ups and Downs* (Abels, de Jong, Meyer, Shew, Burrill, & Simon, 1998) is used to illustrate Level 4 of the composite index for instruction. In the lesson (pp. 23–24), students used data in a table to sketch a periodic graph of tides in San Francisco over a period of three days and compared this graph with similar graphs of tides in The Netherlands.

At the beginning of the lesson, students completed the warm-up activity, which was a puzzle unrelated to lesson content. Heath then asked students to write their own definitions for periodic graph, cycle of a graph, and period of a graph. Their conjectures were based on problems about tides in the previous lesson. Heath introduced the lesson by using maps of Europe and the United States to show the bodies of water in The Netherlands and near San Francisco. Students worked independently on question 5. They sketched a graph of the water level at the Golden Gate Bridge over a period of three days using a table of information about the times and water levels at low and high tides. Heath allowed students to struggle with the graph, and many students were able to complete it. Because they completed the graphs independently, there were few opportunities for students to engage in discussions with their peers. The questions in the lesson were designed for students to talk about the mathematics, but students did not participate in the discussion. Heath explained the graphs on the overhead projector and described the changes in the water level (question 6). She did not elicit discussion with leading questions, nor did she ask students if their graphs were reasonable representations of the data in the table. Students participated in class activities, but they were mildly enthusiastic. Students compared the graph they made with graphs of tides in The Netherlands during a brief discussion. Heath introduced the homework assignment (p. 24, questions 8–10) in which students were asked to interpret a graph displaying the repeating pattern of temperature changes in an air-conditioned room. The concepts periodic graph, cycle of a graph, and period of a graph were explicitly explained on p. 24. Students copied the definitions and compared them to the definitions they had written at the beginning of class.

Heath then introduced an activity she designed in which students collected data on their pulse rates to graph during the next lesson. Students found their resting pulse rate and their pulse rates after multiple segments of activity, alternating running for 1 minute with sitting for 2 minutes. Heath provided the average pulse rate for children of their ages, explained that exercising the heart over time can lower the pulse rate, and talked about the effects of diet on the health of the heart. During this portion of the class period, students were very enthusiastic. Heath had created an activity that not only sparked students' interest, but developed a conceptual basis for a periodic graph of data that was connected to the students' life experiences. In the postobservation interview, Heath stated that her goals for the lesson were met, and she felt students "had a feel for periodic graphs."

This lesson is reflective of instruction at Level 4 because the teacher attempted to teach mathematics for conceptual understanding. The first part of the lesson was more teacher directed and involved less student participation in discussion than at level 5 (in which students and their teacher were actively involved in the lesson) and level 6 (in which the teacher set the stage for students to do the mathematical work on their own). Heath allowed students to complete the graphs but did not elicit students' discussion of the graphs. In the second part of the lesson, however, students were excited about collecting their own data. The future graphing of their pulse rates had the potential to forge some powerful connections between students' own activity and their understanding of the nature of periodic graphs. Heath's desire to teach mathematics for understanding was apparent in creating opportunities for students to explore periodic functions using both contexts presented in the unit and a context she designed for the class.

Level 3. Limited Attention to Conceptual Understanding.

At Level 3, inquiry during class provided limited attention to conceptual understanding of the mathematical content. The main focus of lessons was on building students' procedural understanding. Student conjectures involved making connections between a new problem and problems previously seen or investigating the veracity of particular statements. During some lessons, connections among mathematical ideas and between mathematics and students' lives were apparent, but they were not explained by the teacher or discussed by the students. Students' explanations were focused on procedures used to determine answers rather than on elaboration of reasoning or solution strategies. Teachers' interactive decisions were more reflective of traditional pedagogy than at previous levels. Teachers emphasized understanding of mathematical vocabulary and at times used questioning techniques that focused on the reasonableness of solutions. At times, they added a different context or a review lesson. Teachers' classroom assessment practices were characterized by gathering evidence from homework and classwork to substantiate students' procedural competence. Occasionally, teachers included student explanations as evidence of their skills and procedures, but explanations were generally elicited to generate communication in class rather than as an avenue for assessment of student understanding or as a basis for substantive mathematical conversation. Teacher-directed feedback was indirectly responsive to student needs in that it involved additional whole-class instruction using the same presentation method or the assignment of additional exercises of the same type. For some teachers, feedback was directed toward correct procedures and the format of the answers such as simplifying fractions. The feedback of other teachers, however, included attention to mathematical concepts or the contexts in which problems were situated. Their feedback was mathematically sound and clear to students.

A lesson presented by Lee (Observation 11/12/97) from a class using the MiC sixth-grade algebra unit, *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole, & Burrill, 1998), illustrates Level 3 on the composite index for instruction. In the lesson, students were to investigate ways to give change when purchasing items; analyze an estimation strategy for giving change; read about a "counting on" strategy for making change; use the "small-coins-and-bills-first" method to solve problems; and use arrow language to record counting change. Lee began the lesson with checking the homework assignment, but discussion of the homework transpired at the end of class. The process was described by the observer:

> In the beginning of the period, when checking homework, [Ms. Lee] was only interested in correct answers. If students had correct answers, they were given a transparency so they could present the problem on the overhead projector at the end of the period. Students showed the arithmetic, not the why or any explanation of strategy, and forgot the context of the problem.

Following the initial homework review, Lee held a class discussion about cashiers in stores and making change. She asked whether the students paid attention to how a cashier counted change. The first lesson problem asked students to figure the correct change without using pencil and paper or a calculator, but Lee proceeded to teach them the standard algorithm. The observer noted:

> [Ms. Lee] went right to the algorithm, lined up the decimals, and borrowed. She also forced the students to use arrow language before they had an opportunity to analyze strategies for making change. Since students were directed to certain procedures, they did not have an opportunity to understand the concept. For most of the lesson, [Ms. Lee] directed the way students thought about the problem situation. She focused their attention on procedures. When students gave an answer, she responded either "correct" or "incorrect." There was no further discussion.

Student participation in the lesson varied. Some students were engaged in the classroom activities for most of the time, but most students were not. Students were initially confused about the task and expectations for group work. Most wasted about ten minutes before they got started. Students did not work collaboratively, as noted by the observer: “One student stated, ‘He won’t work with me. He goes ahead.’ Another student reported, ‘We can’t do this. We’ll do it for homework.’”

This lesson is reflective of instruction at Level 3 because limited attention was given to developing conceptual understanding of the mathematical content or linking conceptual and procedural knowledge. For most of the lesson, Lee directed the way students thought about the problem situation. She presented a procedure, and students followed it in a rote manner. No student conjectures were observed. Lee’s questioning focused on given procedures or eliciting correct responses, as evidenced in the homework review. Correct answers were valued to the extent that alternate strategies and supporting explanations were ignored and no further explanations addressed incorrect responses. Lee sought procedural competence as evidence of student learning. Feedback was teacher-directed and frequently occurred during whole-class discussion. However, feedback was often limited to checking whether answers for correct or incorrect. This lesson resembled conventional classroom instruction in mathematics, even when using MiC.

Level 2: Focus on Procedures.

At Level 2, the focus of lessons was on particular procedures or strategies, and students were expected to use the methods presented by their teachers. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students’ life experiences. Teachers’ interactive decisions were predominantly less aligned with teaching for understanding. Limited changes were made in instruction even when teachers or students experienced confusion or misunderstanding.

Teachers’ classroom assessment practices were characterized as gathering evidence from homework and classwork to determine students’ procedural competence. Teacher-directed feedback was indirectly responsive to student needs and was directed toward correct procedures and the format of the answers. Student–student conversation, students’ working relationships, students’ engagement in lessons, and teachers’ planning for teaching instructional units or chapters were generally consistent with Level 3.

A lesson presented by McLaughlin (Observation, 2/6/98) from a class using the seventh-grade conventional textbook *Mathematics: Applications and Connections* (Glencoe/McGraw-Hill, 1997) is used to illustrate Level 2 of the composite index for instruction. In the lesson (p. 319), students solved logic problems in which a matrix was used to record information given in the problem. The class began with a warm-up activity unrelated to the lesson, which included addition and subtraction of integers without context and asked for geometric terms, given their definitions. Students then gave answers to these questions, but their explanations focused on procedures rather than an elaboration on their thinking. The observer noted:

> For one of the warm-up questions –7 + –3, students answered 4 and –4. [Ms. McLaughlin] responded, ‘Who remembers the rules?’ Student explanations were reciting the rules.

In this situation, rules were emphasized, and a conceptual basis for operations with integers was not discussed in class. The instruction did not promote learning mathematics with understanding. McLaughlin then introduced logic problems on p. 319 in the textbook. She demonstrated how to construct a matrix to record the given information: By marking the information given in problem statements pertinent to each person in the problem, the solution became apparent. McLaughlin frequently asked students if they solved the logic problems using other strategies. Eventually, however, she led the class to use one specific strategy for solving the type of problem. The worksheet assigned for group work featured a different type of logic problem than examples and problems in the textbook. Assigned at the end of the class period, students were to complete the work on the next day. Limited discussion occurred during instruction. Most of that discussion was in the form of students responding to their teacher's questions. Students participated in class activities, but their involvement was sporadic and mildly enthusiastic. Inquiry during this lesson was limited to procedures, and the lesson did not promote conceptual understanding. Neither McLaughlin nor the students explored connections among mathematical ideas or between mathematics and students' life experiences.

This lesson is reflective of instruction at Level 2. The focus of the lesson was on particular rules and procedures, and although McLaughlin asked for alternative strategies, students were generally expected to use the method she presented. Limited changes were made in instruction even when student responses were unreasonable. For example, when students stated the results 4 and –4 in response to the warm-up question –7 + –3, McLaughlin requested the rules for addition of integers rather than providing an explanation based on a representation of the situation. Students had no other ways to think about integer operations other than by using the rules.

Level 1: Underdeveloped Lessons.

At Level 1, mathematics was presented in ways that gave students only a surface treatment of the content. Inquiry during class was limited to lower order thinking, and lessons did not promote conceptual understanding. Some teachers devoted a major portion of the class period to review of a previous lesson or homework. The subsequent lesson presentation was not well developed. Consequently, students began independent work or small-group work with little direction. Teachers assisted individuals or groups of students on a one-to-one basis during independent work or small-group work. Teachers presented particular procedures or strategies, and students were expected to use those methods. One of two situations prevailed: Students were unable to solve problems using the presented procedure or strategy, or students practiced the presented procedure or strategy in a rote fashion. Teachers and students did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences. Teachers' interactive decisions were least likely to support students' learning mathematics with understanding. Teachers' questions focused on following particular step-by-step procedures, and their own explanations were preferred over student explanations. No changes were made during instruction to address student questions, difficulties, or unexpected strategies. Teachers' classroom assessment practices were characterized by gathering evidence from homework and classwork to determine students' procedural competence. Teacher-directed feedback was indirectly responsive to student needs. Individualized feedback occurred as teachers responded to questions from specific students during independent seatwork. Feedback provided by students was minimal or nonexistent. For some teachers, feedback was not attentive to student misconceptions, was misleading, or lacked mathematical substance. For other teachers, feedback was directed toward correct procedures and the format of answers rather than to the development of mathematical understanding. Conversation among students was not encouraged. When students did talk with one another, the conversation was social in nature. Most students were engaged in the lesson, but their engagement was inconsistent, mildly enthusiastic, or dependent on encouragement from their teachers. Planning for teaching instructional units or chapters was consistent with Levels 2 and 3.

A lesson presented by Tallackson (Observation, 1/6/98) from a class using the sixth-grade conventional textbook *Mathematics: Applications and Connections* (Glencoe/McGraw-Hill, 1997) is used to illustrate Level 1 of the composite index for instruction. In the lesson (pp. 101–102), students studied addition and subtraction of decimals. At the beginning of class, ten minutes elapsed while students settled and got ready for the lesson. Tallackson initiated a review of the homework problems, pp. 101–102, odd-numbered exercises from 5–21. She asked a student to find the sum of 2.31 and 1.77 and to explain how she found the answer. The student responded with a step-by-step demonstration of the standard algorithm for addition of decimals. When another student queried, "What is a decimal?" Tallackson tried to explain the meaning of 0.08 by drawing a pizza on the board that had one hundred pieces and by shading 8 sections. She then went to the back of the room and pulled out a box of 4" by 4" cards called decimal squares. Each card had a square composed of 100 smaller squares, which were partially shaded to represent a decimal number. In front of the class, Tallackson held up one card at a time and asked a student to write the decimal that corresponded to the representation on the board. Conversations about mathematics were between the teacher and a few students. Many students conversed socially during the half hour devoted to this activity. Tallackson then continued the homework review by asking three students to demonstrate their work for exercises 5, 7, and 9. Reasonableness of responses was never discussed. For homework the class was to complete pp. 101–102, even-numbered exercises from 4–22. Students were directed to do the problems using standard algorithms. In the postobservation interview, Tallackson noted that she had not completed the homework review, and, because students did not understand the meaning of decimal, she "had to teach them."

This lesson is reflective of instruction at Level 1. The lesson provided minimal attention to conceptual understanding, and inquiry during instruction was limited to lower order thinking. The class period was devoted to review of homework, and the standard algorithm was used without meaning. Tallackson's interactive decisions were least aligned with teaching for understanding. For example, her response to a student's question about the meaning of decimals led to a lengthy activity in which decimal cards were used to provide representations of decimals. The decimal cards could have been used as part of a thorough explanation in which a conceptual basis for addition and subtraction of decimals was discussed. Even so, this activity might have been more suitable at the beginning of this chapter. During the lesson, many students were off task and were not engaged in the assigned work. In this instructional context, little mathematical evidence was available for her to assess student understanding. Feedback was minimal and lacked substance.

Results

Levels on the composite index *Instruction* spanned all six levels for teachers using MiC and teachers using conventional curricula.

Year 1

In the first year of data collection, the results for all 34 teachers for the instruction composite revealed differences by grade level, curriculum, and district. By grade level, more fifth-grade teachers were at Levels 4, 5, and 6, indicating that they attempted to teach or taught mathematics for understanding, than sixth- or seventh-grade teachers (see Figure 1a). More sixth-grade teachers were at Levels 1 and 2, indicating that lessons were underdeveloped or focused on procedures. An additional pattern of variation was found when levels of the instruction composite were reviewed by curriculum taught (see Figure 1b). Half of the teachers using MiC were at Levels 4, 5, and 6 in comparison to 2 of the 11 teachers using conventional curricula. Most teachers using conventional curricula were at Levels 1 or 2 compared with one-eighth of the MiC teachers.

When these results were reviewed by district, differences became apparent for MiC teachers (see Figures 1c and 1d). MiC teachers in District 2 were more likely to teach mathematics for understanding than teachers in District 1. These results raise questions about the differences in teacher professional background and professional development opportunities in the two districts. As explained earlier in this chapter, professional development to acquaint teachers of mathematics with reform-based curricula was offered in District 1, and monthly meetings were provided for teachers who were implementing such programs. The district mathematics specialist arranged focus group meetings for all teachers who were implementing reform curricula. Each month teachers explored general pedagogical issues including student-centered instruction, assessment, and use of mathematical tools such as the ratio table. These meetings were held after school hours, and teachers were compensated by the district for their participation. During subsequent years, however, the focus meetings were not held, which led to challenges for three MiC teachers who were new to the study (see Monograph 1, Chapter 4). In District 2, teachers had numerous possibilities for professional development. Each school was given six early-release days for general professional development. In addition, each school received 10 substitute days for professional development in mathematics and/or science, 12–18 days of in-service days in mathematics provided by (USI or Eisenhower) government funding (each involving 2–6 teachers), and 3–5 days of district wide mathematics in-service. Teachers also had opportunities to participate in five days of paid in-service for mathematics during the summer. During the second and third years of data collection, MiC teachers were given one day of release time per month in order to collaborate on planning to teach MiC units. Thus, opportunities for teachers to learn about MiC and methods of teaching mathematics in District 2 far surpassed the opportunities for District 1 teachers. The difference in professional development might account for the some of the differences noted among study teachers in these districts.

