The Longitudinal/Cross-Sectional Study of the Impact of Teaching Mathematics using *Mathematics in Context* on Student Achievement

Monograph 8
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Implications and Conclusions

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The L/CSS Monograph Series

This is the last of eight monographs derived from the National Science Foundation-funded Longitudinal/Cross-Sectional Study of the impact of teaching mathematics using *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997-98) on student achievement.

In 1992, the National Science Foundation (NSF) funded several projects to develop new sets of instructional materials that reflected the reform vision of school mathematics espoused by the National Council of Teachers of Mathematics (NCTM, 1989). One of the funded projects was to the National Center for Research in Mathematical Sciences Education (NCRMSE) at the University of Wisconsin–Madison. The project was organized to develop a comprehensive mathematics curriculum for the Grades 5–8 (NSF Grant No. ESI-9054928). Assisted by the staff of the Freudenthal Institute (FI) at the University of Utrecht in The Netherlands, the *Mathematics in Context* (MiC) curriculum materials were created and field-tested prior to being published in 1997-98 by Encyclopaedia Britannica.

In 1996, as the development of the MiC materials was nearing completion, a proposal was submitted to the National Science Foundation to investigate how teachers were changing their instructional practices in schools whose staffs were using *Mathematics in Context* and how such changed practices affected student achievement. Two NSF grants were awarded to the University of Wisconsin–Madison: first, to conduct a three-year study of the impact of *Mathematics in Context* on student mathematical performance (NSF Grant No. REC-9553889); and second, to analyze the data gathered in that study (NSF Grant No. REC-0087511). This monograph series presents the methodologies used in and the results of scaling the instruction students experienced, their opportunity to learn comprehensive mathematics with understanding, and the capacity of their schools to support teaching and learning mathematics.

As students and teachers begin to use any of the new mathematics materials, district administrators, mathematics educators, teachers, parents, and funding agencies express cogent needs to demonstrate that the curricula have a positive impact on students' understanding of mathematics. They often want to know the bottom line—the results on measures of achievement that confirm improved student mathematical performance. However, while improved student performance is critical, we contend that just relying on outcome measures to judge the impact of a standards-based program is insufficient. In fact, it is not enough to consider student outcomes in the absence of the effects of the culture in which student learning is situated, the instruction students experience, and their opportunity to learn comprehensive mathematics content in depth and with understanding. The dynamic interplay of all these variables has an impact on student learning, and as such, these variables must be considered in making judgments about the impact of any standards-based curriculum.

This monograph series tells the complex story of the variations in how the MiC materials were used by teachers and students in classrooms that vary in location and ecological culture, and the impact of that variation on the achievement of their students. The story unfolds in eight monographs. This last monograph provides the research directors an opportunity to reflect on the study and what we have learned by the effort.

L/CSS Monograph Series on the Impact of Teaching *Mathematics in Context* on Student Achievement

Introduction to Monograph 8

This monograph contains three chapters. Their purpose is to share what we believe we have learned as a consequence of conducting such a longitudinal/cross-sectional study. In Chapter 1, six stories about the ways teachers implemented *Mathematics in Context* are presented. The objective of presenting these stories is to make clear that what happens in the interaction between teachers and students in real classrooms cannot be well captured and presented via the numerous indices used in this study. While we are satisfied with the findings reported in the previous three monographs in this series, there is more to implementing a reform middle-school curriculum than is represented by the quantitative information presented there. The six stories in this chapter portray implementation of MiC by the same teacher over multiple years, teachers in an elementary-middle school feeder pattern, different teachers across grade levels in one middle school, and teachers of students with low abilities and special needs.

In Chapter 2, our purpose is to provide answers to answer questions about what school districts can expect with respect to the variation in how teachers will use a mathematics program that is implemented for the first time and the variation in student performance as a consequence of using the program.

In Chapter 3 we conclude this monograph series with a list of summary statements of what the research staff has learned as a consequence of conducting the study. The statements encompass what we learned about study design, instrument construction, and methods of data analysis; working in large urban districts with administrators, support staff, and teachers; study students; and the teachers who allowed us to gather information from them and from their students. The chapter ends with out answers to the three research questions we posed along with our conclusions.

CHAPTER 1: IMPLEMENTATION STORIES

Mary C. Shafer

In this chapter, stories of the ways teachers implemented MiC are presented to illustrate how differences in teachers' beliefs, school context, teaching and assessment practices, and instructional planning have an impact on student performance. It is clear that what transpires in the interaction between teachers and students in actual classrooms cannot be captured and presented well in the numerous, separate indices used in the study. For the six stories in this chapter classroom achievement scores for students in classes of particular teachers are linked to implementation practices.

High Quality Implementation: Same Teacher Over Two Years

Ms. Keeton taught MiC in Grades 7 and 8 at Guggenheim Middle School in District 2, a large urban school district. A block schedule was in place in her school; class periods were two hours in length and met every other day. Ms. Keeton looped with her classes; that is, she moved from grade level to grade level with her students. She taught multiple classes each day; two of these classes were in the research study. Ms. Keeton's teaching and classroom assessment practices and the opportunity she presented for students to learn mathematics with understanding led to a high quality implementation of MiC, which reflected the pedagogy and comprehensive content that the MiC development team had intended.

Ms. Keeton's vision about mathematics teaching and learning was highly aligned with her principal's vision. Principal Burnett believed the role of his school was to prepare students for the technologically-based job market they would enter in the twenty-first century. He stressed the importance of providing students with the problem solving and analytic skills necessary to compete for higher paying jobs and that would enable them to be flexible life-long learners. Burnett defined mathematics as problem solving and underscored the need for mathematics instruction that capitalized on the connections between mathematics and real life applications. Although Burnett indicated that students have different learning styles and capabilities, he believed that, overall, engaging them in problem solving experiences best develops mathematical ideas and enables students to apply and connect knowledge in new situations (Interview 9/18/97). Similarly, Ms. Keeton felt that students learned best when they used manipulatives and when they applied mathematical ideas in real world contexts. She felt it was crucial that students were actively involved in instruction and that they benefited from exploration of ideas rather than listening to lectures. Ms. Keeton viewed mathematics as problem solving, and she valued students' ability to problem solve over the mastery of basic facts and skills. Ms. Keeton felt that opportunities for students to reflect on their learning were important, and she frequently asked students to summarize what they had learned in class (Interviews 9/18/97, 10/23/98).

School Capacity

Ms. Keeton perceived that the capacity of her school to support mathematics teaching and learning was at a high level. She felt that she received very strong administrative support for changes in instructional practice, and she had a high influence in planning and teaching mathematics. She felt that faculty and staff were committed to academic excellence and that teachers supported one another in their efforts to improve instruction (Interviews 9/18/97, 10/23/98; teacher questionnaires 8/97, 5/99). District and state standardized tests influenced the content and sequence of content she taught during the school year. She felt that the tests were aligned with MiC, but that students still need to practice basic skills. Ms. Keeton commented:

> MiC is very much correlated to the questions on the state test. MiC gives questions that deepen understanding. Students have to prove their understanding. I do pull from other resource activity books or other practice books. I use those as another way of looking at the same thing that they've seen in MiC units. I do pull from other resources to give them practice with algorithms. (Interview 4/12/99)

Throughout the year, this was done on a limited basis, using minimal class time. As the standardized tests approached, she supplemented MiC with commercial test preparation booklets.

By the end of her participation in the study, Ms. Keeton completed a masters degree in mathematics education, which influenced her role as leader of her mathematics teaching team. As the team leader, she made decisions about scope and sequence of mathematics content, innovative curricula, and assessment preparation. During the common planning time available for her teaching team, she led discussions of particular MiC units, mathematics content, instructional and assessment methods, and program evaluation. Ms. Keeton was the first at her grade level to teach units that addressed unfamiliar content, and, if she did not agree with an answer in the teacher guide for a particular task, she brought that up for discussion among teachers during planning time (Interview 4/12/99). She also helped other teachers by sharing her notes about specific MiC units. For each unit she taught, she made notations about implementation strategies such as particular problems in the unit that were best approached in class discussion, questions to pose as students worked in small groups, and problems in the unit that were appropriate for assessment or homework. She copied these notes for the other teachers as they prepared to teach the same unit. During planning time, the teachers always reserved some time to ask questions and share whatever they had learned when teaching various units.

Opportunity to Learn with Understanding

During both years she was in the study, Ms. Keeton presented a comprehensive curriculum. In Grade 7, she taught six units in three content strands (two number, one from Grade 6; two algebra, and two geometry), and in Grade 8, she taught five units in four content strands (one number, two algebra, one geometry, one statistics). In both years, mathematics was explored in enough detail for students to think about relationships among mathematical ideas or to link procedural and conceptual knowledge. Ms. Keeton supplemented MiC with activities disconnected from the curriculum such as practice for district standardized tests and various school or district initiatives.

Instruction

Ms. Keeton held high expectations for her students, and she supported their learning in numerous ways. At the beginning of each class period, she asked students to trace the content they have learned from the beginning of the unit to the previous lesson: "What did we investigate when we began this unit?" "What happened from there?" (Interview 4/12/99). In this way, she helped students look for connections among the lessons, from more exploratory investigations to more explicit content. But she also used this discussion as an opportunity for assessment. For example, in a journal entry after a summary of a unit on number theory, she commented:

> As we reviewed past assignments, students' verbal responses were slow, which led me to believe I needed to place more emphasis on factors and multiples. Students began to respond quickly to my questions for examples of factors and multiples. Then I went ahead with today's assignments. (Journal entry 11/17/98)

After summarizing the unit, Ms. Keeton led a discussion of the day's lesson *before* students opened the unit. She talked about the context used in the lesson, and after reading each task, she posed questions such as "Do you understand what it's asking for?" "What does that mean?" "How are you going to do that?" Then students began their work on the lesson (Interview 10/23/98). This discussion was not designed to funnel students' thinking in a particular direction. Rather, it set the stage for students to rely on their own reasoning as they solved unit problems. Students talked about the meaning of the context, their interpretations of various questions, and the parts of the lesson that might be more challenging for them. In this way, Ms. Keeton made lesson content more accessible to them and allowed them to rely more on their own mathematical reasoning as they solved the problems.

In whole-class discussion and when students worked in small groups, Ms. Keeton pressed students to effectively communicate their thinking processes and defend their answers: "If they can convince me, you know that what they're thinking is appropriate and it does answer the question, it is acceptable. I tell them all the time, defend it. Explain it" (Interview 3/20/98). This kind of interaction promoted inquiry that was sustained during small-group work. For example, when studying about triangles in *Triangles and Beyond* (Roodhardt, de Jong, Brinker, Middleton, & Simon, 1998), Ms. Keeton noted:

> They were very talkative with each other, trying to convince each other whether a particular place in the classroom actually had an example of a triangle. I walked around and heard them say, "That's not a triangle, because this, this, and this." "Yes, it is! You look at that." They've actually gotten up out of their seats to go and convince someone. . . . I say to them a lot, these are not the kinds of assignments that you can just do some arithmetic. . . . These assignments require your thinking and concentration. (Interview 3/20/98)

This kind of discussion was important for students: "They are making some conclusions and sharing information with each other. [Sometimes it's about] their own personal experience and how that fits into the unit. I hear that a lot. It's alive for them" (Interview 4/12/99). For example, in a lesson that involved discussion of the urban and suburban populations, student answers reflected a different perspective on the question "Why would it be important to know whether the population increased or decreased?" Ms. Keeton reported:

> I think the answer in the book said because you may need to build more schools and other buildings, which is the same thing I thought. Well, the kids came up with problems there may be in that city because you wouldn't want to move to that city if there were problems. They were not thinking of a person in the city wanting to know. They were thinking about if the person lived somewhere else and that city had a population change. That was just something that I had not considered. Acceptable answers have logical explanations. (Interview 4/12/99)

Ms. Keeton encouraged active discussion by all students. In order to have each student talk during the class period, she frequently posed questions as she moved among the groups such as "What do you think this means?" "What do you want to know about this?" On other occasions, she asked them to go to the board and demonstrate what they knew. For example, when studying algebraic ideas, she asked them to fill in a table of values on the board and explain how they found each entry (Interview 3/20/98).

The inquiry during instruction promoted learning mathematics with understanding, which is illustrated in a brief description of the lesson from the eighth-grade algebra unit *Patterns and Figures* (Kindt, Roodhardt, Spence, Simon, & Pligge 1998) on 12/11/98. The introductory class discussion focused on organizing data in order to find patterns. Students then worked in groups to describe V- and W-formations of birds and planes, respectively, in which dots were used to represent the sequences. They used tables to organize their work as they explored the relationship between the pattern number and the number of dots in each step in the sequence. They described the patterns they found and generated formulas for the patterns. Ms. Keeton encouraged them to test their formulas to see if the relationships held for all pairs of numbers in the table, and she asked them to relate their formulas to their drawings of the dots in the patterns. In this way, Ms. Keeton promoted connections among the pictorial representations of the context, the table of values, and the generalized formula. The entire lesson focused on conceptual understanding.

Ms. Keeton felt that student work changed as they studied MiC for more than one year. Some students were less reluctant to provide a complete response that included the answer and an explanation. Many realized that even on questions that elicited opinions, a logical defense was important. Because Ms. Keeton taught many of the same students for multiple years, she confidently stated: "I can actually physically point out the students that have changed more in their understanding" (Keeton, Interview 4/12/99). Ms. Keeton also noted differences between students who studied MiC for two years and students who studied MiC for the first time because they were new to the school or were with a different teaching team:

> Students who I've had for the past two years are the ones who sometimes answer these children's questions or explain to them, "Oh, you have to do it this way." They know. They've gone through the "Why do I have to answer all of this?" or "Do I have to read all of that?"

She also noted that new students struggled with "letting go of the traditional algorithm and getting to the understanding." She took these differences into account when she planned instruction.

The summaries at the beginning of every class period helped students look mathematical themes in a series of lessons, and the reading of the current lesson provided opportunities for students to talk about the lesson context, interpretations of tasks, and portions that might be challenging. In this way, the lesson became more accessible to students and allowed them to form reasoned responses. Students' explanations were taken as evidence of their learning, and answers were deemed acceptable when students defended them through logical reasoning. Active participation by all students was a major goal, as every student was given a chance to talk about the mathematics or their reasoning during each class period in whole class or small group settings. Classroom interactions were used to promote making sense of the mathematics.

Instructional Planning

Ms Keeton felt that planning to teach MiC was more time-intensive than planning to teach the traditional texts she had used previously. She stated:

> It forces me to be creative. It forces me to try to think of other context areas where the students can apply whatever skill they're learning about. This is much different than traditional arithmetic that I memorized when I was in middle school. I can, you know, get up there and teach the kids how to reduce a fraction or whatever. But being able to show them where these things are in the real world—for me it requires a lot more time to plan. (Interview 3/20/98)

In planning for teaching a unit, Ms. Keeton worked through the entire unit by herself, including special activities and investigations. She wrote notes to herself about the content, ways to introduce lessons, and the number of pages she wanted students to complete during a class period. She made answer sheets with potential student responses. If something was confusing, she asked other teachers on her teaching team during their common planning time. She also shared other information about the unit, such as the ways the content is developed and new strategies that are presented. When planning to teach a particular lesson, she worked through the lesson tasks again, confirming the notes she had previously written and adding things based on her interaction with students in the previous class period. Directly before teaching she went through the lesson again in order to remember the details of the context and the critical points to attend to in discussion. During teaching, students sometimes showed her different ways of approaching and solving tasks or they offered responses that she had not considered in planning. She recorded this newly discovered information in her notes for the unit (Interviews 3/20/98; 4/12/99).

Classroom Achievement

In the analysis reported here, a teacher group represents all study students taught by a particular teacher in a particular year. For research purposes, students included those who completed both assessments designed for this study. These assessments provided attention to knowledge of mathematics content and application of mathematics in problem solving situations. The results from these assessments were used to create a single proficiency scale, which was used in determining classroom achievement.

Changes in students' performance in Ms. Keeton's classes are shown in progress maps developed from the classroom achievement proficiency scale (see Figure 1-1). The summary data include the number of students, mean, standard deviation, both lower and upper 95% confidence intervals, and score distributions (see Table 1-1). The variation in classroom achievement is displayed in charts that portray the distribution of scores, rather than just the mean (shown by the white line). If the confidence intervals (shown in the black area) do not overlap, the means are considered to be significantly different. The results for 90% of the students are displayed in each progress map.

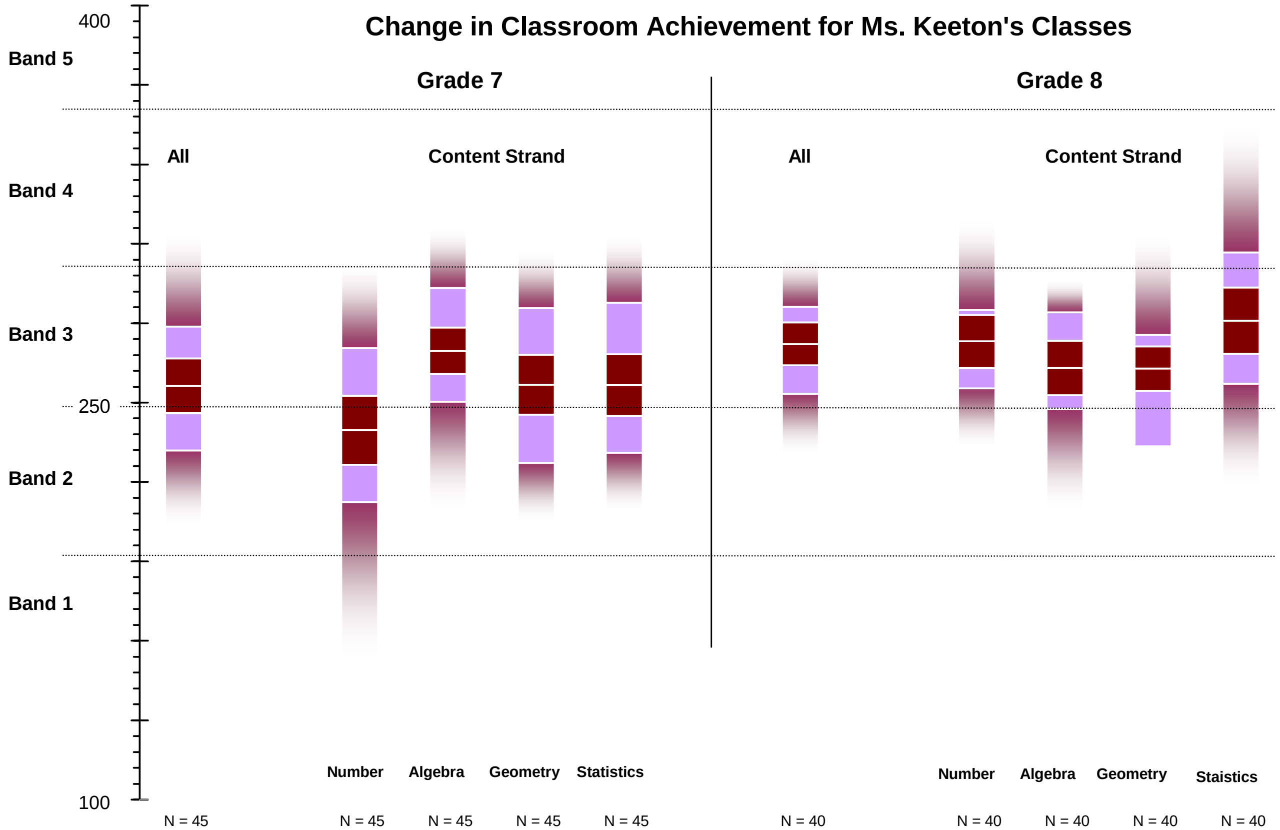

Figure 1-1. Change in classroom achievement from Grade 7 to Grade 8 for students in Ms. Keeton's classes, overall and by content strand.

Table 1-1.
Classroom Achievement for Students in Ms. Keeton's Classes in Grades 7 and 8, Overall and by Content Strand

				95% Confidence Interval		Score distribution percentiles			
	N	Mean	*SD*	Lower	Upper	95%tile	75%tile	25%tile	5%tile
Year 1									
Overall	45	256.3	35.4	246.0	266.7	313.8	278.7	231.9	203.6
Number	45	239.6	44.9	226.4	252.7	302.6	270.6	212.5	153.3
Algebra	45	269.6	30.3	260.7	278.4	315.6	293.4	250.3	209.9
Geometry	45	256.8	38.6	245.5	268.1	307.1	285.6	227.2	205.1
Statistics	45	256.6	40.0	244.9	268.3	312.7	287.8	231.0	209.1
Year 2									
Overall	40	272.2	26.1	264.0	280.3	304.7	286.2	253.3	230.7
Number	40	273.1	32.3	263.1	283.1	318.4	284.9	255.3	233.3
Algebra	40	263.0	33.1	252.8	273.3	295.9	284.1	247.6	209.0
Geometry	40	262.8	27.3	254.4	271.3	312.9	275.5	233.4	233.4
Statistics	40	281.0	40.2	268.5	293.4	354.0	306.8	257.1	218.6

The overall mean score for students in Ms. Keeton's classes increased from 256.3 to 272.2, which is a statistically significant improvement. Their performance showed less variance in Grade 8, with most students in Band 3, and over 25% of the students were in Band 4 in the statistics strand. Significant differences were apparent with respect to the number and statistics content strands. The performance in the number strand suggests that students' abilities to work with numbers grew dramatically, despite only one eighth-grade unit devoted to the number strand. The performance in the statistics strand reflects the important impact of students' opportunity to learn one of the eighth-grade MiC statistics units.

Changes in classroom achievement were likely influenced by the quality of instruction and opportunity to learn with understanding that students experienced in Ms. Keeton's classes. In both Grades 7 and 8, Ms. Keeton taught the units in an in-depth way with attention to conceptual understanding and connections among mathematical ideas. She taught MiC as her primary curriculum, teaching units in multiple content strands with limited class time devoted to practicing basic skills. The combination of teaching for understanding and teaching MiC throughout the year contributed to the performance of Ms. Keeton's students in important ways.

Advantages, Disadvantages, and Implementation Issues

Ms. Keeton talked about many advantages when she taught MiC. She listed the following as primary advantages for students:

- "The kids are able to see the math in real life. They can relate it to a real life situation outside of the math class" (Keeton, Interview 3/20/98).
- "The kids are interacting more with each other on a positive note, which avoids a lot of behavior problems. I don't have to deal with detentions for yelling out or rudeness because they have something to talk about" (Keeton, Interview 3/20/98).
- "I think the retention of skills is better with the MiC. It's better than just standing there telling them an algorithm for how you do it. I think with MiC, because they're actually working with a context about something, the retention of it is better" (Keeton, Interview 3/20/98).
- "The fact that it's a core curriculum [is an advantage]. I mean, kids use this content in other subject areas. The kids will point out to me that they talked about this in science or this in social studies" (Keeton, Interview 3/20/98).
- "[On the state standardized tests] students are now being held responsible for actually understanding their skills rather than just producing the answers for a particular set of questions. So that's one definite advantage of MiC because they get the practice with explanation. There is also a state writing assessment. The essay exam is to give, in a good clear paragraph, an explanation to some question or topic. MiC has been a big help with assisting in that area" (Keeton, Interview 4/12/99).