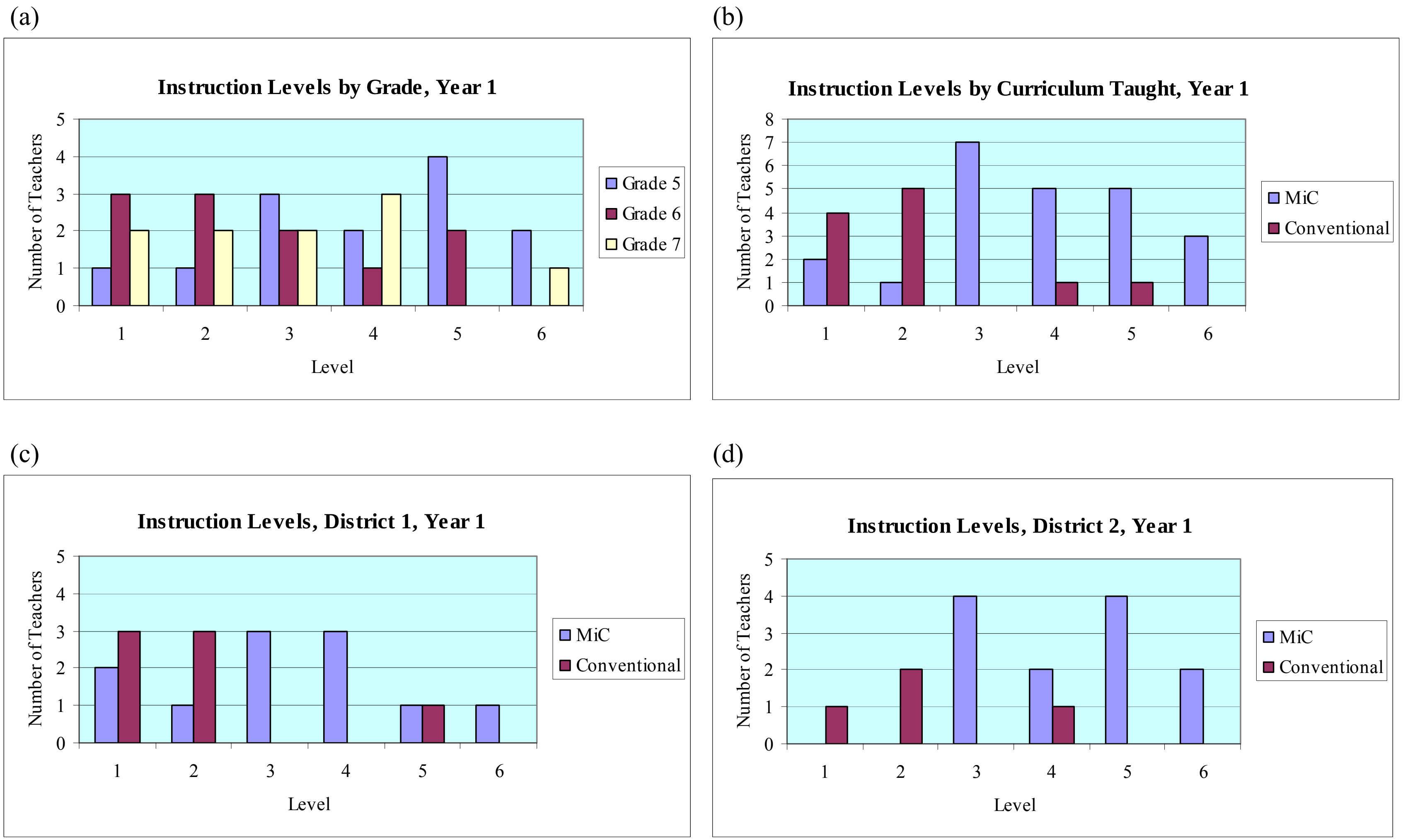
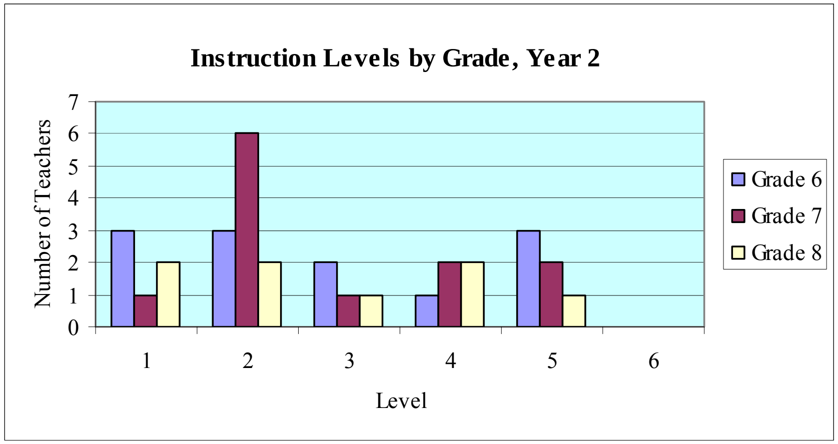


Figure 1-1. Teacher level on the *Instruction* composite index, Year 1: (a) by grade; (b) by curriculum taught; (c) by curriculum taught, District 1; (d) by curriculum taught, District 2

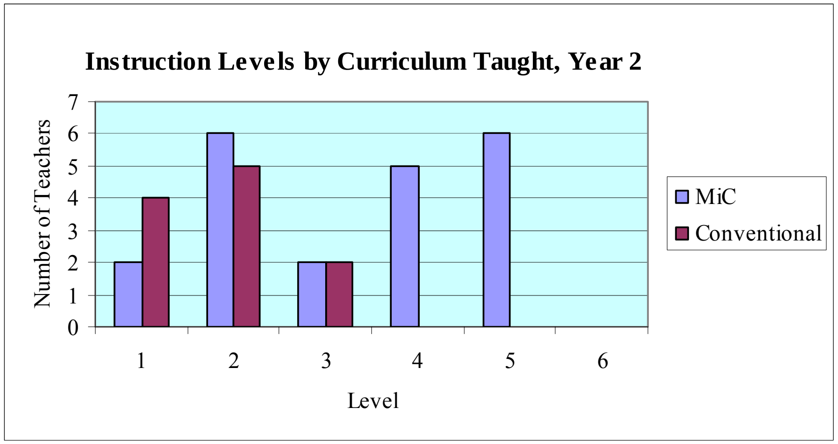
Year 2

In the second year of data collection, the results for all 32 teachers for the *Instruction* composite revealed differences by grade level, curriculum, and district. One-third of the teachers were at Levels 4 and 5, indicating that they attempted to or taught for understanding, but one-half of the teachers were at Levels 1 and 2, indicating that lessons were underdeveloped or focused on procedures, and no one was at Level 6 (see Figure 1-2a). The results for each grade level reflected the overall results. When reviewed by curriculum taught, half of the teachers using MiC and none of the teachers using conventional curricula were at Levels 4 and 5 (see Figure 1-2b). In contrast, 9 of the 11 teachers using conventional curricula were at Levels 1 or 2 compared with 8 of the 21 MiC teachers. When these results were reviewed by district, differences became apparent for MiC teachers. As in Year 1, MiC teachers in District 2 were more likely to teach mathematics for understanding than teachers in District 1 (see Figures 1-2c and 1-2d).

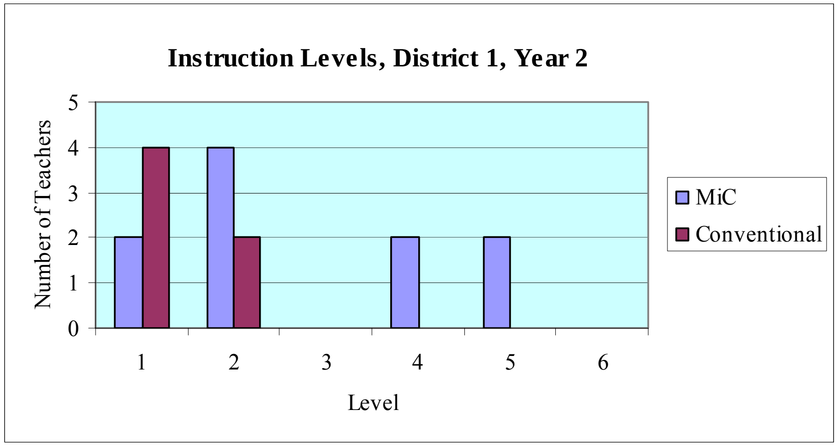
(a)



(b)



(c)



(d)

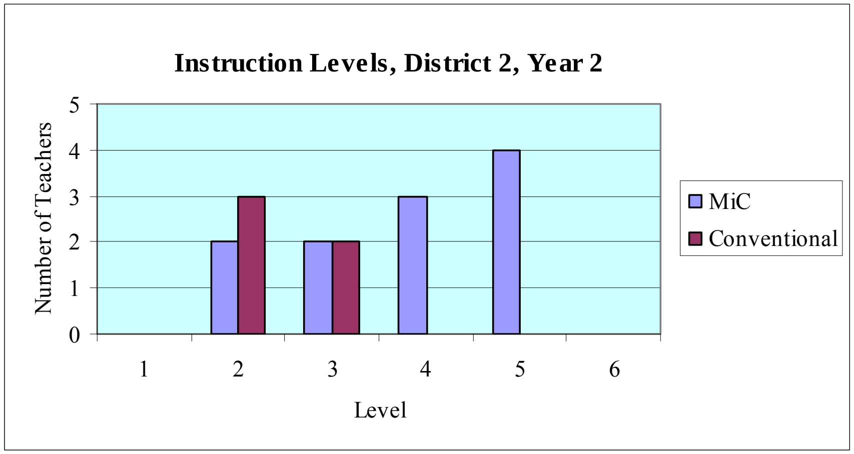


Figure 1-2. Teacher level on the *Instruction* composite index, Year 2: (a) by grade; (b) by curriculum taught; (c) by curriculum taught, District 1; (d) by curriculum taught, District 2

Year 3

The results for 17 teachers in the third year of data collection were similar to previous years. By grade level, 5 of the teachers (4 eighth-grade) were at Levels 4and 5, no one was at Level 6, and 7 of the 17 (3 seventh-grade) were at Levels 1 and 2 (see Figure 1-3a). By curriculum taught, 4 of the 13 teachers using MiC were at Levels 4 and 5, and 4 were at Levels 1 and 2 (see Figure 1-3b). Comparisons by curriculum taught are difficult to make in the third year of data collection due to a small sample size for teachers using conventional curricula (only 4 teachers), and in District 2, no teachers using a conventional curriculum were available in eighth grade. When analyzed by district, differences became apparent for MiC teachers. As in the other years, MiC teachers in District 2 were more likely to teach mathematics for understanding than MiC teachers in District 1 (see Figures 1-3c and 1-3d).

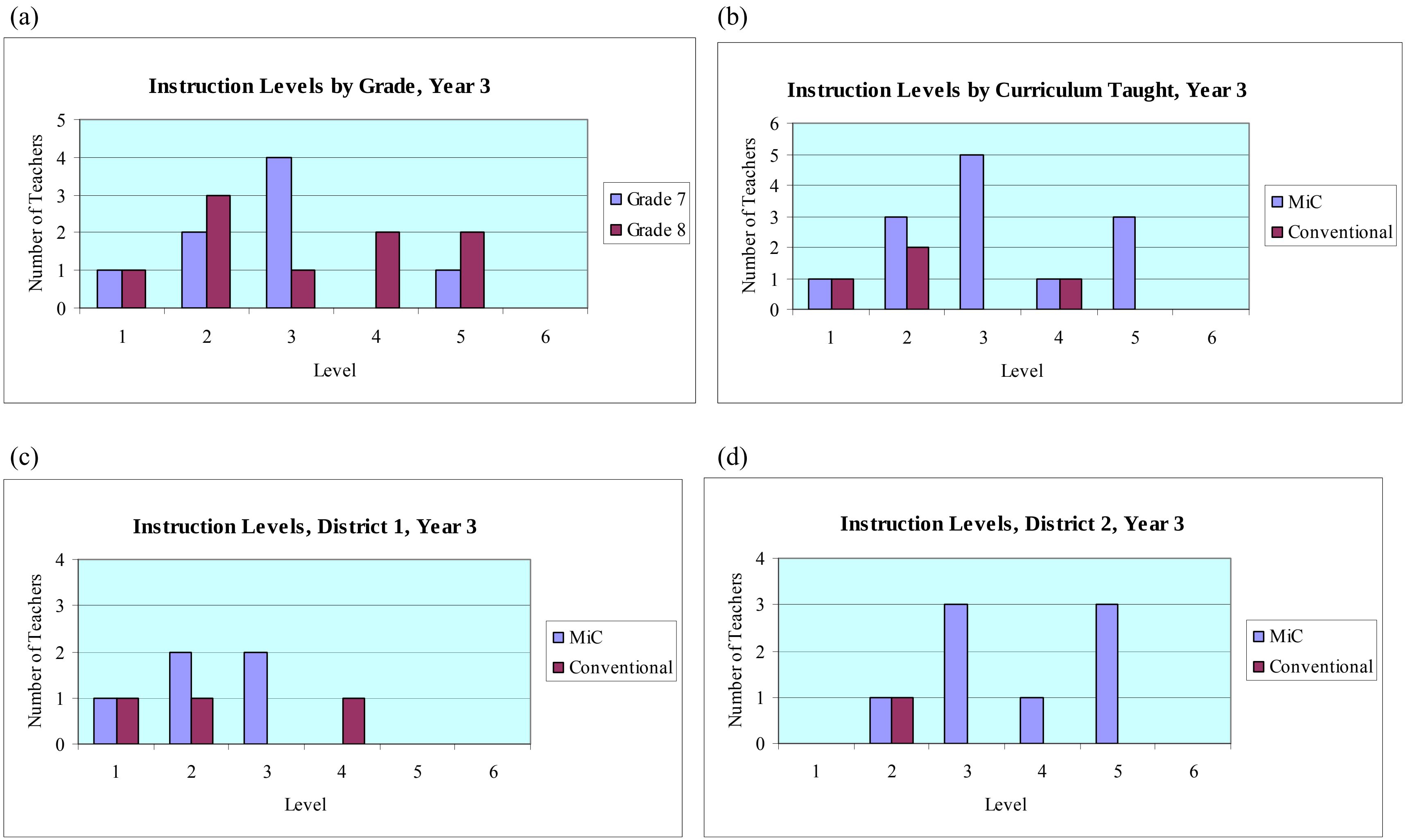


Figure 1-3. Teacher level on the *Instruction* composite index, Year 3: (a) by grade; (b) by curriculum taught; (c) by curriculum taught, District 1; (d) by curriculum taught, District 2

Analysis of Variance

Analysis of the variance was completed to check for significant differences among teachers by grade level, curriculum taught, and district. The Instruction Total was used as the dependent variable. Results suggest that 71% of the variance in the Instruction Total was accounted for by differences in grade level, curriculum taught, and district, and there was an effect for curriculum, grade, and district (see Table 1-6).

Table 1-6.
ANOVA with Instruction Total as the Dependent Variable

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	6425.05	1070.84	33.76	<.0001
Error	80	2537.28	31.71		
Corrected Total	86	8962.33			

R-Square	Coeff Var	Root MSE	OTLu Mean
0.71	15.47	5.63	36.38

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Curriculum Taught	2	4317.18	2158.59	68.06	<.0001
Grade Level	3	424.17	141.39	4.46	0.006
District	1	306.15	306.15	9.65	0.002

When the contrasts were examined (see Tables 1-7 and 1-8), there were significant differences between the means of teachers using MiC and teachers using conventional curricula, between Grade 5 and the other grade levels, and between districts. The results suggest that students experienced significantly different instruction when studying MiC than when studying conventional curricula, in fifth-grade classrooms than in middle-school classrooms, and in District 2 than in District 1.[2]

Table 1-7.
Least Squares Means for Curriculum Taught, Grade Level, and District

Least Squares Means	
Curriculum Taught	
MiC	40.61
Conventional	31.20
Grade Level	
Grade 5	41.86
Grade 6	35.07
Grade 7	36.46
Grade 8	35.44
District	
District 1	35.27
District 2	39.15

[2] Given the power of the statistical tests in this study, p-values <.10 are considered to imply important differences, and p-values <.05 are statistically significant differences (Cohen, 1988).

Table 1-8.
Contrasts in Means for the Instruction Total by Curriculum Taught, Grade, and District

Parameter	Estimate	Standard Error	t Value	Pr > |t|
		Curriculum Taught		
MiC vs Conventional	9.41	1.86	5.05	<.0001
		Grade Level		
Grade 5 vs Grade 6	6.79	1.98	3.43	0.001
Grade 5 vs Grade 7	5.40	1.90	2.83	0.005
Grade 5 vs Grade 8	6.42	2.05	3.13	0.002
Grade 6 vs Grade 7	-1.39	1.53	-0.91	0.364
Grade 6 vs Grade 8	-0.37	1.74	-0.22	0.830
Grade 7 vs Grade 8	1.02	1.65	0.62	0.538
		District		
District 1 vs District 2	-3.87	1.24	-3.11	0.002

Conclusion

Based on 19 subcategories of instruction, the composite index *Instruction* served as a useful tool in developing profiles of the instruction for each teacher. An extensive, diverse set of data from classroom observations, interviews, teaching logs, and journal entries was used to identify and scale variation in the instruction study students experienced. The results suggest significant differences between the instruction experienced by students in MiC classrooms and students who studied conventional curricula, between fifth-grade students and students in other grade levels, and between students in District 2 and students in District 1.

The descriptions of the instruction students experienced are one aspect of the eight grade-level-by-year studies described in Monograph 1, Chapter 2 and are used in Monographs 5, 6, 7 and 8 to further develop our understanding of the achievement results for each class of students. In the next two chapters, I examine in depth the opportunity students had to learn significant mathematics content with understanding and the capacity of their schools to support mathematics teaching and learning.

CHAPTER 2: OPPORTUNITY TO LEARN WITH UNDERSTANDING

Mary C. Shafer

In the structural research model for the longitudinal/cross-sectional study, opportunity to learn with understanding (OTLu) is described through independent and intervening variables (see introduction to this monograph). In the model, the independent variable *curricular content and materials* and the intervening variable *classroom events* are relevant in the analysis of OTLu. The variable *curricular content and materials* represents the teacher's decisions in defining the actual curriculum—the topics and instructional units or chapters covered, the supplementary materials used during instruction, and modifications of the intended curriculum. The variable *classroom events* represents the interactions among teacher and students that promote learning mathematics with understanding. These events arise from a learning environment in which students explore mathematics and are encouraged to make sense of mathematics.

In the simplified research function, variation in classroom achievement (CA) was hypothesized to be attributed to variations in prior achievement (PA), and method of instruction (I), and opportunity to learn with understanding (OTLu). Initially, OTLu included the capacity of schools to support high academic goals. However, as the research team worked with the data, a decision was made to distinguish school capacity (SC) from OTLu. Thus, the research function was expressed as—

$$CA = PA + I + OTLu + SC.$$

As conceived in this study, OTLu consists of three overarching categories: *curricular content*, *modification of curricular materials*, and *teaching for understanding* (see Table 2.1).

Table 2.1.
Characterization of the Composite Index for Opportunity to Learn with Understanding

Category	**Curricular Content**	**Modifications of Curricular Materials**	**Teaching Mathematics for Understanding**
Subcategory			• Development of conceptual understanding • Nature of student conjectures • Connections among mathematical ideas • Connections between mathematics and students' life experiences

In this characterization of OTLu, the category *teaching for understanding* consists of four subcategories: *the development of conceptual understanding, the nature of student conjectures about mathematical ideas, the nature of connections within mathematics*, and *the nature of connections between mathematics and students' daily lives*.

Theoretical Framework

Historically, opportunity to learn (OTL) was described in models of school learning (Carroll, 1963). Subsequently, in assessments of student achievement, OTL was operationalized as a measure for determining differences in content coverage across international samples of students (Gau, 1997). Porter (1991) described OTL as the enacted curriculum experienced by students, arguing that OTL involved more than the content of instruction (concepts, skills, and applications taught) and that instructional strategies should receive attention. When assessing OTL for high school students in mathematics, Porter and colleagues considered modes of instruction (e.g., exposition, field work) and skills (e.g., memorization, understanding concepts, interpreting data, performing procedures, developing proofs; Porter, Kirst, Osthaoff, Smithson, & Schneider, 1993). Although this classification of instructional modes can be interpreted to address methods that promote reasoning in mathematics, they do not directly attend to students' understanding of the mathematics. Gau (1997) expanded the construct of OTL to include other conditions: teachers' mathematical knowledge (mathematics degrees and professional development activities), content and level of instruction (achievement group, content coverage, amount of instructional time, homework), availability of educational resources (e.g., calculators), and extracurricular opportunities. Although these conditions affect OTL, they more directly influence the capacity of schools to support instruction that promotes attainment of high academic standards, rather than students' learning mathematics with understanding.