Ms. Keeton also listed advantages for teachers. She felt that teaching MiC allowed her to be more creative because she was not teaching with the same plans year after year as she had done with the traditional textbook. More importantly, however, she talked about opportunities for teachers to learn mathematics with conceptual understanding:

> It has given me along with my co-workers a better understanding of what we knew. For example, we knew how to teach things backwards and forwards like slope. . . .but I just learned the actual connection that was presented in *Graphing Equations*. . . . That was a very good learning experience for me. I think that's one of the major advantages because now the teachers will have more knowledge of what they're teaching or have taught. (Keeton, Interview 4/12/99)

From Ms. Keeton's perspective, these opportunities for teacher learning were significant as teachers on her team taught and discussed MiC units.

Ms. Keeton outlined several disadvantages and implementation issues when using MiC, but, when possible, she noted ways she addressed them:

- Because the introduction of the lesson was critical for students to pursue the mathematical work on their own, she felt that students who were absent from school had missed important opportunities. In response, she began an after-school program one day a week to give a thorough presentation for students who had missed classes (Keeton, Interview 3/20/98).
- She was concerned that students do not connect their experiences in MiC algebra units with vocabulary they will see when they study a traditional algebra course in high school. For example, students worked with functions in MiC but they are not referred to as functions. Students used tables to organize data, but do not connect that to traditional t-tables of values with values for *x* in one column and values for *y* in another (Keeton, Interview 4/12/99). She added these terms as students studied the units.
- Some unit tasks were vaguely worded. At times, Ms. Keeton had a different approach than the solutions noted in the teacher guide. In that case, she offered to students both her ideas and those in the teacher guide. She commented: "It forces me to constantly think of all the different ways" students might solve a task (Keeton, Interview 3/20/98). But that, in turn, meant that there were more possibilities to interpret for their worthiness as acceptable answers.
- Grading students' written work was time-intensive. Grading was compounded when students responded in ways she did not anticipate. She had to refer back to the original task in the unit in order to check the worthiness of the response. In the second year, she more frequently assessed students using informal methods (asking questions, observation) during whole-class discussion and group work.
- Pacing was very difficult to judge, particularly when factoring in professional days and the need to integrate school and district initiatives into various class periods. (Keeton, Interviews 3/30/98, 4/12/99).

However, Ms. Keeton felt the advantages of students studying MiC far outweighed the disadvantages and difficulties in implementation: comprehensive content, retention of skills, explanation of reasoning, increased interaction centered on mathematics during instruction, and the opportunities for teacher learning that were available as teachers discussed the content of MiC units and their teaching and assessment practices.

Summary

In both Grades 7 and 8, Ms. Keeton's teaching and classroom assessment practices and the opportunity she presented for students to learn mathematics with understanding led to a high quality implementation of MiC, which reflected the pedagogy and comprehensive content that the MiC development team had intended. The significant gains noted in students' performance over time, particularly in number and statistics, were likely influenced by these practices. Ms. Keeton believed that teaching MiC led to advantages for both students and teachers. For students, advantages included improved student interaction, written and oral communication, and retention of skills, and connections between mathematics and real-world experiences and other subject areas. For teachers, MiC provided opportunities to learn mathematics conceptually, to discuss mathematics with other teachers, and to talk about teaching and assessment practices. These benefits outweighed the difficulties in implementation related to student absences, grading student work, and pacing instruction that she experienced.

Attempting to Teach Mathematics for Understanding: The Same Teacher Over Three Years

During the study, Mr. Dillard taught MiC in Grades 6, 7, and 8 at Guggenheim Middle School. He was on the same teaching team as Ms. Keeton. Mr. Dillard also looped with his classes; beginning in Grade 6, he moved with his students from grade level to grade level. Two of his classes were in the research study. Mr. Dillard's teaching and classroom assessment practices and the opportunity he presented for students to learn mathematics with understanding were not as strong as Ms. Keeton's implementation of MiC. However, he taught multiple units each year and attempted to teach mathematics for understanding in ways that MiC development team had intended.

Mr. Dillard believed that students learn mathematics in different ways. While some students feel successful with traditional instruction in which they are expected to follow teacher-presented methods, others prefer using reasoning as they investigated problems or using manipulatives to support their thinking. He noted a change in his thinking about applying mathematics after teaching MiC, now striving for a balance between using mathematics in real-world contexts and learning mathematics procedurally. Mr. Dillard believed that students use basic skills when solving problems. However, because students who did not have a command of these skills became frustrated and gave up more quickly than students who had learned basic skills, he felt that basic skills should be reviewed periodically (Interviews 9/19/97, 10/20/98, 9/20/99).

School Capacity

Mr. Dillard perceived that the capacity of his school to support mathematics teaching and learning was average. He felt that the principal and teachers had visions of teaching and learning mathematics that were aligned on some ideas, but were incompatible on others. He felt that he received an average level of administrative support for changes in instructional practice, and he had a high influence in planning and teaching mathematics. He met regularly with other teachers for collaboration in lieu of professional development at the school. As a member of the same teaching team as Ms. Keeton, Mr. Dillard discussed content, instructional and assessment methods, and program evaluation with other teachers. He believed that MiC was well aligned with the state standardized testing program, and these tests had little influence over curriculum and instruction for his students in Grades 6. But this changed in Grades 7 and 8 as he prepared students to take the additional performance-based assessment as part of the state testing program in

Grade 8. He spent a few minutes during each class period for students to solve practice tasks and talk about possible solutions (Interview 4/20/00; teacher questionnaires 8/97, 5/00).

Opportunity to Learn with Understanding

In the three years he was in the study, Mr. Dillard taught primarily algebra and geometry units. In Grade 6, he taught six units in three content strands (number, algebra, and geometry), and in Grade 7, he taught three algebra units and one geometry unit. In Grades 6 and 7, he supplemented MiC with activities disconnected from the curriculum, such as a district-mandated reading program and practice for standardized tests. In Grade 8, Mr. Dillard taught four units, two each in algebra and geometry, and supplemented MiC with a conventional high school algebra textbook midway through the second semester. At all grade levels, he attempted to teach for conceptual understanding, although he also emphasized students' procedural understanding. Connections among mathematical ideas and connections between mathematics and students' lives were not discussed in detail.

Instruction

When Mr. Dillard began teaching MiC, he struggled with providing time for students to solve and discuss unit tasks, but he gradually developed ways for this to occur. He explained: "I learned to introduce the lesson, ask students to work on a few problems, and reconvene to discuss the content with the whole class, and then repeat that during the lesson" (Personal communication 2/26/98). Mr. Dillard encouraged students to talk about the mathematics they were learning by explaining their ideas to others, asking each other questions, and making sure that everyone had written explanations as well as answers for unit tasks. Occasionally, he read student responses to the class and asked if individual responses were acceptable. Students listed reasons for stating the response was acceptable such as why the argument was valid, and they explained missing elements of the responses that were not acceptable, for example, incomplete arguments, ineffective wording, or lack of more exact answers (Interviews 3/23/98, 4/12/99, 4/20/00). Mr. Dillard felt that students participated more fully in discussions when they studied MiC because there were different ways to solve problems, and he noticed changes in students' discussion from one grade level to another. As they became more comfortable with MiC units, they were able to work for longer periods of time without his intervention, and they became more specific in their arguments and validation of acceptable answers (Interviews 4/12/99, 4/20/00).

Mr. Dillard attempted to teach mathematics for understanding, which is illustrated in a brief description of the lesson from the eighth-grade algebra unit *Graphing Equations* (Kindt, Wijers, Spence, Brinker, Pligge, & Burrill, 1998) on 11/2/99. The class period began with a 50-minute review of homework from page 43 in the unit. The lesson involved writing equations of lines, determining whether lines were parallel, and finding coordinates of points of intersection. Even though this content had been introduced and discussed in prior lessons, Mr. Dillard addressed the topics again in his introductory presentation to the class. He posed questions that attempted to get students conversing about their responses, but most students only gave brief answers. Some students had conjectured that parallel lines have the same slope. However, it became apparent that students still had not made connections between the equation of the line, the slope of the line, and the y-intercept. Trying to encourage discussion, Mr. Dillard mentioned that graphs of lines and their equations should reflect the same information. But he did not ask students who had made conjectures to share their thinking with the class. After this discussion, students began work on the next page in the unit, which was a summary of that section in the unit and required students to use a graphing calculator for finding points of intersection for various lines. As he observed students, Mr. Dillard found that some students did not know how to proceed, and he talked about the content of the lesson with these students individually. Even though they had used the calculators at multiple points during the unit, productivity was hampered by students' inability to use them appropriately. Students sat in groups, but they did not work together on the assignment.

In this lesson, Mr. Dillard attempted to help students develop understanding of the mathematics through his introductory presentation and the questions he posed. However, elaboration of the reasoning that may have promoted such understanding did not transpire. Although some students made conjectures about parallel lines, talking about their reasoning may have sparked others to think about parallel lines in a different way. Student understanding was further affected by lack of ability to use the calculators for the purpose of the lesson. A review of how to use the calculators for the whole class likely would have helped all students as they worked on the assignment.

Instructional Planning

Mr. Dillard felt that teaching MiC units involved more planning time than to teach traditional mathematics curricula. He worked through all unit problems himself, looking for ways different tasks were related. Through this process, he thought about how he would introduce each lesson and how his students might approach and solve the tasks. Even with this preparation, students come up with additional methods that he had to consider on-the-spot during instruction. Mr. Dillard planned by himself, but then discussed ideas for presenting lessons, especially the hands-on activities, with other teachers on his teaching team. They also talked about portions of other units that were important for students to know in order for students to be successful with the unit being planned and any critical portions of the unit that deserved special attention. He found that these discussions helped him present better lessons. When planning to teach individual lessons, Mr. Dillard worked to develop an introduction to the lesson that featured something students would relate to and would get them thinking about the topic of the new lesson before they opened the unit. On some occasions, he planned to work through some examples as a class before students worked on their own. He learned over time to present the lesson, have students begin to work on a few tasks, reconvene for class discussion, and return to small-group work. In this way, he allowed students to check their work and verbalize ideas to the larger group in order to keep them focused on the mathematics and keep them on a successful track (Interviews 3/23/98, 4/12/99, 4/20/00).

Classroom Achievement

Changes in students' performance are shown in the progress maps in Figure 1-2 and the summary data in Table 1-2. From Grade 6 to Grade 7, the overall mean score for students in Mr. Dillard's classes decreased modestly from 267.2 to 263.6. The mean score in the statistics strand increased, and the mean in the number strand remained the same. However, the mean score in the algebra strand decreased, and the decrease in the mean score in the geometry strand was statistically significant. From Grade 7 to Grade 8, the overall mean score increased from 263.6 to 285.1, which is a statistically significant gain. Furthermore, the gains in the number, algebra, and statistics strands also were statistically significant. The performance shows less variance in Grade 8, with most students in Band 3, and nearly 50% of the students in Band 4with respect to the statistics strand. In Mr. Dillard's classes, the combination of content taught in some depth and instruction that attempted to promote conceptual understanding seemed to have a positive effect on student performance.

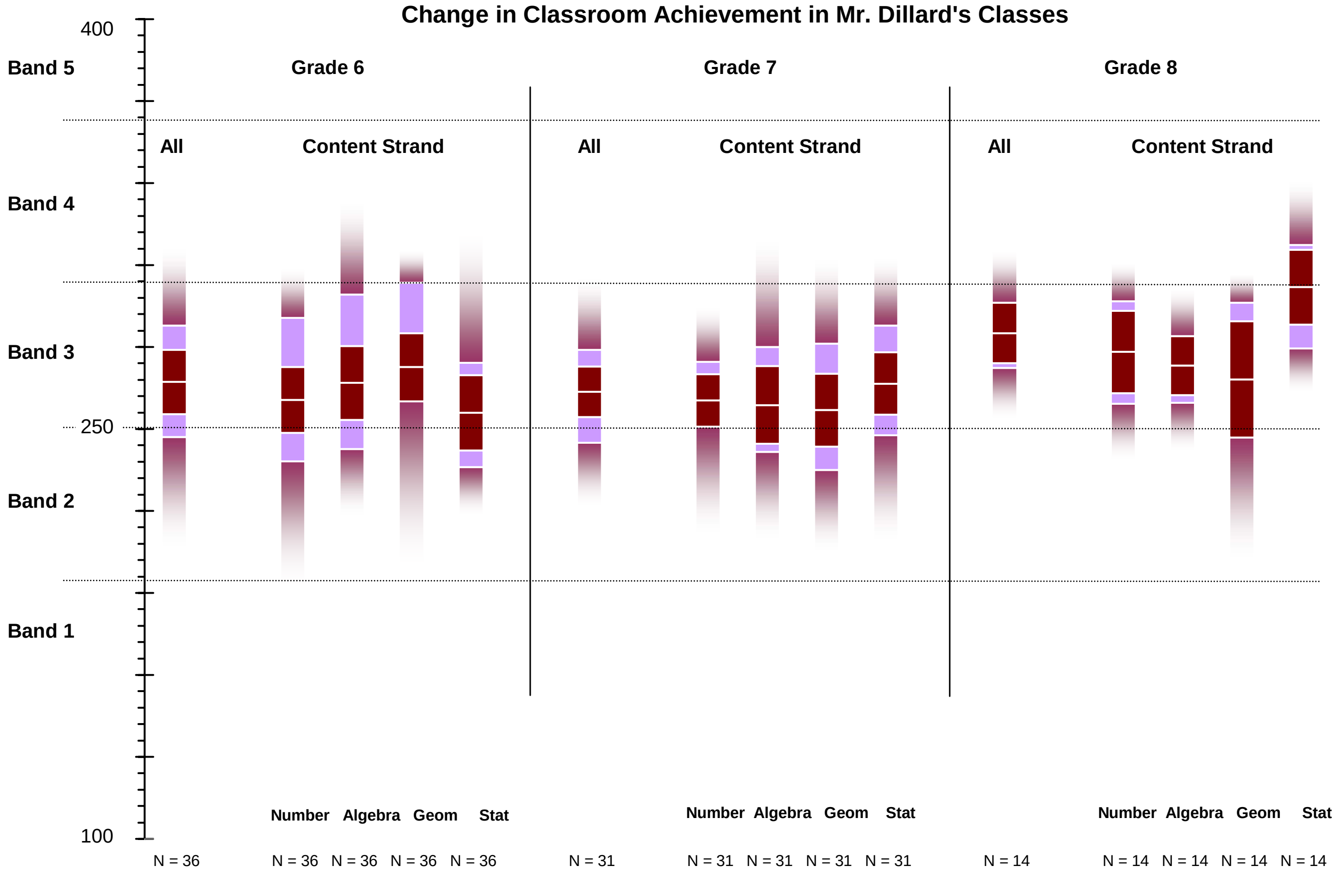

Figure 1-2. Change in classroom achievement from Grade 6 to Grade 8 for students in Mr. Dillard's classes, overall and by content strand.

Table 1-3.
Classroom Achievement for Students in Mr. Dillard's Classes in Grades 6, 7, and 8, Overall and by Content Strand

				95% Confidence Interval		Score distribution percentiles			
	N	Mean	*SD*	Lower	Upper	95%tile	75%tile	25%tile	5%tile
Year 1									
Overall	36	267.2	36.2	255.4	279.0	316.2	287.8	247.1	206.5
Number	36	260.7	36.8	248.6	272.7	308.5	290.6	238.1	192.6
Algebra	36	266.8	41.4	253.3	280.4	333.3	299.2	242.7	218.3
Geometry	36	273.2	38.9	260.4	285.9	316.7	304.7	264.0	202.7
Statistics	36	256.0	42.2	242.2	269.8	321.1	274.3	236.0	218.5
Year 2									
Overall	31	263.6	26.3	254.3	272.8	305.0	279.0	244.9	221.7
Number	31	260.5	27.6	250.8	270.2	294.7	274.8	251.9	212.5
Algebra	31	258.8	40.4	244.5	273.0	319.2	280.1	241.6	209.9
Geometry	31	256.9	37.9	243.6	270.3	312.2	281.3	235.1	205.1
Statistics	31	266.6	32.4	255.2	278.0	312.7	287.8	247.8	209.1
Year 3									
Overall	14	285.1	21.2	274.0	296.2	315.0	296.3	272.4	253.8
Number	14	278.2	28.8	263.1	293.3	310.7	296.7	259.3	238.6
Algebra	14	273.2	20.6	262.4	284.0	301.4	284.1	259.6	242.2
Geometry	14	268.7	42.0	246.7	290.7	308.5	297.7	251.6	205.6
Statistics	14	301.9	26.3	288.1	315.6	340.5	317.3	279.4	263.7

Advantages, Disadvantages, and Implementation Issues

Mr. Dillard talked about important advantages of teaching MiC:

- "Students get a deeper understanding of what they're doing" (Dillard, Interview 3/23/98).

- "The units get students to think more about the mathematics. MiC gives them ways to actually explain why something works. For example, when we do some of the algebra units, you can actually show them the distributive property with the standing and laying bricks, they can actually physically count the pattern out. . . . I gave them a pre-algebra worksheet that had a formal definition of a distributive property, and when they read it they said, 'I have no idea what they're talking about.' We worked a couple exercises on the board, and I said, 'Remember when it was the brick pattern? That's the same thing here.' They said, 'Oh, yeah, that's easy.' So in the transition of going from concrete to abstract, you can use that to kind of build a bridge. MiC does help in that aspect. I think that next year when they start doing algebra with all the abstract concepts that if you can relate it to experiences in MiC, where they did actually physically did it, it will make it easier and they will have a better understanding of it. That's a big advantage for them" (Dillard, Interview 4/12/99).
- "MiC goes more in-depth. I think they learn more about mathematics. I think it's great as an informal preparation for algebra and geometry. I think that it's going to benefit them greatly when they go to take a formal algebra and geometry class where they can see where it came from, why we need variables, and how they're represented" (Dillard, Interview 4/20/00).

Mr. Dillard outlined several disadvantages and implementation issues when using MiC:

- "The students, even this year, seem to fight MiC at the beginning of the year because it's something different" (Dillard, Interview 4/20/00).
- "Some students haven't studied MiC before. Some in eighth grade now didn't have MiC in sixth and seventh grade. It's very difficult for them to walk into a classroom that's been doing MiC. For example, when the new unit refers to something from a previous unit or a unit from last year, new students don't have any idea what the unit is talking about. Then I have to go back and explain it or find a student that remembers it well and let them work with that particular student" (Dillard, Interview 4/20/00).
- "MiC is very time-consuming for teachers in pretty much all aspects. It takes more time to plan, to prepare mid-term exams, and grade student papers" (Dillard, Interviews 3/23/98, 4/12/99, 4/20/00).
- "It's hard to get through all the units. It's one of the reasons why I taught all the algebra units because next year they'll have a traditional algebra book, and they'll have MiC algebra again as their honors algebra curriculum" (Dillard, Interview 4/12/99).
- "The high school has been very upset with us because we don't teach algebra from the traditional textbook. We have argued that they are wrong. They've argued that we are wrong. That's still going back and forth, but it is getting better" (Dillard, Interview 4/12/99).
- "There are a few parents who disapprove of MiC, probably because it's something different from what they did and they don't know how to do the mathematics. They haven't been exposed to it that way" (Dillard, Interviews 3/23/98, 4/12/99).

Despite these difficulties in implementation, Mr. Dillard believed that MiC is more beneficial for students than the traditional middle-school curricula: it allows students to think deeply about the mathematics and develop a good understanding of mathematics, and studying MiC provides a foundation that serves as a bridge to the more abstract study of algebra and geometry in high school. He also felt that the time he spent discussing the content and presentation of MiC units with other mathematics teachers on his teaching team made a difference in his teaching practices.

Summary

Mr. Dillard attempted to teach mathematics for understanding and he provided opportunities for students to learn mathematics in multiple units each year. Although students' performance declined from Grade 6 to Grade 7, the gains in performance from Grade 7 to Grade 8 were statistically significant in number, algebra, and statistics. In Mr. Dillard's classes, the combination of content taught in some depth and instruction that attempted to promote conceptual understanding seemed to have a positive effect on student performance. Mr. Dillard believed that MiC allowed students to think deeply about mathematics and to develop a good understanding of mathematics. He also felt that the discussions he had with other teachers about the content and presentation of MiC units made a difference in his teaching practices. These benefits offset the time expended in planning to teach the units and in assessing student learning.

High Quality Implementation Over Two Years: Elementary into Middle School

In this section, we explore implementation of MiC by two teachers in different schools in the same feeder pattern. Ms. Mitchell taught MiC in Grade 5 at Dewey Elementary School, and Ms. Weatherspoon taught MiC in Grade 6 at Fernwood Middle School in District 1, an urban school district. Both teachers taught mathematics to multiple classes each day. Three of each teacher's classes, two classes of average ability and one of lower ability, were in the research study. The teaching and classroom assessment practices and the opportunities provided for students to learn mathematics with understanding for both teachers led to high quality of implementations of MiC, which reflected the pedagogy and comprehensive content that MiC development team had intended.

Ms. Mitchell thought that the best way for students to learn mathematics was dependent on their ability level and learning style: "I think some children learn faster from using concrete materials and manipulatives. Others learn faster from I think just basic rote memory. A lot of children learn through rote memory. . . . In my opinion, there is just no one best way for children to learn mathematics" (Interview 9/16/97). Ms. Mitchell emphasized problem solving and sharing of different strategies in classroom instruction. Although she felt that mathematics learning proceeds hierarchically and certain skills must be mastered before new ones can be learned, she attempted to place less emphasis on rote memorization and more on activities in which “students are going to get a full understanding of what they are learning” (Interview 9/16/97).

Ms. Weatherspoon thought that students learned mathematics through problem solving and discussion with others. She encouraged students to think about the elements of problems that were important and to disregard unnecessary information. When students solved problems with others, they discussed whether the strategies they developed were reasonable for the situation. Ms. Weatherspoon believed that students developed meaning for the mathematics in the contexts in which problems were situated, and they could fall back on that context whenever they needed help. Ms Weatherspoon felt that students should strive to master some basic skills such as multiplication facts before problem solving, but students should use calculators for computation that otherwise would hinder problem solving (Interview 10/27/98).

School Capacity

Ms. Mitchell perceived that the capacity of her school to support mathematics teaching and learning was low. She felt that the principal’s vision of mathematics teaching and learning was incompatible with the vision held by the teachers. She felt that she received moderately strong administrative support in terms of clearly communicated expectations, but limited support was available for making changes in instructional practice. As team leader, she served as a liaison between the principal and teachers, sharing information and helping the team stay organized. Ms. Mitchell believed that MiC was well aligned with the state standardized testing program, and it had little influence over curriculum for her students because she emphasized problem solving and written explanations of solutions during instruction. She stated that the national standards for teaching and learning mathematics, with its emphasis on problem solving and activities aimed at helping students develop conceptual understanding, had a greater influence on her teaching practices than the state testing program (Interview 9/16/97; teacher questionnaires 8/97, 5/98). Ms. Mitchell and another teacher taught MiC at Dewey Elementary, but they did not have common planning time to discuss MiC units or their teaching and assessment practices.

Ms. Weatherspoon perceived that the capacity of her school to support mathematics teaching and learning was average in the first study year, but in the second year she perceived low school capacity. She felt that principal and teacher visions for mathematics teaching and learning were clearly defined and generally aligned. She felt that she received strong administrative support in terms of clearly communicated expectations, but limited support was available for making changes in instructional practice. However, she felt that she had a high level of influence in planning and teaching mathematics. Professional development at the school included sessions on general teaching methods. Ms. Weatherspoon, who began her teaching career at Fernwood Middle the previous year, was the only MiC teacher at her grade level in the school. Formal meetings for mathematics teachers were held infrequently, and teachers met informally on occasion in lieu of common planning time. Ms. Weatherspoon believed that the state standardized testing program had some influence in her planning. Although she emphasized problem solving and written explanations of solutions during instruction, she did use the commercially-prepared test preparation materials that were mandated in her school (Interview 4/13/99; teacher questionnaires 8/98, 5/99).