Attempts to capture student understanding of mathematics content rarely occur in measures of OTL. However, in the Third International Mathematics and Science Study (TIMSS), both content and instruction were considered (Stigler & Hiebert, 1997). Hiebert (1999) noted that instruction for U. S. students predominately emphasized computational procedures, and conceptual understanding was given little attention. The mathematics curriculum provided "few opportunities for students to solve challenging problems and to engage in mathematical reasoning, communicating, conjecturing, justifying, and proving" (p. 11). Furthermore, the results suggest that students learned what they had the opportunity to learn—simple calculation, terms, and definitions—rather than solving nonroutine problems and using mathematical processes such as reasoning about complex problems and developing mathematics arguments.

In MiC, understanding mathematics, not merely the ability to apply memorized facts and procedures, receives substantial emphasis. In the longitudinal/cross-sectional study, OTL has been interpreted more broadly than as a mere gauge of content coverage and is viewed as a student's opportunity to learn mathematics *with understanding* (OTLu). When learning mathematics with understanding, students need the time and opportunity to develop relationships among mathematical ideas, extend and apply these ideas in new situations, reflect on and articulate their thinking, and make mathematical knowledge their own (Carpenter & Lehrer, 1999). In contrast, when the aim of the lesson is primarily coverage of content, the emphasis is often on unconnected pieces of information and on the practice of procedures or heuristics determined by others (Battista, 1999). Such situations reduce cognitive demands on students.

Because MiC was developed to address the recommendations of the NCTM *Curriculum and Evaluation Standards* (1989), this document was used as an initial foundation for examining curricular and instructional emphases. The predominant theme in the *Standards* is that to develop students' mathematical abilities, all students should have opportunities to engage in problem solving, reasoning, communication, and making connections (both among mathematical ideas and between mathematics and life experiences). Students investigate complex, nonroutine problems, conjecture and invent solutions, formulate conclusions and support their thinking with demonstrations, drawings, and calculations. Therefore, the *teaching for understanding* category for OTLu consists of four subcategories: the development of conceptual understanding, the nature of student conjectures about mathematical ideas, the nature of connections within mathematics, and the nature of connections between mathematics and students' life experiences.

In previous studies of OTL, the content of lessons was documented over time. The type and use of particular curricular materials was not an issue. In studies that contrast the performance on state-mandated standardized tests of students who use standards-based curricula with those who use other curricula, the ways teachers used curricular materials were not considered (Reys, Reys, Lapan, Holliday, & Wasman, 2003; Riordin & Noyce, 2001). The mere use of a particular set of curricular materials does not necessarily dictate that the implementation of the materials was aligned with the intent of the authors. For example, teachers might have included supplementary statements, activities, or lessons that promoted problem solving and reasoning, or they might have supplemented the curriculum with drill-and-practice exercises. Such modifications to daily lessons can strengthen or compromise the intent of the lessons as designed by developers. In the MiC longitudinal study, the nature of the modifications made to curricular materials during instruction was considered, and supplementary activities and instructional emphases were documented.

In summary, as conceived in this study, OTLu consists of three overarching categories: *curricular content, modification of curricular materials,* and *teaching for understanding*. *Teaching for understanding* contains of four subcategories: *the development of conceptual understanding, the nature of student conjectures about mathematical ideas, the nature of connections within mathematics,* and *the nature of connections between mathematics and students' life experiences*.

Methodology

Data were collected through the use of instruments designed to examine *content taught*, *modifications to curricular materials*, and *teaching mathematics for understanding*. Information on *content taught* was gathered through teacher logs and journal entries (Shafer, Wagner, & Davis, 1997), Teacher Questionnaire: Professional Opportunities (Shafer, Davis, & Wagner, 1997), and the Classroom Observation Instrument (Davis, Wagner, & Shafer, 1998). Information on *teaching for understanding* was gathered through the Classroom Observation Instrument.

An index was created for each of the two categories *curricular content* and *modifications of curricular materials*. Indices for the four subcategories of *teaching for understanding* were directly from the Classroom Observation Instrument. All indices were created using qualitative research methodologies. The development of these indices was similar to the development of indices that characterized *instruction Instruction* (see Chapter 1 of this Monograph). The indices for *curricular content* and *modifications of curricular materials* were preliminarily defined by differences noted in research literature and differences the research team had expected to see when investigating teachers' use of curricular materials. The indices for subcategories in *teaching for understanding* were preliminarily defined by identifying differences between conventional approaches and teaching for understanding that were measurable by the research team. For each subcategory, three to six levels were outlined. Further distinctions in the levels for each subcategory were identified through a review of literature that was specific to each one. Models for these indices were from previous research on authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993.

The levels in the index for *curricular content* were positioned along a continuum from vast content studied in little depth to a comprehensive curriculum taught in in-depth ways that emphasized connections among mathematical topics. The levels in the index for *modifications of curricular materials* were positioned along a continuum from haphazard selection of content, seemingly done without a guideline, to modifications that enhanced the development of concepts. The levels in the indices for subcategories of *teaching for understanding* were positioned along a continuum from the least to the most reflective of teaching for understanding. The levels in each index were further refined (or levels were added) as data for fifth-grade study teachers in District 1 were reviewed. This process was based on Strauss' (1987) system of open, axial, and selective coding, which involved repeated coding of the data for interpretive codes. These codes included both external codes identified prior to reviewing the data (e.g., teacher covered only a few topics) and internal codes that were identified through the analysis (e.g., the teacher lingered over content until students demonstrated mastery or the teacher used a new curriculum and slow pacing resulted in coverage of only a few topics). Each index was further revised during review of data from sixth- and seventh-grade teachers in District 1 and all teachers in District 2. As a result, three to six levels were identified for each index in order to capture variation among teachers at different grade levels and from different districts. Occasionally, a level was divided into subcategories to more accurately describe the variation found. For example, one level in the index for *modification of curricular materials* was subdivided further in order to more adequately describe the variation teachers who retreated from using a reform curriculum. Sublevels categorized situations in which (a) teachers supplemented a reform curriculum with conventional materials to the extent that the supplementary materials subsumed the reform curriculum and (b) teachers abandoned the reform curriculum in favor of a conventional curriculum. When a code was added, the entire set of data (including data from the first study year) was reread to see whether previous coding might change in light of the new codes. In this methodology, rich qualitative data were quantified according to the levels described for each subcategory. The qualitative data were then used to illustrate the assigned level for each teacher for each category.

In subsequent sections of this chapter, each category and subcategory of the variable OTLu is described, and the highest rating on the index for each is illustrated with data from a teacher questionnaire, teaching logs, journal entries, and classroom observations.

For descriptions of the level assigned for each teacher on each index, see Shafer, Folgert, & Kwako (2004a, b, and c) and Shafer, Folgert, Wagner, & Kwako (2004).

Curricular Content

The category *curricular content* examines students' exposure to mathematics content. In studying OTLu, it remains critical to examine content because all students should have access to varied and challenging content that will allow them to achieve at high levels and ultimately not restrict future career options (O'Day & Smith, 1993). As such, OTLu characterizes the content taught in terms of the number of units or chapters taught and the specific content presented. For example, some elementary and middle-school teachers may choose to emphasize particular strands of mathematics, such as number. Other teachers may more equitably teach topics from number, algebra, geometry, probability, and statistics.

The index for *curricular content* measured the extent to which all mathematical strands were taught in depth and with an emphasis on connections among concepts. The highest level was assigned when the teacher presented a comprehensive, integrated curriculum with attention to all content strands. For example, all fifth-grade MiC teachers were at this level. For example, Greene taught three number, one algebra, one geometry, and one statistics MiC units during the school year (Fifth grade, MiC, Greene, Observation reports, teacher questionnaire 5/98) and Piccolo taught two number, one algebra, two geometry, and one statistics MiC units (Fifth grade, MiC, Piccolo, Teacher logs 1997-1998, teacher questionnaire 5/98, observation reports 1997-98). Teachers at other grade levels also were assigned Level 5, and because they participated in the study over multiple years, interesting patterns became evident in their use of MiC units. For example, in the first study year, Heath taught two geometry, two algebra, one number, and one statistics unit, whereas in the second study year, she taught an additional algebra unit (Seventh grade, MiC, Heath, Teacher logs 1997-98, 1998-99, teacher questionnaires 5/98, 5/99, observation reports 1997-98, 1998-99). For all teachers at Level 5, teachers did not rush through the units. Rather, the units were taught at sufficient length for students to reason about the mathematics content.

Modification of Curricular Materials

Although examining the content students have experienced is important, a simple inventory of the content covered during lessons or over the course of a year does not provide insight into the opportunities teachers provided for students. Teachers might have included, for example, modifications such as supplemental materials to help students make connections between the mathematics in the text and their lives, to practice prescribed algorithms in conjunction with the text, or to use the text as a supplement to a curriculum in which drilling basic skills is the primary focus. Modifications to the curriculum can, therefore, strengthen or undermine the intent of the lessons, and potentially students' opportunities to learn the content with understanding may be enhanced or compromised. Thus, it is crucial to investigate and characterize the modifications teachers make to the available curriculum.

The index for *modifications of curricula materials* measured the extent to which modifications of curricular materials supported the development of deep understanding of covered concepts. The highest level was assigned when the teacher's modifications enhanced the development of conceptual understanding either through task selection, introduction of models, or emphasis on connections among concepts. For example, when students learned about the effects of the sun's rays on shadows in the seventh-grade MiC geometry unit *Looking At an Angle* (Feijs, de Lange, van Reeuwijk, Spence, & Brendefur, 1998), Gallardo placed a model post (a cylindrical roll of paper about 3 feet tall) in front of the class. He dimmed the lights and demonstrated with a flashlight how shadows "move" or change as the sun passes over objects. Similarly, students investigated blind spots around a student who had crouched down in one area of the room (Seventh grade, MiC, Gallardo, Observation 3/25/99). Through the use of these models, Gallardo supplemented the instructional unit in ways that supported the development of conceptual understanding. Another example of modifications of curricular materials at the highest level is illustrated through Pimm's teaching of writing algebraic expressions (Eighth grade, Conventional, Pimm, Observation 10/27/99). Pimm constantly made reference to students' lives and made the vocabulary come alive. For instance, "When the governor *commutes* a sentence, he gives an order." The discussion of "4 less than a number" centered on a context in which a number of candy bars were eaten. Pimm discussed the commutative property frequently and developed a chart on the board with the expression, its verbal translation, and the expression after the commutative property was applied, when appropriate. In this lesson, Pimm supplemented the textbook with scenarios that supported understanding of the content of the lesson through making connections with students' life experiences and continually emphasizing the relationship between the written symbols and verbal translations.

Teaching Mathematics for Understanding

Although content coverage is an important indicator of students' OTL, it is not sufficient to determine the opportunity a student has to learn that content with understanding. Four subcategories characterize *teaching for understanding* in this study: the extent to which the lesson fosters the *development of conceptual understanding*, opportunities the lesson provides for students to make *conjectures about mathematical ideas*, opportunities for *connections within mathematics* to be explored in the lesson, and opportunities in the lesson for students to forge *connections between mathematics and their daily lives*.

The Development of Conceptual Understanding

Conceptual knowledge is described as the "facts and properties of mathematics that are recognized as being related in some way" (Hiebert & Wearne, 1986, p. 200), or as a network of relationships that link pieces of knowledge (Hiebert & Lefevre, 1986). In the primary grades, for example, students learn the labels for whole-number place-value positions. If this information is stored as isolated pieces of information, the knowledge is not conceptual. If this knowledge, however, is linked with other information about numbers, such as grouping objects into sets of ten or counting by tens or hundreds, then the information becomes conceptual knowledge. The network of relationships about place value grows as other pieces of knowledge related to place value, such as regrouping in subtraction, are recognized and integrated. Procedural knowledge, in contrast, is described as having two parts. One category comprises the written mathematical symbols, which are devoid of meaning and are acted upon through knowledge of the syntax of the system. A second category of procedural knowledge is composed of rules and algorithms for solving mathematics problems, step-by-step procedures that progress from problem statement to solution in a predetermined order. Procedural knowledge is rich in rules and strategies for solving problems, but it is not rich in relationships (Hiebert & Wearne, 1986).

Instruction that fosters the development of conceptual understanding engages students in creating meaning for the symbols and procedures they use. Problems or questions posed by the teacher or in curricular materials may direct students' attention to linking procedural and conceptual knowledge. In addition and subtraction of decimals, for example, lining up the decimal points should be linked with combining like quantities. Instruction might explicitly bring out the relationships between lining up the decimal point in addition and subtraction and lining up whole numbers on the right side for the same operations (Hiebert & Wearne, 1986). Instruction that fosters the development of conceptual understanding provides students with the opportunity to learn mathematics with understanding.

The index for *the development of conceptual understanding* measured the extent to which the lesson promoted conceptual understanding. The highest level was assigned when there was a continual focus in the lesson on building connections or linking procedural knowledge with conceptual knowledge. A lesson presented by LaSalle (Observation, 5/19/98) from a class using the MiC fifth-grade number unit *Measure for Measure* (Gravemeijer, Boswinkel, Meyer, & Shew, 1997) is used to illustrate the highest level of the index for conceptual understanding. In the lesson (pp. 4–6), students were asked to determine the amount of oil on each of several measuring sticks and to express the amounts using both common fractions and Egyptian symbols. At the beginning of the lesson, LaSalle read a definition of castor oil and discussed its uses. She related oil from castor beans to oil from coffee beans. When she introduced measuring sticks, LaSalle talked about how measuring sticks are used in measuring the oil level in cars. She then used two soda cans to demonstrate the use of measuring sticks. Students were asked to measure the part of each can that was filled with soda. One was about 1/4 full while the other was about 2/3 full. When the soda was poured into one can, one student estimated that the amount was 11/12 because it was almost full. LaSalle then distributed whiteboards to each group. A student read #5, and groups were given five minutes to work on the problem. Each group wrote their solution on their whiteboard and presented their solution to the class. LaSalle encouraged students to develop their own strategies to solve problems. For example, in their solutions to #5, the observer noted:

> One group added the fractions by getting common denominators and found they had a sum of 63/64 so they were still missing 1/64. Another group used an area model with rectangles and shaded each fraction. 1/64 of the rectangle was left unshaded.

The classed worked on #6 and 7, in which they used paper strips to determine, and express using fractions and Egyptian symbols, amounts of castor oil shown on measuring sticks that had been dipped into six different buckets. They wrote their solutions on the whiteboards and presented their results to the whole class. In the lesson, students worked with fractions, estimating, and representing quantities. The observer noted that LaSalle worked with students to build discussion: “Classroom discussion is a strength for Ms. LaSalle. Students discuss issues with other students, and she asks meaningful questions to keep the discussion on track.”

On this occasion, LaSalle presented the lesson in ways that supported the development of conceptual understanding. In each part of the lesson, students developed procedures to combine fractional amounts. LaSalle did not present a procedure for students to use in solving problems in this context. Rather, through the demonstration of measuring sticks and combining liquids in two soda cans, she set the stage for students to work on their own. The lesson promoted understanding of measurement, estimation with fractions, and solving problems with fractions. As presented, the lesson focused on linking conceptual and procedural knowledge.

The Nature of Student Conjectures

Conjectures about mathematical ideas relate to recognizing connections among topics, investigating patterns, and forming generalizations about mathematical ideas that are applicable across content strands. There are three types of conjectures that students might make. One type of conjecture involves making a guess about how to solve a particular problem based on experience solving problems with similar situations. For example, students were solving problems in which they used properties of similar triangles. When asked to determine the height of a tree, students conjectured that an appropriate solution strategy would involve similar triangles. The students made a connection between the new problem and problems that they had previously solved. A second type of conjecture occurs when a student makes a guess about the truthfulness of a particular statement and subsequently plans and conducts an investigation to determine whether the statement is true or false. For example, a 12-year-old student disagreed with a statement that she was half as tall as she is now when she was 6-years old, and proceeded to support her argument by comparing her present height with heights of 6-year-old children. A third type of conjecture is a generalization. A generalization is created by reasoning from specific cases of a particular event, is tested in specific cases, and is logically reasoned to be acceptable for all cases of the event.

The index for *the nature of student conjectures* measured the extent to which the lesson provided opportunities for students to make conjectures about mathematical ideas. The highest level was assigned when students made generalizations about mathematical ideas during the lesson. A lesson presented by Fiske (Observation, 1/23/98) using the fifth-grade MiC number unit *Some of the Parts* (van Galen, Wijers, Burrill, & Spence, 1997) is used to illustrate the highest level on the index for the nature of student conjectures. In this lesson, the ratio table was used to support students’ reasoning about fractional amounts, specifically in finding the amounts of ingredients if the number of servings in a recipe decreases. Fiske began with a review of a recipe for Potato Chip Sandwiches, which she found in a cookbook. She then led a whole-class discussion around the recipes students turned in as their homework assignment. The class reviewed the amounts of ingredients for one recipe and doubled the amounts. Fiske then read the recipe from the MiC unit and led the class in halving the recipe. Fiske continually drew pictures that depicted the fractions and shaded half of the represented amount. Students expressed the generalizations they made, as noted by the observer:

> At one point, after several drawings, students were asked what was half of 1/4. Some students were able to respond without the help of a drawing. They had generalized the situation mentally. By the time they got to half of 1/6, all hands were waving to

answer without a drawing to guide them. With respect to taking half of a fraction, one student said, "You can keep multiplying the denominator by 2." She was so excited about sharing her generalization that she could hardly be restrained from yelling out her discovery. This contribution on her part was done without the teacher asking for a generalization. (Fifth grade, MiC, Fiske, Observation 1/23/98)

During this lesson, students formed generalizations about finding half of a fraction. Fiske set the stage for students to formulate conjectures by sequencing the tasks and presenting them in ways that supported their reasoning. Rather than provide a generalized rule for them, Fiske allowed students to work toward their own generalizations.

Connections among Mathematical Ideas

Traditionally, mathematics has been taught as a series of discrete skills to be memorized rather than as a sense-making endeavor. Ideally, however, instruction addresses mathematical topics thoroughly enough to explore relationships and connections among them. When connections are drawn between and among mathematical ideas, students can link procedural and conceptual knowledge, recognize relationships between representations, and recognize the interconnectedness of mathematical topics. Students look for and discuss relationships among mathematical ideas, express understanding of mathematical topics, or provide explanations of their solution strategies for relatively complex problems in which two or more mathematical ideas were integrated (Newmann, Secada, & Wehlage, 1995). When these connections are promoted, students are provided with an opportunity to learn mathematics with understanding. Sometimes, however, topics are treated in isolation of other mathematical topics, and are covered in ways that give students only a surface treatment of their meaning.

Topics can be thought of in two different ways. First, topics can be broad areas of mathematics such as probability, area, and ratios, as in the following problem. Students are asked to determine the probability of a frog jumping from a cage and landing on white or black floor tiles and to express this probability as a fraction or percent (Jonker, et al., 1997). In solving this problem, students use area, number, and probability concepts. Second, connections can be made among more narrowly defined areas such as a lesson involving the solution of quadratic equations. In this lesson, connections can be made between factoring, completing the square, or using the quadratic formula. Even though these problems connect mathematical topics, instruction may not focus on discussing or developing these connections. The index for this subcategory reflects both the problems and instruction.

The index for *connections within mathematics* measured the extent to which connections among mathematical ideas were explored in the lesson. The highest rating was assigned when the mathematical topic of the lesson was explored in enough detail for students to think about relationships among mathematical topics. A lesson presented by Gallardo (Observation 3/13/00) from a class using the MiC eighth-grade algebra unit *Get the Most Out of It* (Roodhardt, Kindt, Pligge, & Simon, 1998) illustrates lessons that emphasized connections among mathematical ideas. In the lesson (pp. 21–26), students interpreted equations and graphed constraints and feasible regions. The lesson began with a review of solving and graphing a constraint. Using the inequality $3y + 9x \leq 6$, Gallardo led the class in a discussion of substituting values for x and solving for y values, graphing two points and drawing a line, and substituting coordinates of a point to determine the side of the line to shade. Students applied their prior knowledge and experiences to the constraint problems. In order to predict the values of y, they studied patterns in the data tables they constructed for particular situations. Gallardo encouraged students to listen to each other and build shared understanding of the content. For example, when a student gave an explanation of her solution for the inequality, it was difficult to understand. Gallardo invited other students to think about and rephrase her solution. Gallardo then led a whole-class discussion of some problems from pp. 21–24. He continually asked students about the reason for each step of the procedure as they worked through problems. He also focused on conceptual understanding by constantly reminding students that the solution pairs were points on the line and each solution pair made a true statement with respect to the constraint. Students discussed connections between graphs of lines and graphs of inequalities and talked about how to decide the side of the line to shade for graphing inequalities. Gallardo continually promoted connections among three representations of the same situation by recording and discussing the inequality, the table of solution pairs, and the graph. Throughout this time, students volunteered to verbally explain their solutions, record on the whiteboard their substitutions of values for x to determine the corresponding y values, and graph the lines. Students corrected themselves when they wrote incorrect symbols (e.g., = instead of ≤ or ≥). Gallardo valued students' work and encouraged them to teach each other. In this lesson, the mathematical content was explored in enough detail for students to think about and explore relationships between graphs of lines and graphs of inequalities and among three representations (equation, table, and graph) of the same situation.

Connections between Mathematics and Students' Life Experiences

This index measures whether connections between mathematics and students' daily lives were apparent in text problems or discussed by the teacher or students. Examples of problems that foster such connections are estimating the sale price of an item or determining the amount of ingredients required to serve four people when a recipe serves seven. When connections are made between mathematics and students' daily lives, they will see its usefulness and tend not to view mathematics as an academic pursuit confined within the four walls of the classroom. Rather, mathematics can be viewed as a problem solving endeavor in which informal procedures can be used efficiently, a subject that they should and can learn with understanding since it will be used as part of their daily lives.

The index for *connections between mathematics and students' life experiences* measured the extent to which connections between mathematics and students' daily lives were apparent in the lesson. The highest rating was assigned when connections between mathematics and students' daily lives were clearly apparent in the lesson. To illustrate the highest level on this index, a lesson taught by Heath (Observation 11/17/98) is described. In the first section of the seventh-grade MiC geometry unit *Triangles & Beyond* (Roodhart, de Jong, Brinker, Middleton, & Simon, 1998), students investigated triangles that were used in building a geodesic dome. The observer noted that the class discussed where they had seen geodesic domes, and they found triangles in the room and in a picture of a geodesic dome. For homework, students were to find triangles in magazines and newspapers or to draw ones they saw somewhere

(Seventh grade, MiC, Heath, Observation 11/17/98). In this lesson, connections between mathematics and students' daily lives were clearly apparent.

The Composite Index Opportunity to Learn with Understanding

Although teachers in all four research sites completed interviews and questionnaires, in Districts 1 and 2 classroom observations were conducted and teachers completed teaching logs and journal entries. The composite index *for OTLu*, therefore, was created only for teachers in Districts 1 and 2. Thirty-four teachers were involved in the analysis for the first year of data collection, 32 teachers in the second year, and 17 teachers in the third year. Some teachers were in the study multiple years. For example, Broughton, Dillard, and St. James participated in the study during all three years of data collection.

The composite index *OTLu* was created in a multiple-step process in the same manner as the instruction composite index was developed. Because each index contained from three to six levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the OTLu Total.[3] Using SAS (SAS Institute, 2000), a correlation matrix was created to examine the strength of the correlations between the subcategories and the OTLu Total (see Table 2.2).

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[3] The sum of the weighted results was taken as a measure of quality of OTL_u. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of the quality of OTL_u.

Table 2.2.
Correlations between the Opportunity to Learn with Understanding Total and the Categories and Subcategories

Subcategory	Curricular Content	Modification of Curricular Materials	Teaching for Understanding			
			Development of Conceptual Understanding	Nature of Student Conjectures	Connections within Mathematics	Connections to Life Experiences
Curricular Content						
Modification of Curricular Materials	0.352***					
Development of Conceptual Understanding	0.606***	0.139				
Nature of Student Conjectures	0.573***	0.231*	0.706***			
Connections within Mathematics	0.514***	0.040	0.775***	0.685***		
Connections to Life Experiences	0.580***	-0.006	0.623***	0.524***	0.563***	
OTLu Composite	0.865***	0.433***	0.842***	0.796***	0.756***	0.731***

*p<.05
**p<.01
***p<.001

One category was not well correlated with the other category and the subcategories, *modification of curricular materials*, but it was highly correlated with the OTLu Total. To verify these results, a principle component factor analysis was completed using SAS (SAS Institute, 2000). Factors 1 and 2 accounted for a significant amount of the variance among the subcategories (see Table 2.3). *Modification of curricular materials* was an important contribution in Factor 2, and the other category and the subcategories were influential in Factor 1. Therefore, it was decided that the OTLu Total for each teacher contained the weighted sum of all categories and subcategories. Therefore, the total for each teacher was not altered.

Table 2.3.
Contribution of Categories and Subcategories to Principle Component Factors 1 and 2

Category/Subcategory	Factor 1	Factor 2
Curricular Content	79*	27
Modifications to Curricular Materials	25	94*
Teaching for Understanding		
Development of Conceptual Understanding	89*	-11
Nature of Student Conjectures	85*	5
Connections among Mathematical Ideas	85*	-23
Connections to Life Experiences	77*	-26

* Values were multiplied by 100 and rounded to nearest integer; values greater than 0.4 were flagged with *, indicating an important contribution

Using the OTLu Total for each teacher, cluster analysis was conducted, which permitted the classification of teachers into four groups. For each group of teachers, common characteristics from the categories and subcategories of OTLu were sought and identified. Descriptions of each group of teachers were then created by using the qualitative evidence that supported the rating for each category and subcategory. These descriptions became the levels of the composite index *for OTLu*. By using these levels, the research team was able to capture variation among study teachers at different grade levels, in different treatments, in different districts, and in different years of data collection. The four levels are summarized in Table 2.4.

Table 2.4.
Summary of the Composite Index for Opportunity to Learn with Understanding

Level 4: High Level of Opportunity to Learn with Understanding	**Level 3: Moderate Level of Opportunity to Learn with Understanding**	**Level 2: Limited Opportunity to Learn with Understanding**	**Level 1: Low Level of Opportunity to Learn with Understanding**
• Curriculum with attention to all content areas • Few modifications to curricular materials • Portions of lessons focused on conceptual understanding • Student conjectures related to validity of particular statements • Connections among mathematical ideas clearly explained by the teacher • Connections between mathematics and students' life experiences apparent	• Content taught in depth, but limited to one or two content areas • Supplementary activities occasionally used • Limited development of conceptual understanding • Student conjectures related to making connections between a new problem and problems previously seen • Connections among mathematical ideas briefly mentioned • Connections between mathematics and students' life experiences reasonably clear if explained by the teacher	• For teachers using MiC: Few content areas taught due to slow pacing • For teachers using conventional curricula: Vast content as disparate pieces of knowledge, laden with prescribed algorithms • For teachers using MiC: Supplementary materials subsumed the curriculum • For teachers using conventional curricula: Few modifications to curricular materials; supplementary activities occasionally used • Limited development of conceptual understanding • Student conjectures related to making connections between a new problem and problems previously seen • Connections among mathematical ideas briefly mentioned • Experiences reasonably clear if explained by the teacher	• Vast content as disparate pieces of knowledge, laden with prescribed algorithms • For teachers using MiC: Supplementary materials subsumed the curriculum • For teachers using conventional curricula: Haphazard presentation of content; no adherence to textbook as guideline • Conceptual understanding not promoted • None observed; connections not encouraged • Connections among mathematical ideas not discussed • Connections between mathematics and students' life experiences not evident

Each of the four levels of the *OTLu* composite index *OTLu* is described below and is illustrated with information from classroom observations, teaching logs, teacher journal entries, and teacher questionnaires.

Level 4: High Level of Opportunity to Learn with Understanding

At Level 4, teachers presented a comprehensive, integrated curriculum with attention to all content areas. They followed the adopted curriculum faithfully with few, if any, modifications. Some lesson questions fostered conceptual development of mathematical ideas or some aspects of the lessons focused on conceptual understanding. Observed student conjectures consisted mainly of investigating the veracity of statements. Connections among mathematical topics were discussed by teachers and students or connections were clearly explained by teachers. Connections between mathematics and students' life experiences were clearly apparent in the lesson.

Data from Piccolo are used to illustrate Level 4 of the composite index *for OTLu*. Ms. Piccolo taught two number, one algebra, two geometry, and one statistics MiC units, and she presented the units with few modifications (Fifth grade, MiC, Piccolo, Teacher logs 1997-1998, teacher questionnaire 5/98, observation reports 1997-98). A lesson presented by Piccolo (Observation 5/8/98) from a class using the MiC fifth-grade geometry unit *Figuring All the Angles* (de Lange, van Reeuwijk, Feijs, Middleton, & Pligge, 1997) is used to illustrate the subcategories in teaching for understanding. In the lesson (pp. 15–17), students investigated the distortion caused by representing curved surfaces with flat maps. They looked at flattened curved sections of grapefruit peel and lines of latitude and longitude on flat maps and globes. (The terms latitude and longitude were not used formally in the unit.) In previous lessons in this unit, students used flat maps to locate places using distances and directions. Piccolo began the lesson with informal assessments of students' prior knowledge about the shape of the Earth and history of theories about that shape. Throughout the lesson, Piccolo connected class discussions to the information she learned through this assessment. Students completed question 1 on p. 15, "Describe the location of the United States [on pictures of the Earth in the unit]." After this, Piccolo placed large laminated maps of the United States on their desks. She led students in a discussion of the map in the unit and the text that described the location of St. Louis, Missouri as 38ºN and 90ºW. Students then began work in small groups on questions 2-4, in which they used the map to locate several U. S. cities and investigated whether planes flying in the same direction would ever meet. Piccolo valued students' statements about mathematics and used them to work toward shared understanding for the class. For instance, students were convinced that airplanes flying north would never meet. She brought out the classroom globe and encouraged students to trace northerly flights with their fingers. This process helped them reconstruct their ideas.

The lesson continued with questions 5-7 about the distortion caused by creating flat maps of curved surfaces. Piccolo asked students what would happen to the map if it were pressed onto a large grapefruit that she had brought to class. She cut the peel into sections (along lines of longitude) and flattened the pieces on the overhead projector. In this way, Piccolo provided a visual support for students' thinking, one that students could refer back to when solving problems. Students completed the questions in their groups and shared their answers in subsequent whole-class discussion.

This lesson is reflective of a high level of the *teaching for understanding* subcategories in OTLu. The focus of this lesson was on conceptual understanding. The questions in the unit provided opportunities for students to understand differences in representations of the world. Piccolo introduced activities but allowed students to solve the problems on their own. Students elaborated their thinking in both oral and written ways. Feedback was ongoing, addressing student misconceptions through alternative representations. For example, after letting students struggle with the question about whether planes on northerly routes would meet, Piccolo introduced a strategy for students to use that gave them access to the problem and allowed them to pursue a solution. In this way, she did not reduce the mathematical work for them. Rather, she opened opportunities for students to think about and explore the mathematical ideas. Throughout the lesson, Piccolo provided the necessary tools (maps, globe) and visual clues (flattened grapefruit peels), but she did not interject comments that might reduce students' cognitive activity. The lesson promoted connections among mathematical ideas (flat vs. spherical maps, number lines, lines of latitude and longitude, geometry, and measurement), and connections between mathematics and students' life experiences (maps, globes, and compass directions).

Level 3: Moderate Level of Opportunity to Learn with Understanding

At Level 3, teachers taught mathematical concepts in depth, but restricted content primarily to one or two content strands such as number and algebra. They generally followed the adopted curriculum, but occasionally supplemented the text with activities that were disconnected from the text. Development of conceptual understanding, however, was limited. Few lesson questions fostered conceptual development of mathematical ideas or conceptual understanding was a small part of the lesson design. Observed student conjectures consisted mainly of making connections between a new problem and problems already seen. Connections among mathematical ideas might have been briefly mentioned, but these connections were not discussed in detail. Although the lesson did imply connections between mathematics and students' daily lives, these connections were not immediately apparent to students. Such connections, however, would have been reasonably clear if teachers brought them into discussion.

Data from Teague are used to illustrate Level 3 of the composite index for *OTLu*.OTLu. Teague taught four MiC units during the school year—one number and three algebra (one of which was a seventh-grade unit). During the last month of the school year, she used a traditional pre-algebra textbook as the basis of instruction (Seventh grade, MiC, Teague, Teacher logs 19987-1999, teacher questionnaire 5/99, observation reports 1998-99). A lesson presented by Teague (Observation 1/19/99) from a class using the MiC eighth-grade algebra unit, *Reflections on Number* (Wijers, et al., 1998), is used to illustrate the subcategories in teaching for understanding. In the unit, students learn about number theory, and in the first section, they explore number patterns in Pascal's Triangle and use Pascal's Triangle to determine the routes to a given point on a grid. In the lesson (pp. 4-5), students investigate codes consisting of 0's and 1's that are represented as routes on a grid, draw different routes on a grid and determine their length, and draw all possible routes representing codes of length 2. This class period began with a 15-minute quiz that was not related to MiC. During the MiC portion of the class period, Teague led the class in a lengthy 35-minute checking of previous classwork related to number patterns in Pascal's Triangle and routes on a grid. During this time, students called out answers as Teague wrote them in a grid displayed on the chalkboard. Although the numbers in Pascal's Triangle were the same as the numbers of routes from given points on the grid, few students were able to articulate this relationship on their own. Teague showed the class how to rotate their worksheets so that the numbers in Pascal's Triangle were more visible. Throughout this review, students were given time to write complete answers to the open-ended questions posed on page 4. Teague also checked the answers for page 5 in which students determined the number of people who can share 24 hard candies. Teague then assigned page 6; during the next 20 minutes, students were to find all possible rectangular arrays of 24 square pictures. The period ended with a half-hour session during which students prepared for the performance-based statewide assessment by using practice worksheets. \

This lesson is reflective of a moderate level of the *teaching for understanding* subcategories in OTLu. The lesson itself focused on conceptual understanding and opportunities for students to make conjectures. Although half of the two-hour class period was devoted to the MiC lesson, much of it was used for checking student answers to questions posed in the previous lesson; the new lesson was limited to one page. In this class, students seemed unprepared to complete unit questions on their own, as evidenced by their lack of answers to unit problems assigned during the previous class period. Students were allowed to find their own solutions, yet they seemed to wait until the questions were checked by their teacher. Student conjectures observed by the observer were limited, and connections from one context to the next were not apparent to students.

Level 2: Limited Opportunity to Learn with Understanding

At Level 2, teachers covered only a few topics. Because many experimental teachers used MiC for the first time during the whole school year, slow pacing resulted in coverage of only a few topics. Some MiC teachers supplemented the curriculum with conventional materials to the extent that these materials subsumed MiC. Teachers who used conventional curricula generally followed the adopted curriculum with few modifications, but tended to linger over content until students demonstrated mastery. For both MiC teachers and teachers using conventional curricula, conceptual understanding was a small part of the lesson design; lessons focused on building students' procedural understanding without meaning. Observed student conjectures and connections were consistent with Level 3.

Data from McLaughlin are used to illustrate Level 2 of the composite index for OTLu. McLaughlin taught chapters primarily on number but did include lessons on calculating area of rectangles, identifying definitions of geometric terms, problem solving strategies, and logical reasoning. She presented vast content treated as disparate pieces of knowledge heavily laden with vocabulary and prescribed algorithms, and used various textbook chapters with few modifications (Seventh grade, conventional, McLaughlin, Teacher logs 1997-1998, teacher questionnaire 5/98, observation reports 1997-98). A lesson presented by McLaughlin (Observation 2/6/98) from a class using the seventh-grade conventional textbook *Mathematics: Applications and Connections* (Glencoe/McGraw-Hill, 1997) is used to illustrate the subcategories in teaching for understanding. In the lesson (p. 319), students solved logic problems in which a matrix was used to record information given in the problem. The class began with a warm-up activity unrelated to the lesson, which included addition and subtraction of integers without context and asked for geometric terms, given their definitions. Students then gave answers to these questions, but their explanations focused on procedures rather than an elaboration on their thinking. The observer noted: "For one of the warm-up questions –7 + –3, students answered 4 and –4. [Ms. McLaughlin] responded, 'Who remembers the rules?' Student explanations were reciting the rules." In this situation, rules were emphasized, and a conceptual basis for operations with integers was not discussed in class. The instruction did not promote learning mathematics with understanding. McLaughlin then introduced logic problems on p. 319 in the textbook. She demonstrated how to construct a matrix to record the given information: By marking the information given in problem statements pertinent to each person in the problem, the solution became apparent. McLaughlin frequently asked students if they solved the logic problems using other strategies. Eventually, however, she led the class to use one specific strategy for solving the type of problem. The worksheet assigned for group work featured a different type of logic problem than examples and problems in the textbook. Assigned at the end of the class period, students were to complete the work on the next day.