Opportunity to Learn with Understanding

Ms. Mitchell presented a comprehensive curriculum, teaching eight units: three number, one algebra, two geometry, and two statistics. She supplemented MiC with resources on fraction operations in order to better prepare students for work in sixth grade. On many occasions, Ms. Mitchell worked toward helping students develop conceptual understanding through problem solving and sharing answers in whole-class discussion. Procedural understanding was also important in her instruction, especially with respect to fractions.

Ms. Weatherspoon presented a comprehensive curriculum, teaching four units in three content areas: one number, four algebra, and one geometry. She supplemented the units with activities that emphasized connections among mathematical ideas and connections to students' lives. Lessons also promoted linking conceptual and procedural understanding.

Instruction

Ms. Mitchell taught mathematics for understanding, which is illustrated in a brief description of the lesson from the fifth-grade number unit *Some of the Parts* (van Galen, Wijers, Burrill, & Spence, 1997) in 11/98. In the lesson students studied different ways to compute with fractions using a ratio table, and they chose their own strategies for completing ratio tables. Mitchell noted that some students had difficulty completing ratio tables, especially in situations in which they needed to take half of a fraction. Resisting the temptation to introduce algorithms to address this, Ms. Mitchell added more work with ratio tables, missing an opportunity to help students make connections with the fraction bar and fraction strips presented earlier in the unit. However, students began to find their own solution strategies for working with the ratio table. For example, in determining the amount of ingredients to use for 6 servings, some students multiplied entries in the column for 2 servings by 3, while other students added the entries for 2 servings and 4 servings. Ms. Mitchell continually asked all students to write down their strategies, share their strategies in whole-class discussion, and prepare to react to all presented strategies. She did not state whether strategies were correct or incorrect. Rather, she encouraged students to evaluate the strategies without her intervention. Students explained to one another why their strategies worked, and when they discovered an incorrect strategy, they worked together to revise the strategy. For example, students noted when addition was used when it should have been subtraction. In keeping with the philosophy of the curriculum, Ms. Mitchell resisted the temptation to teach algorithms for fraction operations, and she encouraged students to find their own solution strategies. When difficulties arose in calculating half of fractional amounts, however, reminding students of the fraction models used earlier in the unit may have provided another avenue for students to use in developing understanding of fractions in this context.

Ms. Weatherspoon worked toward developing conceptual understanding, and she shared the mathematical work with her students, which is illustrated in a lesson from the MiC sixth-grade number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole & Brendefur, 1998) on 2/22/99. In this lesson (pp. 36–38), students were introduced to a method for calculating a fraction of a fraction. For example, for 1/5 of 2/3 of $30, 2/3 of $30 = $20, and 1/5 of $20 = $4. Ms. Weatherspoon introduced the lesson with an activity that she designed. She distributed 20 bags to her students and instructed them not to look into the bags. She told the students that half of the bags had play money in them and 1/5 of those bags had more than $5 in them. She asked what fraction of the bags had more than $5 in them and how they determined the answer. Students suggested the following:

- I would count them.
- Take 1/5 of 20.
- First figure out how many have money in them by taking 1/2 of 20 to get 10.
- Only 1/5 of the 10 have more than $5 in them or 1/5 of the 1/2.
- I think it is 2 because 1/5 of 10 is 2.

Ms. Weatherspoon then asked students to name the fraction of the whole number of bags that had more than $5 in them. A student answered, “2 out of 20 or 1/10 or 10%.” When Ms. Weatherspoon asked, “What did you just figure out?” the student answered, “1/5 of 1/2 is 1/10.” Students then checked the bags to verify this result. In the unit, students worked together to understand such calculation in the context of recycling aluminum cans in a park. As a class, they worked through pp. 36-37; they calculated 9/10 of 4/5 of 250 kg of aluminum cans that would be recycled. Ms. Weatherspoon led the class through each part of lesson by asking them what they would do to solve the problem and why they chose that method. As they worked through the problem, their conjectures consisted of making connections between the new problem and the teacher-designed activity. Ms. Weatherspoon emphasized equivalent representations of numbers, and students expressed different representations in their answers. For example, when students changed a decimal to a percent, they stated that percent involved hundredths, and they looked for a number out of 100. Ms. Weatherspoon also emphasized the method for multiplying fractions introduced in the lesson. By doing so, she provided a conceptual foundation for taking a fraction of a fraction. Multiple strategies were encouraged and valued. For example, when students calculated 4/5 of 250, strategies included using a calculator to divide 250 by 5, then multiplying by 4; using a ratio table; or using a bar to represent 250, dividing it into five segments, and finding the amount for four segments. This lesson illustrates the high level of inquiry that frequently occurred during instruction in Ms. Weatherspoon’s classes. In this lesson, she continually pressed students to develop conceptual understanding, and she shared the mathematical work with her students. She emphasized connections among fractions, decimals, and percent throughout the lesson.

Instructional Planning

Ms. Mitchell felt that planning to teach MiC was more time intensive than planning to teach from a traditional text. As she planned for teaching a unit, Ms. Mitchell considered the information in the unit overview, in particular, pacing for lessons. She also compared the student and teacher editions. This process enabled her to decide which things she needed to clarify when she taught a lesson. Ms. Mitchell considered students' prior knowledge related to unit content and used that to decide whether to include additional instruction, particularly for the lowest ability group of the three classes she taught. Ms. Mitchell also checked whether the MiC units met the new state standards and felt comfortable that MiC was aligned with those standards. She had reservations, though, that fractions were not addressed completely enough for expectations in sixth grade, and she resolved this issue by using MiC ancillary materials (*Number Tools*) and supplementary worksheets from other resources (Interview 4/9/98).

When planning to teach an individual lesson, Ms. Mitchell solved some of the problems in the student edition herself or at least reviewed them to get "a general idea of how they should be solved" (Interview 4/9/98). She also read the comments in the teacher guide for each problem. This process allowed her to anticipate difficulties her students might have with the content and to decide whether modifications were warranted, especially for lower-achieving students. For example, on one occasion Ms. Mitchell used a formal assessment that included symmetry and symmetric patterns. She reported that students were unable to do the work individually, but she had anticipated the trouble spots and was prepared with specific ways to help the students (Observation 10/20/97). Ms. Mitchell's planning went beyond becoming familiar with curricular content. She made decisions for students based on potential student questions or misunderstandings.

When planning to teach a MiC unit, Ms. Weatherspoon read through the unit, identifying the development of the mathematical content, the major concepts presented, and the skills needed. She sorted the problems into two groups, those that required her to guide the students and those that students could solve more independently when working with partners. Ms. Weatherspoon divided the unit into sections to be completed in one class period and determined problems that could be eliminated or used as homework assignments. She also identified the skills with which her students lacked proficiency and planned warm-up activities for extra practice. She noted places suitable for students to use calculators. Ms. Weatherspoon planned to use MiC ancillary materials (*Number Tools*) and student activity sheets that accompanied MiC units. When planning to teach individual MiC lessons, Ms. Weatherspoon worked through each problem to become familiar with the lesson content (Interview 4/13/99).

Classroom Achievement

Changes in students' performance are shown in the progress maps in Figure 1-3 and in the summary data in Table 1-3.

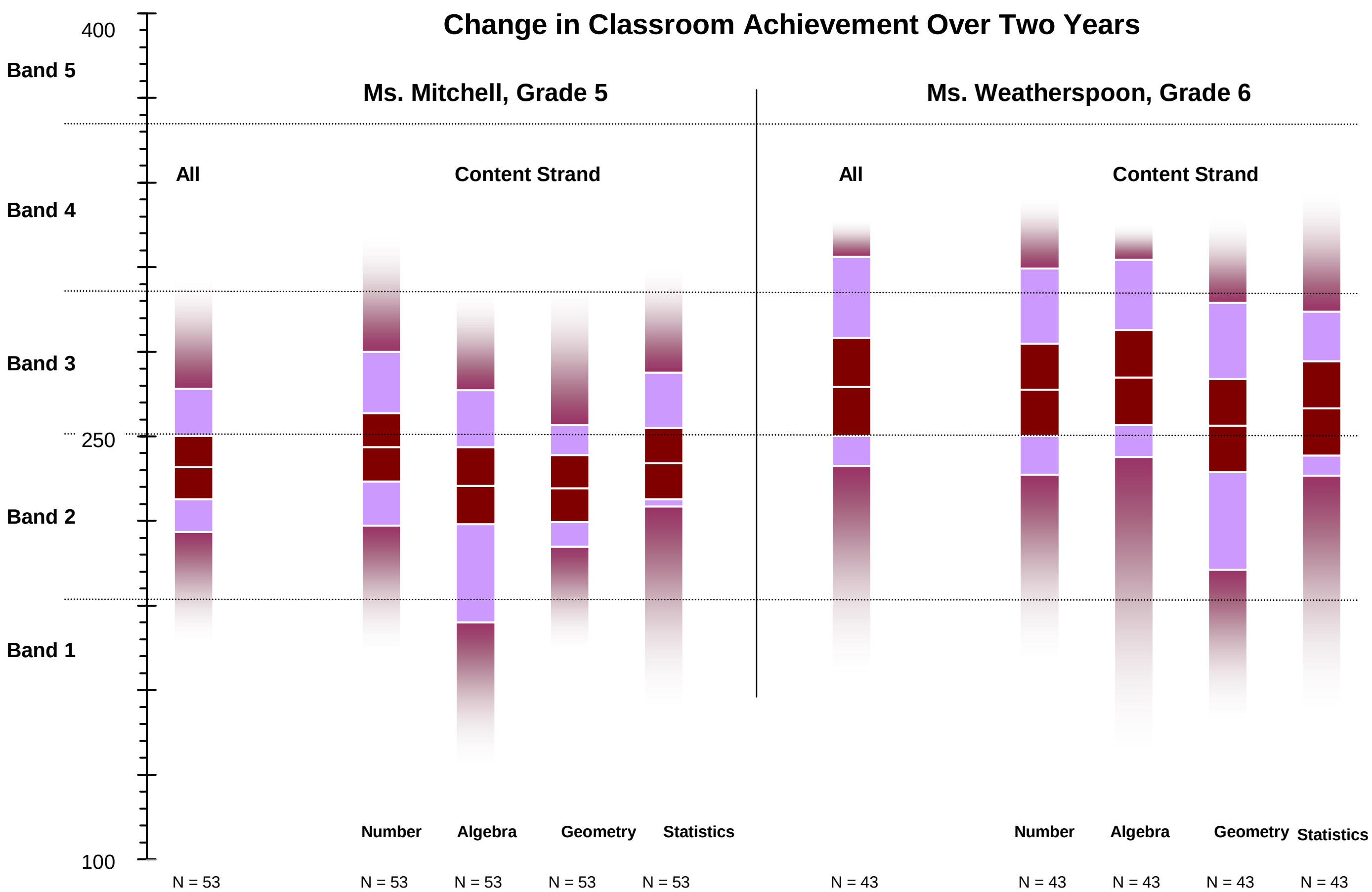

Figure 1-3. Change in classroom achievement from Grade 5 to Grade 6 for students in Ms. Mitchell's and Ms. Weatherspoon's classes, respectively, overall and by content strand.

Table 1-3.
Classroom Achievement for Students in Ms. Mitchell's and Ms. Weatherspoon's Classes in Grades 5 and 6, Respectively, Overall and by Content Strand

				95% Confidence Interval		Score distribution percentiles			
	N	Mean	*SD*	Lower	Upper	95%tile	75%tile	25%tile	5%tile
Ms. Mitchell, Grade 5									
Overall	53	239.0	41.7	227.7	250.2	306.4	266.9	216.1	176.1
Number	53	246.1	44.8	234.0	258.2	321.0	280.0	218.4	173.3
Algebra	53	232.4	50.7	218.8	246.1	300.9	266.3	184.1	133.0
Geometry	53	231.5	44.3	219.6	243.5	303.3	254.0	210.9	173.5
Statistics	53	240.4	46.8	227.8	253.0	309.9	272.7	225.1	153.5
Ms. Weatherspoon, Grade 6									
Overall	43	267.5	58.1	250.2	284.9	326.7	313.8	239.5	164.9
Number	43	266.5	54.7	250.2	282.9	334.7	309.5	236.4	170.6
Algebra	43	271.0	56.4	254.1	287.8	324.9	312.7	242.7	137.7
Geometry	43	253.8	55.5	237.3	270.4	327.3	297.3	202.7	149.1
Statistics	43	259.9	56.1	243.2	276.7	337.5	294.2	236.0	153.0

From Grade 5 to Grade 6, the overall mean score for students in these classes increased from 239.0 to 267.50, which is a statistically significant gain. The increases in mean scores for each content strand were also statistically significant. At the end of Grade 6, the overall mean and means for each content strand were in Band 3, and the performance of more students was in Band 4 compared to at the end of Grade 5.

As students from Ms. Mitchell's classes advanced to middle school, some students were assigned to Ms. Weatherspoon's classes. While the analysis of students' performance with these teachers is not a longitudinal analysis, it suggests that classes of MiC students who have experienced teaching that attended to conceptual understanding and opportunity to learn comprehensive mathematics content in depth showed growth in performance as they moved from studying MiC in elementary school through Grade 6 in middle school.

Advantages, Disadvantages, and Implementation Issues

Ms. Mitchell listed the quality of discussion and availability of hands-on activities as advantages of teaching MiC:

- "I really think discussion improves because you're posing the questions to the children but you're sometimes letting them generate the discussion. Sometimes it's very good because they might question each other, you know. They might question each other's strategy. They share a lot. There's a lot of interaction between the students simply because they might say, 'I did mine the same way,' 'I did mine just a little differently,' 'Can I explain mine?' 'Mine is very similar'" (Mitchell, Interview 4/9/98).
- "You're given the opportunity to use hands-on activities in every unit. And it does not have to be manipulatives that you bought from a company. They are things that you can make yourself" (Mitchell, Interview 4/9/98).

Ms. Mitchell also noted the importance of having external support for times when she needed to discuss a difficult lesson. On one occasion she called the research center, which she found very helpful: "She [the researcher] even faxed me an explanation of the lesson, which was great. I made a transparency of that sheet and explained it to the children that way" (Mitchell, Interview 4/9/98). This underscores the need for opportunities for teachers to have times when they can collaboratively work with other teachers to understand the content they have not previously taught and to have consultants or district specialists available for teachers when they need support from others.

Ms. Mitchell stated that she was so pleased with MiC that she was unable to think of disadvantages. However, in past years she was well known for helping students master operations with fractions by the time they finished fifth grade. She expressed concern that there was not enough practice in the fifth-grade number units for students to master these skills. Mastery of fraction operations at this grade level conflicts with the intent of MiC to build on students' intuitive knowledge of fractions, introduce tools such as the fraction bar and ratio table to support their thinking about fraction operations, and revisit and extend these operations to more abstract forms in sixth- and seventh-grade units. Ms. Mitchell hoped that students retained the understandings about fractions that they had an opportunity to learn in fifth grade with MiC as they moved into middle school.

As advantages of using MiC, Ms. Weatherspoon listed understanding of mathematics and readiness for the state standardized testing program:

- "Students understand the meaning of, for example, fraction operations, and I think that's a big difference with MiC. I also think that it takes them through different levels of thinking" (Weatherspoon, Interview 4/13/99).
- "I think my kids had more of an advantage than the other children when it came to the standardized tests because they were used to seeing problems that were in a context and writing answers that required explanations" (Weatherspoon, Interview 4/13/99).

At the same time, Ms. Weatherspoon expressed concern that students who had not learned operations with whole numbers such as multi-digit multiplication may never have the opportunity to learn those skills. She added, however, "But they're getting other things. So I guess there's a balance. But you, as a teacher, have to decide" (Weatherspoon, Interview 4/13/99). The dilemma is whether to explore mathematical content in number and other strands and encourage conceptual understanding, reasoning, and communication, or to make mastery of basic skills the focus of instruction. Ms. Weatherspoon decided to keep the focus on conceptual understanding, but added a brief review of basic skills at the beginning of each class period before the MiC lesson. Because class periods were 90 minutes and met each day, she felt that a brief review would not interfere with students completing one MiC lesson per day. Ms. Weatherspoon listed two difficulties in implementation of MiC. First, she found it difficult to help all students develop explanations of high quality that "really are at the heart of whatever they're trying to explain" (Weatherspoon, 4/13/99). Second, some students had poor reading skills, and although many of these students did well in class because of the discussions, she was concerned that they may not succeed when working independently.

Both teachers thought that students benefited from studying MiC, but both were concerned about basic skills. In the past Ms. Mitchell helped students to learn procedures for operations with fractions by the end of fifth grade. Ms. Weatherspoon was pleased with the understanding of fraction operations students were developing by studying MiC, but she was concerned about skills that students should have known prior to sixth grade such as operations with whole numbers, a dilemma she was trying to address by adding brief reviews each day. For both teachers, the deep understanding, reasoning, and discussion promoted by using MiC were substantial benefits for their students.

Summary

These two fifth- and sixth-grade teachers had similar teaching and classroom assessment practices and provided similar opportunities for students to learn mathematics with understanding. Their practices led to a high quality of implementation of MiC. These teachers thought that the understanding, reasoning, and discussion promoted by using MiC were substantial benefits for their students. Although only some of Ms. Mitchell's students were assigned to Ms. Weatherspoon's classes, the analysis presented here suggests that classes of MiC students who have experienced teaching that attended to conceptual understanding and opportunity to learn units from multiple content strands in depth showed growth in performance as they studied MiC in Grade 5 in elementary school and Grade 6 in middle school.

Implementation across Three Years in the Same Middle School

In this section, we explore the implementation of MiC by three teachers who taught mathematics in the same middle school. Mr. Brown, Ms. Muldoon, and Ms. Reichers taught Grades 6, 7, and 8, respectively, at Von Humboldt Middle School in District 1. They taught multiple classes each day; two of each teacher's classes were in the study. These teachers varied in their teaching and assessment practices and opportunities for students to learn mathematics with understanding. Mr. Brown's practices led to a low quality of implementation of MiC, and Ms. Muldoon's practices were more reflective of traditional mathematics instruction. Ms. Reichers' practices led to a high quality of implementation of MiC, which reflected the pedagogy and comprehensive content that MiC development team had intended.

Mr. Brown felt that students learn mathematics best when it is presented in real-world contexts because they might see a purpose for studying mathematics. He felt basic facts and procedures were critical tools for problem-solving and conceptual development, but recognized that these skills could be developed in a problem-solving situation as needed or as rote procedures learned at one time and applied later. Brown viewed problem solving as separate from the mathematics presented in textbooks and alluded to the faculty-designed course on problem solving as a way for students to develop specific problem-solving skills (Interview 9/11/97).

Ms. Muldoon believed that students should develop conceptual understanding before they learn abstract procedures. However, if they learn alternative methods for calculation, at some point efficient algorithms should be emphasized. Ms. Muldoon thought that there were two reasons for students to use calculators during instruction: for calculations with large numbers and checking results of alternative strategies for calculation. She felt that situating lessons in contexts made a difference for students. She believed that problem solving was important, and it should be used throughout the curriculum (Interview 10/22/98).

Ms. Reichers had transferred to Von Humboldt Middle in the final year of the study. Ms. Reichers believed that the best way for students to learn mathematics is to immerse them in thinking about it and discussing it. To her, learning mathematics is a process: "It's about learning some skills and practicing them in different situations so that when they move on to the next place, they have a foundation and they have confidence in the process so that they can apply mathematics in a new situation" (Interview 10/28/99). She was not an advocate of introducing algorithms for students to memorize and practice without thinking. She believed that linking conceptual and procedural knowledge is essential. In keeping with her belief that "math is about life," she felt that making connections between mathematics and life experiences was very important (Interview 10/28/99).

School Capacity

Mr. Brown, Ms. Muldoon, and Ms. Reichers perceived that the capacity of their school to support mathematics teaching and learning was low. They felt that the principal and teacher visions of teaching and learning mathematics were aligned on some ideas, but were incompatible on others. They felt that they received strong administrative support in terms of clearly communicated expectations and changes in instructional practice, and moderate to high influence over mathematics planning and teaching. These teachers felt thought that the faculty and staff were committed to academic excellence. Formal meetings with other mathematics teachers in the school were held infrequently, and teachers varied in the amount of times they met informally with other teachers to discuss mathematics curriculum, instruction, and assessment. District and state standardized tests had varying degrees of influence for these teachers. Ms. Muldoon included more opportunities for journal writing and work with open-ended questions, multiple-choice tasks, and short-answer items. Ms. Reichers added practice on computation, but less extensively than Mr. Brown who regularly supplemented MiC with drill-and-practice computer programs.

Opportunity to Learn with Understanding

Mr. Brown used five sixth-grade MiC units throughout the school year, two each in number and algebra and one in geometry. He regularly supplemented MiC with computation practice from a traditional textbook and provided opportunities for students to use drill-and-practice computer programs on a regular basis. Inquiry during instruction was limited because students generally worked through MiC lessons without the benefit of introductory discussion of the content and context of the lesson. Connections among mathematical ideas were not promoted, and connections between the mathematics and students' life experiences were not discussed in detail.

Ms. Muldoon taught three seventh-grade units, one in number and two in geometry. However, she supplemented the seventh-grade MiC units with sections of fifth- and sixth-grade MiC units that contained prerequisite skills for the seventh-grade units. She also supplemented with MiC computation practice from the traditional textbook used in previous years. The lessons presented provided limited attention to conceptual understanding, and making conjectures was generally not encouraged. Connections among mathematical ideas and between mathematics and students' lives were not discussed in detail. In Ms. Muldoon's classes, instruction was more reflective of conventional pedagogy.

Ms. Reichers presented a comprehensive curriculum, teaching five units: three algebra (one seventh-grade unit), two geometry (one seventh-grade unit), and one statistics. Ms. Reichers worked toward developing conceptual understanding, and she shared the mathematical work with her students. She posed questions that encouraged students to explain their thinking, and helped students make connections among mathematical ideas. Ms. Reichers sought student explanations in addition to procedural understanding as evidence of student learning.

Instruction

Lessons in Mr. Brown's classes were underdeveloped. Frequently, no formal lessons were presented. Lesson content was not discussed prior to independent work, and students lacked the support they needed to understand the mathematics on their own. After assigning student work, he moved from student to student to explain the lesson, and students depended on him to do the mathematical work for them. The nature of these lessons is illustrated in a brief description of the lesson from the sixth-grade MiC algebra unit *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole, & Burrill, 1998) on 3/16/98. In this unit students were introduced to informal forms of mathematical expressions and formulas. In this lesson, students used arrow strings to show order of operations. To begin the lesson, Brown asked students to turn to pp. 54–58 and to spend the class period working on these pages. No expectations were given as to how far they should get by the end of the period, who they should work with, when they could ask a question, and so on. Students were confused because they were not near p. 54 prior to the lesson. Five hands went up immediately, and Mr. Brown started moving from student to student. The classroom was very quiet. Students worked individually and talked only with the teacher when he came to their desks. They waited for Mr. Brown to tell them how to do a problem, and then they did it that way. After that, they asked how to do the next one, and so on. Students depended on Mr. Brown to answer all questions and did not move ahead in the lesson until he came to their desks. In some cases, this took a long time. The only strategy used was the one that Mr. Brown explained individually to each student. Students had very little motivation for, or knowledge of, the mathematics they were supposed to be learning. They left incorrect answers in their notebooks after talking to Mr. Brown, and they skipped other questions. To students and their teacher, it seemed that the focus was on each question in the lesson, rather than on the mathematics they were studying. Each question was treated as a new and isolated topic.