This lesson is reflective of a limited level of the *teaching for understanding* subcategories in OTLu. The focus of the lesson was on particular rules and procedures, and although McLaughlin asked for alternative strategies, students were generally expected to use the method she presented. Limited changes were made in instruction even when student responses were unreasonable. For example, when students stated the results 4 and –4 in response to the warm-up question –7 + –3, McLaughlin requested the rules for addition of integers rather than providing an explanation based on a representation of the situation. Students had no other ways to think about integer operations other than by using the rules.

Level 1: Low Level of Opportunity to Learn with Understanding

At Level 1, teachers presented vast content as disparate pieces of knowledge, heavily laden with vocabulary and prescribed algorithms. Consistent with Level 2, MiC teachers covered few topics and tended to supplement the curriculum with conventional materials to the extent that they subsumed MiC. Teachers who used conventional curricula presented the content in a haphazard way that did not adhere to a text and did not emphasize connections among mathematical topics. For both experimental and control teachers, lessons did not promote conceptual understanding, and student conjectures were not observed. Connections between mathematics and students' lives were not apparent during lessons.

Data from Brown are used to illustrate Level 1 of the composite index *for OTLu*. Brown taught two number, one algebra, and one geometry MiC units, and he supplemented the units with computer-assisted drill-and-practice programs. This combination resulted in a dual emphasis on basic skills and some conceptual understanding (Sixth grade, MiC, Brown, Teacher logs 1997-1998, teacher questionnaire 5/98, observation reports 1997-98). A lesson presented by Brown (Observation 3/16/98) from a class using the MiC sixth-grade algebra unit *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole, & Burrill, 1998) is used to illustrate the subcategories in teaching for understanding. In this unit students are introduced to informal forms of mathematical expressions and formulas. In this lesson, students used arrow strings to show order of operations. To begin the lesson, Brown asked students to turn to pp. 54–58. Mr. Brown told the class to spend the period working pp. 54–58. No expectations were given as to how far they should get by the end of the period, who they should work with, when they could ask a question, and so on. Students were confused because they were not near p. 54 prior to the lesson. Five hands went up immediately, and Mr. Brown started moving from student to student. The classroom was very quiet. Students worked individually and talked only with the teacher when he came to their desks. They waited for

Mr. Brown to tell them how to do a problem, and then they did it that way. After that, they asked how to do the next one, and so on. Students depended on Mr. Brown to answer all questions and did not move ahead in the lesson until he came to their desks. In some cases, this took a long time. The only strategy used was the one that Mr. Brown explained individually to each student. Students had very little motivation for, or knowledge of, the mathematics they were supposed to be learning. They left incorrect answers in their notebooks after talking to Mr. Brown, and they just skipped other questions. To students and their teacher, it seemed that the focus was on each question in the lesson, rather than on the mathematics they were studying. Each question was treated as a new and isolated topic.

On this occasion no formal lesson was presented. Students were given an assignment, but the content was not discussed prior to students completing it. Students lacked the support they needed to understand the mathematics on their own. This lesson is reflective of a low level of the *teaching for understanding* subcategories in OTLu. Mathematics was presented in ways that gave students only a surface treatment of the content. Inquiry during class was limited to lower order thinking, and the lesson did not promote conceptual understanding. Students began independent work or small-group work with little direction. The class did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences.

Results

Levels on the composite index *OTLu* spanned all four levels for teachers using MiC and teachers using conventional curricula. Patterns of variation are described and illustrated for each study year. Analysis of the variance found is then reported.

Year 1

The results for all 34 teachers for the *OTLu* composite revealed differences by grade level and type of curriculum taught. By grade level, more fifth-grade teachers were at Levels 3 and 4 of *OTLu*, indicating that students experienced moderate or high levels of *OTLu*, than sixth- or seventh-grade students (see Figure 1a). Most sixth- and seventh-grade teachers were at Level 2, indicating that students experienced limited OTLu. An additional pattern of variation was found when the levels of the *OTLu* composite were reviewed by curriculum taught (see Figure 1b). Twelve of the 23 teachers using MiC were at Level 4, in comparison to none of the teachers using conventional curricula. Also, 4 teachers using MiC were at Level 3 in comparison to 2 of the 11 teachers using conventional curricula. In contrast, most teachers using conventional curricula were at Level 2 compared with about 7 teachers using MiC. When these results were reviewed by district, the results reflected the results by curriculum taught (see Figures 1c and 1d).

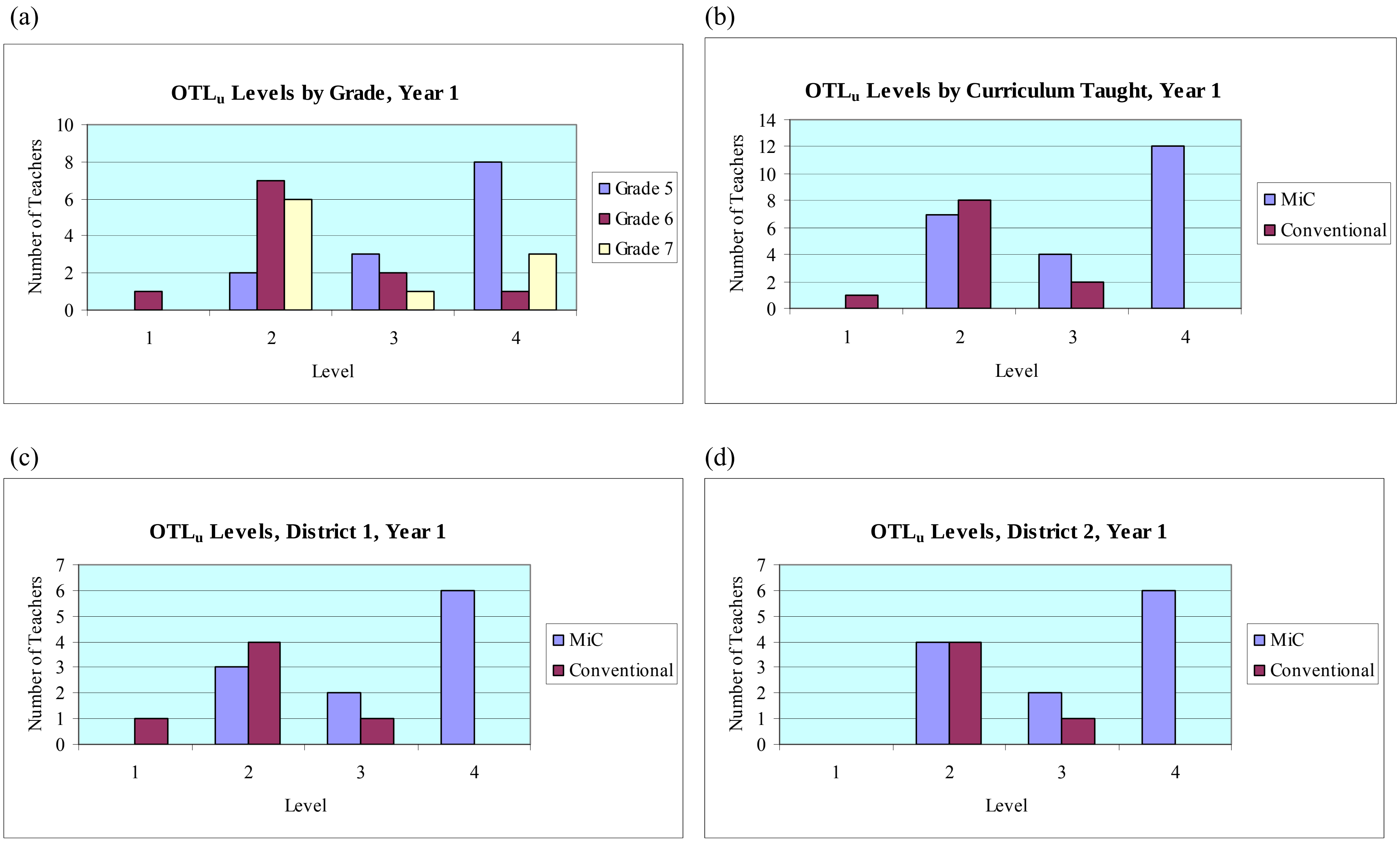

Figure 1. Teacher level on the composite index *for OTLu*, Year 1: (a) by grade; (b) by curriculum; (c) by curriculum, District 1; (d) by curriculum, District 2

Year 2

The results for all 32 teachers for the *OTLu* composite revealed differences by grade level and type of curriculum taught. By grade level, more seventh- and eighth-grade teachers were at Levels 3 and 4, indicating moderate or high levels of *OTLu*, than sixth-grade teachers (see Figure 2a). An additional pattern of variation was found when the levels of *OTLu* were reviewed by curriculum taught (see Figure 2b). Five of the 21 teachers using MiC were at Level 4 and 11 were at Level 3, in comparison to 3 of the 11 teachers using conventional curricula at Level 3 and none at Level 4. Most teachers using conventional curricula (8) were at Level 2 compared with 4 teachers using MiC. When these results were reviewed by district, the results reflected the results by curriculum taught (see Figures 2c and d).

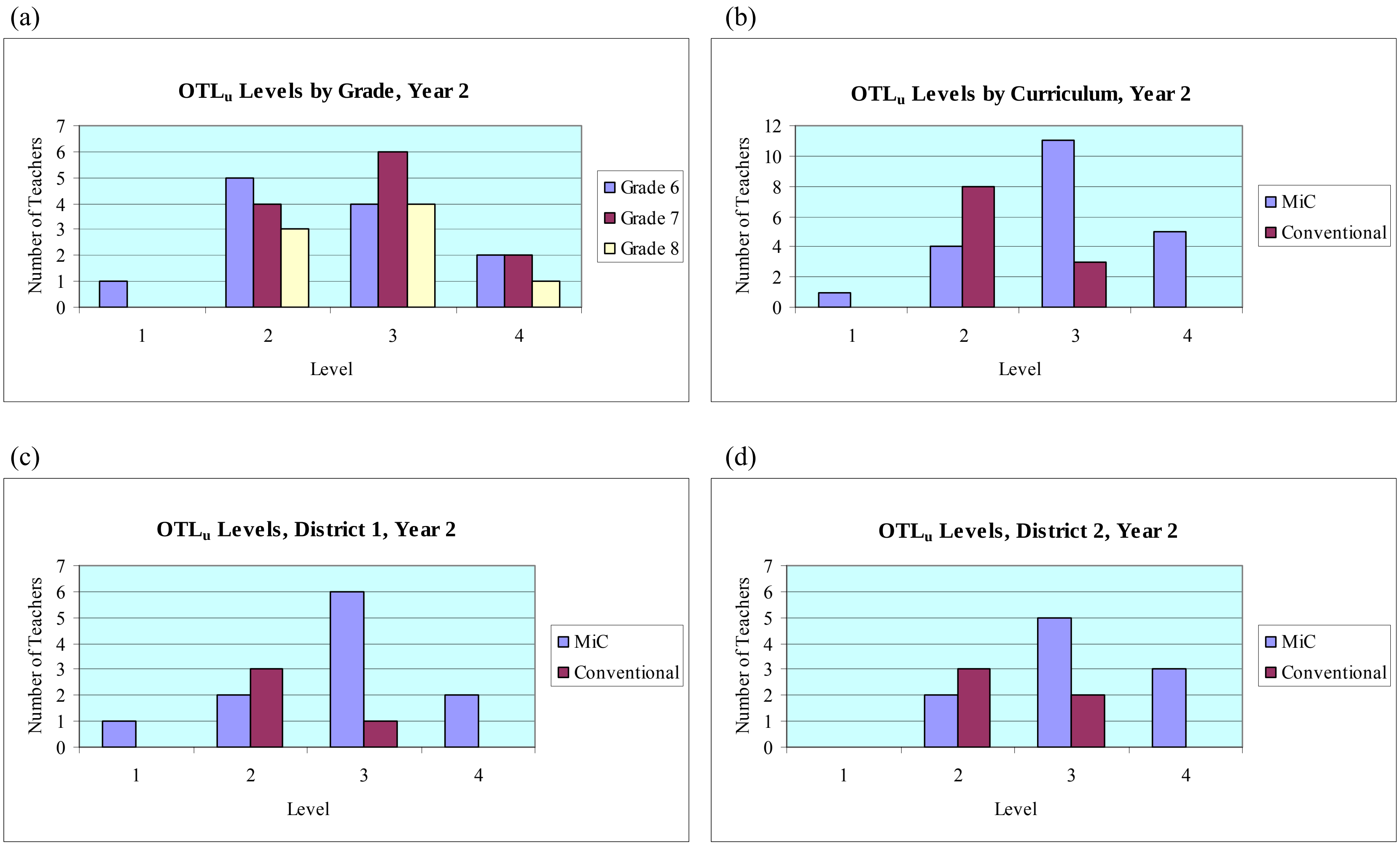

Figure 2. Teacher level on the composite index *for OTLu*, Year 2: (a) by grade; (b) by curriculum; (c) by curriculum, District 1; (d) by curriculum, District 2

Year 3

The results for all 17 teachers for the *OTLu* composite revealed differences by grade level, type of curriculum taught, and by district. By grade level, more eighth-grade teachers were at *OTLu* Levels 3 and 4 than seventh-grade teachers (see Figure 3a). The number of teachers using conventional curricula was very small (only 3 seventh-grade and 1 eighth-grade), making comparisons by curriculum difficult. Four of the 13 teachers using MiC were at Level 4, and 3 MiC teachers were at Level 3 (see Figure 3b). However, 5 of the MiC teachers were at Level 2 compared with 3 of the 4 teachers using a conventional curriculum. When these results were reviewed by district, the results reflected the results by curriculum (see Figures 3c and d).

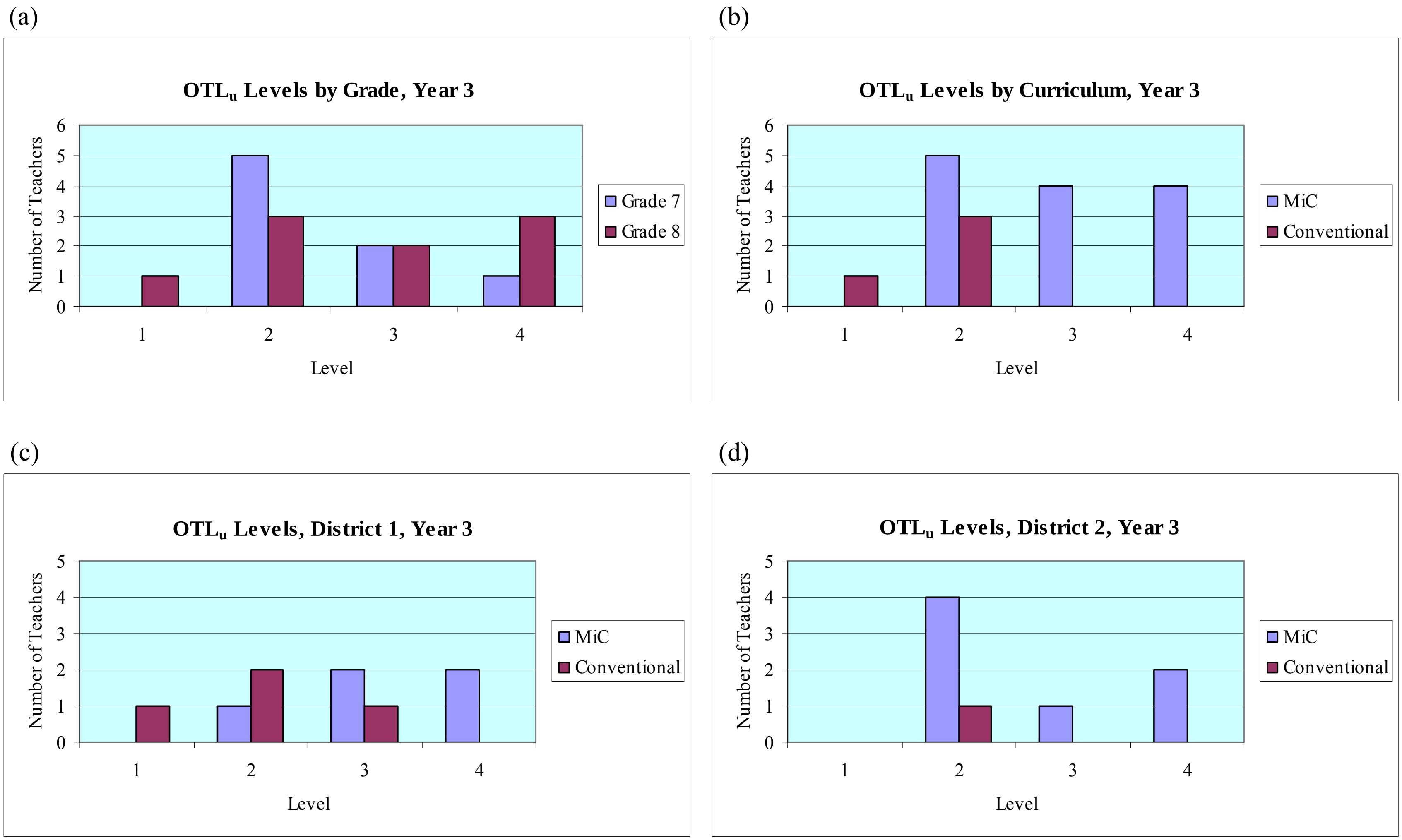

Figure 3. Teacher level on the composite index *for OTLu*, Year 3: (a) by grade; (b) by curriculum; (c) by curriculum, District 1; (d) by curriculum, District 2

Analysis of Variance

Analysis of the variance was completed to check for significant differences among teachers by grade level, curriculum taught, and district. The OTLu Total was used as the dependent variable. Results suggest that 57% of the variance in the OTLu Total was accounted for by differences in grade level, curriculum taught, and district, and there was an effect for curriculum and grade.