On this occasion no formal lesson was presented. Students were given an assignment, but the content was not discussed prior to students completing it. Students began independent work with little direction. They lacked the support they needed to understand the mathematics on their own. Only a surface treatment of the content occurred at best. Inquiry during class was limited to lower order thinking, and the lesson did not promote conceptual understanding. The class did not discuss connections between the content of particular lessons and other mathematical content, nor did they explore connections between mathematics and students' life experiences.

Lessons in Ms. Muldoon's classes provided limited attention to conceptual understanding. The main focus of lessons was on building students' procedural understanding through direct teaching methods. When Ms. Muldoon posed questions, students responded with answers only or steps in particular procedures, rather than elaboration of reasoning. Few questions addressed the reasonableness of answers. The focus of Ms. Muldoon's classroom assessment practices was gathering evidence from homework and classwork to substantiate students' procedural understanding. Occasionally, Ms. Muldoon asked for explanations, but she did not attempt to build substantive discussions of the mathematics. The feedback she gave to students was indirectly responsive to their needs in that it involved additional whole-class instruction using the same method. In short, lessons were more reflective of traditional teaching practices.

A brief description of a lesson from the seventh-grade MiC number unit *Cereal Numbers* (Abels, Gravemeijer, Cole, Pligge, &Meyer, 1998) on 4/15/99 illustrates the instruction students in Ms. Muldoon's class experienced. In this lesson (pp. 8–10), students learned about absolute and relative comparisons through comparison of corn production by state per square mile and corn production relative to state populations. The lesson began with a discussion of p. 8, #10, listing the top five corn-producing states from a table with data by state for corn production, population, and land area in square miles. A student gave the answer for #10, which was accepted without question or further elaboration. Ms. Muldoon said, "On p. 9, you will have a chance to practice long division." She talked about the meaning of a square mile. A student read #11, "Wilson claims that Nebraska produced more corn per square mile than Indiana. Do you agree or disagree with Wilson? Explain." Ms. Muldoon then showed the class how to divide the corn production for Nebraska by the land area (934,400,000 ÷ 76,644) using the division algorithm, rather than number sense and estimation to avoid calculating with such large numbers. When a student gave an incorrect answer, Ms. Muldoon explained why it was incorrect without asking the class for input. Ms. Muldoon set up the division on a calculator, and a student read the answer from the display. She suggested that students check their answer with the calculator when they did the work for Indiana. When asked about the next item, "If you consider corn production relative to the population of the state, how would you reorder the top five corn-producing states?" one student made a suggestion for finding a solution. But this suggestion was not acceptable to Ms. Muldoon, who proceeded to tell students to divide the amount of corn produced by the population, and she did the work on the board for them. She also demonstrated the division for the next task. A student gave a correct answer for the last task, but no explanation was elicited. The assignment was to finish p. 10.

In this lesson, Ms. Muldoon presented a particular way to think about the problems, and she did the calculations on the board for students. Students then practiced the calculations in a rote fashion. The lesson was focused on the algorithm for long division, not on using number sense or estimation, and reasonableness of answers was not discussed. Inquiry during class included limited attention to understanding absolute and relative comparisons. When questions were posed, students stated answers, and they were not expected to elaborate on their reasoning. Ms. Muldoon sought procedural understanding as evidence of student learning. Feedback was teacher-directed and was limited to checking for accurate answers.

Ms. Reichers worked toward developing conceptual understanding, and she shared the mathematical work with her students, which is illustrated in a lesson from the MiC eighth-grade algebra unit *Graphing Equations* (Kindt, Wijers, Spence, Brinker, Pligge, & Burrill, 1998) on 1/13/00. In the lesson, students were introduced to slope as a measure used to describe the direction or steepness of a line. The lesson began with a review of the previous lesson, which included how to find the slope of a line. The remainder of the lesson, as presented by Ms. Reichers, promoted conceptual understanding of slope and its relation to the steepness of a line. For example, students stated, “The line with the slope of 1 is not as steep as the line with the slope of 2,” and “If the lines are not parallel, the slopes are not the same.” Many students made conjectures during the lesson. For instance, students offered and explained these conjectures: “As the slope gets larger, the line gets steeper,” “As the line gets closer to the *y*-axis, it gets steeper,” “If lines have opposite slopes, they go in opposite directions,” and “If lines are parallel, they have the same slope.” Ms. Reichers posed questions that promoted connections among mathematical ideas, such as various representations of slope (ideas explored in other MiC units: height/distance, glide ratio, tangent) and when slope is written as a ratio, equivalent ratios can be used to find other ordered pairs, which, in turn, can be used to determine the direction of the line. Throughout the lesson, students explained their responses. One student went to the overhead projector to show the class how he determined a slope of $^{-}1/2$ for a particular line. Ms. Reichers was pleased that students wanted to offer many conjectures for investigation, and she stopped the class so that every student had an opportunity to talk. Even though the discussion was a series of conjectures and explanations, rather than building on one another’s ideas, the class definitely worked toward an understanding of the mathematics. During the subsequent independent work, students checked answers with one another, but did not discuss the mathematics with each other. Some students waited for Ms. Reichers to answer their questions. However, students took their work seriously.

In this lesson, Ms. Reichers began with a review of how to find slope, which prepared them for the explorations in the current lesson. As they worked on finding the slopes of ten lines, they began to make and test conjectures about the slope of a line and the steepness of a line, the direction of a line given its slope, and the slopes of parallel lines. Ms. Reichers encouraged these conjectures and stopped the class so that every student had the opportunity to have input in the discussion. Although the discussion did not develop in ways for students to build on one another’s ideas, the discussion did promote shared understanding of the mathematics. Ms. Reichers posed questions that encouraged students to make connections among mathematical ideas. She asked for explanations as well as answers as evidence of student learning.

Among the three teachers, there were clear differences in teaching and classroom assessment practices. In Mr. Brown’s case, lessons were underdeveloped and students began work without the support they needed to understand the mathematics on their own. Inquiry was limited, which made little evidence available for the purposes of classroom assessment. Ms. Muldoon’s teaching practices were more traditional in nature through showing students how to do the calculations. These practices reduced the thinking required by students in solving unit tasks. She sought briefly-stated answers and steps in procedures as evidence of student learning. In comparison, Ms. Reichers’ practices were more reflective of teaching mathematics for understanding. She encouraged all students to make and explain conjectures, promoted connections among mathematical ideas, and allowed students to work toward shared understanding of the mathematics. She sought explanations as well as accurate statements as evidence of student learning.

Instructional Planning

At the beginning of the school year, Mr. Brown administered a diagnostic test to get a general sense of the range of abilities in the class. This knowledge helped him anticipate the topics that required more practice than provided in MiC and enabled him to select appropriate activities from his own resources for extra practice during class time or for homework. Mr. Brown found that his students usually worked through a unit at a slower pace than suggested by MiC because they asked many questions. In planning individual lessons, Mr. Brown reviewed the teacher's guide to organize the necessary materials and identify any skills for which he needed supplementary activities. However, he did not work through the unit problems. He commented:

> I'll let the children explore all the questions. . . . Sometimes the children will come up to me and say, "What does this mean?" and I'll say that I didn't see that one coming. And sometimes I have trouble understanding what it is the unit is looking for. I just look it up in the teacher's guide and say, "Okay, they're looking for this, or this is the angle the question is working toward." (Interview 4/27/98)

Without thorough planning, including working lesson tasks by himself, Mr. Brown did not anticipate students' difficulties. Rather, he planned to allow students to work through the tasks and ask him about tasks they did not understand. He referred to the teacher guide when he could not immediately answer students' questions. As a consequence, lessons were underdeveloped in nature throughout the school year.

When planning to teach a MiC unit, Ms. Muldoon read through the unit to become familiar with the content and to know the goal "so I know what actually the thrust of the book is going to be about. I don't want any surprises in the middle" (Interview 4/15/99). She identified potential problems students might have with the unit and made appropriate adjustments for them including mini-lessons on key concepts, extra skill practice, whole-class discussions, and additional time for the unit. When planning to teach individual MiC lessons, Ms. Muldoon looked over the notes she made during unit planning, and she considered the students' performance on the previous day's lesson, noting whether she needed to reteach or review a particular topic or skill or skip repetitious activities (Interview 4/15/99).

When planning to teach a MiC unit, Ms. Reichers read the teacher guide to get an overview of the unit. She explained:

> I look at the front of the teacher's manual at the NCTM *Standards*, the mathematical topics that are covered. I look at the suggested pacing. I look at how often the authors think there should be assessments, what kind of assessments there are, what the main ideas are, and, of course, all of the equipment I'm going to need. (Interview 5/9/00)

Ms. Reichers did not formally or informally assess students' prior knowledge before she planned a unit. Instead, she informally assessed as students worked through the unit during whole-class discussions and observation, taking time to provide instruction on missing concepts, skills, and procedures. Because MiC was designed with a spiral format (mastery over time), the development of the concepts studied in eighth-grade units had begun years earlier. Students whose mathematics instruction was not consistently MiC or had not included all of the units for each year from Grades 5–7 might not have prerequisite conceptual understanding or skill in procedures unique to MiC such as using a ratio table for the division of fractions and decimals. When planning to teach individual MiC lessons, Ms. Reichers worked through all of the problems in the lesson and read the accompanying material in the teacher's guide. She noted the mathematics involved, any struggles she had, and possible questions she could ask to lead students' thinking about the mathematics. Ms. Reichers considered how students performed during the previous lesson and planned a warm-up activity or oral questioning to determine what they had learned (Interview 5/9/00).

Classroom Achievement

Changes in students' performance over the three years are shown in the progress maps in Figure 1-4 and in the summary data in Table 1-4. At the end of Grade 6, the mean performance for students in Mr. Brown's classes was in Band 2 with few students in Band 3. Performance in the algebra and statistics strands was higher than in other content strands, and these differences were statistically significant. From Grade 6 to Grade 7, the overall mean scores for students in these classes increased from 209.1 to 257.7, which is a significantly significant difference. At the end of Grade 7, the mean performance for students in Ms. Muldoon's classes was in Band 3. Over 50% of the students were in Band 3, some students were in Band 4, and fewer students were in Band 1 than in the previous year. The significant increase in performance was apparent in all content strands. From Grade 7 to Grade 8, the overall mean scores for students showed important gains from 257.7 to 268.3. At the end of Grade 8, nearly 75% of the students were in Band 3, and less variance was apparent in student performance. The increase in the number strand was statistically significant.

While the analysis of students' performance with these teachers is not a longitudinal analysis, it suggests that students who studied MiC but had experienced a low quality of instruction and opportunity to learn mathematics with understanding recovered dramatically when MiC was taught using traditional methods, and students' performance continued to increase as they experienced instruction that reflected teaching for understanding and opportunity to learn comprehensive mathematics content in depth. In addition to changes in the quality of instruction each year, changes in the content taught likely contributed to these gains in performance. Ms. Muldoon supplemented the seventh-grade units with portions of units from previous grade levels, and Ms. Reichers taught two seventh-grade units to support learning of eighth-grade units. This combination of a higher quality of instruction and more in depth study of MiC units likely influenced the substantial gains for students from one grade level to the next in middle school.

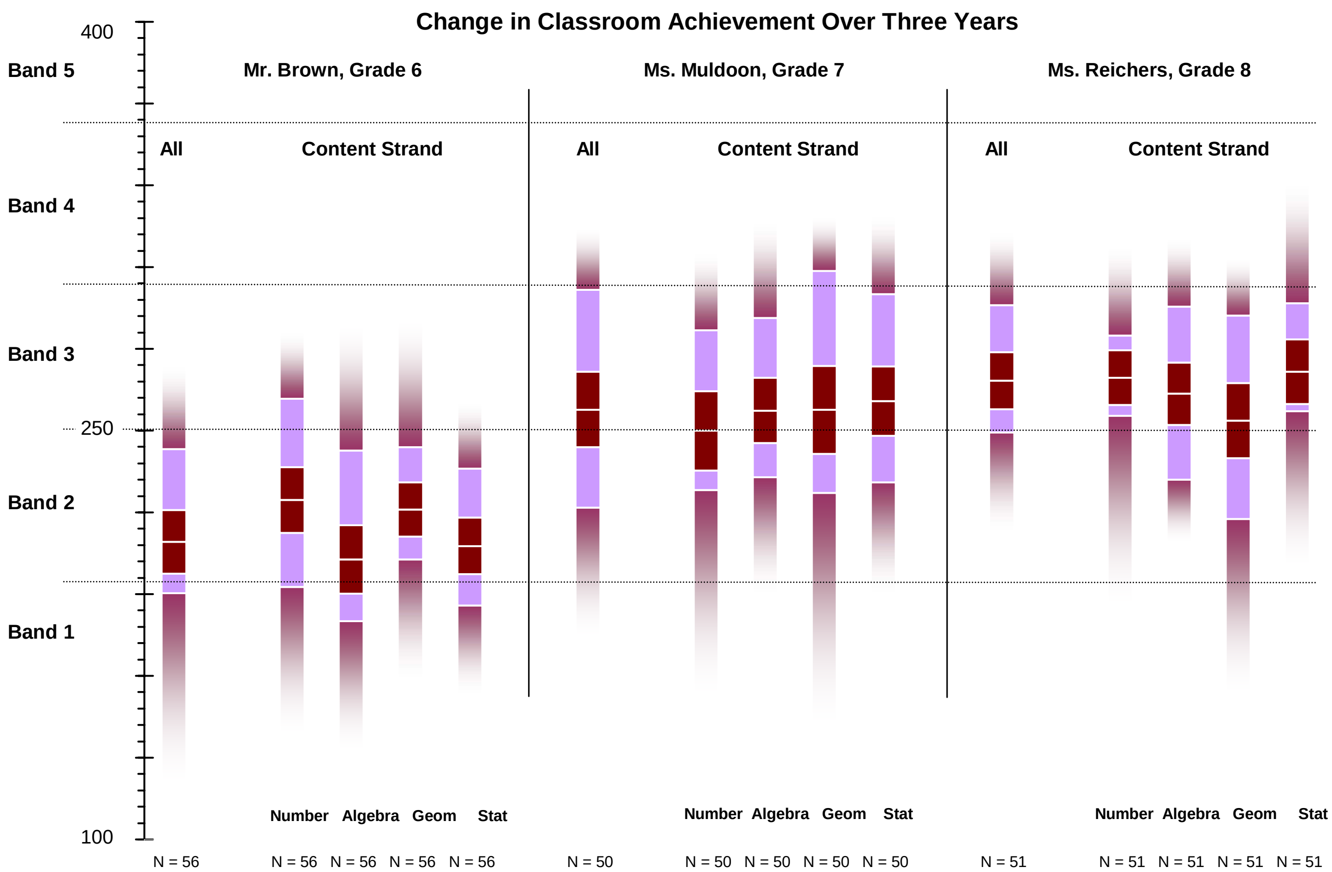

Figure 1-4. Change in classroom achievement for Grades 6, 7, and 8 classes taught by Mr. Brown, Ms. Muldoon, and Ms. Reichers, respectively, overall and by content strand.

Table 1-4.
Classroom Achievement for Grades 5, 6, and 7 Classes Taught by Mr. Brown, Ms. Muldoon, and Ms. Reichers, Respectively, Overall and by Content Strand

				95% Confidence Interval		Score distribution percentiles			
	N	Mean	*SD*	Lower	Upper	95%tile	75%tile	25%tile	5%tile
Mr. Brown, Grade 6									
Overall	56	209.1	44.6	197.5	220.8	273.5	243.2	190.4	121.2
Number	56	224.5	46.0	212.4	236.5	286.0	261.7	192.6	139.1
Algebra	56	202.8	48.2	190.1	215.4	288.1	242.7	180.1	133.0
Geometry	56	221.0	37.9	211.1	231.0	289.9	243.9	202.7	158.7
Statistics	56	207.7	39.8	197.3	218.1	260.0	236.0	185.9	153.0
Ms. Muldoon, Grade 7									
Overall	50	257.7	50.1	243.8	271.6	323.9	301.6	221.7	174.9
Number	50	249.9	52.7	235.3	264.5	314.8	286.9	228.2	153.3
Algebra	50	257.4	43.1	245.4	269.3	326.4	291.3	233.0	189.8
Geometry	50	257.6	58.3	241.4	273.7	328.2	308.6	227.2	142.4
Statistics	50	260.8	46.0	248.0	273.5	328.4	300.1	231.0	190.3
Ms. Reichers, Grade 8									
Overall	51	268.3	38.2	257.9	278.8	322.8	296.0	249.4	212.7
Number	51	269.4	36.8	259.3	279.5	317.4	284.9	255.3	186.3
Algebra	51	263.5	41.7	252.1	275.0	320.3	295.6	232.1	209.0
Geometry	51	253.7	50.1	239.9	267.4	312.9	292.3	217.6	153.9
Statistics	51	271.6	43.4	259.7	283.6	340.5	296.7	257.1	200.6

Advantages, Disadvantages, and Implementation Issues

The teachers in this group talked about the advantages in using MiC in middle school:

- "I definitely see advantages in teaching this way in terms of having the children understand the thinking process that's involved in their answers, in trying to come up with answers instead of just trying to be a human calculator. Those are the main strengths of the program" (Brown, Interview 4/27/98).
- "The questions ask students to think" (Muldoon, Interview 4/15/99).
- "Being able to look at a real life situation and see math in it" (Reichers, Interview 5/9/00).
- "I really think the best way for kids to learn mathematics is to talk about it, to immerse them in it as if they're were in a particular situation. That's why I think MiC is so neat" (Reichers, Interview 10/28/99).
- "I'm beginning more and more to really appreciate the depth that you can get into with MiC because you can always refer back to something they've studied before. So it's neat that you can tie things in and say, 'Remember when?'" (Reichers, Interview 10/28/99).

These teachers also outlined some disadvantages and difficulties in implementing MiC:

- "My concern still stays the same: How is MiC going to affect how students are being evaluated, especially in light of the issue in our state of teachers' pay being connected to student performance? If the instrument they're using for accountability involves computation, then I'm going to make sure that in my classroom there's an allowance for computation, too" (Brown, Interview 4/27/98).
- "If there were more kids in each class that could read and help the others in a group, then it would be ever so much easier" (Muldoon, Interview 4/15/99).
- "I can't look back and say we did this, this, and this because topics are all embedded. You know, I could say, 'We graphed inequalities,' but the kids don't know they graphed inequalities, and if they ever saw it on a test, they wouldn't know what to do. My problem is that the mathematics is so much connected to the context that I'm not sure they've abstracted the mathematics" (Reichers, Interview 5/9/00).

For Mr. Brown, MiC provided students with the opportunity to learn mathematics on their own with little teacher intervention. At the end of the year, he was still concerned about students' performance on the state standardized tests in light of discussions about teacher salaries, and he intended to continue the use of supplementary materials for basic-skills practice. Despite the difficulties in implementation, Ms. Muldoon and Ms. Reichers thought the advantages of using MiC in helping students develop reasoning skills in mathematics were important, and Ms. Reichers appreciated that students studied mathematics in depth and looked for connections among mathematical topics.

Summary

These teachers who taught different grade levels in the same middle school had different teaching and classroom assessment practices and provided different opportunities for students to learn mathematics with understanding. The implementation of MiC ranged from low quality in Grade 6 to high quality in Grade 8. Although a core of the same students were in each of these teachers' classes, the results suggest that students who experienced a higher quality of implementation over time substantially improved in performance as a result of more interaction during instruction, more in-depth study of MiC units, and increasing levels of support from their teachers.

Teaching Students with Low Ability in Grade 5

Ms. Linne taught at Beethoven Elementary, a school for Grades 3–5 in District 1. Students at Beethoven Elementary were grouped homogeneously for mathematics instruction. Ms. Linne taught a small class of students who were in the lowest group; some of the students were mainstreamed from special education classes. She taught multiple units and attempted to teach mathematics for understanding in ways that MiC development team had intended.

Ms. Linne felt that students learned mathematics best when they were interested in lessons. She explained that, especially with the low group, "[students] learn best with manipulatives and concrete approaches, working together, and playing games" (Linne, Interview 9/9/97). Ms. Linne considered problem solving as different from the mathematics in traditional textbooks, and her class "[worked] on problem solving every day at the beginning of class" (Linne, Interview 9/9/97). She felt that success with problem solving was more dependent on conceptual understanding than on mastery of algorithms. As such, calculators were readily available to help students with cumbersome computations during problem solving.

Ms. Linne's vision for mathematics teaching and learning was inconsistent with the principal's vision. Principal Armour viewed basic facts as the foundation for mathematics learning and emphasized the need for mastery of basic facts before engaging in problem solving. He considered the lack of basic fact acquisition as the main stumbling block to success in mathematics "with the thought being that once you learn your times tables, math is over in terms of really struggling. Then it becomes more of a context thing" (Interview 9/9/97). He believed that knowing procedures superceded understanding concepts.

School Capacity

Ms. Linne perceived that the capacity of her school to support high expectations was limited. She felt that incompatible visions of teaching and learning mathematics were held by the principal and teachers in her school. She received little administrative support regarding choice of instructional materials and change in instructional practice, and little support was available as teachers implemented changes in policy. Ms. Linne reported that she had a high influence in planning and teaching mathematics and moderately high influence over educational policy such as curriculum. She also felt that the support for innovation from the principal, faculty and staff was very strong. As the Grade 5 leader, Ms. Linne served as a liaison between the principal and fifth-grade teachers. She also led weekly grade-level meetings. At times, the three MiC teachers at Beethoven talked about the pace of instruction, but they did not discuss the mathematics or teaching and assessment practices. Ms. Linne believed that the district and state standardized testing programs affected the content she taught, and she was concerned that students may not have enough time to learn the content on these tests (Interview 9/9/97; teacher questionnaires 8/97, 5/98).

Opportunity to Learn with Understanding

Ms. Linne taught six MiC units: two each in number and geometry, and one each in algebra and statistics. She supplemented MiC with daily warm-up activities, practice for standardized tests, and homework generally from external resources. Ms. Linne attempted to teach mathematics for understanding. However, connections among mathematical ideas were not discussed in detail, and discussion of connections between mathematics and students' lives at times overshadowed the mathematics.

Instruction

Ms. Linne began every lesson with a warm-up activity that included problem solving or practice of basic skills. In her introduction to the MiC portion of the lesson, she talked about the lesson to give her students an idea of what they would be learning that day. She explained:

> We'll do an activity and we'll talk a lot about it to give them some ideas so they're going in the right direction. Then when we break up in smaller groups, they should already have an idea of where they're headed. (Interview 9/9/97)

Some students paired up with others who understood the content: "The kids who aren't sure or that are on the edge will tend to lean toward the kids that they feel have a good grasp of what's going on" (Interview 4/22/98). Others checked their results with Ms. Linne.