Table 2-4.
ANOVA with OTLu *Total as the Dependent Variable*

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	1485.57	247.59	18.03	<.0001
Error	80	1098.66	13.73		
Corrected Total	86	2584.23			

R-Square	Coeff Var	Root MSE	OTLu Mean
0.57	18.20	3.70	20.36

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Curriculum Taught	2	1102.43	551.21	40.14	<.0001
Grade Level	3	139.58	46.52	3.39	0.0220
District	1	1.02	1.02	0.07	0.7857

When the contrasts were examined (see Tables 2-6 and 2-7), there were significant differences between the means of teachers using MiC and teachers using conventional curricula and between fifth-grade and the other grade levels, but not between the means of teachers by district. The results suggest that students experienced significantly different instruction when studying MiC than when studying conventional curricula and when in fifth-grade classrooms than when in middle-school classrooms.[4]

Table 2-6.
Least Squares Means for Curriculum Taught, Grade Level, and District

Least Squares Means	
Curriculum Taught	
MiC	22.73
Conventional	17.07
Grade Level	
Grade 5	23.36
Grade 6	19.43
Grade 7	20.33
Grade 8	19.71
District	
District 1	20.60
District 2	20.82

[4] Given the power of the statistical tests in this study, p-values <.10 are considered to imply important differences, and p-values <.05 are statistically significant differences (Cohen, 1988).

Table 2-7.
Contrasts in Means for the OTLu Total by Curriculum, Grade, and District

Parameter	Estimate	Standard Error	t Value	Pr > |t|
		Curriculum Taught		
MiC vs Conventional	5.73	1.03	5.56	<.0001
		Grade Level		
Grade 5 vs Grade 6	3.92	1.30	3.01	0.003
Grade 5 vs Grade 7	3.03	1.25	2.41	0.018
Grade 5 vs Grade 8	3.64	1.35	2.70	0.008
Grade 6 vs Grade 7	-0.89	1.00	-0.89	0.375
Grade 6 vs Grade 8	-0.28	1.14	-0.25	0.806
Grade 7 vs Grade 8	0.61	1.08	0.56	0.573
		District		
District 1 vs District 2	-0.22	0.82	-0.27	0.785

Conclusion

Furthering the traditional description of OTL, two additional categories, modification of curricular materials and teaching mathematics for understanding, were scaled and included in the composite index *for OTLu*. The composite index was designed to capture differences in OTLu among study teachers in the two districts in which teachers used MiC or conventional curricula. Classroom observations, teacher logs, journal entries, and questionnaire data were used to triangulate and describe the variation, which was found by curriculum and by grade level. The results suggest significant differences between the opportunity to learn with understanding experienced by students in MiC classrooms and students who studied conventional curricula and between fifth-grade students and students in other grade levels. The curricular content is comprehensive, with its attention to geometry, algebra, and statistics in addition to number, and the curriculum is rich in developing connections among mathematical ideas. Contexts in which lessons are situated provide a basis for exploring mathematical ideas and applying mathematics in daily life experiences.

The descriptions of OTLu for each teacher are one aspect of the eight grade-level-by-year studies described in Monograph 1, Chapter 2, and they are used in Monographs 5, 6, 7, and 8 to further develop our understanding of the achievement results for each class of students. In the next chapter, we examine in depth the capacity of study schools to support mathematics teaching and learning.

CHAPTER 3: SCHOOL CAPACITY

Mary C. Shafer

Although an individual teacher's knowledge and skills indisputably affect his/her practices, an important complement to idiosyncratic teacher characteristics are the multiple layers of teaching contexts—students in the classroom, peers in the department and/or school, administrators, local policies, and state standards for education and testing. Talbert & McLaughlin (1993) argued that "to understand teaching in context is to understand the interplay of the multiple,multiple embedded contexts of education in the daily lives of teachers" (p. 190). These contexts influence the likelihood that a teacher will embrace a vision of teaching for understanding, strive to implement this vision, and then persist in teaching for understanding. For example, a strong, supportive, unified schoolwide professional community is more likely to engage teachers and students in pursuing meaningful learning than a community in which no common goals are set for student achievement and no time is provided for teacher collaboration. Through examining school capacity, broad social and institutional factors that influence students' opportunity to learn mathematics with understanding are identified and described.

School capacity is the collective power of the school staff to improve student achievement (Newmann, King, & Youngs, 2000). Through the work of the school's professional community, a vision of student learning, professional inquiry, and control over school policy and activities unfolds. School capacity is strong when all programs and initiatives are focused on the vision for student learning and professional inquiry. The principal nurtures school capacity through effective leadership that aligns school policy with the vision and sets the tone for professional development opportunities for faculty and staff. Quality technical resources such as curricula, assessments, and equipment contribute to obtaining the established goals.

Following Newmann & Associates (1996), in this study, the composite variable school capacity includes two major categories: cultural conditions and structural conditions. Often implicit, the culture of a school is revealed through "a pattern of shared beliefs, values, and competencies that tend to guide the nature of [the professional community's] work and their expectations for success" (Newmann & Associates, 1996, p. 206). Official policies and practices in the school undergird the distribution of authority, organization of staff, tasks of teachers, and allocation of resources. These policies and organization systems form school structure. In reality, cultural and structural conditions interact, with one influencing the other.

The composite variable for school capacity is specified from the variables in the original research model: *school context* (prior variable) and *support environment* (independent variable; see the introduction to this monograph). In the original simplified research function, variation in classroom achievement (CA) was hypothesized to be attributed to variations in opportunity to learn with understanding (OTLu), which included school capacity, prior achievement (PA), and method of instruction (I). However, as the research team worked with the data, a decision was made to distinguish school capacity (SC) from OTLu. Thus, the research function was expressed as—

$$CA = SC + OTLu + PA + I.$$

Four subcategories characterized *cultural conditions*: *shared vision for mathematics teaching and learning between principal and teacher, administrative support, school as a workplace,* and *support for innovation* (see Table 3-1). Three subcategories characterized *structural conditions*: *collaboration among teachers, work structure,* and *influence of standardized testing*. A single index, which is a composite of multiscaled information from the seven subcategories of school capacity, represents school capacity in the simplified research function.

Table 3-1.
Characterization of the Composite Index for School Capacity

Category	**Cultural Conditions**	**Structural Conditions**
Subcategory	• Shared Vision • Administrative Support • School as a Workplace • Support for Innovation	• Collaboration • Work Structure • Influence of Standardized Tests

Methodology

An index was created for each of the seven subcategories of school capacity. Four indices were created using qualitative research methodologies: *shared vision for mathematics teaching and learning between principal and teacher, support for innovation, work structure,* and *influence of standardized testing*. The development of these indices was similar to the development of indices that characterized *instruction Instruction* (see Chapter 1 of this monograph). For each subcategory, three to five levels were outlined by the research team. Further distinctions in the levels for each subcategory were identified through a review of literature that was specific to each one. Models for these indices were from previous research on authentic instruction, tasks, and assessment (Newmann, Secada, & Wehlage, 1995); Cognitively Guided Instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996); instruction that included teachers' understanding and beliefs about constructivist epistemology (Schifter & Fosnot, 1993); and utilization of particular instructional innovations (Hall, Loucks, Rutherford, Newlove, 1975, quoted in Schifter & Fosnot, 1993.

Data on other subcategories of school capacity were gathered through two teacher questionnaires. These questionnaires were adapted from the Center for Educational Work (1997); Center on Organization and Restructuring of Schools (1996); Institute for Advanced Study (1995); National Center for Improving Student Learning & Achievement in Mathematics & Science (1997); Porter, Kirst, Osthoff, Smithson, & Schneider (1993); Urban Mathematics Collaborative (1990); and Webb & Dowling (1993). The number of questionnaire items for each of these subcategories varied from two to five statements. A Likert scale indicating the teacher's level of agreement accompanied each statement. For example, the subcategory of *administrative support* was characterized through five sets of teacher perceptions: administrative support for staff; administrative support for instruction and class management; administrative support for instructional practice; professional development climate in the school; and professional collaboration. The Likert-scale ratings for these items were summed, and teachers were separated into five groups that captured the variation among the teachers at different grade levels and in different districts.

Cultural Conditions

Four subcategories of school capacity characterized *cultural conditions* in this study: *shared vision for mathematics teaching and learning between principal and teacher, administrative support, school as a workplace,* and *support for innovation*. In this section, each subcategory is described, and the highest rating on the index for each subcategory is illustrated with interview and questionnaire data. For descriptions of the level assigned for each teacher on each index, see Shafer, Folgert, Webb, Kwako, Lee & Wagner (2003) and Shafer, Folgert, Webb, & Kwako (2003a, and b).

Shared vision for student learning. A schoolwide professional community that seeks to attain high intellectual quality nurtures high expectations and supports students in achieving those expectations (Marks, Doane, & Secada, 1996). When a staff shares a vision for student learning and develops a clear plan to attain these objectives, teachers are able to comfortably work to achieve the goals. Talbert and McLaughlin (1993) contend that “regardless of what is being learned, students and teachers are much more engaged in school work when staff members share educational goals” (p. 175). It is important to recognize, however, that consensus may not only ignore the principles of teaching mathematics for understanding, but may undermine them or reinforce traditional teaching practices and goals. Nevertheless, the unease that often accompanies the implementation of novel teaching practices is clearly minimized when teachers feel they are working in concert with their principal’s vision, and they can more freely dedicate themselves to these pedagogies. When teachers and administrators share visions of high intellectual quality, it is more likely that teachers use curricula that sustain high cognitive demands for students and hold high expectations for high achievement for all students (Newman & Associates, 1996).

The index for *shared vision for student learning* measured the extent to which principal and teacher visions were aligned. The highest level was assigned when principal and teacher visions for mathematics teaching and learning were clearly defined and generally aligned. For example, Burnett, the principal at Guggenheim Middle School, stressed the importance of providing students with the problem solving and analytic skills necessary for the technologically-based job market of the twenty-first century (Principal, Burnett, Interview 9/18/97). He emphasized that these skills would enable students to be flexible and life-long adult learners. Although Burnett indicated that students have different learning styles and capabilities, he believed that engaging in problem solving experiences best develops mathematical ideas and enables students to apply knowledge in new situations. Keeton, a teacher at Guggenheim Middle, felt that students learned best when they applied mathematical ideas in real world contexts (Seventh grade, MiC, Keeton, Interview 9/18/97). She felt student engagement was a critical component of instruction and that students benefited from participating in exploration of ideas rather than listening to lectures. Keeton viewed mathematics not as a litany of basic facts, but as a system for problem solving, believing that basic skills were embedded in problem solving. Connecting mathematics to students' everyday experiences was another important consideration in her instruction. In this case, visions for student learning of mathematics for Principal Burnett and Teacher Keeton were closely aligned.

Administrative support. Just as a shared vision strengthens a teacher's commitment to implementing new practices, the support and encouragement a teacher perceives from the principal enables him/her to take risks without fear of reprisal. Principals can demonstrate their commitment to new practices, for example, by discussing instructional strategies with teachers, providing opportunities for them to observe exemplary teachers, and structuring time for collaboration (Newmann & Wehlage, 1996). Effective principals hold high expectations for intellectual quality and establish ways for staff collaboration to attain those goals.

In this study, the subcategory of *administrative support* was characterized through five sets of teacher perceptions: administrative support for staff; administrative support for instruction and class management; administrative support for instructional practice; professional development climate in the school; and professional collaboration. The highest overall rating was assigned when the teacher perceived very strong support from school administrators. For example, Fulton believed that expectations for student learning were clearly articulated by his principal (Fifth-grade, conventional curriculum, Fulton, Teacher questionnaires 8/97). He was encouraged and supported by the principal to teach effectively. Fulton felt that he was encouraged to try new ideas, talk to administrators about teaching practices, and observe other teachers. He also noted that teachers collaborated regularly. Furthermore, the principal obtained the resources he requested. However, even with this strong support, Fulton thought the professional development climate in the school could be improved, citing that professional development did not accompany changes in school policy, professional development did not build on teachers' experiences, and teachers' suggestions regarding professional development were not elicited.

School as a workplace. Promoting high intellectual quality for students and the professional growth of teachers requires an "ethos of caring, sharing, and mutual help among staff, and between staff and students" (Newman & Wehlage, 1996, p. 289). This culture develops through trust, respect, and shared responsibilities. The climate for such a vision for the school is enhanced when teachers feel their voices are being heard and taken seriously by the administration. Although the principal typically assumes ultimate responsibility for decision making, in schools in which clear channels are established for communication about ideas related to teaching and learning, teachers support reform more than schools in which teachers' input is not sought (Newmann, Kings, & Young, 2000).

The subcategory of *school as a workplace* was characterized through five sets of teacher perceptions: teacher influence over school policy regarding educational and administrative decisions; teacher control over school policy related to planning and teaching; teacher perception of faculty and staff commitment and cooperation; and level of support from other teachers for improvement of practice. The highest rating was assigned to teachers who perceived that they exerted influence over school policy and decision making. For example, Murphy acknowledged that she had little control over evaluation and hiring of teachers and school budgets, but she was actively involved in educational decisions regarding discipline, curricular content, instructional materials, grading, and professional development (Fifth-grade, MiC, Murphy, Teacher questionnaires 8/97). Murphy also felt that high standards were held for faculty and staff at her school and that there was a focused commitment to student learning in mathematics. She thought that teachers worked together toward school goals, and new ideas were continually sought and evaluated. She felt supported by other teachers in her school and in her district in her efforts to improve practice.

Support for innovation. Although individual teachers are often able to implement reform practices successfully, their endeavors are eased when conditions favorably support innovation. These supports can be as concrete as funding of needed materials and as abstract as risk-taking without impunity. Professional development opportunities are important for encouraging innovative practices. Critical concerns in increasing school capacity include the ways in which professional development is used to go beyond teachers' individual skills and knowledge and the funding priorities for professional development conducted locally or externally (Newmann, King, & Youngs, 2000).

In this study, the subcategory of *support for innovation* was characterized through five sets of teacher perceptions: opportunities for professional development at local, state, and national levels; monetary support for local and external staff development; and observation of teaching. The highest level was assigned to teachers who perceived a high level of support for innovation in their schools. For example, Perry noted that she participated in district and state professional development opportunities through funds provided by her principal for registration fees, travel, and substitute coverage (Seventh grade, MiC, Perry, Interview 10/9/97). She attended workshops on her own time as well. Perry felt that teachers in her school were comfortable discussing their ideas for in-school professional development with their principal. The principal frequently stopped in Perry's classroom for informal observations and provided constructive feedback for her. Perry felt confident to ask the principal for advice on teaching particular mathematics content, as he was a mathematics teacher before holding administrative positions in the district.

Structural Conditions

Three subcategories characterized *structural conditions* in this study: *collaboration among teachers, work structure,* and *influence of standardized testing*. In this section, each subcategory is described, and the highest rating on the index for each subcategory is illustrated with interview and questionnaire data. For descriptions of the level assigned for each teacher on each index, see Shafer, Folgert, Webb, Kwako, Lee & Wagner (2003) and Shafer, Folgert, Webb, & Kwako (2003a, b).Shafer, Folgert, Webb, Kwako, Lee & Wagner (2003) and Shafer, Folgert, Webb, & Kwako (2003a and b).

Collaboration among teachers. Newmann, King, and Youngs (2000) found that schools that were able to raise and sustain student achievement had a collaborative structure that had two primary features: high priority to schoolwide, cohesive professional development and common planning time for subject matter teachers. Time for teacher collaboration permitted development of standards and expectations, reflection on teaching practices, and more comprehensive assessment of students' knowledge and understanding. Furthermore, professional networks among teachers from other schools can play a pivotal role in learning to implement new forms of teaching: "[I]t may be that professional networks and discourse communities outside the school are more important than, or at least an influential complement to, school-based community for diffusing and enabling teaching for understanding in American classrooms" (Talbert & McLaughlin, 1993, p. 177).

The subcategory of *collaboration* was characterized through four subcategories: subject and frequency of formal faculty meetings; subject and frequency of planning with other mathematics teachers; subject and frequency of discussions about mathematics with other mathematics teachers; and principal's perception of teacher collaboration. Results of teacher questionnaires suggest that study teachers did not have access to sustained collaboration on a broad range of topics. To illustrate this, we describe the responses of McFadden (Seventh-grade, MiC, McFadden, Teacher Questionnaires 8/97). In formal meetings in the mathematics department, McFadden reported that teachers regularly discussed curriculum, pedagogy, and assessment of student learning, and they evaluated new curricula. During daily planning time, however, the following were less likely to happen: discussion of topics, methods, and exercises emphasized during instruction and ideas for assisting specific students. Teachers were even less likely to talk during planning time about assessing student understanding and preparation of course goals, lessons, projects, tests, or grades. Although teachers regularly discussed parental issues, they less frequently discussed professional literature, their own experiences in teaching mathematics, and interesting mathematical ideas. In an interview, comments by McFadden's principal supported her questionnaire responses, stating that teachers had developed informal ways to collaborate regularly in lieu of common planning time (Principal, Stone, Interview 10/13/97).

Work structure. Although the principal is an indisputably key figure in leadership, the department head can play an equally influential role in modeling change and supporting teachers' efforts to implement innovative practices. Subject area departments can become strong professional discourse communities that can support teacher learning and risk-taking. Department heads can engender enthusiasm of the subject, bolster commitment to innovative pedagogies, and promote reflection on daily practice. Talbert and McLauglin (1993) found that "teachers in strong, collegial departments are…more likely to maintain strong commitment to teaching, even to reject traditional teaching methods in favor of teaching for understanding to enable students' success" (p. 190) than their counterparts in weak departments.

In this study, *work structure* was examined through the responsibilities of a teacher leader and the extent to which the duties related with that position supported instruction. Both the principal's perception and teacher's perception were investigated and a rating was assigned to each. Each rating included the responsibilities of a teacher leader as mentor (highest ranking), instructional decision maker, liaison between the principal and teachers, person with administrative duties, and no teacher leader available (lowest ranking). Each rating also included whether (a) the position was specifically related to mathematics, such as a mathematics specialist; (b) the position was general, such as a grade-level leader; or (c) the person operated in an unofficial capacity, such as a teacher who was perceived to have more expertise than others in the department. To illustrate, we describe the role of the mentor Wells for teachers of mathematics in District 3. All teachers complemented Wells in her efforts to support implementation of new mathematics curricula. For instance: "She is very well respected by all the teachers. There's a lot of confidence in her teaching as far as her presentation of math and her overall view of where the kids are going" (Fifth grade, MiC, Edgebrook, Interview 10/1/97). Teachers mentioned Wells' support in providing professional development opportunities as they began to teach MiC for the first full school year:

> She's very enthusiastic about mathematics, and she's the one that started the meetings [about MiC] in her home for all of us to get together. She also worked on getting the fifth-grade teachers involved, and then the sixth-grade teachers….She's just always been a big help. (Seventh grade, MiC, Perry, Interview 10/9/97)

When the district adopted MiC, teachers at the elementary school led a presentation for parents about the curriculum during Back-to-School Night. One teacher commented: "Even though [Ms. Wells] didn't do the presentation, she attended. She was very supportive and very visible" (Fifth grade, MiC, Edgebrook, Interview 10/1/97). Wells noted that her role also included supporting teachers' efforts to understand and cope with new state requirements for standardized testing. For example, Wells reported: "The state has asked us to come up with another form of assessing children, and we are in the process of talking about what that looks like" (Eighth grade, MiC, Wells, Interview 12/3/98). Thus, for both the elementary and middle school teachers, Wells served as a mentor who sought professional development opportunities for teachers of mathematics, supported their efforts in implementing curricula, and led discussions related to state mathematics requirements.

Influence of standardized testing on instruction. Darling-Hammond and Wise (1985) asserted that mandated testing systems affected both curricular content and pacing of instruction and led teachers to teach to the tests. Talbert and McLaughlin (1993) explain: "This form of instruction…linked to specific curricular objectives, prompts teachers to regard students' learning as test results and to adopt fast-paced teaching directly to the test" (p. 175). Clearly, testing is increasingly important in the context of teaching in America's public schools. These practices, however, stand counter to pedagogy that enables students to learn with understanding. A critical aspect of supportive school capacity in an era of reform is the ability of a principal to buffer his/her staff from initiatives that prompt teachers to regard test results as a proxy for learning, to perceive reform instruction as antithetical to testing success, and to question the efficacy of reform (Talbert and McLaughlin, 1993).

In this study, the subcategory of *influence of standardized testing* describes the extent to which the state testing program influenced teachers' instruction. The highest rating was assigned when teachers felt that standardized tests had little influence on curricular or instructional decisions. Qualitative methodologies revealed four reasons for the low level of influence: teachers already included problem solving and written explanations during instruction; they felt their curriculum and/or instructional practices were aligned with the tests; they were unfamiliar with the tests; or they perceived that their daily practices were sufficient for preparing students for standardized tests. For example, teachers in District 3 reported that the state testing program did not influence curriculum and instruction in major ways. One sixth-grade teacher stated that this philosophy was supported by district administrators: "We've been reassured not to let that bother us….The administration reassured us, don't worry about it. Just teach what you need to teach, and it'll show up [in student scores]" (Sixth grade, MiC, Schleuter, Interview 9/29/97). Some teachers added a few exercises, but not an extensive review before standardized tests:

> I don't think [the test] influences what I teach. I really believe in and support what I'm teaching now through MiC. However, when I know that it's coming, I will concentrate a little bit more on a few problems each day, on just math problems, you know, four or five. I call it "practice makes perfect" or just a reminder to jog their memory on certain things. [The test] doesn't really influence me, but yet I'd like the kids to do well. (Seventh grade, MiC, Perry, Interview 12/18/98)

Other teachers did not worry about the state tests because they believed their curriculum was more rigorous, and as a consequence, students will do well:

> I looked at our state standards and then I looked at the standards that we wrote, [which align with] *Mathematics in Context*, actually. We feel that our standards are more rigorous than the state. So we are not—or I am not—changing my instruction at all. I think our students will do just fine on the state test. I'm not concerned about it. I really think that what we're providing for our children is so much more than what the state is requesting right now that I'm not worried about it. I wasn't worried about it last year, and the kids did just fine. (Eighth grade, MiC, Wells, Interview 12/3/98)

The Composite Index School Capacity

The composite index *school School capacity Capacity* was created in a multiple-step process. Because each index contained four or five levels, the indices were weighted so they would have equal emphasis. The weighted sum is referred to as the School Capacity Total.[5] Using data from all teachers in Districts 1 and 2, a correlation matrix was created to examine the strength of the correlations between the subcategories and the School Capacity Total (see Table 3-2). The results suggest that *administrative support* was well correlated with *school as a workplace*, and three subcategories were well correlated with the School Capacity Total, *administrative support, school as a workplace,* and *collaboration.*

[5] The sum of the weighted results was taken as a measure of school capacity. Torgerson (1958) pointed out that, although the sum of the results of individual indices is ordinarily calculated for interval or ratio scales, inherent in all scales is the presumption that distance has meaning. Therefore, measurement on an ordinal scale is done either explicitly or implicitly as if it were an interval scale whose characteristics of order and distance stemmed from a priori grounds (p. 24). Thus, the weighted sum was taken as a measure of school capacity.

Table 3-2.
Correlation of the Subcategories of the School Capacity Composite Variable

Category	Cultural Conditions				Structural Conditions		
Subcategory	Shared Vision	Administrative Support	School as a Workplace	Support for Innovation	Collaboration	Work Structure	Inflence of Standardized Tests
Shared Vision							
Administrative Support	0.115						
School as a Workplace	0.085	0.632**					
Support for Innovation	0.003	0.207	0.084				
Collaboration	0.010	0.574**	0.390	0.111			
Work Structure	-0.034	0.113	0.153	0.143	0.070		
Inflence of Standardized Tests	0.023	-0.198	0.036	-0.121	-0.333	-0.090	
School Capacity Total	0.352	0.719**	0.740**	0.392	0.511**	0.400	0.112

**p<.01

Using the School Capacity Total, cluster analysis was then conducted, which permitted the classification of teachers into five groups or levels: Level 5 in which teachers perceived a high level of school capacity; Level 4—moderately high level of school capacity; Level 3—average level of school capacity; Level 2—low level of school capacity; and Level 1—very low level of school capacity. Ratings for both teachers using MiC and teachers using conventional curricula spanned all five levels.

Results

Year 1

In the first study year, teachers' perceptions of the capacity of their schools to support mathematics teaching and learning varied considerably (see Figure 3-1a). While half of the teachers were at Levels 4 and 5, indicating moderately high or high levels of school capacity, nearly 30% perceived a low level of school capacity and 20% perceived an average level of school capacity. Of the 17 schools that participated in the study in Year 1, 15 had more than one study teacher. Teachers in 7 of these schools were consistent in the levels of school capacity they perceived. In the other schools, levels of school capacity varied to different degrees. For example, at Guggenheim Middle School in District 2, teachers were at Levels 1, 2, 3 and 5, whereas at Calhoun North in District 3, teachers were at Levels 2, 4, and 5. (Complete data sets can be found in Shafer, Folgert, Webb, Kwako, Lee & Wagner [2003] and Shafer, Folgert, Webb, & Kwako [2003a, b].Shafer, Folgert, Webb, & Kwako, 2003a, b and Shafer, Folgert, Webb, Kwako, Lee, & Wagner, 2003.))

The results revealed differences by grade level, curriculum taught, and district. By grade level, fifth- and seventh-grade teachers were more likely to be classified at Levels 4 and 5, indicating that they perceived a moderate or high level of school capacity (see Figure 3-1b). In comparison, sixth-grade teachers were more likely to be assigned Level 2, indicating that they perceived low levels of school capacity. An additional pattern of variation was found when the levels of the school capacity composite were reviewed by curriculum taught. Half of the 40 MiC teachers were at Levels 4 and 5, in comparison to 3 of the 11 teachers using conventional curricula (see Figure 3-1c). One-fourth of the MiC teachers were at Level 3 in contrast to none of the teachers using conventional curricula, and nearly one-fourth of the MiC teachers and 6 of the 11 teachers using conventional curricula were at Level 2. When these results were reviewed by district, additional variations became apparent. MiC teachers in Districts 2, 3, and 4 were more likely to perceive a moderate to high level of school capacity (Levels 4 and 5) than in District 1 in which the largest group of MiC teachers perceived average school capacity (Level 3; see Figure 3-2).

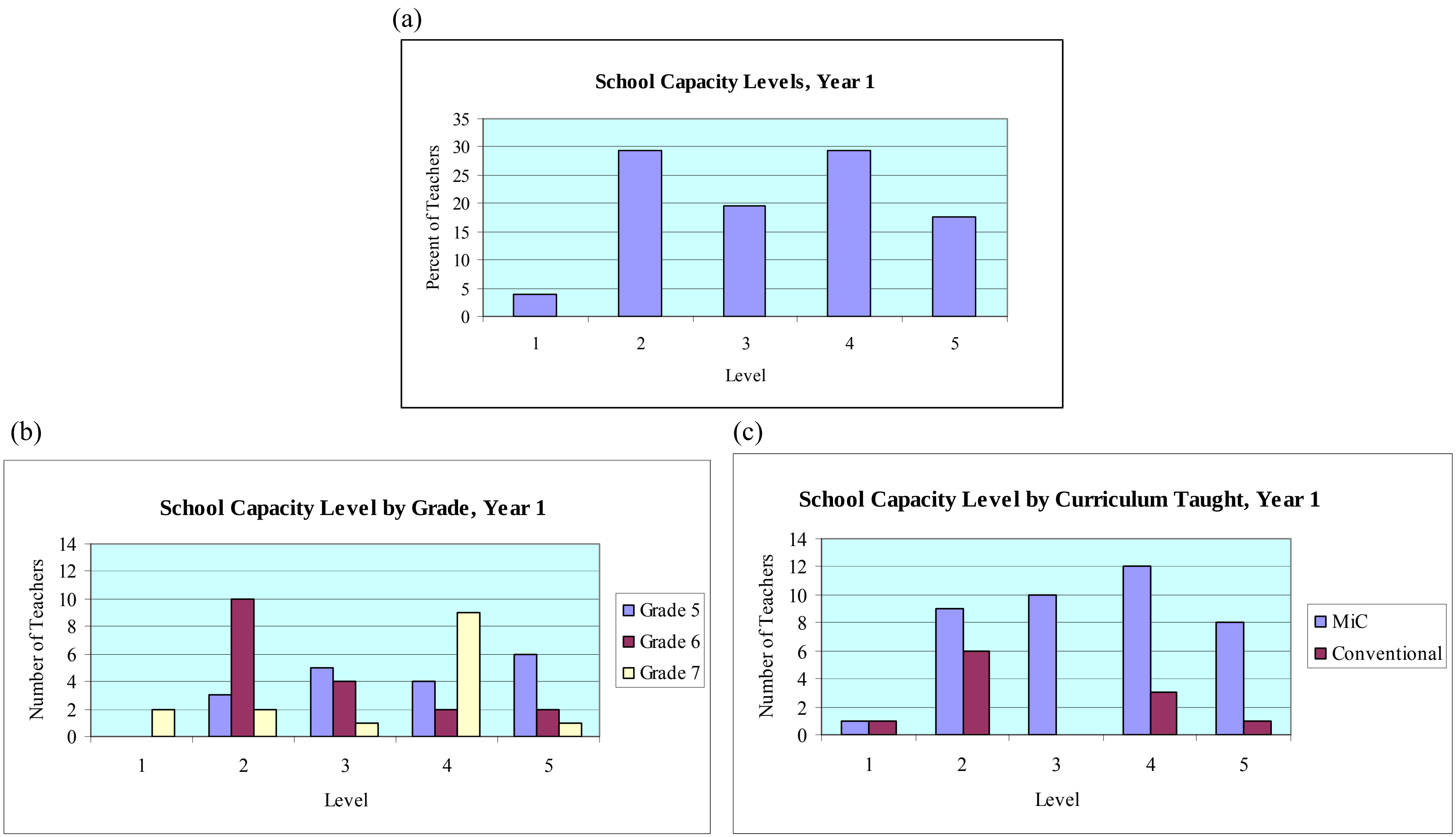

Figure 3-1. Teacher level on the composite index for *School Capacity*school capacity, Year 1: (a) by percent of teachers; (b) by grade; and (c) by curriculum.

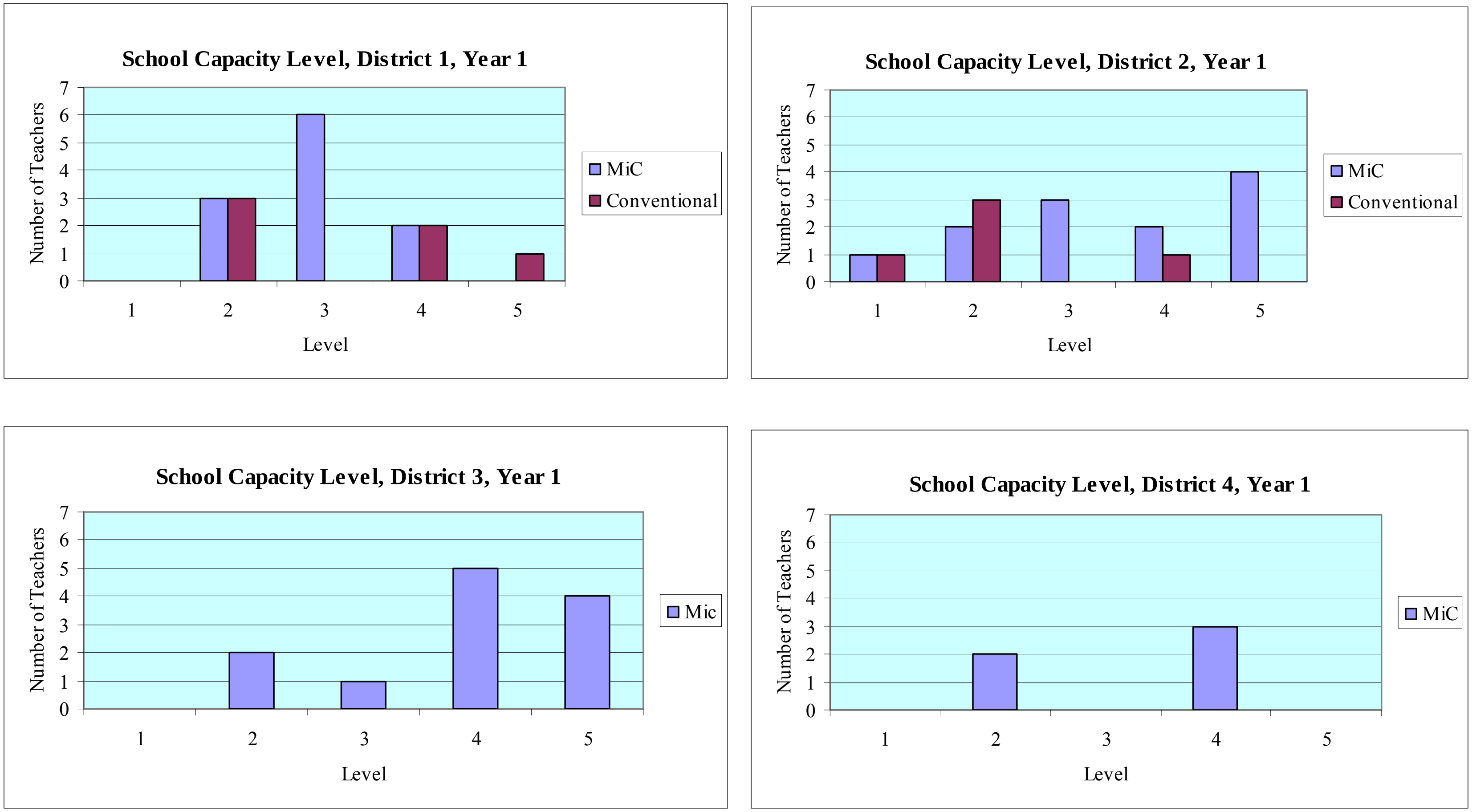

Figure 3-2. Teacher level on the composite index *School Capacity* for school capacity by district, Year 1.

Year 2

In the second study year, the results for all teachers for the *School Capacity* school capacity composite revealed differences within school, by grade level, curriculum taught, and district. Half of the 46 study teachers were at Levels 4 and 5, indicating that they perceived a moderately high or high level of school capacity, and nearly one-half were at Levels 2 and 3, indicating low or average levels of school capacity (see Figure 3-3a). Of the 10 schools that participated in the study, 9 had more than one study teacher. In 5 of these schools, teachers were generally consistent in the levels of school capacity they perceived.

By grade level, teachers were widely spread across the levels (see Figure 3-3b). An additional pattern of variation was found when the levels of the *School Capacity* school capacity composite were reviewed by curriculum taught. More than half o the MiC teachers were at Levels 4 and 5 compared to 1 of the 11 teachers using conventional curricula, and one-third of the MiC teachers in comparison to 10 of the 11 teachers using conventional curricula were at Levels 2 and 3 (see Figure 3-3c). When these results were reviewed by district, additional variations became apparent. Consistent with Year 1 results, MiC teachers in Districts 2, 3, and 4 were more likely to perceive a moderate to high level of school capacity (Levels 4 and 5) than in District 1 in which nearly all MiC teachers perceived low to average school capacity (see Figure 3-4).