Ms. Linne attempted to teach for understanding, which is illustrated in a lesson from the MiC fifth-grade statistics unit, *Picturing Numbers* (Boswinkel et al., 1997) on 1/14/98. In this unit, students were introduced to graphical representations of simple numerical data. The lesson began with a warm-up activity in which students displayed and analyzed their own data. Each student decorated a piece of pizza with his/her favorite topping. Students organized their pizza slices into a bar graph on the chalkboard and drew conclusions. For example, students observed that pepperoni was the favorite because it had the most pieces, and there was a tie between pizza with everything on it and pizza with onions. In the MiC portion of the lesson, students compared the relative size of whales by combining information from four photographs displayed in the unit. Students conjectured about the relative size of the whales and whether the Minke whale would fit in their classroom. Ms. Linne helped students make connections between the length of the whale and the length of the classroom. During this discussion, students were asked to determine whether the lengths they determined were reasonable. For example, when students measured the width of the room, they eventually had to add a meter because they began their measurement away from the wall. Some students noted connections among meters, yards, and feet. For instance, "A meter looks like a yard stick. There are three feet in a yard stick." Students were very interested in the whales. One student commented, "That whale is bigger than an elephant. Its tail is the same size as an elephant!" As students cut photographs of the whales from the first student activity sheet, they chatted about how people use whale fat. After talking through problems 1, 2, and 3 in the lesson, they put the units away. Ms. Linne demonstrated how to place the cut-out whales on the graph on the student activity sheet. She asked students to consider the scale on the graph: "If each mark is 1 meter, let's check the length of the Minke whale to make sure that it is 9 meters." The discussion, however, did not lead students to think more critically about the scale. Students then worked independently to measure the whales' lengths by using the graph. They took their work seriously, although no exchanges occurred among peers during class discussions. Interactions were between students and their teacher.

In this lesson, the contexts were inviting to the students, and they were actively involved in the lesson. With encouragement from Ms. Linne, students measured the classroom and made conjectures about the sizes of the whales. Although Ms. Linne attempted to teach for understanding by encouraging students to do the mathematical work in the lesson, the discussions might have placed more emphasis on the mathematics. For example, a brief discussion of what is involved in measuring might have helped students to more accurately measure the classroom, and a more complete discussion of the scale on the graph of the whales was important in helping students interpret the graph.

Instructional Planning

Ms. Linne thought that planning to teach MiC units was more time-intensive than planning to teach a chapter from a conventional text. She planned whole-class presentation and discussion, investigation of problems, small-group activity, and a homework assignment for each lesson. The first step in planning was to read through the whole unit, including assessments and student activity sheets, to understand the expectations of the unit. She commented:

> The content is much more open-ended, and you have to figure out what you're going to accept and how you're going to lead the kids to where you want them to go, how to take the kids' ideas and get them to focus on what the lesson wants them to do. That definitely is different from planning to teach from traditional textbooks. (Interview 4/22/98)

In planning to teach individual lessons, she did not work each task out prior to instruction. She explained:

I look the lessons over. But I'm just looking at the general direction that it's going. That way if the kids have a question, it's easier for me to divert and focus on their question. If I planned for every question, I think it would be harder for me because I might want to change the question in the unit. (Interview 4/22/98)

Ms. Linne considered worksheets for homework from a traditional text because her class included students with the lowest ability among the fifth-grade students in her school. Ms. Linne taught units that contained content on the standardized tests, and she integrated MiC units with various science and social studies units. Students' performance on previous lessons determined the pace of instruction and the choice of supplementary materials (Interview 4/22/98; Teacher Log 1997–1998).

Classroom Achievement

At the beginning of Grade 5, Ms. Linne's class had the lowest mean percentile score on the standardized tests, 25.11 ($N = 9$) among the fifth-grade study teachers in Districts 1 and 2. For comparison purposes, the mean percentile score at the beginning of the study for the three study classes taught by Ms. Mitchell was 43.04 ($N = 46$). At the end of Grade 5, the overall mean score in classroom achievement for Ms. Linne's class was 242.47 ($N = 9$), and Ms. Linne's class modestly outperformed students in Ms. Mitchell's classes (mean score 238.96, $N = 53$). Ms. Linne's class of students with low ability, some mainstreamed from special education classes, performed well in classroom achievement at the end of Grade 5. The combination of Ms. Linne's attempt to teach mathematics for understanding, the opportunity to learn mathematics from six MiC units in four content strands throughout the school year, and practice on basic skills likely contributed to their strong performance in classroom achievement.

Advantages, Disadvantages, and Difficulties in Implementation

Ms. Linne listed two advantages of teaching MiC, opportunities to problem solve and increased participation in discussion:

- "Kids have to know how to problem solve. They have to figure things out" (Interview 4/22/98).
- "For the most part, they've come a long way in their willingness to give an answer and go out on the limb. What I find more interesting is the fact that there are so many different ways to do it and each kid can say, 'Well, here's how I got it,' and they're right. They've become comfortable with it. There's more than one way to get to where you're going. They listen to each other. They say to each other 'Here's how I did it. But I like your way or I don't like your way and here's why'" (Interview 4/22/98).

The disadvantages Ms. Linne listed centered on the need for practice and concern about prerequisite skills:

- "I think that for certain kids, you definitely have to use MiC in conjunction with some practice, which isn't incorporated into the book. I mean, the book is great, but at some point I have to go back and be sure that they have the practice they need" (Interview 4/22/98).
- "MiC really assumes that the kids have a lot of prior knowledge. For example, in the fraction unit, [the authors] already assume a lot of things there, and there wasn't any room, for example, for multiplying and dividing fractions. Some of the kids in my class didn't know their multiplication facts, and they're having problems in division. So there are some places where you have to be familiar enough with the units to supplement them" (Interview 4/22/98).

For students with low abilities, reinforcement and practice are important elements of instruction. Ms. Linne was concerned that there was not enough practice in the MiC fifth-grade number units, for example, with operations with fractions, a concern also voiced by Ms. Mitchell. As noted before, mastery of fraction operations at this grade level conflicts with the intent of MiC to build on students' intuitive knowledge of fractions, introduce tools to support students' thinking about fractions, and revisit and extend these operations to more abstract forms in sixth- and seventh-grade units. Ms. Linne did not use the MiC ancillary unit, *Number Tools*, which was designed to support her students' learning in the number strand. Rather, she supplemented MiC units with worksheets from the traditional text she used in previous years. Nevertheless, Ms. Linne felt that her students benefited from the open-ended unit tasks, problem solving, and discussion of ideas that were opened through the study of MiC units.

Summary

Ms. Linne taught a small class of students with low abilities; some students were mainstreamed from special education classes. She attempted to teach mathematics for understanding, and she provided opportunities for students to learn mathematics in units in each content strand. These students showed a strong performance in classroom achievement at the end of Grade 5. The combination of her attempt to teach mathematics for understanding, the opportunity to learn mathematics from six units throughout the school year, and practice on basic skills likely contributed to their performance in important ways. Although she integrated extra skill practice into lessons daily, Ms. Linne felt the benefits students experienced from studying MiC stemmed from the problem solving and discussion promoted in the units.

Special Education Class in Middle School

Ms. Schroeder taught mathematics to a special education class at Calhoun North Middle School in District 3. Ms. Schroeder taught this class in a resource room, but also helped students when they were mainstreamed into regular classes taught by seventh-grade teacher, Ms. Perry, and eighth-grade teacher, Ms. Wells. Ms. Schroeder taught the same students in Grades 7 and 8. Her teaching and assessment practices led to a high quality implementation of MiC. She taught mathematics for understanding, maintaining the problem solving and inquiry that the MiC development team had intended while also integrating practice of basic skills.

Ms. Schroeder believed that her special education students learned mathematics through actively solving problems that are set in real-world contexts, rather than completing pages of practice with algorithms. She felt that her students needed more repetition than students in regular mathematics classes, and she incorporated a review of basic skills in her lessons daily. She stated that even though some of her students needed to learn multiplication facts, for example, this did not deter her from asking students to solve problems that involved multiplication. By stressing problem solving in the years before MiC was available and using MiC throughout the school year for multiple years, "we've pushed them farther than we thought we could with the [traditional] textbook" (Interview 1/15/98). Ms. Schroeder's beliefs about the importance of problem solving in mathematics instruction were consistent with those espoused by the principal of her school:

> I don't think there's any purpose in learning basic skills if you're not applying those to problem solving. . . . We try to present problems that require basic skills to solve problems, and then in the context of solving the problems, we make sure that students have mastered their basic skills. (Principal Adler, Interview 9/29/99)

Principal Adler supported the mathematics faculty at Calhoun North in the change to problem solving and inquiry in mathematics instruction, and Ms. Schroeder believed in this emphasis for her special education students.

School Capacity

Ms. Schroeder perceived that the capacity of her school to support mathematics teaching and learning was at a high level. She felt that she received strong administrative support in terms of clearly communicated expectations and changes in instructional practice. She had a high influence in planning and teaching mathematics. Ms. Schroeder felt that faculty and staff were committed to academic excellence and that teachers supported one another in their efforts to improve instruction. The mathematics department chair, Ms. Wells, acted as a mentor for other teachers, and Ms. Schroeder had the opportunity to learn about teaching MiC as she worked with special education students who were mainstreamed into one of Ms. Wells' classes. Ms. Schroeder commented: "That's the number one way I'm learning about MiC" (Interview 1/15/98). By observing this exemplary teacher, Ms. Schroeder learned about the content in MiC units and ways to promote discussion about mathematics (Interviews 1/15/98, 6/8/99, 1/5/00; teacher questionnaires 9/97, 6/00).

Opportunity to Learn with Understanding and Instruction

Classroom observations were not completed for teachers in District 3. The information in this section was gathered through interviews and, to a lesser extent, questionnaires. Ms. Schroeder taught multiple units in the number, algebra, and geometry strands each year. At times, she supported students' learning by teaching entire MiC units or portions of units from the previous grade level before teaching units at the current grade level. When MiC was implemented in the school for the first time, Ms. Schroeder helped students learn how to solve problems by actually doing every problem together as a class. After a few units, however, she began to see them develop confidence that they could solve the problems in small groups. Over time, Ms. Schroeder began lessons with talking about the content of the new lesson *before* students opened the unit and followed the introduction with a problem similar to the unit problems. This introduction has helped students become more successful when they solved unit tasks in small groups because it addressed the difficulties they had in interpreting the text:

> Before I even begin, we do a lot of discussion and then some parallel problems before they ever, ever do the problems in the book, because it's difficult. The reading, the understanding of what's being asked of them, is hard for them. (Interview 6/8/99)

This type of lesson introduction, multiple contexts in the unit, and manipulatives in unit activities helped students learn content that may have been inaccessible in the past. For example, Ms. Schroeder reported that one of the students' favorite units was a sixth-grade algebra unit *Operations* (Abels, Wijers, Burrill, Cole, & Simon, 1998), in which they learned about operations with integers. She reported: "They thought they were doing algebra, which they were! They've got a mindset to think about it" (Interview 6/8/99). Over time, Ms. Schroeder noticed changes in students' work: "They're deriving a lot more. Their answers are much more complete" (Interview 6/19/00). In one seventh-grade class, four students worked in pairs on unit tasks and became very competitive: "It was a tremendous class. They worked very independently, and then when we finished a section, they were very eager to hear answers and they argued about one answer in particular" (Interview 6/19/00). Ms. Schroeder encouraged discussion and solving problems with multiple approaches:

> They talk about how they got their answers. MiC lets them choose different ways of solving the problems. That's one thing I think they've really realized, that not everybody has to do everything the same lock step way. They found out that they are smart. . . . They do have different learning patterns, and they aren't able to always use them. Their talents aren't always obvious. So when they can defend their answer, and they've gotten it in a totally different way than others—you know, it's kind of nice for them. (Interview 6/8/99)

Ms. Schroeder felt that the content of MiC was more appealing to students because of the real-life contexts. She did not shy away from presenting units that focused on integer operations, calculation with powers of ten, ratio, three-dimensional figures, and Euler's Formula—content that formerly was not even approached with special education students in middle school. She commented: "They're very impressed with their abilities" (Interview 6/19/00). Ms. Schroeder supported their learning in significant ways through special lesson introductions, small group work, and class discussion of solutions to unit tasks.

Instructional Planning

Ms. Schroeder was aware of the recommended sequence of MiC units, but modified it to meet the needs of her students who were in special education: "I have to decide first of all what units we will do. They don't necessarily do the grade level that the students are in. If they haven't had the previous units, we might go back and do a sixth-grade unit" (Interview 6/19/00). When planning to teach a MiC unit, Ms. Schroeder reviewed the unit, especially the section reviews, to become familiar with the content of the unit, unit goals, vocabulary, and kinds of problems.

When planning to teach individual MiC lessons, Ms. Schroeder worked through all of the problems to become familiar with the content of the lesson. Since most of Ms. Schroeder's students had difficulties with language and reading, she especially noted the vocabulary and sentence structure and thought of ways to restate the problems without changing their meaning. To help give students a frame of reference, Ms. Schroeder prepared a problem that was very similar to the problems in the lesson, sometimes from the MiC ancillary materials *Number Tools*, which she worked through with them prior to the lesson. Ms. Schroeder valued small-group work because it gave students the opportunity to talk about how to solve problems, and it gave her the opportunity to listen to their discussions. She commented: "I can keep track of what [students] are actually doing, because there are only four of them [in a group]. I was observing them every single day, seeing what they were writing, and listening to them" (Interview 6/19/00). Ms. Schroeder's classroom assessment practices began during the parallel problems posed at the end of the lesson introduction and continued through group work. These informal methods of assessment were focused on students' learning of the mathematics, their reasoning, and their communication.

Classroom Achievement

Changes in students' performance over the two years are shown in Table 1-5. The mean overall classroom achievement score increased substantially from 182.01 at the end of Grade 7 to 219.11 at the end of Grade 8. Substantial increases were also evident in the number, algebra, and statistics strands. Ms. Schroeder chose to provide opportunities for her special education students at both grade levels to learn comprehensive content in ways that promoted understanding, rather than limit them to practicing algorithms for entire class periods day after day. She believed that students needed to experience problem solving along with basic skills practice. Her teaching and classroom assessment practices focused on students' understanding of the mathematics and the development of reasoning and communication skills. This combination of a high quality of instruction and opportunity to learn mathematics with understanding contributed to substantial gains in student performance.

Table 1-5.
Classroom Achievement for Students in Ms. Schroeder's Classes in Grades 7 and 8, Overall and by Content Strand

	N	Classroom Achievement Mean
Grade 7		
Overall	7	182.01
Number	7	175.51
Algebra	7	167.91
Geometry	7	224.15
Statistics	7	201.78
Grade 8		
Overall	5	219.11
Number	5	239.73
Algebra	5	219.73
Geometry	5	216.82
Statistics	5	219.84

Advantages, Disadvantages, and Difficulties in Implementation

Ms. Schroeder listed some advantages of using MiC with her special education students:

- "It gives them real-world experiences that they don't get from a textbook. For example, *Operations*. That unit gave them a lot of encouragement. They really understood the time differences" (Interview 6/9/99).
- "It's far more than just calculating, and it's closer to things they're involved in doing every day—having to measure things that are around them, rather than just doing arbitrary problems, and in *Number Tools*, the problems with how many

individual plants in the containers and containers in a cart. Then they all made up their own problems and solved them using ratio tables" (Interview 6/19/00).

Ms. Schroeder talked about how difficult MiC was for both herself and her students:

- "It's hard. It's hard, especially when they don't get it. As a teacher, it would be much easier to open a textbook and assign the odd ones on this page" (Interview 6/9/99).
- "The math is different. There is math that I didn't know how to do myself" (Interview 6/19/00).

However, when asked whether she thought it was worthwhile to stick with MiC, Ms. Schroeder stated emphatically: "I'd definitely stay with it. Absolutely!" (Interview 6/19/00). The benefits for her students outweighed the difficulties she had in learning and presenting mathematics that formerly was not expected for special education students.

Summary

Ms. Schroeder taught mathematics in Grades 7 and 8 to a special education class in a resource room. Her teaching and assessment practices led to a high quality implementation of MiC. She taught mathematics for understanding, maintaining the problem solving and inquiry that the MiC development team had intended while also integrating practice of basic skills. Ms. Schroeder taught multiple units in the number, algebra, and geometry strands each year, and at times, supported students' learning by teaching units or portions of units from the previous grade level. Substantial increases in student performance were evident in the number, algebra, and statistics strands. The combination of a high quality of instruction and opportunity to learn mathematics with understanding contributed to substantial gains in student performance.

Conclusion

The stories in this chapter illustrate that, when viewed together, relationships among teachers' beliefs, school context, teaching and assessment practices, and instructional planning provide a different perspective in looking at student performance than when separately scaled in the indices developed for this study. It is clear that when teachers implemented MiC with instruction that attended to conceptual understanding and a higher level of discussion than in typical mathematics classrooms, students' performance changed in powerful ways over years of schooling, whether with the same teacher or different teachers. It is also evident that poor implementation of MiC led to low achievement. However, large gains occurred when the quality of instruction improved for these students, even if it was more reflective of traditional instruction in mathematics classrooms. Furthermore, the performance of these students continued to increase when they experienced a higher quality of instruction and opportunity to learn with understanding. Teachers of students in a low ability group and in a special education class used MiC as their primary mathematics curriculum and taught units from multiple content strands, even though they added practice on basic skills regularly. These teachers held high expectations for their students, as did other teachers in these stories. They did not shy away from keeping the focus on conceptual understanding and letting students tackle difficult content. These practices influenced the important changes in classroom achievement demonstrated by their students.

The teachers in these stories reported that more time was necessary for planning to teach MiC units. Their reasons varied from thinking about the mathematical goals of the units, working through all unit problems, learning new mathematics content, learning mathematics conceptually, and planning lesson introductions. Some teachers had the benefit of common planning time with other mathematics teachers during which they discussed particular MiC units, mathematics content, and instructional and assessment methods. One teacher reported that observing an exemplary teacher teaching MiC was an important learning experience for her, and another teacher reached out for help from a contact outside the school. These situations underscore the need for teachers to have times when they can collaboratively work with one another to understand the content they have not previously taught and to have district specialists or consultants available for teachers when they feel the need for support from others.

The teachers in these stories talked about difficulties in implementation such as helping students develop explanations of high quality, working with absentees or students new to MiC, and grading student work. They mentioned that MiC provided opportunities for reasoning and depth of understanding, improved student interaction, explanations of thinking and written communication, and connections between mathematics and life experiences. And they thought the benefits they found in teaching MiC offset the difficulties they experienced in implementation.

CHAPTER 2. INSIGHTS ABOUT IMPLEMENTING A STANDARDS-BASED CURRICULUM IN SCHOOLS

Mary C. Shafer and Thomas A. Romberg

The purpose of this chapter is to provide answers to questions about the implementation of the standards-based curriculum, *Mathematics in Context* (MiC), a NSF-supported middle-school curriculum. If a school district decides to adopt such a program what can be expected? In particular, how might teachers implement the program? Can improved student performance be expected?

Because MiC expects changes in both the mathematical content that is taught and the way that content is taught, one can expect considerable variation in how the new materials are implemented in classrooms by teachers. Fidelity to the content and pedagogical intent of MiC is a serious issue. Some teachers may not use all the units at a particular grade level, others may select only units in particular content strands, and still others may sequence units based on contexts common in other subjects. Some teachers trust that students will develop and practice basic skills as they study MiC units. Other teachers will supplement MiC with other resources such as worksheets and lessons from traditional curricula, in some cases to the point that the supplementary materials subsume MiC. Furthermore, some teachers will teach MiC in the spirit of the intended curriculum, as they encourage students to investigate problems, compare solutions, and make connections among mathematical ideas. Other teachers will teach MiC with traditional methods of instruction by telling students how to solve problems, which diminishes the intended inquiry. Therefore, when adopting a standards-based curriculum such as MiC, some teachers will implement the new curriculum well, while others will nominally implement MiC by continuing to use lessons and worksheets from their prior programs, and/or teach the new program as if no changes in instructional methods were expected.

There are several reasons to expect variation in the way MiC is implemented. First, to implement MiC well, teachers need professional development prior to teaching MiC and on-going support during implementation. This support requires administrative assistance and includes opportunities for teachers to meet and share on a regular basis. Prior to teaching MiC, teachers find it helpful to learn about the philosophy of the curriculum and to work through each unit with other teachers. In this process, they become familiar with the presentation of the content, the kinds of tasks students are expected to complete, homework assignments, and assessments. As they teach MiC throughout the school year, their needs for professional development often change. Teachers request sessions about developing effective strategies for group work, class discussion, and informal classroom assessment. If they receive limited professional development or if they are isolated as the only MiC teacher at their grade levels or in their schools, opportunities for reflective thinking and discussion are limited, and difficulties in implementation are likely to occur.

A second reason to expect variation in implementation of MiC is related to the content of the instructional units. In contrast to the focus on arithmetic skills in conventional middle-school mathematics curricula, MiC also includes geometry, measurement, pre-algebra, data analysis, and probability. Some teachers lack experience teaching these content areas. For example, topics involving percent are included in fifth-grade MiC units, topics traditionally taught in middle school; sixth-grade algebra units include solving systems of linear equations, content generally taught in a ninth-grade algebra course; seventh- and eighth-grade units include using the Pythagorean Theorem and proving that triangles are similar, topics usually taught in a tenth-grade geometry course. Because they had never taught this content, some teachers simply show students how to do the work without letting them investigate the content as presented in MiC units. Others revert to teaching the content they are familiar with, the basic skills content that is the theme of traditional texts, or they might structure lessons so that more time is devoted to skill development than to teaching MiC. Thus, some teachers will regress to focusing teaching only on the content they are familiar with and are comfortable teaching.

The changes in pedagogy required for teaching MiC will vary among teachers as they implement the curriculum. Conventionally, mathematics instruction has followed a five-step procedure focused on covering the content of a textbook: check homework, explain today's lesson, collectively work similar problems, assign problems to do in class, and assign homework. In contrast, MiC focuses on teaching for understanding. This often involves posing contextual problems, and having students investigate ways of representing and solving the problems, discussing and comparing solution strategies. Expected by the developers of MiC, this pedagogical approach is not easy for teachers who themselves have never experienced mathematics in this way. Many teachers recognize that opportunities to use different pedagogical approaches are opened and are expected when teaching MiC. But few teachers have experienced, or discussed with other teachers, what these changes might look like in action. Therefore, teachers will be at different levels of teaching mathematics for understanding.

A fourth reason to expect variation in implementation of MiC is differences in teachers' classroom assessment practices. In traditional mathematics classrooms, student performance is judged in terms of the number of correct answers to a set of procedural tasks. In addition, however, MiC expects teachers to also judge the fruitfulness and quality of students' solutions to complex tasks. Some teachers capitalize on the opportunities for classroom assessment practice that are opened as they observe students complete tasks and listen to them talk about their ideas. These teachers seek evidence of students' approaches to problem solving, the reasoning they used, and accuracy of solutions. If students have trouble explaining their ideas, they use questioning techniques to help students express their ideas more clearly. Since "what gets tested is what gets learned," if teachers fail to incorporate new assessment strategies, students will not see problem posing and solving as important. In this case, teachers seek only evidence of procedural understanding. They attend more to the accuracy of answers than to the thinking involved in determining solutions, and the quizzes and tests they use focus on particular skills at the expense of learning about process. Thus, the nature of teachers' classroom assessment practice will vary among MiC teachers.

Differences in the settings in which student learning is situated also lead to variation in implementation of MiC. Districts and schools are complex social organizations and the adoption of a new mathematics program, such as MiC, is often only one of several initiatives. For example, initiatives such as mandatory silent reading in every class period, supplementary materials that feature application of mathematics in the workplace, and computer-assisted drill-and-practice programs take significant amounts of class time and compromise the time necessary for students to investigate MiC unit activities. Another initiative in many districts is for eighth-grade students to complete a traditional ninth-grade algebra course. For students who do not meet the criteria for these programs, teachers may choose to use a traditional algebra text during eighth grade rather than teaching the MiC algebra units or continue teaching units from various content strands. Thus, other initiatives may take precedence over MiC during class time, compromising students' opportunity to learn comprehensive mathematics with understanding. Another aspect of the settings in which MiC implementation takes place involves the support of district personnel. In some districts, particularly in large urban areas, implementation of MiC begins on a small scale, involving a few schools in a particular region, before adoption by more schools in the district. In this case, district mathematics specialists are responsible for supporting teachers who are implementing MiC along with all the other programs and initiatives in the district. Thus, teachers receive minimal attention and support as they face the challenges of teaching a new curriculum, and their needs for specific types of professional development, such as focusing on conceptual understanding, leading student discussion, and developing new ways to assess student learning, are not addressed.

In summary, some teachers will implement MiC well, while others will only nominally implement MiC in the intended ways because they extensively add supplemental materials or they teach MiC with traditional methods. Several factors influence the ways teachers implement MiC, which include ongoing professional development and support, experience teaching particular content, and changes in teaching and assessment practices. Complications occur when district- or school-mandated initiatives compromise the time needed for students to investigate and reflect on the mathematics they are learning in MiC units.

Reasons for Variation in Implementation by Study Teachers

Most study teachers who used MiC found that implementing the curriculum was difficult work, and several reasons account for the variation in the implementation by study teachers. First, teachers taught content they had not previously taught. Some teachers learned unfamiliar content from another teacher. For example, two fifth-grade teachers taught the same sequence of units. When one did not understand the content, she asked the other teacher, the grade-level mathematics expert, to explain it to her. In one middle school, teachers taught different units because there were only enough units for one teacher's classes. The team leader, who was completing a masters degree in mathematics education, was the first to teach units that addressed unfamiliar content. For each unit she taught, she made notes about the content and presentation of the content in the teacher guide and a notebook. When other teachers taught the unit, they used her notes and discussed difficult portions with her during team meetings. Similarly, another teacher who was working toward a secondary teaching certificate in mathematics helped other teachers learn unfamiliar content. In addition, emphasizing connections among mathematical ideas, a design feature of MiC, was new to some teachers. For example:

> I am personally surprised at the difficulty in teaching the interrelated concepts of ratio, fractions, decimals, and percent. Since I personally learned all of these concepts by algorithms, it is difficult for me to rethink how to present this to students. (Sixth grade, Gollen, Journal entry 2/11/99)

Furthermore, mathematical tools that support students' thinking, such as fraction bars and the ratio table, were new to teachers. For example:

> I have found this unit wonderful for my students—they are able to make relationships among fractions while working with fraction bars that would have been difficult with the use of formulas. They are able to make connections and come up with great strategies, and they appear to have a clear understanding of the concepts presented. (Fifth grade, Murphy, Journal entry 11/14/97)

On the other hand, the same teacher reported that she and her students did not completely understand how to use a ratio table, and she asked the research team for help in this regard:

> Please tell me how to explain p. 52 Section B "The Ratio Table" in *Number Tools* [a MiC ancillary resource]. The students and I understand doubling and adding. However, we want to know which columns to add if they are not in bold type. (Fifth grade, Murphy, Journal entry 1/6/98)

Teachers also had access to a toll-free phone number to talk with members of the research team or members of the MiC development team, and teachers felt this was a beneficial resource. For example:

> [Finding the measures of exterior angles of polygons] was difficult for the students. I must admit, I was a little confused myself in the beginning. It did help talking to [a member of the research team]. (Fifth-grade, Mitchell, Journal entry 2/27/98)

In these cases, study teachers worked together or took the initiative to reach out to the research team to help them learn unfamiliar content and ways to use mathematical tools. Other teachers dismissed unfamiliar content in geometry and statistics units by teaching only units with a primary focus on number or algebra. Others added worksheets from supplementary resources for students to practice skills, such as algorithms for dividing fractions through exercises that were devoid of contexts, rather than letting them explore the mathematics in MiC units.

Second, implementation of MiC was affected when teachers used portions of MiC units from earlier grade levels to support students' thinking. For example, one seventh-grade teacher taught three MiC units in two content strands. However, she also taught sections of fifth- and sixth-grade MiC units that contained prerequisite skills for the seventh-grade units. Even though the instruction in her classes was more reflective of conventional pedagogy, supplementing the seventh-grade units with portions of units from previous grade levels may have influenced the substantial gains in achievement for her students in her classes.

A third reason implementation varied among study teachers was differences in pedagogy. Some teachers were effective leaders of substantive discussions that promoted making sense of the mathematics, and they worked toward helping students link conceptual and procedural understanding. Other teachers expressed concern about how to help students when they could not find their own solutions, wondering, for example, whether “it is the unknown and unfamiliar that is preventing spontaneous behavior or something else” (Fifth grade, Murphy, Journal entry 1/22/98). They worked at guiding students to complete unit tasks and found that they needed to provide time for students to reason out solutions: “It’s so hard to get the kids to think through things, to take enough time to relax with the information, so they can see patterns emerge” (Eighth grade, Reichers, Journal entry 1/6/99). Teachers also struggled with providing time for students to discuss unit tasks. They gradually developed ways for students to do the mathematical work *and* discuss various solutions, for example: “I learned to introduce the lesson, ask students to work on a few problems, reconvene to discuss the content with the whole class, and repeat that during the lesson” (Sixth grade, Dillard, Personal communication 2/26/98). In addition, teachers frequently talked about their inadequacies in discerning difficulties students might encounter as they learned new topics and ways of helping students overcome these potential difficulties. On the other hand, some teachers who taught MiC told students their own strategies for solving tasks, which stunted opportunities for students to think through their own methods. For example, a task the sixth-grade MiC number unit *Fraction Times* (Keijzer, van Galen, Gravemeijer, Shew, Cole, & Brendefur, 1998) involved describing the amount each child should receive when groups of children divided a given amount of money. The lesson as presented, however, provided limited attention to conceptual understanding. The observer noted:

> Some students (not many) understood conceptually that division and fair share were involved. Mr. Harvey gave the class *his* methods for solving the problem: “I have $2.00 to split 8 ways, so [each gets] 1/4 [of a dollar] or 25 cents,” and “To split $9.00 among 5 people, I used $1.75. I’ll keep the extra 25 cents.” (Sixth grade, Harvey, Observation 3/16/98)

In this situation, the teacher expounded his own strategies, but he did not provide a way for students to make sense of them. Other teachers felt that they should not present formal lessons with MiC because they perceived that students were to do the work themselves. Students were given an assignment, but the content was not discussed prior to independent work, and students lacked the support they needed to understand the mathematics on their own. In these cases, teachers moved from student to student to explain the lesson. Confusion frequently occurred, and inquiry was reduced to waiting for teachers to do the work for them. Thus, teaching practices varied greatly among study teachers, ranging from those who promoted conceptual understanding and substantive discussion of mathematics to those who did not present formal lessons because it was perceived that students should complete the instructional units with little input from their teachers.

The time teachers had for planning instruction varied, which had an impact on implementation. Study teachers benefited from collaborative planning sessions. Teachers in one middle school had common planning time during which they discussed the mathematics and presentation of the mathematics in particular MiC units, teaching and assessment practices, and program evaluation. In addition, these teachers were given one day of release time per month in order to concentrate on planning to teach MiC. Evidence of the success of these efforts was noted in observation reports and in journal entries written by these teachers. For example:

“Interesting fact: The more comfortable I am teaching MiC, the better the kids like it. Having the MiC days to prepare, discuss, and work through is *very* beneficial” (original emphasis, eighth grade, Teague, Journal entry 4/29/99). In contrast, teachers who taught MiC in isolation in their schools or at their grade levels struggled with implementing MiC, especially at Grade 6. At this grade, students began their middle-school years, and the structural and cultural conditions were different from what they experienced in elementary grades. In most cases, student performance declined over the year. However, significant improvement in student performance was evident for students in the middle school in which the MiC teachers had substantive discussions about mathematics content, instruction, and assessment during common planning times. This type of collaboration likely supported the quality of instruction and opportunity to learn mathematics with understanding that students experienced. This, in turn, led to improved performance and mitigated the impact of students’ transition from elementary to middle schools.

A fifth reason implementation varied is the extent of support teachers received as they taught MiC. For instance, one study teacher, the only MiC teacher in her school, requested mentoring in the classroom on a regular basis so that she could discuss unit goals, plan reactions to potential student difficulties, and develop skills in leading classroom discussions. Another teacher, the only MiC teacher at her grade level, requested that a consultant teach her class while she observed the lesson, a request that was accommodated by the district mathematics specialist. The teacher found this to be a worthwhile experience that influenced her teaching. In addition, both of these teachers felt supported by the study’s on-site observer. Even though the observer was not in the classroom in the capacity of a mentor, the teachers appreciated opportunities to get some general feedback from the observer. However, both teachers expressed the need to discuss their teaching in greater depth regularly. These requests went beyond the professional development opportunities provided in their district. In the years prior to the study, the focus of professional development was to review reform-based curricula and subsequently teach some units as replacements for particular sections in their current curricula. During the first study year, the district mathematics specialist arranged focus group meetings for all teachers who were implementing reform curricula. Each month teachers explored general pedagogical issues including student-centered instruction, assessment, and use of mathematical tools such as the ratio table. These meetings were held after school hours, and teachers were compensated by the district for their participation. During subsequent years, however, the years these two teachers were in the study, focus meetings were not held, and these teachers were left with limited support to make changes in their teaching practices. In comparison, in another district, during the summer before the first year of teaching MiC, teachers participated in a district-funded weeklong camp in which they looked for conceptual development across the MiC units at all grade levels, determined the sequence of teaching the units, and discussed instructional approaches for effectively teaching MiC. In addition, fifth-grade teachers met weekly before school without pay to collaborate on teaching MiC. Throughout the school year, school administrators provided paid monthly evening meetings for the teachers during which they discussed implementation issues and continued their in-depth review of specific units in preparation for teaching them. The monthly meetings took place in subsequent years, although the focus varied each year. The contrast between professional development opportunities in these districts demonstrates the variation in ongoing professional development for study teachers.

In summary, study teachers varied in their implementation of MiC. This variation stemmed from teaching unfamiliar content in ways that promoted conceptual understanding, teachers' knowledge of ways to support learning of units at a particular grade level with units from previous grade levels, pedagogical methods that supported student reasoning and communication, time for increased instructional planning and collaborative experiences in planning, and the extent of administrative support in changing curriculum and instruction in mathematics at these grade levels.

Need for Long-Term Professional Development

The professional development provided for teachers by the research team was critical in supporting their changes in teaching and assessment practices. During the first four-day summer institute in 1997, the research team selected major themes or substrands as the focus of the selected activities in each content strand. Teachers worked through student lessons in cooperative groups, learned about methods for generating classroom discussions, and talked about linkages among the lessons. The philosophy of MiC was illustrated with the algebra strand, and work in the number strand focused on developing rational number concepts and using the bar model and the ratio table to support students' thinking. Sessions on the geometry and statistics units featured lessons from substrands in each area. Throughout this institute, teachers openly used multiple ways to solve problems and talked about explicit and implicit concepts in the selected lessons. Teachers raised and discussed a variety of issues including the conflict between mastery of concepts over time and grading; use of representations to support reasoning; prerequisite knowledge for using MiC number units; managing students who work at different paces; meaningful feedback during whole-class discussions; and calculator use. They learned about MiC end-of-unit assessments through scoring and discussing student responses on the assessment for a sixth-grade number unit. Teachers also discussed ideas for homework when teaching MiC.

The summer institutes in 1998 began with a session during which teachers discussed implementation issues. Teachers in both districts recognized that planning was imperative in teaching MiC, as opposed to the limited planning necessary when they taught conventional curricula. The issues mentioned by teachers in District 1 were more related to the units themselves such as understanding the mathematics, unit goals, and vocabulary. They were concerned with identifying the most important concepts and tasks in each lesson, portions that could be omitted, and items for homework. Issues such as cooperative groups, assessment, and working with parents were discussed, but plans of action for developing effective instructional strategies were in the beginning stages. The issues raised by teachers in District 2 were more concerned with pedagogy, particularly working to develop the quality of student responses. For example, they noted that modeling explanations was crucial. They established guidelines for good answers and talked about written answers as a class. This type of classroom interaction opened possibilities for teachers to learn about student thinking and to provide feedback that targeted particular needs and strove to help students develop mathematical understanding. In addition, teachers had clear ideas for assessment. They planned frequent written assessments based on the summaries of each section in an instructional unit, rather than only the formal assessment at the end of the unit. In this way, they responded to student needs on a regular basis. The institutes continued with a demonstration lesson on geometry. In District 1, this lesson was presented by a seventh-grade study teacher. Other sessions included attention to substrands in number, algebra, and statistics units. These sessions were different from the 1997 institutes in that teachers at particular grade levels traced development of particular content for each strand, and they presented the findings for their grade level to the larger group. Teachers also discussed possibilities for informal classroom assessment, homework, and the newly developed parent materials. Teachers and consultants worked with feeder-pattern groups to list units taught in the previous school year and to suggest order of teaching MiC units for the current school year.

MiC teachers expressed the need for learning about quality tasks for formal assessments and scoring rubrics for these tasks. This became the focus of the third professional development institute. Teachers discussed what to assess and how to assess; assessments that attended to the three levels in the Dutch assessment model (c.f., de Lange & Romberg, 2004); grading such assessments; designing assessment tasks; and developing scoring rubrics. Teachers worked individually or together to create and refine assessment tasks. They brought in examples of general scoring rubrics to discuss and compare: one strictly focused on mathematics in student responses; one expressed in "student-friendly" language with attention to mathematical understanding, thinking, and communication; and one attending to the correctness of the solution, the formulation and execution of a solution plan, and the clarity of explanation. Teachers independently scored a set of twenty student assessments from a task on the end-of-unit assessment for a seventh-grade MiC algebra unit according to their own scoring principles. The results were compared during an intense discussion of the consistency among scorers, the types of items that should receive more score points, attention to what students were trying to convey in their responses, and translating the scores into grades. Teachers were encouraged to record their ideas about the characteristics of good assessment tasks and things to remember when scoring assessments. This type of professional development provided opportunities for middle-school teachers to think about and discuss assessment issues and work with each other to create or select quality assessment tasks and scoring rubrics.

Scoring institutes presented by the research team each spring also provided opportunities for teachers to learn about quality assessment tasks and item-specific scoring rubrics. In each district, teachers scored some items from each grade-specific Problem Solving Assessments developed for the study. During scoring, study teachers noticed differences in the ways students expressed their thinking, and they talked about how they might work with their students to develop more complete responses. Teachers expressed the value of this experience for them. For example: "It was very interesting. It's such an eye opener to see student work. I really enjoyed the opportunity to see problems from all three grade levels" (Eighth grade, Wells, Personal communication 5/99) and "Very beneficial in understanding how students think about solving math problems" (Seventh grade, Perry, Personal communication 5/99). The remaining study assessments were scored during institutes held at the research center by elementary, middle school, and high school teachers, who felt this experience had a significant impact on their teaching. They commented that they emphasized mathematical communication, included lessons that promoted more complex reasoning, and integrated various types of problems designed to elicit student thinking at more complex levels.

These types of professional development opportunities are not available to most teachers as they implement MiC over time. District mathematics specialists raise important questions about professional development when teachers implement a new curriculum. For example, the mathematics specialist in District 1 listed four issues that need attention (Quincy, Personal communication 7/13/05, 7/20/05). First, what are effective ways for teachers to learn about and trace the development of particular concepts across MiC units in settings that are available in schools? Might this be the focus of mathematics department meetings at the middle-school level? If so, who would lead such sessions? Second, what are effective ways to help teachers learn the mathematics they need to teach MiC units? Third, given the mobility of teachers from one school year to another, what can be done for teachers who are new to the system each year? Finally, after teachers become more comfortable with the goals, content, and presentation of the content in MiC units, the focus needs to change to refining teaching practices in order to bring instruction to a higher level of discourse. What are effective ways at the district level to engage teachers in substantive discussions that promote in-depth changes in teaching and assessment practices? Thus, the needs for professional development are extensive and long-ranged as teachers strive to implement MiC in ways that are consistent with the spirit of the curriculum and that lead to teaching mathematics for understanding.

Expectations about Student Performance

Assessment of student performance as a consequence of implementing a new middle-school curriculum, such as MiC, depends on answers to three questions:

1. Have the students had an opportunity to learn the content and processes emphasized in the new curriculum?
2. Were the students adequately prepared to study the content of the new curriculum?
3. Is the district's method of assessing student performance aligned with the new curriculum?

The first question raises the issue of the quality of implementation of MiC: Did students have the opportunity to learn comprehensive content with an emphasis on connections among ideas? Did they have the opportunity to develop mathematical processes such as reasoning and communication? In this study, we found that when MiC is implemented well, improved student performance is likely to occur. For example, student performance increased over time when teachers taught MiC units through the entire school year in ways that promoted conceptual understanding, investigation, reflection, and discussion. Even when teachers used conventional methods but taught MiC for the entire year, student performance was positively affected, especially when teachers used portions of units from previous years to support students' thinking in grade-level units. On the other hand, we found that when MiC is not well implemented, expectations about student performance are unlikely to be realized. When only a few MiC units were taught during the school year because MiC was substantially supplemented with skill practice or MiC was replaced with a conventional textbook, student performance remained at the same level as the previous year or declined considerably. In fact, when MiC was not implemented well, student performance was not distinguishable from performance of students who studied conventional curricula.

The second question raises the issue of skills that were assumed prerequisite for studying MiC units: Were the students adequately prepared to study the content of MiC? Prior achievement in elementary school mathematics strongly influences the level of student achievement when using a new curriculum in middle school. The authors of MiC expected that students would begin Grade 5 with knowledge of whole number operations, understanding of fractions such as ½ and ¼, understanding of money, measurement with customary units, and beginning ideas of properties of shapes. As students began Grade 6, it was expected that they had beginning ideas of operations with fractions and decimals, an understanding that one whole is equivalent to 100%, and a greater understanding of properties of shapes. We learned from study teachers, however, that students often did not have these prerequisite skills. For example, teachers in District 2 routinely noted weaknesses: in Grade 5, basic facts; in Grade 6, multiplication facts; simple addition, subtraction, and multiplication with decimals; differences between doubling a quantity and taking half of a quantity; differences between shapes such as quadrilateral, rectangle, square, and parallelogram; and the measure of a right angle; and in Grade 7, subtraction of whole numbers with regrouping, multiplication of two-digit numbers, and division; comparing and ordering basic fractions such as ½, ¼, and ¾; using rulers to measure length; and beginning notions of area and volume. Nevertheless, many students were able to learn these skills as they studied MiC units. For example, although particular units did not have a primary focus on operations with whole numbers, these operations were embedded in lesson activities in numerous units across content strands. There was also concern about whether students who began their study of MiC in Grades 6 and 7 would succeed without having studied the Grade 5 units. But this concern was unfounded for groups of students who studied MiC as their primary curriculum for entire school years. For example, students who studied MiC in both Grades 6 and 7 experienced gains in performance in classroom achievement from 3% to 16% over the two-year period. Gains were also evident for students who studied MiC over two years beginning in Grade 7, even though students did not have the benefit of studying MiC units at Grades 5 and 6. On the other hand, when middle-school teachers taught some MiC units and supplemented heavily with computational drills, gains in performance were minimal or performance declined substantially. The push for learning particular procedures did not have the impact these teachers had hoped.

The third question involves the alignment of MiC with district or state mandated standardized tests: Were the districts' methods of assessing student performance aligned with the goals and content of MiC? Using traditional standardized tests limits the judgments about student performance when students study MiC. Because MiC emphasizes different mathematical topics and strategies, assessing student achievement requires summative assessments that reflect these emphases. In Districts 1 and 2, one component of the state testing system was a state-developed performance-based assessment. These assessments, administered at Grades 5 and 8, were aligned with the content and mathematical processes emphasized in MiC. However, the other component in each testing system was a traditional test, which often determined placement of students in summer school and in classes at the next grade level. In District 2, district funding was directly tied to student results on the traditional portion of the testing system, which fueled needs for test preparation on basic skills. In Grades 6 and 7, these tests did not give students a chance to demonstrate the mathematics they could apply when solving novel problems, the reasoning they could employ, and their communication of approaches and results of problem solving.

Nevertheless, information from the longitudinal/cross-sectional study makes it clear that the quality of MiC implementation has a substantial impact on student performance. When MiC was well implemented, students' performance was good and it grew over time. When teachers encouraged students to investigate problems, compare solutions, and make connections among mathematical ideas, and when students studied MiC units in three or four content strands during the entire school year, they showed gains in performance. The combination of instruction that attempted to promote conceptual understanding and comprehensive content taught in some depth had a positive effect on student performance. The type of collaboration experienced by teachers also made a difference. When teacher collaboration involved discussion of mathematics curriculum, instruction, and assessment, student performance increased. This type of collaboration likely supported the quality of instruction and opportunity to learn with understanding students experienced. Also, when the quality of instruction students experienced surpassed the instruction they experienced in the previous year, students showed substantial gains in performance. In one case, these gains may also have been influenced by supplementing seventh-grade MiC units with portions of units from different grade levels. Furthermore, when the quality of instruction was high and students studied MiC the entire school year, considerable gains in performance were evident from Grade 7 to Grade 8. The results were evident whether students had the same teachers in consecutive grade levels or different teachers throughout their middle-school years.

Information from the longitudinal/cross-sectional study also suggests that when MiC is well implemented, student performance surpasses performance of students who studied conventional curricula, and the difference grows over time. In fact, student performance when MiC is well implemented increases during middle school while the performance of students who study conventional curricula declines during these years. Furthermore, when MiC was nominally implemented, student performance was comparable to or less than that of students who studied conventional curricula.

The results of this study suggest that over time students who studied MiC and experienced high quality of instruction and opportunity to learn with understanding showed gains in performance. Student performance remained the same or was negatively affected when teachers implemented MiC in isolation in their schools and when students transitioned from elementary to middle schools. However, when teachers collaborated in meaningful discussions about mathematics curriculum, instruction, and assessment, the influence of these factors was mitigated and student performance increased. On the other hand, students who studied conventional curricula showed declines in performance. Moreover, when teachers nominally implemented MiC by substantially supplementing with skill practice or replacing MiC with a conventional textbook, student performance matched the performance of students who studied conventional curricula.

Conclusion

If a school district decides to adopt MiC, variation in the ways teachers implement the curriculum will be encountered. In the longitudinal/cross-sectional study, variation in implementation occurred because teachers taught unfamiliar content with different instructional methods and had different levels of planning and professional development. Some teachers will implement MiC well. That is, they will teach the curriculum as intended by emphasizing investigation of problems, substantive discussion, and connections among mathematical ideas, and teaching multiple units in three or four content strands during the entire school year. When implemented well, student achievement grows over time, whether students have the same or different teachers in consecutive grade levels. Other teachers will nominally implement MiC. In these cases, the intended inquiry in MiC is diminished as teachers tell students how to solve problems or present underdeveloped lessons because of perceptions that teacher input is expected to be minimal. In addition, supplementary materials are given priority over MiC. When implemented nominally, student performance decreases over time and parallels the declining performance of students who studied conventional curricula. As teachers begin to teach MiC, their needs for professional development and collaborative discussions with other teachers are important and extensive. Long range professional development that focuses on the curriculum itself, knowledge of mathematical content, and teaching and assessment practices are crucial to successful implementation of MiC.

CHAPTER 3: WHAT WE LEARNED

Thomas A. Romberg and Mary C. Shafer

The purpose of this final chapter in the series of monographs is to summarize the findings of the longitudinal/cross-sectional study of the impact of teaching mathematics using *Mathematics in Context.* This proved to be a very complex study in which we gathered copious information that has been difficult to sort, scale, and write about. The first thing we learned was that we did not have the resources, staff, and time to do everything that could, or probably should, have been done. Nevertheless, to summarize what we have learned, we have chosen to focus on four aspects of the study: (1) the study design, instrument construction, and methods of data analysis; (2) working in large urban districts with administrators and support staff; (3) the students in the study; and (4) the teachers who allowed us to gather information from them and in their classrooms. The chapter concludes with the answers to our three research questions.