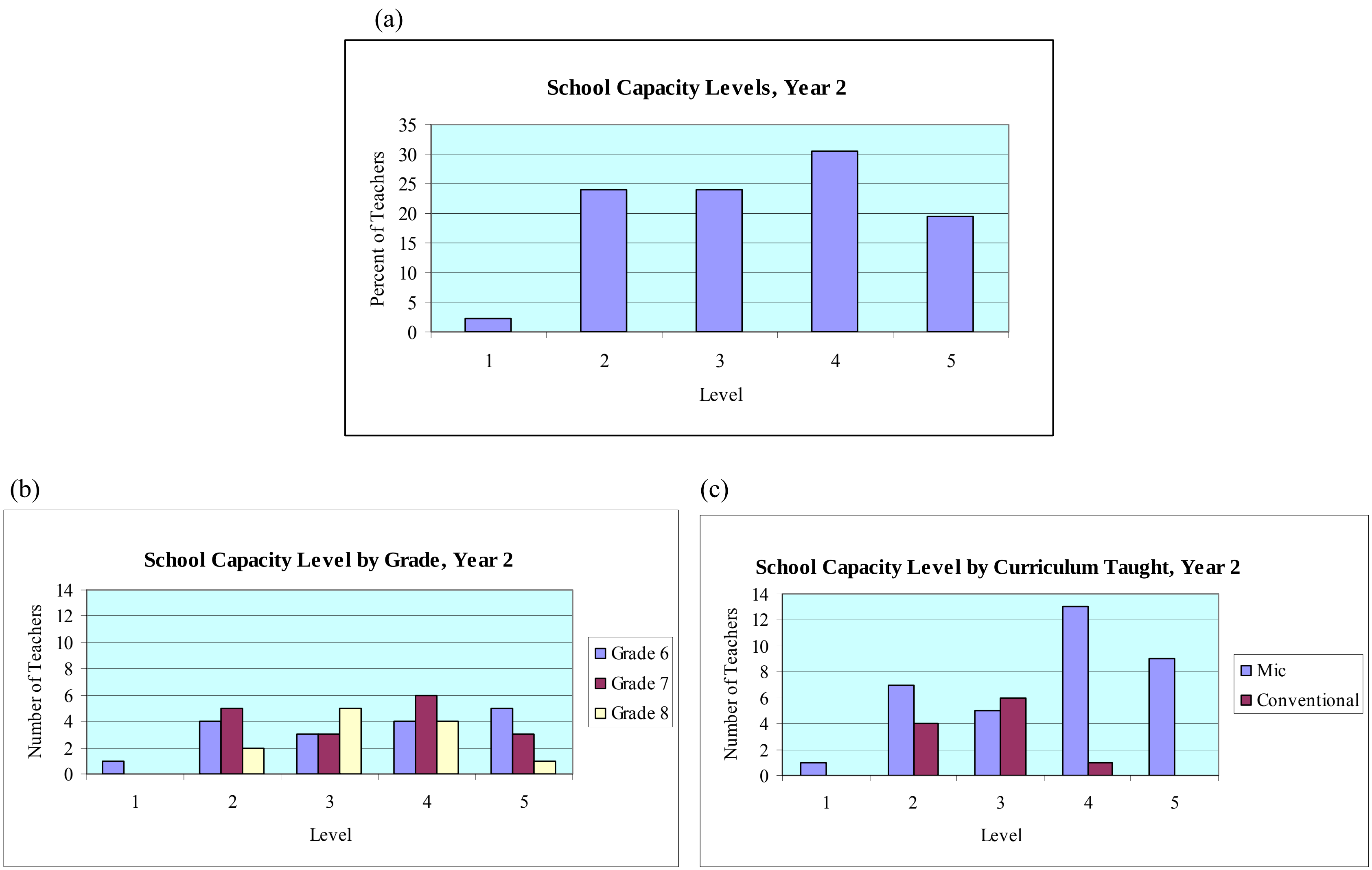

Figure 3-3. Teacher level on the composite index *School Capacity*for school capacity, Year 2: (a) by percent of teachers; (b) by grade; and (c) by curriculum.

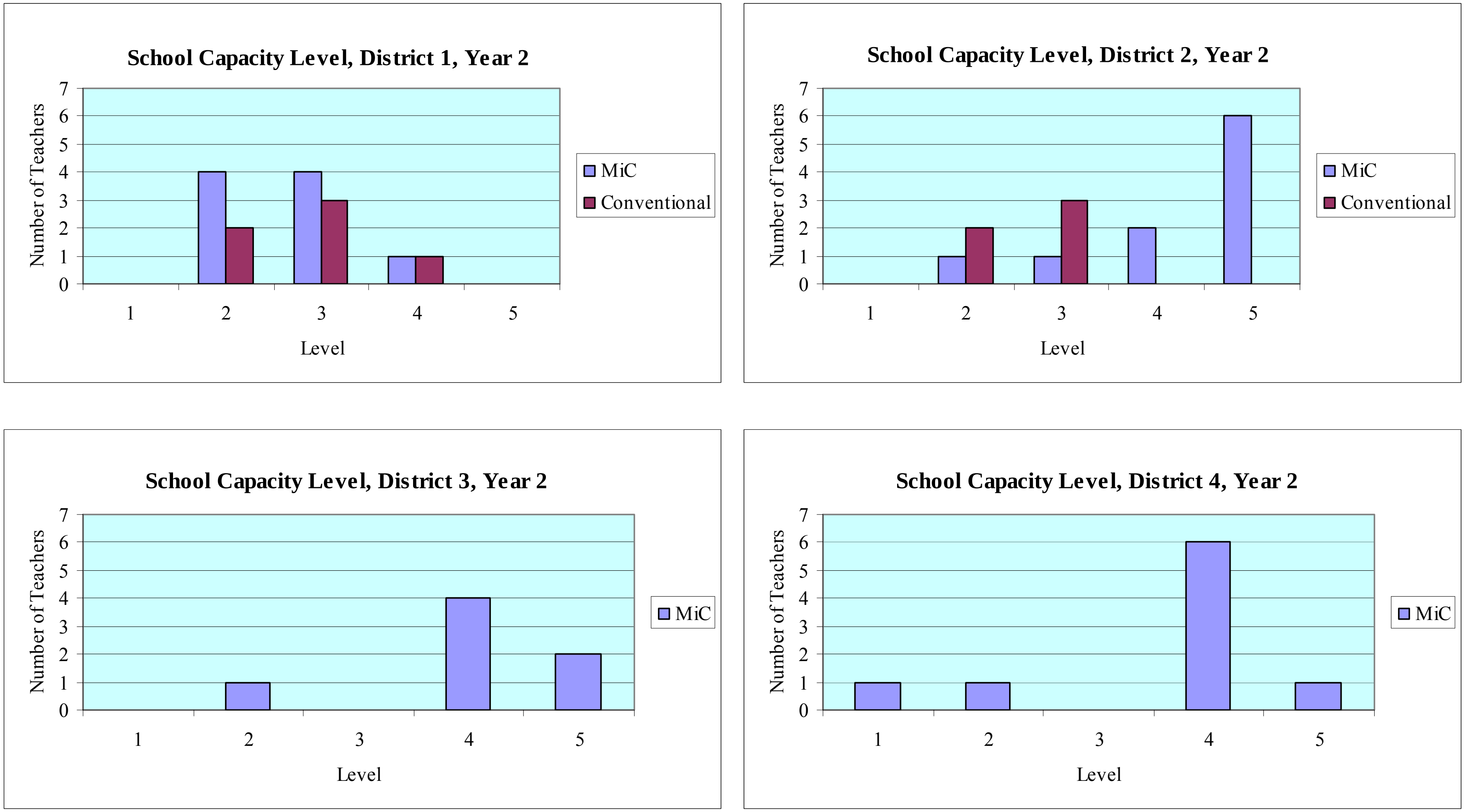

Figure 3-4. Teacher level on the composite index *School Capacity* for school capacity by district, Year 2.

Year 3

In Year 3, the results for all teachers for the *School Capacity* school capacity composite revealed differences within school, by grade level, curriculum taught, and district. Over one-third of the 24 study teachers were classified at Levels 4 and 5, indicating that they perceived a moderately high or high level of school capacity, one-third perceived an average level of school capacity (Level 3), and one-fourth perceived low school capacity (Level 2; see Figure 3-5a). Of the 8 schools that participated in the study, 7 had more than one study teacher. In 5 of these schools, teachers were consistent in the levels of school capacity they perceived.

By grade level, eighth-grade teachers were more likely to be assigned Levels 4 and 5, in contrast to the seventh-grade teachers, most of whom were at Levels 2 and 3 (see Figure 3-5b). With respect to curriculum taught, 9 of the 21 MiC teachers were at Levels 4 and 5, and one-third were at Level 3 (see Figure 3-5c). Comparison with teachers of conventional curricula was difficult due to the small sample. When these results were reviewed by district, additional variations became apparent. Consistent with results in Years 1 and 2, MiC teachers in Districts 2, 3, and 4 were more likely to perceive a moderate to high level of school capacity (Levels 4 and 5) than in District 1 in which nearly all MiC teachers perceived low to average school capacity (see Figure 3-6).

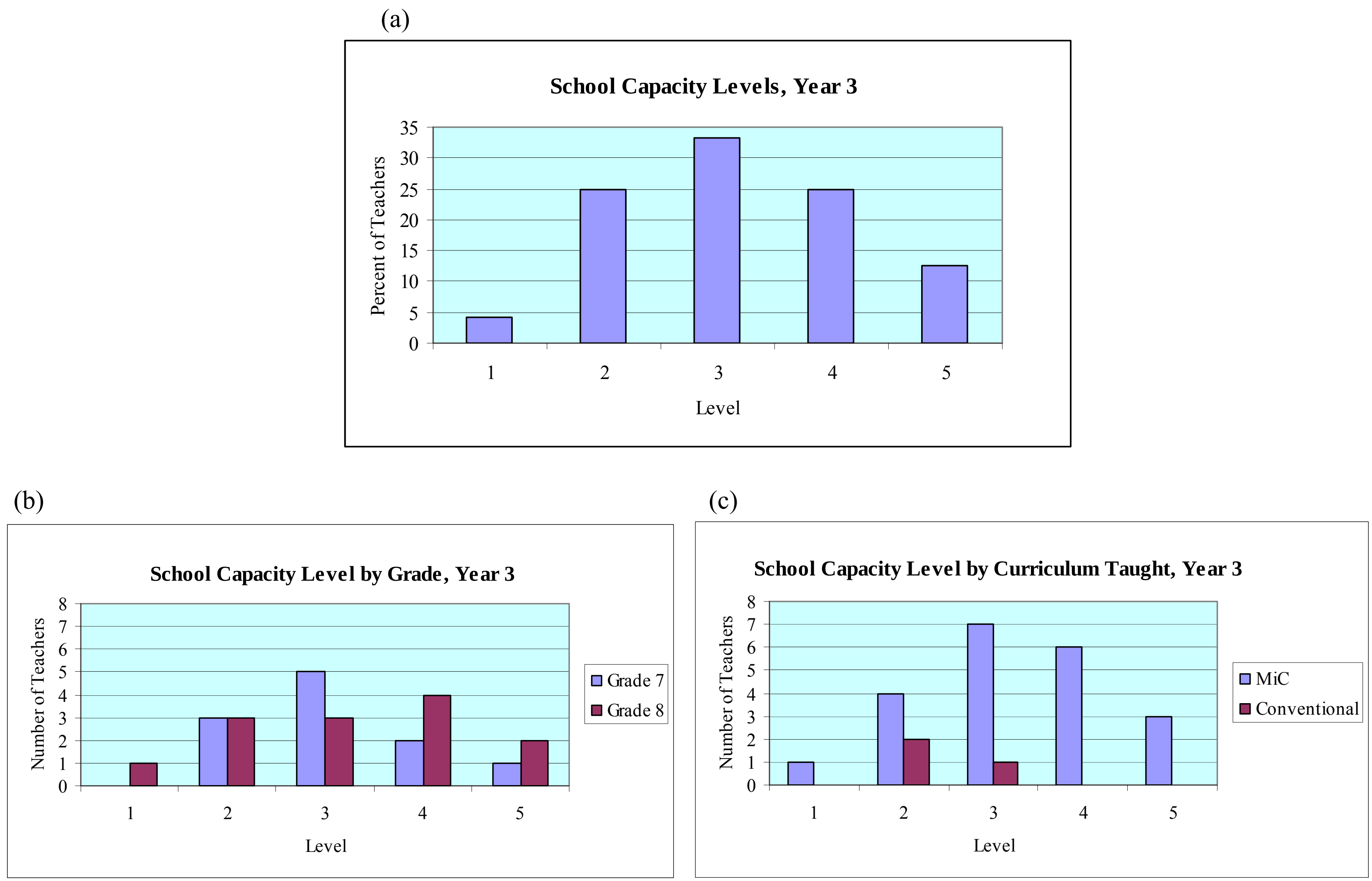

Figure 3-5. Teacher level on the composite index *School Capacity*for school capacity, Year 3: (a) by percent of teachers; (b) by grade; and (c) by curriculum.

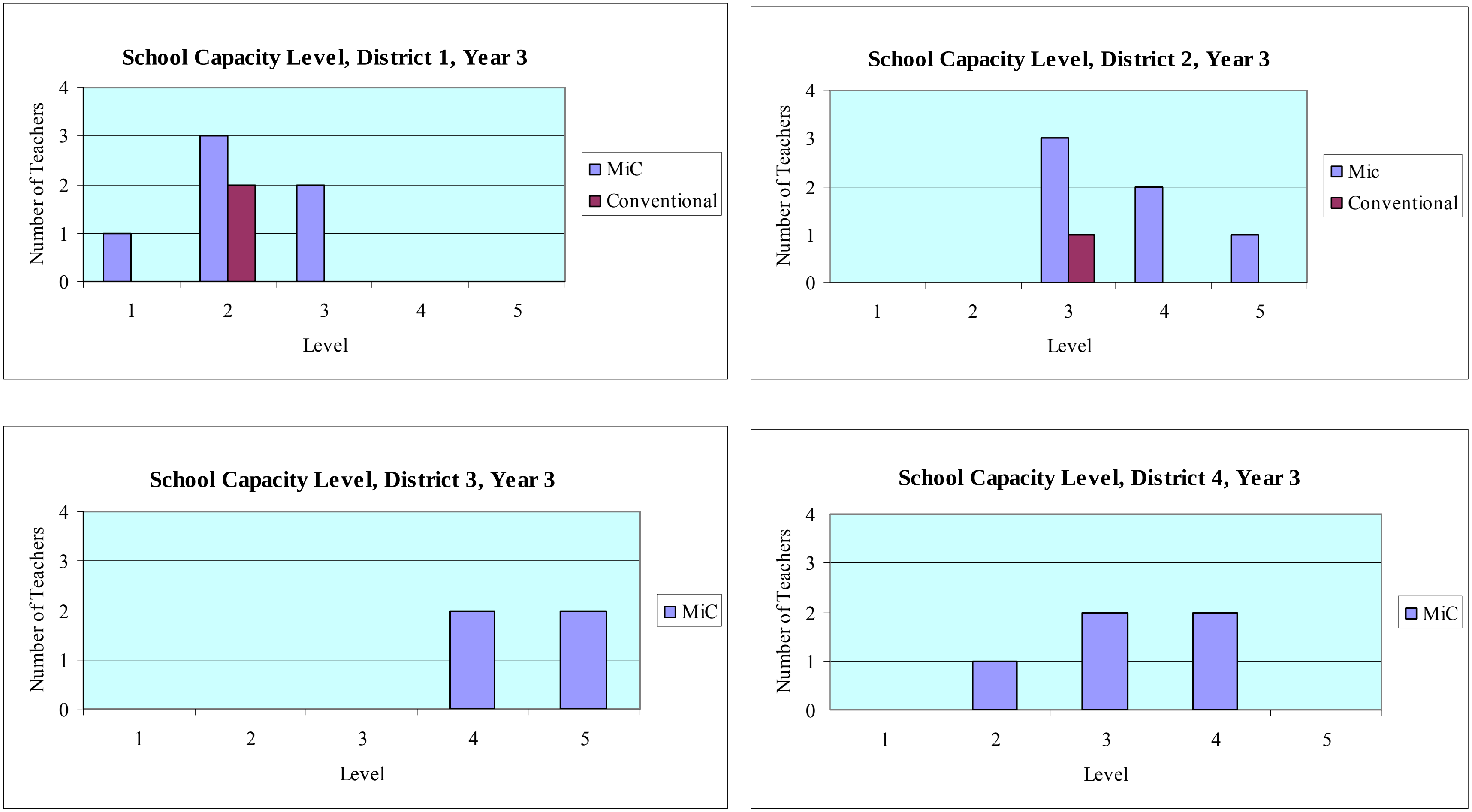

Figure 3-6. Teacher level on the composite index *School Capacity* for school capacity by district, Year 3.

Analysis of Variance

Analysis of the variance was completed to check for significant differences among teachers by grade level, curriculum taught, and district. The School Capacity Total was used as the dependent variable. Because both teachers using MiC and conventional curricula were in Districts 1 and 2, the analysis of variance was completed only for those districts. Results suggest that only 18% of the variance in the School Capacity Total was accounted for by differences in grade level, curriculum taught, and district (see Table 3-3). Nevertheless, the results show an effect for curriculum taught.

Table 3-3.
ANOVA with School Capacity Total as the Dependent Variable

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	176.81	29.46	2.87	<.014
Error	74	760.95	10.28		
Corrected Total	80	937.76			

R-Square	Coeff Var	Root MSE	School Capacity Mean
0.18	17.78	3.20	18.02

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Curriculum Taught	2	71.55	36.77	3.48	0.036
Grade Level	3	24.64	8.21	0.80	0.498
District	1	39.01	39.01	3.79	0.055

When the contrasts were examined (see Tables 3-4 and 3-5), no significant differences between the means of the teachers using MiC and the teachers using conventional curricula, between grade levels, and between the two districts were apparent. However, important differences were noted between curriculum taught and Districts 1 and 2. The results suggest that there were important differences between MiC teachers' perceptions of school capacity and conventional teachers' perceptions, and important differences between teachers' perceptions of school capacity in District 2 and in District 1.[6]

Table 3-4.
Least Squares Means for Curriculum Taught, Grade Level, and District

Least Squares Means	
Curriculum Taught	
MiC	18.62
Conventional	17.30
Grade Level	
Grade 5	19.29
Grade 6	17.66
Grade 7	17.97
Grade 8	17.67
District	
District 1	17.43
District 2	18.87

[6] Given the power of the statistical tests in this study, p-values <.10 are considered to imply important differences, and p-values <.05 are statistically significant differences (Cohen, 1988).

Table 3-5.
Contrasts in Means for the School Capacity Total by Curriculum, Grade, and District

Parameter	Estimate	Standard Error	t Value	Pr > |t|
		Curriculum Taught		
MiC vs Conventional	1.39	0.77	1.80	0.075
		Grade Level		
Grade 5 vs Grade 6	1.63	1.53	1.42	0.160
Grade 5 vs Grade 7	1.32	1.09	1.21	0.229
Grade 5 vs Grade 8	1.62	1.20	1.34	0.184
Grade 6 vs Grade 7	-0.31	0.90	-0.34	0.731
Grade 6 vs Grade 8	-0.01	1.05	-0.01	0.990
Grade 7 vs Grade 8	0.29	0.99	0.03	0.765
		District		
District 1 vs District 2	-1.43	0.73	-1.95	0.055

Conclusion

Based on seven subcategories of school capacity, the composite index *School Capacity* served as a useful tool in describing teachers' perceptions of the capacity of their schools to support mathematics teaching and learning. The results suggest a relationship among the *cultural conditions* of *administrative support* and *school as a workplace*. The results are consistent with previous research that was conducted in schools in which conventional curricula were used and confirm the significance of these subcategories in schools implementing MiC, a standards-based middle-school mathematics curriculum. Although not statistically significant, the results suggest that there were important differences between MiC teachers' perceptions of school capacity and conventional teachers' perceptions, and important differences between teachers' perceptions of school capacity in District 2 and in District 1.

The descriptions of school capacity for each teacher in Districts 1 and 2 are one aspect of the eight grade-level-by-year studies described in Monograph 1, Chapter 2, and they are used in Monographs 5, 6, 7, and 8 to further develop our understanding of the achievement results for each class of students.

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