The MiC materials consist of 40 curriculum units (10 at each grade level 5-8), a teacher's guide for each unit, including assessment materials, and two sets of supplementary materials, *News in Numbers* and *Number Tools*. The materials differ from most conventional mathematics texts in both content and expected pedagogy. (See Monograph 1, Chapter 1 for details about the differences.) The content of MiC includes more than just the focus on arithmetic skills found in conventional materials. MiC is organized by mathematical strands based on NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989).

- number (whole numbers, common fractions, ratios, decimal fractions, percents, integers);
- algebra (creation of expressions, tables, graphs, and formulas from patterns and functions);
- geometry (measurement, spatial visualization, synthetic geometry, coordinate and transformational geometry); and
- statistics and probability (data visualization, chance, distribution and variability, and quantification of expectations).

The pedagogy underlying MiC was based on the Dutch "Realistic Mathematics Education" (RME), an approach to school mathematics deemed consistent with the NCTM *Standards*. This approach to mathematics instruction includes four components:

- goals that reflect an assumption that students need to participate in the mathematization of reality and can do that by exploring aspects of several mathematical domains. In so doing, they gradually shift from creating "models of" problem situations to "models for" mathematical reasoning and problem solving. Through this process, they should come to understand both how mathematics has developed and how it is used in the world in which they live;
- the design of a structured set of instructional activities in each of the four mathematical domains that reflect those goals;
- the provision of strategies for teachers to guide and support students' investigations of reality; and
- the development of an assessment system that monitors both group and individual student progress.

Given the differences in the MiC materials and instructional approach from conventional middle school mathematics content and instruction, we deemed it important to study the process of implementing the new curriculum materials and instructional approach in a variety of school settings. In particular, we wanted to see how the program would work in urban, high poverty school districts. During the development of the materials, considerable pilot- and field-test data were gathered, and the data provided us with encouraging information. In addition, standardized test score data reported by districts established a positive trend of substantial gains in schools using MiC. However, there was no long-term information on the implementation process, nor sufficient information about student achievement over time. Thus, the study was designed to answer three questions about the implementation of MiC. The research questions were:

1. What is the impact of the MiC instructional approach on student performance?
2. How is this impact different from that of traditional instruction on student performance?
3. What variables associated with classroom instruction account for variation in student performance?

What we Learned about the Study Design

The research model used to gather information to answer these three questions was an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). The model is composed of variables and their theoretical interrelationships (represented by arrows in the model). This model, illustrated in Figure 3-1, includes 14 variables in five categories (prior, independent, intervening, outcome, and consequent). (See Monograph 1, Chapter 2 for details on each variable.) In this section, we describe what we have learned about using a modeling approach to examine the impact of curricular materials and an intended instructional approach, the schemes we developed for gathering, scaling, and analyzing the data for this study, and utility of quasi-experimental research designs.

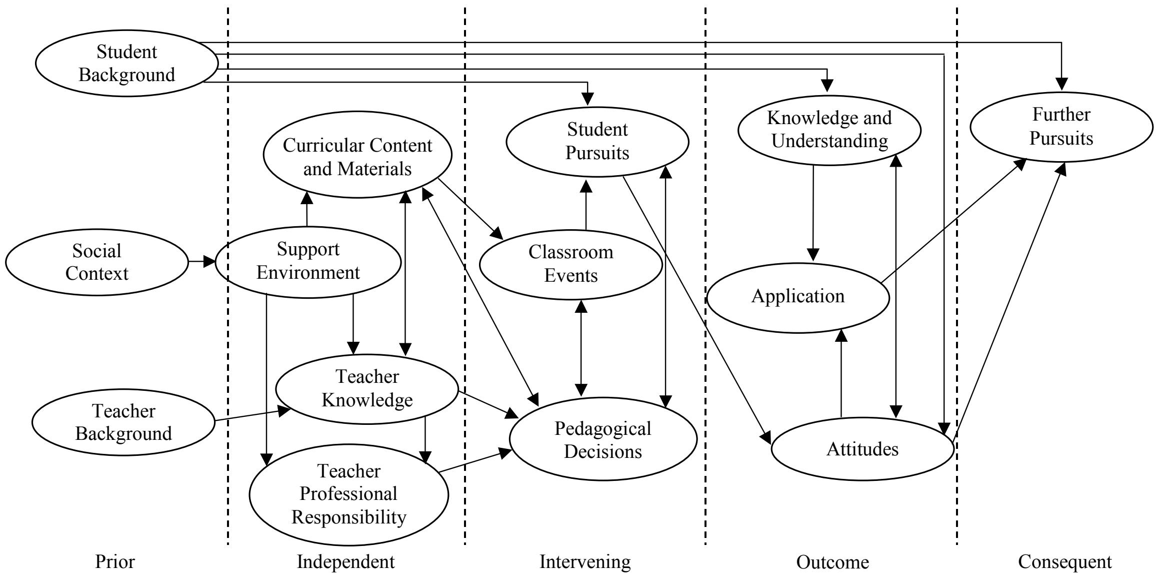

Figure 3-1. Model for the monitoring of school mathematics.

Using a Modeling Approach

The research model used to gather information to answer the three research questions was a structural model. We found that basing the study on a structural model and quantifying the variables in the model was a reasonable way to conduct studies of the implementation of curricula in schools. In particular, the logic of the sequence of variables from “Prior” to “Consequent” was reasonable. Furthermore, the identification of potential variables gave us a starting point for the development of measures for the study. However, we found that many of the “Independent” and “Intervening” variables were composites of related sub-variables. For example, the variable *Classroom Events* represented the interactions among the teacher and students that promoted learning mathematics with understanding. These events arose from a learning environment in which students explored mathematics and were encouraged to make sense of mathematics. Data were therefore gathered on six sub-variables: lesson presentation and development; nature of mathematical inquiry during instruction; teachers’ interactive decisions; nature of students’ explanations; elicitation of multiple strategies; and lesson closure, reflection, or summary. For analysis purposes, measures of the variables and sub-variables were usually statistically collinear. Nevertheless, intellectually, the sub-variables were different and, although information about them was aggregated into a single index, thereby losing some of the distinct character of events in particular classrooms, the measures of the sub-variables could also be disaggregated in order to describe particular events or aspects of instructional approaches.

Quantifying the variables in a structural model was hard work. We are proud that we were able to create good measures for many of the variables. In the year prior to the study, we identified and developed all the instruments we planned to use and spent considerable resources in piloting their use, in developing scoring routines, and in scaling the resultant data. However, we were unable to develop adequate measures for the mathematical background of both students and teachers in the model. For students, we decided to use the data from district standardized tests as a measure of student prior mathematical knowledge. However, the schools used different tests and no item data were available. In fact, the best we could do was to use each student's percentile rank as a score, assuming that the norm-populations of the various tests were similar. Also, assessing a teacher's mathematical background and knowledge proved to be politically difficult. While we could test students, we could not test teachers. The questionnaires we used failed to capture the essential features of their mathematical knowledge. Finally, our measure of student attitude toward mathematics, while carefully developed and well situated in the research literature, proved to be of limited use. Overall, the initial responses were very positive, which limited opportunity for capturing growth. Initial responses and responses at the end of each school year were similar and very positive, and little variance was evident in class means. Only one change in attitudes was statistically significant among the treatment groups.

For analysis purposes, because co linearity across indices for many of the variables posed a serious interpretation problem, a simplified model was designed with five composite variables. The simplified model proved to be important in this study. It described the relationship between variation in classroom achievement (CA), aggregated by strand or total performance and variations in preceding achievement (PA), method of instruction (I), opportunity to learn with understanding (OTL*u*), and school capacity (SC). This relationship can be expressed as: $CA = PA + I + OTLu + SC$. These composite indices, based on one or more sub-indices developed for each variable in the original model, were then created and used in further subdividing and describing treatment groups and in regression analysis to determine the impact of the variables on student achievement.

The creation of composite classroom achievement scale (CA) via a subcontract with the Australian Council for Educational Research as the primary outcome measure for the longitudinal study was critical. (See Monograph 4, Chapter 2 for information on the development of this scale.) CA was based on the degree of mathematization in student responses to items from the eight different assessments (two at each of four grade levels) developed for the study in collaboration with the Freudenthal Institute. Ratings for individual students were calculated and achievement bands were determined. As a result, six levels of achievement were derived from the typical set of knowledge, skills, and understandings students demonstrated when they engaged with the test items. The descriptions of the CA achievement bands were consistent with both the Program for International Student Assessment (PISA) definition of mathematization (Adams & Wu, 2002) and Romberg's view of mathematical literacy (Romberg, 2001). The CA progress maps provided pictorial representations of the variable of "mathematical competence." Progress maps were useful in monitoring changes in student achievement over time and in comparison of achievement for various groups such as all students at a particular grade level in a particular year, or by district, curriculum taught, teacher/student groups, gender, and ethnic group.

In this study, instruction was described through the three intervening variables in the research model: *pedagogical decisions* (the teacher's decisions in defining the actual curriculum), *classroom events* (interactions that promote learning mathematics with understanding), and *student pursuits* (the nature of students' involvement in learning). Using the underlying single dimension of teaching mathematics for understanding, the instruction composite variable was composed of five major categories: unit planning, lesson planning, mathematical interaction during instruction, classroom assessment practice, and student pursuits during instruction. The composite index for *Instruction* was created and used to classify teachers' instruction into six levels from underdeveloped lessons to most reflective of teaching mathematics for understanding. (See Monograph 3, Chapter 1 for information on the development of this index.)

Opportunity to learn has been interpreted more broadly in this study as a student's opportunity to learn mathematics *with understanding* (OTL*u*). OTL*u* was described through the independent variable *curricular content and materials* (the content and materials used in defining the actual curriculum) and the intervening variable *classroom events* (the interactions among teacher and students that promote learning mathematics with understanding). As conceived in this study, OTL*u* was composed of three overarching categories: curricular content, modification of curricular materials, and teaching for understanding. Teaching for understanding contained four subcategories: the development of conceptual understanding, the nature of student conjectures about mathematical ideas, the nature of connections within mathematics, and the nature of connections between mathematics and students' life experiences. The composite index for *OTLu* was then created and used to classify the opportunities students experienced into four levels from low OTL*u* to high OTL*u*. (See Monograph 3, Chapter 2 for information on the development of this index.)

The composite variable for school capacity was specified from the variables in the original research model: *school context* (prior variable) and *support environment* (independent variable). School capacity is the collective power of the school staff to improve student achievement (Newmann, King, & Youngs, 2000). Through the work of the school's professional community, a vision of student learning, professional inquiry, and control over school policy and activities unfolds. School capacity is strong when all programs and initiatives are focused on the vision for student learning and professional inquiry. The principal nurtures school capacity through effective leadership that aligns school policy with the vision and sets the tone for professional development opportunities for faculty and staff. Quality technical resources such as curricula, assessments, and equipment contribute to obtaining the established goals. In this study, school capacity was characterized through four subcategories of cultural conditions in the school (shared vision for mathematics teaching and learning between principal and teacher, administrative support, school as a workplace, support for innovation) and three subcategories of structural conditions (collaboration among teachers, work structure, influence of standardized testing). The composite index *School Capacity* was used to classify teachers' perceptions of school capacity into five levels from low school capacity to high school capacity. (See Monograph 3, Chapter 3 for information on the development of this index.)

Structure of Data Collection and Analysis

Data to test the relationships between variables in the research model were collected over three years, and analysis included eight grade-level-by-year studies, three cross-grade comparisons, three cross-year comparisons, and longitudinal studies for three groups of students over the three years of the study (see Figure 3-2). We found that we were overly ambitious. We simply did not have the resources to adequately gather data on all the variables in several school districts over three years. All data used to answer Question 2 were gathered in just two urban school districts. It would have been better if we had focused on one group of students in

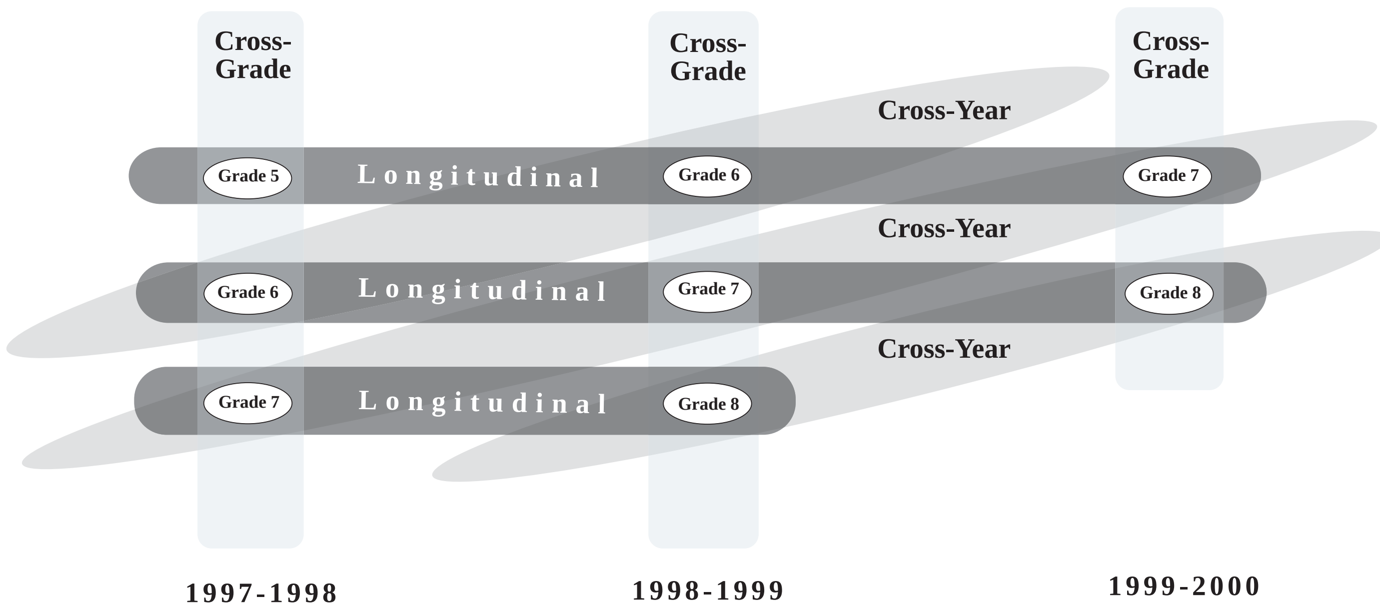

Figure 3-2. Structure of the multiple cohort study design.

several districts starting at Grade 6 rather than three cohorts starting at Grades 5, 6, and 7. Starting a cohort at Grade 5 in elementary schools made the cross-sectional and longitudinal studies difficult. Several factors limited our ability to follow Grade 5 students in later years. For example, in District 1, 90% of the students who completed both study assessments in Grade 5 did not participate in the study for three years, despite working with the on-site district coordinator to locate study students who did not appear on class rosters in the second and third years. Although district and school administrators agreed to schedule study students into classes of study teachers, this request was not honored. Some students who matriculated into study middle schools were not assigned to study teachers, and the on-site coordinator arranged for more teachers to participate in the study in order for the research team to follow these students longitudinally. Other study students were sprinkled among all classes for a particular study teacher. In this case, the teacher was asked to administer study assessments to those students, with the research team providing full class sets of assessments so that study students remained anonymous. Another factor played into the loss of students in District 1—an initiative in which parents chose the schools their children attended each year. Many Grade 5 students went to nonstudy middle schools, even though they were in traditional feeder patterns from elementary to middle schools.

The cross-grade comparisons, cross-year comparisons, and longitudinal studies were flawed due to student and teacher attrition. For example, we hypothesized that we would be able to see a shift in the pedagogy of Grade 7 teachers, and in their student's mathematical achievement in the third year of the study, as a consequence of teaching MiC the previous two years. Unfortunately, only 1 of the 10 teachers that taught Grade 7 in the first year of data collection was still teaching that grade in the third year. Another complicating factor that made comparisons difficult was that many study students participated in district initiatives for Grade 8 students to enroll in algebra classes. This was most pronounced in District 3 where district administrators asked that these students be withdrawn from the study.

To answer Question 1, our analysis plans had to be modified. Initially, we had planned to carry out structural equations to examine the relationship of the variables in the structural model with the classroom as the unit of analysis for the eight grade-level-by-year studies in classes using MiC across all four districts. Then from a similar structural analysis including data from classes using conventional mathematics curricula, we could answer Question 3. Instead, because we only had complete data on all the variables in two districts, we chose to make descriptive comparisons using student performance on the CA scale and its content subscales as the dependent variables. The comparisons were done for districts, grades, schools, teachers, gender of students, and ethnicity of students. These comparisons made it very obvious that there was considerable variation in student performance in every comparison made.

The cross-grade and cross-year studies were designed to use the data from the grade-level-by-year studies to portray general student progress in performance as a consequence of instruction using MiC. This was done, but these studies were compromised by student and teacher transiency in three of the four districts. The comparability of the teacher/student groups in different grades and in different years was not obvious. In the cross-grade studies, there were at least four mitigating circumstances that made the assumption that the groups of students at different grade levels have similar background characteristics and similar instructional experiences in the same schools problematic. First, the shift in schooling as Grade 5 students moved from elementary classrooms to middle school classrooms involved new schools with students from different feeder schools and a quite different school culture. Second, the background, experience, and quality of instruction between elementary school teachers and middle school teachers were different. Third, the movement of some Grade 7 students into Grade 8 algebra classes made the group of students using MiC in Grade 8 different from those at earlier grades. Fourth, data gathered in the first year of implementation of a new curriculum were likely to be different than data from the second and third years because both teachers and students had become more comfortable with MiC.

The cross-year studies involved students at the same grade level at different points in the study. The results of these studies were confounded by differences in the student sample in District 2. In the first year, students were from Guggenheim and Hirsch Metro Middle Schools, but Hirsch Metro teachers withdrew from the study and a new feeder pattern was chosen at the request of the district for the second year. Therefore, the sample in the second year included students from a different middle school. The longitudinal studies were designed to use the data from the grade-level-by-year studies to examine growth over time. However, because the student sample had to be restricted to those students who participated in two or three years of the study, students had to be the unit of analysis for the cohorts. Again, because student attrition was so high the results of any comparisons are suspect.

Quasi-Experimental Design

To answer Question 2, data were to be gathered in classrooms in which MiC was implemented and in classrooms in which conventional curricula were used in the same school districts via a quasi-experimental design. We learned that quasi-experimental studies, while an attractive alternative to true experiments, are unsatisfactory because there is no way to control all the sources of variation in classrooms. As illustrated in this study, no matter how one examined the experimental units, they were not equivalent. Second, identifying treatment groups in real classrooms proved to be difficult. As illustrated in this study, many of the MiC teachers nominally implemented MiC in their classrooms, and in a few conventional classrooms, instruction was hardly conventional. This led to restricting comparisons to three treatment groups based on teacher levels for the Instruction and OTL*u* composite variables. One treatment group included the 28 MiC teacher/student groups that reflected the intended pedagogy and content of MiC. Another treatment group, referred to as the MiC (Conventional) group, included 28 teacher/student groups that used MiC with conventional pedagogy or MiC materials supplemented with conventional materials. The third treatment group included 23 teacher/student groups that used conventional mathematics materials. In addition, there were three outlier conventional teacher/student groups that exhibited reform characteristics. These treatment groups demonstrate that any contrast between MiC and conventional classrooms is more than the label associated with the text materials being used. In comparing the treatment groups, we used analysis of covariance to control for the most significant source of differences, prior achievement. We found this to be a reasonable alternative to controlling for such variation in a true experiment.

While the power of true experimental studies is undeniable, we are convinced that in implementation studies involving many teachers over more than one year, one cannot randomly assign units (schools, teachers, or students) to treatments. This leads us to the conclusion that using a structural model and quantifying the variables in the model is the best way to conduct studies of the implementation of curricula in schools.

What we Learned by Conducting Research in Large Urban School Districts

Three of the four school districts with whom we worked were large districts in urban settings, two of which served a predominantly low-income population. When we designed this study, we wanted to work in such school districts because of the political pressure to help these struggling schools. Also, the National Science Foundation asked us to work where they had invested in mathematics and science reform. Additionally, it is considered too easy to study educational phenomena in affluent suburban districts. Most programs will work in that environment, but many programs do not transfer to an impoverished urban environment. We were anxious to demonstrate that reform could happen in such schools. In this section, we describe what we learned in large urban schools.

Classes of students in large urban schools incur multiple changes each school year. For example, during the second and third years of data collection, two seventh-grade MiC classes had two teachers during the school year and one eighth-grade class using a conventional curriculum had three teachers in one school year. As the data collection progressed, only 37% of the sixth-grade teachers, 14% of the seventh-grade teachers, and 46% of the eighth-grade teachers participated in the study as envisioned. Although we expected normal occurrences such as changes in personnel due to smaller enrollments or parental leaves, we did not anticipate teachers moving from one grade level to the next with their classes in District 2, which involved 11% of the sixth-grade teachers and 14% of the seventh-grade teachers. Teachers left the study for various reasons including moving from study schools to nonstudy schools, being on family leave, accepting an administrative position, or resigning from participation in the study. Teachers also varied in their use of MiC. In some cases, MiC units were supplemented with other materials to the extent that MiC was subsumed by the supplementary materials. For example, a sixth-grade MiC teacher of low ability students in District 2 reported in journal entries for an entire month that she taught a series of lessons on fraction operations with varying success. A seventh-grade MiC teacher in District 1 regularly reported in journal entries that his lessons were supplemented with daily worksheets on basic computation. Parental concerns also influenced teachers' use of supplementary materials for practicing basic skills. In addition, when teachers were absent due to illness or participation in workshops, they felt that it was inappropriate to ask substitute teachers to continue MiC lessons. Instead, they prepared lessons that reinforced computational skills. These practices made the learning environments unstable for students. Teachers also compromised data collection by administering tests on days of scheduled classroom observations, neglecting to cancel planned observations when school schedules changed or when they were absent due district professional development or to illness, and not completing teaching logs that added significantly to our understanding of what transpired on a daily basis in study classrooms.

It was our intent to study intact groups of students who stayed together over the three years of the study. However, practically this proved to be impossible in schools in these districts. For example, in District 2, 96% of the students who completed both study assessments in Grade 5 did not participate in the study for three years. In this very large urban district, mobility was a significant factor in the loss of study students. Students transferred to nonstudy classes in the same school or to different schools inside and outside the district throughout the school year, which compromised our ability to follow students longitudinally. Also, students were absent on one or more days in which the study assessments were administered. Because the analysis of classroom achievement was based on student responses to both study assessments, only those students who completed both study assessments were included in the analysis. Thus, student absenteeism also created serious problems in the study.

In the large urban research districts, commitment to our study often compromised data collection. In fact, there was an unwillingness of many school districts to be involved in such a study. Many districts expressed the concern that they might be seen as withholding opportunities for learning mathematics in more powerful ways from students who continued to study conventional mathematics curricula. Also, schools using conventional curricula were difficult to find because principals did not want to be perceived as disregarding efforts to reform mathematics curriculum and instruction. Furthermore, it was difficult for large urban districts to live up to contractual commitments of keeping intact groups of students, assigning incoming fifth-grade students to classes of study teachers, selecting classes that were of average ability, and procuring standardized test scores for study students. Moreover, the responsibility of coordinating this study at the district level was assigned to the district mathematics specialist, which proved problematic in District 2. The transition from the initial contact persons to the mathematics specialist in District 2 occurred in late July, shortly before we met study teachers during the first August summer institute, and details of the study were unfamiliar to the mathematics specialist. In fact, a different coordinator was selected in District 2 each year of data collection, partially as a result of restructuring district administrative positions. This meant that in each study year, the coordinator in District 2 was unfamiliar with research goals and the complex issues that had arisen in relation to past data collection. Another example of difficulty in maintaining contractual agreements involved a student questionnaire used to assess the impact of MiC as students transitioned to high school mathematics, the data from which was to be used in measuring the consequent variable in the structural research model, *Further Pursuits*. High school counselors were asked to distribute the questionnaires to study students who were in Grade 9, and students were asked to complete the questionnaires under the supervision of their parents and subsequently mail the questionnaires to the research center in postage-paid envelopes. Because the response rate was very low, these questionnaires were not distributed during the final year of the study. On the other hand, administrators in Districts 1 and 2 were very helpful in recommending retired mathematics teachers to gather classroom observation data, and our ability to contract with these observers was critical in this study. In fact, in any study of implementation, classroom observations in study schools in all research districts should be conducted, which was not possible in this study. The classroom observation instrument developed for the study allowed the observer to mark ratings on separate indices, but also to provide written evidence supporting the rating, a list of activities that transpired during the lesson and the time allotted to each, and general descriptions of the lesson. Observation reports provided pictures of the mathematical interaction that transpired in study classes based on a consistent set of variables considered during each observation. Throughout the study, classroom observation reports provided information about the content of instructional materials, the ways lessons were presented and developed, the nature of the mathematical inquiry during instruction, teachers' interactive decisions, and students' involvement in lessons.

In these large urban districts, our study was just one of many well-meaning initiatives, and implementing MiC began on a small scale, involving a few schools, before adoption by more schools in the district. District mathematics specialists were responsible for supporting teachers who were implementing MiC, along with all the other programs and initiatives in the district. Study teachers received minimal attention and support as they faced the challenges of teaching a new curriculum, and their needs for specific types of professional development, such as focusing on conceptual understanding, leading student discussion, and developing new ways to assess student learning, were not addressed. We did not have adequate resources to help study teachers, and the districts were unable to provide consistent long-term support. Study teachers were also inundated with initiatives that left them little time to devote to our research. For example, initiatives such as mandatory silent reading in every class period, supplementary materials that feature application of mathematics in the workplace, and computer-assisted drill-and-practice programs took significant amounts of class time and compromised the time necessary for students to investigate MiC unit activities. Another initiative in many districts was for eighth-grade students to complete a traditional ninth-grade algebra course. For students who do not meet the criteria for these programs, some teachers chose to use a traditional algebra text during the second semester of eighth grade rather than teaching the MiC algebra units or continue teaching units from various content strands.

Data collection was also compromised by the lack of seriousness about study assessments by teachers and students. The significance teachers and students assign to study assessments is an important consideration in conducting educational research, particularly when these assessments are used to measure growth in student performance over time. Students in this study seemed not to put forth the effort to develop reasoned responses. Many attempted to solve assessment tasks, but few responses earned full credit. Realizing that study assessments were not going to affect their grades, students seemed not to reason through the tasks and develop thoughtful responses that the research team had hoped. Study assessments did not affect students' grades or promotion from one grade level to the next. We learned that it is important for study teachers to encourage students to accept the challenges of rigorous assessments and take seriously the commitments they have made to participate in educational research. Increasing the attention given to study assessments may involve working closely with teachers to schedule administration of the assessments and discussing appropriate conditions for administering them. Furthermore, research teams might take a more active role in talking directly with students about the importance of these assessments and in answering students' questions about particular tasks after assessments are completed.

Teaching mathematics happens in communities. Several social conventions such as the beliefs about the mathematics content that should be taught to all students, ability grouping as an efficient way of providing for individual differences, the validity and utility of standardized testing, and the utility of conventional grading practices were being challenged by MiC. For example, Ms. Teague, a MiC teacher in District 2, held beliefs about mathematics instruction that were more traditional, tending to bemoan perceived student weaknesses rather than focus on instructional issues that might address student needs. In the first year of data collection, Ms. Teague identified repetitive drill as an important way to students master needed skills. She believed that students could engage in problem solving without having mastered basic skills, but non-mathematical factors, such as reading comprehension and discipline, often mediated against successful problem solving. In the second year of data collection, Ms. Teague noted that students were capable of doing more mathematics without having mastered basic skills, an awareness that developed as she looped with her classes from one grade level to the next. Other teachers reported that they were becoming more facilitative during instruction, and they noted differences in student engagement during instruction because they had studied mathematics with MiC during the previous school year. Teachers had developed ways to allow students to do more mathematical thinking in class and to express their ideas in writing. Teachers also cut back on supplemental activities such as problems of the week, instituted different methods for grading such as partial-credit scoring, and assessed students more frequently during classroom discussions. Teachers requested times when they could collaborate with one another. For example, MiC teachers from the same teaching team in District 2 were given one day of release time per month in order to collaborate on planning to teach MiC units. Ms. Teague reported that these collaborative times were effective for changing instruction because she was able to discuss and work through a MiC unit with other teachers before teaching it. In fact, these meetings allowed teachers on this team to discuss mathematics content as well as teaching and classroom assessment practices and program evaluation.

In spite of the hurdles, many teachers were able to implement MiC well and their students profited by the experience. For example, Ms. Keeton presented a comprehensive curriculum in two consecutive years in which she looped with her classes from Grade 7 to Grade 8. In Grade 7, she taught six units in three content strands (two number, one from Grade 6; two algebra; and two geometry), and in Grade 8, she taught five units in four content strands (one number, two algebra, one geometry, and one statistics). In both years, mathematics was explored in enough detail for students to think about relationships among mathematical ideas or to link procedural and conceptual knowledge. Ms. Keeton supplemented MiC with activities disconnected from the curriculum such as practice for district standardized tests and various school or district initiatives. Ms. Keeton held high expectations for her students, and she supported their learning in numerous ways through class discussion, group work, and feedback given as a result of classroom assessment practice. Inquiry during instruction promoted learning mathematics with understanding. Ms. Keeton felt that student work changed as they studied MiC for more than one year. Some students were less reluctant to provide a complete response that included both the answer and an explanation. Many realized that even on questions that elicited opinions, a logical defense was important. The increase in the overall mean classroom achievement (CA) score for students in Ms. Keeton's classes was statistically significant from Grade 7 to Grade 8. Changes in classroom achievement were likely influenced by the quality of instruction and opportunity to learn with understanding that students experienced in Ms. Keeton's classes. The combination of teaching for understanding and teaching MiC throughout the year contributed to the performance of Ms. Keeton's students in important ways.

The challenges we faced in conducting comparative longitudinal research in the reality of school life seemed daunting at times. Student and teacher attrition, various interpretations of commitment, treatment fidelity, and teachers' needs for professional collaboration affected data collection. These variations draw attention to the need to study the effects of the culture in which student learning is situated when analyzing the impact of standards-based curricula. Controlling potential sources of variation, as is done in laboratory experiments, is more difficult in classroom settings. But this does not mean that comparative research cannot be done in today's schools. Rather, as this study demonstrates, impact studies of high quality can be conducted in classroom settings when data collection and analysis are designed to take into consideration the variations encountered in these settings.

What we Learned about Students

Despite the difficulties in data collection, we were able to make some interesting conclusions about study students. First, we underestimated the capability of students to learn mathematics with understanding. Given the opportunity to explore some problem situations through a set of structured activities in MiC units, students did learn important mathematics with understanding. For example, fifth-grade students investigated the distortion caused by representing curved surfaces with flat maps, and sixth-grade students solved systems of linear equations. Seventh-grade students investigated periodic graphs and the tangent ratio, and eighth-grade students interpreted equations and inequalities and graphed constraints and feasible regions for inequalities. In analyses of groups of students who began a particular school year with comparable prior achievement, the results suggest that over time students who studied MiC and experienced high quality of instruction and opportunity to learn with understanding showed gains in performance. In some cases, significant increases in classroom achievement (CA) were also noted when teachers taught MiC throughout the school year, but whose instruction was more reflective of conventional pedagogy, especially when grade-level units were supplemented with portions of units from previous grade levels. In longitudinal analysis of same students from Grade 5 through Grade 7, gains in performance were evident across the grade levels on 19 of 20 items repeated on a set of assessments designed for the study, with increases ranging from 8 to 33 percent correct from Grade 5 to Grade 7, and an average increase of 22% over the three grade levels. The results for this cohort were comparable to or greater than the original Grade 8 samples from national and international tests on 80% of the items.

Students with low ability also were capable of learning mathematics with understanding. They studied number, geometry, algebra, and statistics units. For example, Ms. Linne's class of fifth-grade students with low ability in District 1, some mainstreamed from special education classes, had higher mean CA scores at the end of the school year than other classes with higher prior achievement scores. The combination of Ms. Linne's attempt to teach mathematics for understanding, the opportunity to learn mathematics from six MiC units in four content strands throughout the school year, and supplemental practice on basic skills likely contributed to their strong CA performance. For Ms. Schroeder's special education students in District 3, overall CA means jumped 20% from Grade 7 to Grade 8, and substantial increases were also evident in the number, algebra, and statistics strands. Ms. Schroeder chose to provide opportunities for her special education students at both grade levels to learn comprehensive content in ways that promoted understanding, rather than limit them to solely practicing algorithms daily. She believed that students needed to experience problem solving along with basic skills practice. Her teaching and classroom assessment practices focused on students' understanding of the mathematics and the development of reasoning and communication skills. This combination of a high quality of instruction and opportunity to learn mathematics with understanding contributed to substantial gains in student performance.

Learning the concepts and skills in a mathematical domain requires that students be engaged in a rich set of structured activities over time. To learn the ideas in a domain, students should have the opportunity to investigate problem situations that encourage mathematization. Such situations include those that are subject to measure and quantification, that embody quantifiable change and variation, that involve specifiable uncertainty, that involve our place in space and the spatial features of the world we inhabit and construct, and that involve symbolic algorithms and more abstract structures. Also, such situations embody systematic forms of reasoning and argument to help establish the certainty, generality, consistency and reliability of one's mathematical assertions. Some study students were able to engage in such activity. For example, in District 3, the CA performance of students was significantly higher from one year to the next, and over 50% of the students in the third year of data collection demonstrated a moderate level of mathematization, as they were able to translate either a contextualized or a non-contextualized, generally non-routine problem into mathematical terms for solution.

Student learning should be seen as a product of involvement in a classroom culture. Learning with understanding is a product of interactions over time with teachers and students in a classroom environment that encourages and values exploration of problem situations, modeling, argumentation, and other mathematical processes. This type of rich mathematical interaction transpired in some study classrooms. For example, when students in Mr. Gallardo's eighth-grade MiC class studied a lesson from the algebra unit *Get the Most Out of It* (Roodhardt, Kindt, Pligge, & Simon, 1998, pp. 21–26) on 3/13/00, students interpreted equations and graphed constraints and feasible regions. The lesson began with a review of solving and graphing a constraint. Using the inequality $3y + 9x \leq 6$, Gallardo led the class in a discussion of substituting values for x and solving for y values, graphing two points and drawing a line, and substituting coordinates of a point to determine the side of the line to shade. Students applied their prior knowledge and experiences to the constraint problems. In order to predict the values of y, they studied patterns in the data tables they constructed for particular situations. Mr. Gallardo encouraged students to listen to each other and build shared understanding of the content. For example, when a student gave an explanation of her solution for the inequality, it was difficult to understand. Mr. Gallardo invited other students to think about and rephrase her solution. Mr. Gallardo then led a whole-class discussion of some problems from pp. 21–24. He continually asked students about the reason for each step of the procedure as they worked through problems. He also focused on conceptual understanding by constantly reminding students that the solution pairs were points on the line and each solution pair made a true statement with respect to the constraint. Students discussed connections between graphs of lines and graphs of inequalities and talked about how to decide the side of the line to shade for graphing inequalities. Mr. Gallardo continually promoted connections among three representations of the same situation by recording and discussing the inequality, the table of solution pairs, and the graph. Throughout this time, students volunteered to verbally explain their solutions, record on the whiteboard their substitutions of values for x to determine the corresponding y values, and graph the lines. Students corrected themselves when they wrote incorrect symbols (e.g., = instead of ≤ or ≥). Mr. Gallardo valued students' work, and students worked together to develop shared understanding of the relationships between graphs of lines and graphs of inequalities and among three representations (equation, table, and graph) of the same situation. Students' active involvement in the lesson was nurtured in this class.

Learning with understanding in a domain is acquired by students gradually over time as a consequence of active engagement in structured activities designed to help students evolve from their informal ideas about a domain to more formal and abstract ways of representing and reasoning in that domain. This notion was challenging for MiC teachers. They had to resist the temptation to teach abstract procedures before building on the informal knowledge students had acquired, and they had to trust that MiC would help students become proficient in mathematical skills over time. For example, fifth-grade MiC teacher Ms. Mitchell emphasized problem solving and sharing of different strategies in classroom instruction. Although she felt that mathematics learning proceeds hierarchically and certain skills must be mastered before new ones can be learned, she attempted to place less emphasis on rote memorization and more on activities in which "students are going to get a full understanding of what they are learning" (Interview 9/16/97). In years prior to the study, Ms. Mitchell was well known for helping students master operations with fractions by the time they finished Grade 5. She expressed concern that there was not enough practice in the fifth-grade number units for students to master these skills. Mastery of fraction operations at this grade level conflicts with the intent of MiC to build on students' intuitive knowledge of fractions, introduce tools such as the fraction bar and ratio table to support their thinking about fraction operations, and revisit and extend these operations to more abstract forms in sixth- and seventh-grade units. Ms. Mitchell hoped that students retained the understandings about fractions that they had an opportunity to learn in Grade 5 with MiC as they moved into middle school. Ms. Mitchell voiced some of the concerns expressed by other teachers about students' performance in number as a consequence of studying MiC units. However, in our analyses of classroom achievement (CA) by content strand, students who were in the MiC treatment group showed increased performance in the number strand. In fact, the MiC treatment group's achievement was significantly higher in number, algebra, geometry, and statistics than that of the MiC (Conventional) group's achievement. With respect to number, these results are contrary to the voiced intent of many MiC (Conventional) teachers who deliberately supplemented the MiC materials with conventional materials on number because they believed that MiC was weak on number. Students in the MiC (Conventional) treatment group whose teachers supplemented MiC with extensive practice on basic skills demonstrated less gains or declines in performance than the MiC treatment group, and their performance was similar to students who used conventional curricula with the exception of geometry. The implication of this finding is that supplementing a reform curriculum with conventional materials is not likely to result in different student performance. What matters is that students actively engage in lessons that build on their informal ideas in particular domains and gradually move toward more abstract ways of reasoning in the domain. Students in our study have shown that they can reason about problematic situations, extend their knowledge of mathematics, and apply skills in multiple content strands.

What we Learned about Teachers

Convincing principals and teachers to participate in the study in the conventional treatment group proved to be difficult. Only two districts agreed to be involved in a comparative study. Principals questioned incentives for their teachers to participate as teachers using conventional curricula. They did not perceive as adequate compensation the professional development opportunities for teachers provided by the study. Teachers also expressed negative opinions about participating in groups using conventional curricula. In District 1, some study teachers using conventional curricula who taught multiple classes wanted to be perceived as reform-oriented. As a result, they used conventional curricula with study classes and MiC with other classes. In District 2, when an eighth-grade teacher in the conventional group was involved in an accident, no other teachers in the school agreed to participate in the study so we could continue to follow the students longitudinally. Consequently, an eighth-grade conventional group was not available in that district during the third year of data collection. Fewer teachers in the conventional group participated in the study over time, which compromised the comparative analyses.

Teachers felt constrained by a variety of rules and expectations that restricted their pedagogical ability. In particular, external tests, practice for such tests, and established grading practices proved to be a major impediment to changes in instruction. For example, district and state standardized tests significantly influenced the instruction of the teachers in one middle school, as large amounts of time were devoted to test preparation. In fact, this became such a serious issue that they withdrew from the study in the second semester. The principal was not convinced that mathematics education reform should be supported wholeheartedly. His interpretations of state standards led to a different approach for reaching expectations of improved student achievement. He stated that district funding for his school was directly tied to student results on traditional portion of the state testing program, and he perceived that students would not perform well if they studied MiC. He strongly encouraged the teachers to withdraw from participation in the study. Consequently, they withdrew from the study in the second semester, and a conventional curriculum was used for the remainder of the school year. In contrast, other teachers reported the benefits of using MiC for standardized tests because students had opportunities to solve contextualized problems and write explanations.

For teachers, the MiC instructional approach represented, on the whole, a substantial departure from their prior experience and established beliefs. The content of MiC units was sometimes a challenge for teachers. In MiC, mathematical content is introduced in a different instructional sequence than in conventional middle-school curricula. For example, concepts related to percent are introduced in fifth- and sixth-grade MiC units, rather than more conventionally in eighth grade, and content traditionally reserved for high-school students such as topics in algebra and geometry is introduced in fifth- through eighth-grade units. Teachers also had to learn how to introduce and work with new mathematical tools that supported students' thinking such as the ratio table. In addition to the challenges of teaching new mathematical content and new ways to support student thinking about mathematics, few MiC teachers had experience teaching mathematics that emphasized the development of conceptual understanding and student reasoning rather than algorithms and procedures. Furthermore, during instruction, teachers draw on pedagogical content knowledge, but MiC teachers frequently talked about their lack of pedagogical content knowledge for new content. Teachers also expressed concern about how to help students when they could not find their own solutions to particular problems. They began to work with guiding students to complete mathematical tasks and found that they needed to develop ways to provide time for students to think about instructional tasks, reason out strategies, and determine solutions. They gradually developed ways for students to do the mathematical work and discuss various strategies, and they worked at improving the quality of group work. Teachers' own understanding of the mathematics, ways the mathematics is presented, pedagogical content knowledge for new content, and instructional strategies, all of which are central to effective instruction, were being developed as teachers taught MiC for the first time for the whole school year. In addition, teachers struggled with grading student work. For example, Ms. Keeton reported that grading students' written work was time-intensive. Grading was compounded when students responded in ways she did not anticipate. She had to refer back to the original task in the unit in order to check the worthiness of the response. In the second year, she more frequently assessed students using informal methods (asking questions, observing) during whole-class discussion and group work.

Teachers using MiC developed a changed view of their students and their students' capabilities. For example, after one year of teaching MiC, Ms. Teague noted that students were capable of doing more mathematics without having mastered basic skills. Teachers noted that students think more about the mathematics and develop a deeper understanding of mathematics when they study MiC. Ms. Linne, who taught MiC to students with low ability, reported that students were more willing to offer answers and strategies in class because they knew that there were multiple ways to solve problems. Ms. Keeton also stated that she and other teachers on her teaching team developed a better understanding of the mathematics they already knew as a consequence of teaching MiC. That is, teachers as well as students learned mathematics with conceptual understanding. From Ms. Keeton's perspective, these opportunities for teacher learning were significant.

Answers to the Three Research Questions

In our proposal to the National Science Foundation, three questions were raised. In concluding this monograph, we give answers to each of the questions based on the evidence gathered in the study.

Question 1. What is the impact of the MiC instructional approach on student performance?

Based on our analyses of classroom achievement (CA) in the eight grade-level-by-year studies, the cross-sectional comparisons, and the longitudinal studies, the MiC instructional approach, as practiced in four school districts in four grades, yields considerable variation in CA scores when implemented in classrooms. In over half of the comparisons of student performance, important differences were found on CA. Furthermore, about half of the differences also reflect a constriction of variation within some teachers' classrooms due to practices of homogeneous grouping. Moreover, CA was affected by the implementation of MiC, judged by the quality of instruction and opportunity to learn with understanding that students experienced. Overall, there was considerable variation in CA when MiC was implemented in classrooms, but there were ample examples of gains in CA scores. Several factors influenced these results including, in large part, each student's prior mathematical knowledge and each teacher's level of implementation of MiC.

Question 2. How is this impact different from that of traditional instruction on student performance?

Using the composite indices of both *Instruction* and *Opportunity to Learn with Understanding*, we found that there were three distinct treatment groups. The first treatment group included classes that reflected the intended pedagogy and content of MiC. Another treatment group used MiC with conventional pedagogy or MiC materials were supplemented with conventional materials, and the third treatment group used only conventional materials. The three treatment groups were labeled MiC, MiC (Conventional), and Conventional, respectively. This demonstrates that any contrast between classrooms in which MiC or conventional curricula are used is more than the label associated with the text materials being used.

Using analysis of covariance with prior achievement as the covariate and CA as the dependent variable, we examined the differences between the three treatment groups. When differences for prior achievement that favored the MiC group were taken into account, the overall achievement for the MiC group was significantly higher than the achievement of the Conventional group and higher than the MiC (Conventional) group. Although this overall analysis masks the within-group variation due to differences in school districts, grade levels, or teachers using the different treatments, this finding implies that if one is going to teach MiC, it is important to implement the program as intended. Regardless of other differences, if MiC is implemented as intended, (a) students perform better than when MiC is taught with conventional pedagogy or supplemented extensively with skill practice, and (b) students perform better than those using a conventional program.

Question 3. What variables associated with classroom instruction account for variation in student performance?

In this set of studies, considerable information was gathered on a large set of variables in addition to the use of curricular materials. Overall, the achievement of students in the year prior to being studied was the best predictor of student performance in a given year. When prior achievement was accounted for, the degree of implementation of MiC was a significant predictor of student achievement. Clearly, variations in the quality of instruction, the opportunity to learn with understanding, and the capacity of schools to support mathematics teaching and learning affected reform implementation, and, therefore, student achievement.

Conclusion

Conducting research in schools is complex and difficult. The complexity of clarifying instructional patterns in mathematics classrooms as exhibited in the daily interaction of teachers and students when a new and quite different standards-based curriculum is implemented is not easy to examine. We note that the reform view of mathematics is, above all, integrative: Every element is seen as part of a larger whole, with each part sharing reciprocal relationships with other parts. This approach stresses the acquisition of understanding by all, including the traditionally underprivileged, to the highest extent of their capability, rather than the selection and promotion of an elite. The philosophy underlying reform mathematics simultaneously stresses erudition and common sense, integration through application, and innovation through creativity. Most importantly, it stresses the student creation of knowledge. Against this broad and ambitious view of mathematics, traditional school mathematics appears thin, artificial, and isolated.
We note as well that assessment, if intended to measure those mathematical abilities students will need in their adult lives, should move away from a focus on whether students can reproduce and use algorithms in contexts they have used continually in class drills and, instead, toward providing reliable evidence that a student can apply knowledge, *in reasonable and flexible ways*, to new, unfamiliar problem contexts. Over the course of years, students should be able to show, on those assessments, evidence of growth in the level or complexity of tasks he or she can *solve*. The complexity of instructional issues involved in creating and sustaining classrooms that support this type of achievement include the interconnected roles of tasks; student-teacher interactions involving mathematics concepts; the reasonable, appropriate, and flexible use of technological tools in the classroom; classroom norms of collaborative and individual work; sustained professional development and teacher community; enhanced organizational support; and community and parent involvement and education. Implementing this reform vision of student achievement should mean, at a minimum, that students become mathematically literate.

To the extent that our research supports this vision, we conclude that Realistic Mathematics Education as exemplified in *Mathematics in Context* is a viable instructional approach that produces high level of learning when implemented well.

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